Prof. RNDr. Jiří Anděl, DrSc.
proděkan pro studijní zále«itosti
Dekanát MFF UK
Ke Karlovu 3
12116 Praha 2

Evaluation of Mgr. Petr Častulík and His Doctoral Thesis entitled

Analysis of Kinetic Equations and their Fluid Dynamic Limits

There are plenty of different models of incompressible and compressible fluid continuum mechanics and thermodynamics that have been proposed and used to model certain processes in liquids. More specifically, we can name the incompressible fluids with the viscosity and the heat conductivity depending on the quantitites as the shear rate, the pressure, the temperature, the concentration of another species, the density (if the fluid is inhomogeneous), etc.; we can also mention the multipolar fluids that take into account the dependence of material moduli on higher spatial derivatives of the velocity gradient and the temperature, or the Rivlin-Eriksen fluids where the dependence on higher material derivatives is considered, or the rate type and integral type fluids used in order to model stress relaxation, nonlinear creep and some other viscoelastic effects in liquids. Fluids physicists have been also using various approximations of the complete Navier-Stokes-Fourier system in order to simplify computational costs and still obtain the main features exhibited by the liquids: the Reynolds equation (flows in thin layers) or the Prandtl equations (boundary layer flows) can serve as good examples.

Petr Častulík task was to contribute towards understanding of the links between some of these models and their counterparts on the kinetic level. Need to say that this project inludes not only the understanding the models on the continuum level but also the identification of the appropriate model on the level of kinetic equations. In his thesis, Častulík deals with three problems of a quite different type.

In the first chapter, the classical Boltzmann equation are considered in the half-space equipped with three type of boundary conditions. Under appropriate scaling relevant to the boundary layer theory developed by Prandtl, it is shown that solution of compressible Prandtl equation can be approximated by the rescaled Boltzmann equations.

The second part focuses on the multipolar compressible fluids. The theoretical foundations as introduced by Green and Rivlin in the sixties, and up-dated by Nečas and Šilhavý in the eighties are recalled in the fourth section: the main disadvantages of this approach consist in unclear (missing) higher order boundary conditions and too many material coefficients that appear even in the linear theory and that are not easy to experimentally specify. The method of macroscopic limit thus can play a remarkable role in identifying both of them. Petr Častulík's approach as presented in Sect. 2.3, where he derives a kinetic equation that takes into account nonlocal and noninstant binary collisions, and in Sect. 2.4, where the fluid dynamic limit of this kinetic equation is investigated and the models with higher order velocity and temperature gradients are successfully obtained, represents the first remarkable step in this direction.

The final part deals with two generalizations of the Boltzmann equation. These two generalizations differ by the sign before the "new" higher order term. The model with the negative sign has been studied by Alexeev who originally proposed to investigate this type of generalization and investigate its limit towards the macroscopic equations of continua. Petr Častulík provides arguments supporting the choice of the term with the opposite sign. On the kinetic level, he also investigates rigorously the well-possedness of the linearized problem (using the Lax-Milgram theory to treat the case with negative sign (the problem is elliptic), and using the semigroup technique to study the case with the positive sign (the problem remains evolutionary)). In both cases, the fluid dynamic limits are established and two types of generalizations of the macroscopic acoustic equations are obtained.

In his doctoral thesis, Petr Častulík obtained in each chapter original results worth of publication (in fact, Chapter 1 coincides with his publication "The Compressible boundary layer and the Boltzmann equation", *Applied Math. Letters*, **15**(1) (2002), 17–23), proving his very good understanding of the subject and a deep physical insight into the topic that enables him to improve existing methods and achieve new results. His style of writing is very compact, and thrifty.

In Prague mathematical community, Petr Častulík has studied the fluid mechanics on the level of the kinetic equation almost isolated, and the fact that he could spend three times 6 months in Paris (University of Cachan) under the guidance of Claude Bardos (influenzed also by discussions with Y. Sone and K. Aoki) has played, of course together with his abilities, the most significant role in the development of his research.

To summarize, I am convinced that the doctoral thesis in consideration is suitable for its successful defense in the area "Mathematical and Computational Modeling" and that Petr Častulík has proved his ability of performing senior scientific research.

Prague, August 8, 2007 doc. RNDr. Josef Málek, CSc.