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## Evaluation of Doctoral Thesis of Petr Častulík

*Analysis of Kinetic Equations*
*and their Fluid Dynamic Limits*

The Thesis of P. Častulík addresses three rather different topics from the point of view of fluid dynamic limits of kinetic equations. The first part is devoted to the fluids at high Reynolds number studied in a tiny region close to the boundary, where the relevant macroscopic description is provided by the Prandtl equations. The second section, forming definitely the bulk of the work, is devoted to the study of multipolar viscous fluids. An original approach is developed in order to derive the macroscopic equations from the kinetic ones. The third part deals with modifications of the Boltzmann equation introduced by Alexeev and the author.

- In Chapter 1, the compressible Prandtl equations are identified as a formal limit of the Boltzmann equation under a suitable scaling. Although the analysis is purely formal based on comparison of different terms in the asymptotic expansion with respect to a small parameter proportional to the thickness of the boundary layer and the vertical component of the velocity, the result is interesting as it completes several recent studies. The argument is very sketchy and some details would certainly deserve more comments. In particular, the choice of the boundary conditions (1.13) as well as the assumptions concerning the asymptotic expansion of the collision operator.

- Chapter 2 represents the heart of the present work. Here the candidate examines the multipolar viscous fluids from the point of view of "macroscopic" limits of the kinetic equation. In contrast with the standard Boltzmann equation, a new term is introduced related to the non-local as well as non-instant binary interactions. The principal idea asserts that this term should give rise, being considered in the asymptotic limit in the spirit of Chapman and Enskog, to the extra terms appearing in the macroscopic description of the so-called multipolar fluids. The results are compared with those obtained in the framework of classical continuum fluid mechanics. Although the final result is not completely satisfactory as some crucial terms possess the opposite sign to what have been expected, the analysis definitely sheds some light on the complexity of the problem and opens new directions for future discussion.

- In Chapter 3, a modified version of the Boltzmann equation introduced by Alexeev is studied. After a thorough discussion, the author claims that the leading term in the model proposed by Alexeev should have an opposite sign - the fact that changes drastically the character of the resulting equation. Both models are then studied analytically, and relevant existence results are obtained. Here the author uses two rather different methods relevant to the elliptic and hyperbolic character of the problem, respectively.

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On the point of conclusion, I would like to say that the present Thesis represents a significant piece of work. Although most of the results are of purely formal nature, the author demonstrated a deep insight into the problem combined with a great piece of physical intuition. Last but not least, to the best of my knowledge, this kind of problems have been neglected by most of the domestic research groups so that Častulík's work opens new possibilities for future research activities. I warmly recommend the candidate P. Častulík to obtain the PhD degree in the branch Mathematical and computer modeling.

In Prague, 15 August 2007

Eduard Feireisl
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