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August 20th, 2007

Report on Petr Castulik's PhD

Petr Castulik's thesis deals with various issues in the problem of hydrodynamic limits of the kinetic theory of gases.

Hydrodynamic limits are the process by which the PDEs of fluid mechanics are derived from more microscopic models — such as molecular dynamics, or the equations of the kinetic theory of gases or plasmas (such as the Boltzmann, or the Fokker-Planck-Landau equations).

This is a very hard subject, since the tremendous difficulties in the analysis of all the nonlinear PDEs of fluid mechanics (such as the Euler or Navier-Stokes systems) also appear in their derivation from kinetic models and impose very stringent limitations on the validity of these derivations.

As a result, most mathematicians have studied this problem of hydrodynamic limits in the simplest physical situation, i.e. in the case of an ideal, monatomic gas. By contrast, the originality in Petr Castulik's project is in his choice of other physical models that go far beyond the ideal gas case and try to capture more complicated interactions.

Petr Castulik's thesis is divided into three independent chapters devoted to three very different problems.

Chapter 1 bears on the derivation of Prandtl's compressible boundary layer theory directly from the kinetic theory of gases.

Prandtl's boundary layer theory makes a transition from the no-slip boundary condition that is prescribed on the solution of the Navier-Stokes equations (compressible or not) and the corresponding solution of the Euler equations in the inviscid limit — we recall that, in the inviscid case, only the normal component of the velocity field at a solid boundary can be set equal to 0, at variance with the no-slip condition where both the tangential and normal components are 0. Prandtl's boundary layer theory leads to a very unstable system of PDEs which is usually not well-posed except in very specific situations.

Petr Castulik explains how these boundary layer equations can be derived directly from the Boltzmann equation under convenient scaling assumptions, well adapted to zoom in on a thin layer of fluid of the order of the square-root of the Knudsen number.

His derivation of Prandtl's equations from the Boltzmann equation is formal — i.e. it assumes the existence of smooth solutions of both the kinetic model and the boundary layer equations — and obtained by appropriate *modifications of the usual Chapman-Enskog expansion method for various* boundary conditions at the kinetic level.

Chapter 2 deals with the derivation of the Navier-Stokes models for multipolar fluids from kinetic models more complicated than the Boltzmann equation — in the same spirit as the Enskog equation.

We recall that the Boltzmann equation is obtained from molecular gas dynamics under some very specific scaling assumption (called the Boltzmann-Grad limit) in which the molecular radius is sent to 0. As a result

(a) only binary collisions are considered in the Boltzmann equation;

(b) each such collision is instantaneous and localized in a single point in space.

Petr Castulik proposes modifications of the classical Boltzmann equation that take into account the molecular radius as well as the duration of each molecular collision. This leads to perturbations of the usual Boltzmann equation by terms of the order of the mean free path. As a result, the leading order term in the hydrodynamic limit is still a Maxwellian, but the perturbation leads to a more complicated stress tensor field, and the resulting Navier-Stokes type equation includes higher order derivatives of the solution (up to order 4, for instance). In this case again, the derivation *of the hydrodynamic equation is obtained by the Chapman-Enskog expansion method, thereby leading to a formal result — assuming smoothness on the solution of both the limiting fluid equations and the kinetic model.* Petr Castulik compares this derivation from a kinetic model with more classical *derivations based on thermodynamics and symmetries in in the constitutive relations for the fluid.*

Chapter 3 considers a modification of the classical Boltzmann equation due to Alexeev, which consists of adding to the streaming operator the second order term in the Taylor expansion of the solution restricted to a characteristic line.

Petr Castulik studies the linearized equation about some Maxwellian state. Depending on the sign of the second order perturbation, the resulting problem is well-posed either as a Cauchy-problem or as a boundary value problem in the time variable. He discusses the respective merits of the theories corresponding to both signs for this perturbation, starting from Alexeev's argument based on the Boltzmann H Theorem.

Finally he establishes the acoustic limits for this model, for both signs of the Alexeev perturbation term. In this limit, this perturbation term is *not* scaled with the mean free path, thereby leading non classical acoustic equations.

In my opinion, Petr Castulik's thesis is a highly original and significant contribution to a very difficult class of mathematical problems, which opens door to further questions in applied analysis and modeling. Undoubtedly, this contribution is at the level of a very good PhD.

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