The goal of this thesis is to study real quadratic number fields  $\mathbb{Q}(\sqrt{D})$  such that, for a given rational integer m, all m-multiples of totally positive integers are sums of squares. We prove quite sharp necessary and sufficient conditions for this to happen. Further, we give a fast algorithm that verifies this property for specific m, D and that for a fixed m finds all such fields in polynomial time.