

The aim of this work is to study the number of variables of universal quadratic forms in number fields. In particular, we provide the whole proof of the following theorem: In each degree $2n$, there are infinitely many totally real number fields that require universal quadratic forms to have arbitrarily large rank. The key step in this proof is to estimate the trace of an algebraic integer using one of the Stieltjes's bounds of discriminant. We focus on these bounds and introduce tools for proving them. Furthermore, we deal with some elementary estimates of the number of algebraic integers whose trace is bounded and summarize the relevant theory of traces and discriminants.