An ordering on bases in Banach spaces is defined as a natural generalization of the notion of equivalence. Its theory is developed with emphasis on its behavior with respect to shrinking and boundedly-complete bases. We prove that a bounded operator mapping a shrinking basis to a boundedly-complete one is weakly compact. A well-known result concerning the factorization of a weakly compact operator through a reflexive space is then reinterpreted in terms of the ordering.

Next, we introduce a class of Banach spaces whose norm is constructed from a given two-dimensional norm N. We prove that any such space X_N is isomorphic to an Orlicz sequence space. A key step in obtaining this correspondence is to describe the unit circle in the norm N with a convex function φ. The canonical unit vectors form a basis of a subspace Y_N of X_N. We characterize the equivalence of these bases and the situation when the basis is boundedly-complete. The criteria are formulated in terms of the norm N and the function φ.