We study the behaviour of logarithmically convex combinations of operators given by $Tf = |S_1 f|^{\frac{1}{\theta}} |S_2 f|^{1-\frac{1}{\theta}}$, where S_1, S_2 are some (usually quasi-linear) operators acting on spaces of measurable functions and $\theta \in (1, \infty)$ is a parameter. We develop two, quite different in nature, interpolation theories, each of which enables us to obtain a rather comprehensive information about the behavior of such operators on function spaces. The first one is completely general and is based on abstract interpolation and Calderón spaces. We illustrate the theoretical results by a wide variety of examples of pairs of spaces X, Y such that $T\colon X \to Y$ is bounded, these in particular include the so-called Calderón-Lozanovskiĭ construction. The second theory departs from pointwise estimates by Calderón operators and is particularly tailored for obtaining boundedness results between Orlicz spaces given weak-type estimates that arise in applications. A common feature of both theories is an approach, apparently new, involving interpolation of four spaces. The input data in each case consists of two reasonable separate endpoint estimates for the operators S_1 and S_2.