Report on bachelor thesis by Ondrej Bínovský:

Imaginary quadratic fields with class number 1

Summary: One of the basic phenomena encountered in algebraic number theory is that unique factorization can fail over number fields. The *class number* can be colloquially described as a quantitative measure of such a failure. It is a positive integer and is equal to one if and only if we have unique factorization.

The thesis focuses on the family of imaginary quadratic fields, for which it is known that exactly nine cases have class number one. This result is one of the major achievements of the 20th century in the subject and the credit sould go essentially to Heegner and Stark, as it is described in the nice historical introduction of this work.

The candidate explains two proofs of the main result: the imaginary quadratic fields with class number one are those of the form

$$\mathbb{Q}(\sqrt{-n}), \quad n \in \{1, 2, 3, 7, 11, 19, 43, 67, 163\}.$$

The first proof follows Stark's work, whereas the second one is Siegel's proof. Both methods require the study of modular functions, concepts from algebraic number theory, and a few results on elliptic curves. All the necessary tools are explained in Sections 2-4.

In addition, the candidate describes in Section 3 a sufficient criterion for class invariants, giving a different proof from the existing one due to Gee, and describes explicitely Weber's transformation equations for modular functions, with an application to two particular cases.

Evaluation: The thesis is well organized in the presentation of the material. It is well written and mathematically correct. Almost all the claims are either proved or a reference is given where the proof can be found. The variety of the sources shows that the candidate has familiarity with the literature on the subject. Moreover, the discussion in Sections 3 and 4 shows that the candidate has mastered well the tools and the techniques related to modular functions.

Recommendation for the thesis: `Pass, with grade 1.`

Comments: I have three minor comments:

- There is a typo in Proposition 2.1.1, where $\overline{\overline{\mathcal{F}}}$ should be $\overline{\mathcal{F}}$;
- On p.16, Lemma 2.3.7, the set s_N has not been defined yet. It is defined on p.31, whereas it should be defined the first time it is used.
- On p.29, before (3.17): there is a typo, $\Phi_{\mathfrak{p}^{24}}$ should be $\Phi_{\mathfrak{f}^{24}}$.

Giacomo Cherubini