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Principles of information processing in neuronal models

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1 Introduction and motivation

Neurons communicate via chemical and electrical synapses, in a process known as synaptic transmission. The crucial event that triggers synaptic transmission is the action potential (or spike), a pulse of electrical discharge that travels along the axon excitable membrane. The shapes and durations of individual spikes generated by a given neuron are very similar, therefore it is generally assumed that the form of the action potential is not important in information transmission. The series of action potentials in time (spike trains) can be recorded by placing an electrode close to or inside the soma or axon of a neuron. Since individual spikes in a spike train are usually well separated, the whole spike train can be described as a series of all-or-none point events in time (Gerstner & Kistler, 2002). The lengths of interspike intervals (ISIs) between two successive spikes in a spike train often vary, apparently randomly, both within and across trials (Gerstner & Kistler, 2002; Shadlen & Newsome, 1998; Stein *et al.*, 2005). In order to describe and analyze the way information about stimuli is represented in spike trains (Perkel & Bullock, 1968; Softky, 1995; Strong *et al.*, 1998), methods to compare different spike trains are needed first (Bhumbra *et al.*, 2004; Buracas & Albright, 1999; Nemenman *et al.*, 2004; Paninski, 2003; Rieke *et al.*, 1997; Victor & Purpura, 1997).

In the rate coding scheme information sent along the axon is encoded in the number of spikes per observation time window (the firing rate) (Adrian, 1928). In most sensory systems, the firing rate increases, generally non-linearly, with increasing stimulus intensity (Kandel *et al.*, 1991). Any information possibly encoded in the temporal structure of the spike train is ignored. Consequently, rate coding is inefficient but highly robust with respect to the ISI 'noise' (Stein *et al.*, 2005). The question whether the temporal structure of ISIs is due to unavoidable fluctuations in spike generation or whether it represents an

informative part of the neuronal signal is not yet fully resolved (Gerstner & Kistler, 2002; Shadlen & Newsome, 1994; Stein *et al.*, 2005) and leads to the idea of temporal coding.

Temporal codes employ those features of the spiking activity, that cannot be described by the firing rate. For example, time to first spike after the stimulus onset, characteristics based on the second and higher statistical moments of the ISI probability distribution, or precisely timed groups of spikes (temporal patterns) are candidates for temporal codes (Buracas & Albright, 1999; Gerstner & Kistler, 2002; Rieke *et al.*, 1997). Possibility of information transmission by changes in ISIs serial correlation has been reported in crayfish interneurons (Sugano & Tsukada, 1978; Wiersma & Adams, 1950). For a classic overview of temporal coding see Perkel & Bullock (1968), for a more recent discussion see Abeles (1994); Rieke *et al.* (1997); Shadlen & Newsome (1994); Stein *et al.* (2005); Theunissen & Miller (1995).

2 Aims of the thesis

While the description of neuronal activity from the rate coding point of view is relatively straightforward, the temporal coding allows infinite number of possibilities. Spike trains which are equivalent from the rate coding perspective may turn out to be different under various measures of their temporal structure. The aim of this work is to introduce and apply a measure of randomness of the neuronal activity. We discuss properties of this measure with respect to rate and temporal coding schemes and its application to experimental data. We show, that spiking randomness is capable to capture characteristics that would otherwise be difficult to obtain with conventional methods. The notion of randomness is very different from that of variability, even though these terms are sometimes interchanged. Furthermore, since the definition of randomness is based on the concept of entropy (Shannon & Weaver, 1998), relation with other

information-theoretic quantities can be established.

3 Theory and Methods

3.1 Probabilistic description of neuronal activity

Spike train consists of times of spike occurrences $\tau_0, \tau_1, \dots, \tau_n$. For the purpose of further analysis it is advantageous to describe such spike train equivalently by a set of n ISIs $t_i = \tau_i - \tau_{i-1}$, $i = 1 \dots n$. Arguably the most important characteristics calculated from t_i is the estimate \bar{t} of the mean ISI,

$$\bar{t} = \frac{1}{n} \sum_{i=1}^n t_i. \quad (1)$$

Since $\sum_{i=1}^n t_i = \tau_n - \tau_0$, the average \bar{t} is computed without recourse to particular interval lengths and thus presents the lowest level of ISI analysis (Moore *et al.*, 1966). Other common parameters, coefficient of variation and standard deviation of ISIs, require all measurements, t_i , and both rely on the estimate s^2 of the ISI variance,

$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (t_i - \bar{t})^2. \quad (2)$$

However, \bar{t} and s^2 are meaningful only if the spiking activity is stationary, i.e., if the major probability characteristics of the firing are invariant in time (Cox & Lewis, 1966; Landolt & Correia, 1978). Stationary neuronal firing is typically observed in the spontaneous activity, or under constant stimulus conditions (Gerstner & Kistler, 2002; Moore *et al.*, 1966; Tuckwell, 1988).

The probabilistic description of the spiking results from the fact, that the positions of spikes cannot be predicted deterministically, only the probability that a spike occurs can be given (Gerstner & Kistler, 2002). By far the most common probabilistic descriptor is the ISI probability density function $f(t)$, where

$f(t) dt$ is the probability that spike occurs in an interval $[t, t + dt)$ (Moore *et al.*, 1966). Probability density function does not depend on the ordering of ISIs, i.e., it completely describes the firing if ISIs are mutually independent realizations of a positive random variable T , with mean ISI $E(T)$ and variance $Var(T)$ estimated by formulas (1) and (2). Such firing is called renewal process of ISIs (Cox & Lewis, 1966; Gerstner & Kistler, 2002). The plausibility of renewal models under steady-state stimulus conditions is supported by observation, that after a spike is emitted, the membrane potential of the cell returns to its (approximately) constant resting value (Gerstner & Kistler, 2002; Landolt & Correia, 1978; Stein, 1967; Tuckwell, 1988). Sometimes, however, there might be a dependency structure between the observed ISIs (Chacron *et al.*, 2001; Lansky & Rodriguez, 1999; Lindner, 2004; Longtin & Racicot, 1997; Ratnam & Nelson, 2000; Sakai *et al.*, 1999). The dependence may arise, for example, due to incomplete resetting of the membrane potential after the spike is emitted, which is experimentally observed especially in the distal parts of the neuron (Abeles, 1982). Such type of neuronal firing is not a renewal process, although the ISI probability distribution is invariant in time (due to the stationarity of spiking).

3.2 Spiking variability

One of the most frequently used characteristics of renewal neuronal firing is the ISI variability. The variability may be measured simply using the ISI variance, $Var(T)$, but variance depends on the mean ISI. Usually, it is required to characterize the spike train differences from the temporal coding point of view, in other words to describe properties which are distinct from the mean ISI. To achieve this the ISI lengths are rate-normalized, i.e.,

individual ISIs are divided by the mean ISI,

$$\Theta = \frac{T}{E(T)}, \quad (3)$$

so we obtain a new dimensionless random variable Θ with mean $E(\Theta) = 1$. Variance of Θ is equal to the coefficient of variation of the original random variable T , $Var(\Theta) = C_V$, where

$$C_V = \frac{\sqrt{Var(T)}}{E(T)}. \quad (4)$$

The main advantage of C_V as a measure of spiking variability (compared to variance) is that C_V is dimensionless and its value does not depend on the choice of units of ISIs (e.g., seconds or milliseconds) and thus ISI probability distributions with different means can be compared meaningfully (Softky & Koch, 1993). Furthermore, the C_V of ISIs is related to the variability coding hypothesis (Perkel & Bullock, 1968). The coding characterized by C_V has been proposed as a possible mechanism to transmit information about light intensity as well as adaptation state in the same spike train. A dark adapted cell has a larger C_V of ISIs than a light-adapted cell (for a given mean ISI) (Ratliff *et al.*, 1968). Changes in the level of bursting activity, characterized by values $C_V > 1$, are reported to be the proper code for edge detection in certain units of visual cortex (Burns & Pritchard, 1964) and also in hippocampal place cells (Fenton *et al.*, 2002). The variability of ISIs generated by the leaky integrate-and-fire model (Burkitt, 2006; Gerstner & Kistler, 2002) was recently a topic for a very extensive discussion initiated by Softky & Koch (1993).

Nevertheless, $E(T)$ and C_V are not sufficient to describe all possible differences between spike trains. The spike trains described by the renewal processes of equal variability may have ISI probability distributions that differ in higher than second statistical moments. Additionally, C_V does not account for

statistical dependency between ISIs (by definition), and thus spike trains with the same marginal probability distributions of ISIs have the same variability. Instead of employing characteristics based on higher statistical moments of the probability distributions involved and serial correlation coefficients of the ISIs we propose to measure the randomness of the spiking activity. Spiking randomness accounts automatically for differences in both marginal probability distributions and serial dependence of ISIs.

3.3 Spiking randomness

The randomness of spiking can be defined as the measure of 'choice' of different ISI lengths that appear in the spike train and the measure of 'freedom' in their serial ordering. Bigger choice of ISIs and more freedom in their ordering results, intuitively, in greater randomness of spiking. We first overview the concept of entropy (Shannon & Weaver, 1998), on which the measure of spiking randomness is based.

For a discrete random variable X with the set of possible states $\{x_1, x_2, \dots, x_n\}$ and the corresponding probability mass function $p_i = \text{Prob}\{X = x_i\}$, the entropy $H(X)$ is defined as (Shannon & Weaver, 1998)

$$H(X) = - \sum_{i=1}^n p_i \ln p_i. \quad (5)$$

The entropy $H(X)$ is positive or equal to zero with equality if only one option is possible (no randomness). Maximum randomness (maximum $H(X)$) is reached when all p_i 's are the same. If the logarithm base in formula (5) is 2, $H(X)$ can be interpreted as the average length in bits of the shortest description of X (Cover & Thomas, 1991). The entropy is a unique measure of randomness satisfying a set of intuitive conditions (Jaynes & Bretthorst, 2003; Shannon & Weaver, 1998), however, it is applicable to discrete systems only.

The extension of formula (5) for continuous probability

distributions is impossible because the value of $H(X)$ diverges (Cover & Thomas, 1991). Therefore, the differential entropy $h(T)$ of the ISI probability density function $f(t)$ is defined as

$$h(T) = - \int_0^{\infty} f(t) \ln f(t) dt. \quad (6)$$

Differential entropy $h(T)$ does not have all the properties and intuitive interpretation of the entropy $H(X)$. The value of $h(t)$ changes with coordinate transforms, e.g., depends on the time units of ISIs. Probability density function $f(t)$ has a physical dimension (it is a derivative of probability with respect to time), therefore $h(T)$ has the dimension of its logarithm, e.g., logarithm of a milisecond. These facts show, that the differential entropy cannot be used to quantify the randomness of spiking activity. To overcome this problem, a discretization method has been adopted in literature (Rieke *et al.*, 1997; Strong *et al.*, 1998) which converts the task back to formula (5), however, the results depend on the discretization factor (Chacron *et al.*, 2001).

Here we proceed in a different way, avoiding the discretization. We want the randomness to characterize the spike train differences from the temporal coding point of view, in a similar way to C_V . Thus formula (3) is employed to rate-normalize the ISI lengths and the spiking randomness η is defined as the differential entropy of the random variable Θ . The following relation holds (Kostal *et al.*, 2007)

$$\eta = h(T) - \ln E(T). \quad (7)$$

Before discussing the properties of η we mention another approach to defining randomness. The exponential probability density function, $f_{\text{exp}}(t)$, is given as

$$f_{\text{exp}}(t) = \lambda \exp(-\lambda t), \quad (8)$$

where $\lambda > 0$ is the inverse of its mean, $\lambda = 1/E(T)$. An important property of $f_{\text{exp}}(t)$ is, that it achieves maximum differential entropy among all ISI probability distributions with the same mean ISI (Cover & Thomas, 1991). The exponential model $f_{\text{exp}}(t)$ represents the 'zero point' on the differential entropy scale for all ISI probability density functions with the same means. Kullback-Leibler (KL) distance $D(f, f_{\text{exp}})$ given by formula (Cover & Thomas, 1991)

$$D(f, f_{\text{exp}}) = \int_0^{\infty} f(t) \ln \frac{f(t)}{f_{\text{exp}}(t)} dt \quad (9)$$

measures the deviation between probability density functions $f(t)$ and $f_{\text{exp}}(t)$. Therefore, $D(f, f_{\text{exp}})$ can be used to quantify the randomness of probability density function $f(t)$, if $f(t)$ has the same mean as $f_{\text{exp}}(t)$ (Kostal & Lansky, 2006c). It can be shown that η is related to $D(f, f_{\text{exp}})$ by a simple formula (Kostal *et al.*, 2007)

$$\eta = 1 - D(f, f_{\text{exp}}) \quad (10)$$

and thus both proposed measures of randomness, η and $D(f, f_{\text{exp}})$, are equivalent in their properties because their values differ only in a sign and a constant. Definition (7) can be naturally extended to account for non-renewal spiking activity (Kostal & Lansky, 2006a).

3.4 Properties of spiking randomness

Here we summarize basic properties of the spiking randomness η , and compare it with the properties of variability as measured by the coefficient of variation C_V .

- Due to rate-normalization of the ISI probability distribution, the randomness η is a dimensionless quantity and does not depend on coordinate transformations (Kostal *et al.*,

2007). Consequently η allows to compare different stationary spiking activities in the same way as C_V .

- Maximum spiking randomness is generated only by the renewal process with exponential probability distribution of ISIs (Poisson process). Substituting formula (8) into formula (7) gives $\eta = 1$. Any non-renewal spiking activity with exponential marginal probability distribution of ISIs must have $\eta < 1$, since less freedom in serial ordering of ISIs results in smaller randomness (Kostal & Lansky, 2006a).
- Coincidentally, both $\eta = 1$ and $C_V = 1$ for exponential distribution. Many non-exponential probability distributions can have $C_V = 1$, but their randomness is always $\eta < 1$. The equality $\eta = 1$ completely characterizes the exponential distribution of ISIs.
- Equally variable spike trains may differ in their randomness. However, the same spiking randomness may be achieved with different spiking variabilities. Thus, randomness provides an alternative rather than superior characteristic of neuronal firing compared to variability (Kostal *et al.*, 2007).
- C_V is limited from below by $C_V = 0$ (regular spiking) but there is no maximum spiking variability. Values $C_V > 1$ are characteristic of bursting activity. On the other hand, there is no unique minimal randomness probability distribution, because $\eta = -\infty$ for any discrete random variable. However, discrete probability distributions are not valid models of spiking activity (ISI 'noise' is always present), and the fact that η may not be finite is of little practical consequence (Kostal & Lansky, 2006a).
- Spiking randomness is an information-theoretic measure, related to entropy and KL distance. The strength of information-theoretic measures lies in their ability to reveal

non-linear dependencies (Cover & Thomas, 1991; Rieke *et al.*, 1997; Yamada *et al.*, 1993). Recently, KL distance has been used in the field of neuronal coding from the classification theory point of view Johnson *et al.* (2001) and as a predictor of purely rate coding models (Johnson & Glantz, 2004). Renormalized entropy (a special case of KL distance) has been shown to provide additional information over traditional tools in EEG record analysis (Kopitzki *et al.*, 1998; Quiroga *et al.*, 2000; Thakor & Tong, 2004).

4 Summary of main results

4.1 Model spiking activity

Probabilistic models of stationary spiking activity may be divided in two categories: statistical and biophysical. The statistical models are described by probability density functions which are simple enough to manipulate and adequately describe experimentally observed data but no other connection with neurophysiological reality is required. The biophysical models, on the other hand, result from attempts to describe the behavior of real neurons at different levels of abstraction (Gerstner & Kistler, 2002; Tuckwell, 1988). However, mathematical expressions for biophysical models are rarely given in a closed form and one has to rely on numerical approximations.

The analysis of several statistical and biophysical renewal process models was performed in Kostal & Lansky (2006b,c); Kostal *et al.* (2007). Here we present an overview of the main results. The statistical models are represented by gamma and log-normal ISI probability distributions, both are commonly used for experimental data description (Duchamp-Viret *et al.*, 2005; Levine, 1991; Mandl, 1992; McKeegan, 2002; Rospars *et al.*, 1994). The inverse Gaussian distribution (Chhikara & Folks, 1989) results from a simple point stochastic neuronal

model (perfect integrator) where the approach of the membrane potential towards the threshold is described by the Wiener process with a positive drift (Berger *et al.*, 1990; Levine, 1991). The inclusion of leakage current into this model results in the more realistic diffusion leaky IF model (Burkitt, 2006; Tuckwell, 1988) where the membrane potential evolution is described by the Ornstein-Uhlenbeck process. The parameters of the leaky IF model determine two firing regimes, depending on the ratio $\xi = S/(\mu\tau)$ of the threshold membrane potential S to the neuronal input μ and membrane time constant τ (Burkitt, 2006; Kostal *et al.*, 2007). The sub-threshold regime is characterized by $\xi > 1$, supra-threshold by $\xi < 1$. The gamma, lognormal and inverse Gaussian distributions are completely characterized by $E(T)$ and C_V and therefore it is possible to calculate unique value of η for each value of C_V (note that η is independent of $E(T)$). Similar calculation is possible for the leaky IF model once μ, τ and S are known (determining the supra- or sub-threshold regimes) because the amount of input 'noise' σ^2 controls the actual value of C_V (Kostal *et al.*, 2007). The following inference can be made based on Fig. 1, where the randomness of each model is plotted against the corresponding variability.

- The randomness-variability curves of the investigated models are often U-shaped with high randomness values distributed around $C_V = 1$. The notable exceptions are the sub-threshold regime of the leaky IF model and the statistical Pareto model (not included, Kostal & Lansky (2006a)).
- While small variability generally implies low randomness, high variability in the firing may result in both low as well as high randomness.
- It is well known that the lognormal, inverse Gaussian and leaky IF supra-threshold ISI distributions never become exponential, but in addition their maximum randomness

(minimal KL distance from the exponential model) is not located at $C_V = 1$.

- The behavior of the leaky IF model in the supra-threshold regime is comparable to the perfect integrator (inverse Gaussian model). In the sub-threshold regime, the effect of coherence resonance (Lindner *et al.*, 2002) is demonstrated by local decrease of C_V for $\eta \approx 0.96$. However, there is no corresponding local decrease in spiking randomness, i.e., the coherence resonance for certain (high) values of ξ is observable in C_V but not in η . Though it is known that the degree of coherence resonance depends on the measure employed (Lindner *et al.*, 2004), the apparent disappearance of the effect on the η scale raises the question of what is the proper measure of ISI coherence (Kostal *et al.*, 2007).

Several statistical models of non-renewal spiking activity described by first-order Markov chains (Cox & Lewis, 1966) were examined in Kostal & Lansky (2006a). Markov structure in experimental data is reported in literature (Ratnam & Nelson, 2000), and even the first-order case makes the existence of certain (short) sequences of ISIs more probable than others (basic temporal pattern formation, Kostal & Lansky (2006a)). Although the examined models were not used in data analysis previously, the results show that the serial correlation coefficient (Cox & Lewis, 1966) is a weak indicator of the true ISI serial dependence. For example, the Lawrance and Lewis model with exponential marginal probability distribution of ISIs (Lawrance & Lewis, 1977) can achieve either randomness $\eta = 0.97$ or $\eta = 0.82$ for the same value of first-order serial correlation $\rho = 0.17$. It follows, that randomness η or mutual information between ISIs (Cover & Thomas, 1991) should be employed when deciding on renewal or non-renewal character of experimental data. However, estimation of these information-theoretic quantities requires large amounts of data which are usually not available in experimental

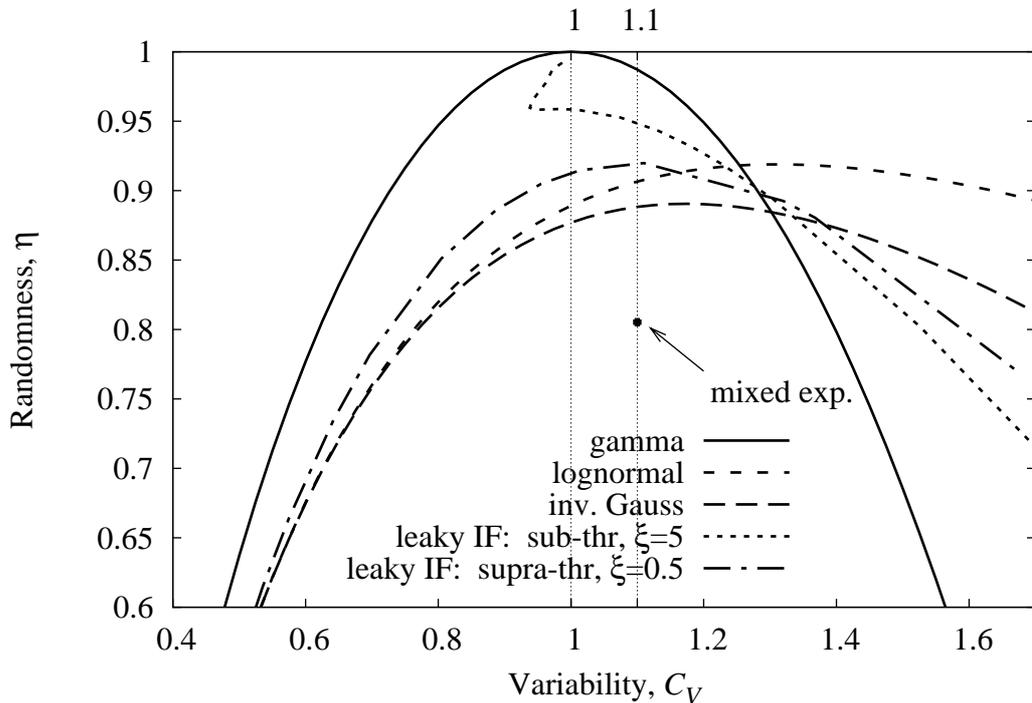


Fig. 1: Randomness vs. variability for some widely used renewal models of neuronal activity. For $C_V = 1$ the gamma distribution becomes exponential and thus $\eta = 1$. The inverse Gaussian and lognormal models never become exponential their maximum randomness (minimal KL distance from the exponential model with the same mean ISI) is not located at $C_V = 1$. The behavior of leaky IF model in the supra-threshold case is similar to the behavior of inverse Gaussian and lognormal models. The sub-threshold activity exhibits the effect of coherence resonance (local decrease of C_V for $\eta \approx 0.96$). The dot shows, that for $C_V = 1.1$ the randomness of the exponential mixture model is lower than that of any other model considered here.

recordings. Finally we note, that the maximum order of non-zero serial correlation coefficient does not coincide with the dimension of the joint probability density function describing the activity. For example, first-order moving average process is non-renewal with all second- and higher-order serial correlations equal to zero. However, joint probability density function of two adjacent ISIs does not describe such process, since the Markov property,

$\text{Prob}\{T_n \leq t_n | T_{n-1} = t_{n-1}, \dots, T_1 = t_1\} = \text{Prob}\{T_n \leq t_n | T_{n-1} = t_{n-1}\}$ (Cox & Lewis, 1966), does not hold in this case.

4.2 Experimental data

Duchamp-Viret *et al.* (2005) estimated the randomness from the spontaneous activity recordings done on olfactory receptor neurons of freely breathing and tracheotomized rats. The recordings were obtained under steady-state conditions and it was shown that in the majority of cases the firing can be considered renewal. It was demonstrated, that the spontaneous activity is less variable but more random in the case of tracheotomized animals than in those freely breathing. This effect is further enhanced if η is adjusted for different spiking rates by considering instead the ratio $\eta/E(T)$, i.e., the 'amount' of randomness per time unit (Kostal & Lansky, 2006a).

5 Conclusions

Comparison of neuronal spiking activity under different conditions plays a key role in resolving the question of neuronal coding. The spiking activity of a neuron is usually not deterministic, therefore ISI is described by means of probability distributions. We proposed an information-theoretic measure of spiking randomness, η , which can be related to the differential entropy or KL distance from the maximum entropy distribution. Conceptually, the spiking randomness can be best compared with the more often used ISI variability measured by the coefficient of variation, C_V . However, the properties of randomness and variability are different. Namely, small variability generally implies low randomness, but high variability of firing may not result in high level of randomness. Simultaneously, the same level of randomness can be reached by different values of variability, depending on the probabilistic model of the spiking.

References

- Abeles, M. (1982) *Local cortical circuits: Studies of brain function*, vol. 6. Springer-Verlag, Berlin.
- Abeles, M. (1994) Firing rates and well-timed events in the cerebral cortex. In Domany, E., Schulten, K. & van Hemmen, J. L. (eds.), *Models of Neural Networks II*. Springer, New York, pp. 121–138.
- Adrian, E. (1928) *The basis of sensation*. W. W. Norton & Co., New York.
- Berger, D., Pribram, K., Wild, H. & Bridges, C. (1990) An analysis of neural spike-train distributions: determinants of the response of visual cortex neurons to changes in orientation and spatial frequency. *Exp Brain Res*, **80**, 129–134.
- Bhumbra, G., Inyushkin, A. & Dyball, R. (2004) Assessment of spike activity in the supraoptic nucleus. *J Neuroendocrinol*, **16**, 390–397.
- Buracas, G. & Albright, T. (1999) Gauging sensory representations in the brain. *Trends Neurosci*, **22**, 303–309.
- Burkitt, A. (2006) A review of the integrate-and-fire neuron model: I. homogeneous synaptic input. *Biol Cybern*, **95**, 1–19.
- Burns, B. & Pritchard, R. (1964) Contrast discrimination by neurons in the cat's visual cerebral cortex. *J Physiol*, **175**, 445–463.
- Chacron, M., Longtin, A. & Maler, L. (2001) Negative interspike interval correlations increase the neuronal capacity for encoding time-dependent stimuli. *J Neurosci*, **21**, 5328–5343.

- Chhikara, R. & Folks, J. (1989) *The Inverse Gaussian Distribution: theory, methodology, and applications*. Marcel Dekker.
- Cover, T. & Thomas, J. (1991) *Elements of information theory*. Wiley, New York.
- Cox, D. & Lewis, P. (1966) *The statistical analysis of series of events*. Chapman and Hall, London.
- Duchamp-Viret, P., Kostal, L., Chaput, M., Lansky, P. & Rospars, J. (2005) Patterns of spontaneous activity in single rat olfactory receptor neurons are different in normally breathing and tracheotomized animals. *J Neurobiol*, **65**, 97–114.
- Fenton, A., Lansky, P. & Olypher, A. (2002) Properties of the extra-positional signal in hippocampal place cell discharge derived from the overdispersion in location-specific firing. *Neuroscience*, **111**, 553–566.
- Gerstner, W. & Kistler, W. (2002) *Spiking Neuron Models: Single Neurons, Populations, Plasticity*. Cambridge University Press, Cambridge.
- Jaynes, E. & Bretthorst, G. (2003) *Probability Theory: the logic of science*. Cambridge University Press, Cambridge.
- Johnson, D. & Glantz, R. (2004) When does interval coding occur? *Neurocomputing*, **59**, 13–18.
- Johnson, D., Gruner, C., Baggerly, K. & Seshagiri, C. (2001) Information-theoretic analysis of neural coding. *J Comput Neurosci*, **10**, 47–69.
- Kandel, E., Schwartz, J. *et al.* (1991) *Principles of neural science*. Elsevier, New York.

- Kopitzki, K., Warnke, P. & Timmer, J. (1998) Quantitative analysis by renormalized entropy of invasive electroencephalograph recordings in focal epilepsy. *Phys Rev E*, **58**, 4859–4864.
- Kostal, L. & Lansky, P. (2006a) Classification of stationary neuronal activity according to its information rate. *Network: Comp Neur Sys*, **17**, 193–210.
- Kostal, L. & Lansky, P. (2006b) Similarity of interspike interval distributions and information gain in a stationary neuronal firing. *Biol Cybern*, **94**, 157–167.
- Kostal, L. & Lansky, P. (2006c) Variability and randomness in stationary neuronal activity. *Biosystems*. doi:10.1016/j.biosystems.2006.05.010.
- Kostal, L., Lansky, P. & Zucca, C. (2007) Randomness and variability of the neuronal activity described by the ornstein-uhlenbeck model. *Network: Comp Neur Sys*, in print.
- Landolt, J. & Correia, M. (1978) Neuromathematical concepts of point process theory. *IEEE Trans Biomed Eng*, **25**, 1–12.
- Lansky, P. & Rodriguez, R. (1999) Two-compartment stochastic model of a neuron. *Physica D*, **132**, 267–286.
- Lawrance, A. & Lewis, P. (1977) An exponential moving-average sequence and point process (ema1). *J App Prob*, **14**, 98–113.
- Levine, M. (1991) The distribution of the intervals between neural impulses in the maintained discharges of retinal ganglion cells. *Biol Cybern*, **65**, 459–467.
- Lindner, B. (2004) Interspike interval statistics of neurons driven by colored noise. *Phys Rev E*, **69**, 22901.

- Lindner, B., Garcia-Ojalvo, J., Neiman, A. & Schimansky-Geier, L. (2004) Effect of noise in excitable systems. *Phys. Rep.*, **392**, 321–424.
- Lindner, B., Schimansky-Geier, L. & Longtin, A. (2002) Maximizing spike train coherence or incoherence in the leaky integrate-and-fire model. *Phys Rev E*, **66**, 31916.
- Longtin, A. & Racicot, D. (1997) Assessment of linear and nonlinear correlations between neural firing events. In Cutler, C. & Kaplan, D. (eds.), *Nonlinear Dynamics and Time Series: Building a Bridge between the Natural and Statistical Sciences*. Fields Institute Communications, pp. 223–239.
- Mandl, G. (1992) Coding for stimulus velocity by temporal patterning of spike discharges in visual cells of cat superior colliculus. *Vision Res*, **33**, 1451–1475.
- McKeegan, D. (2002) Spontaneous and odour evoked activity in single avian olfactory bulb neurones. *Brain Res*, **929**, 48–58.
- Moore, G., Perkel, D. & Segundo, J. (1966) Statistical analysis and functional interpretation of neuronal spike data. *Annu Rev Physiol*, **28**, 493–522.
- Nemenman, I., Bialek, W. & de Ruyter van Steveninck, R. (2004) Entropy and information in neural spike trains: Progress on the sampling problem. *Phys Rev E*, **69**, 056111.
- Paninski, L. (2003) Estimation of entropy and mutual information. *Neural Comput*, **15**, 1191–1253.
- Perkel, D. & Bullock, T. (1968) Neural coding. *Neurosci Res Prog Sum*, **3**, 405–527.
- Quiroga, R., Arnhold, J., Lehnertz, K. & Grassberger, P. (2000) Kulback-leibler and renormalized entropies: Applications to electroencephalograms of epilepsy patients. *Phys Rev E*, **62**, 8380–8386.

- Ratliff, F., Hartline, H. & Lange, D. (1968) Variability of interspike intervals in optic nerve fibers of limulus: Effect of light and dark adaptation. *Proceedings of the National Academy of Sciences*, **60**, 464–469.
- Ratnam, R. & Nelson, M. (2000) Nonrenewal statistics of electrosensory afferent spike trains: Implications for the detection of weak sensory signals. *J Neurosci*, **20**, 6672–6683.
- Rieke, F., Steveninck, R., Warland, D. & Bialek, W. (1997) *Spikes: Exploring the Neural Code*. MIT Press, Cambridge.
- Rospars, J., Lansky, P., Vaillant, J., Duchamp-Viret, P. & Duchamp, A. (1994) Spontaneous activity of first- and second-order neurons in the frog olfactory system. *Brain Res*, **662**, 31–44.
- Sakai, Y., Funahashi, S. & Shinomoto, S. (1999) Temporally correlated inputs to leaky integrate-and-fire models can reproduce spiking statistics of cortical neurons. *Neural Networks*, **12**, 1181–1190.
- Shadlen, M. & Newsome, W. (1994) Noise, neural codes and cortical organization. *Curr Opin Neurobiol*, **4**, 569–579.
- Shadlen, M. & Newsome, W. (1998) The variable discharge of cortical neurons: Implications for connectivity, computation, and information coding. *J Neurosci*, **18**, 3870–3896.
- Shannon, C. & Weaver, W. (1998) *The Mathematical Theory of Communication*. University of Illinois Press, Illinois.
- Softky, W. (1995) Simple codes versus efficient codes. *Curr Opin Neurobiol*, **5**, 239–247.
- Softky, W. & Koch, C. (1993) The highly irregular firing of cortical cells is inconsistent with temporal integration of random epsps. *J Neurosci*, **13**, 334–350.

- Stein, R. (1967) Some models of neuronal variability. *Biophys. J*, **7**, 37–68.
- Stein, R., Gossen, E. & Jones, K. (2005) Neuronal variability: noise or part of the signal? *Nat. Rev. Neurosci.*, **6**, 389–397.
- Strong, S., Koberle, R., de Ruyter van Steveninck, R. & Bialek, W. (1998) Entropy and information in neural spike trains. *Phys. Rev. Lett.*, **80**, 197–200.
- Sugano, N. & Tsukada, M. (1978) Effect of correlated adjacent interspike interval sequences of the excitatory motor axon on the opening movement of the crayfish claw opener muscles. *Biol Cybern*, **29**, 63–67.
- Thakor, N. & Tong, S. (2004) Advances in quantitative electroencephalogram analysis methods. *Annu. Rev. Biomed. Eng.*, **6**, 453–495.
- Theunissen, F. & Miller, J. (1995) Temporal encoding in nervous systems: A rigorous definition. *J Comput Neurosci*, **2**, 149–162.
- Tuckwell, H. (1988) *Introduction to Theoretical Neurobiology*. Cambridge University Press, New York.
- Victor, J. & Purpura, K. (1997) Metric-space analysis of spike trains: theory, algorithms and application. *Network: Comp Neur Sys*, **8**, 127–164.
- Wiersma, C. & Adams, R. (1950) The influence of nerve impulse sequence on the contractions of different crustacean muscles. *Physiol Comp*, **2**, 20–33.
- Yamada, S., Nakashima, M., Matsumoto, K. & Shiono, S. (1993) Information theoretic analysis of action potential trains. *Biol Cybern*, **68**, 215–220.

List of publications

- [I] Duchamp-Viret, P., Kostal, L., Chaput, M., Lansky, P. & Rospars, J. (2005) Patterns of spontaneous activity in single rat olfactory receptor neurons are different in normally breathing and tracheotomized animals. *J Neurobiol*, **65**, 97–114. (IF: 4.170)
- [II] Kostal, L. & Lansky, P. (2006a) Classification of stationary neuronal activity according to its information rate. *Network: Comp Neur Sys*, **17**, 193–210. (IF: 2.055)
- [III] Kostal, L. & Lansky, P. (2006b) Similarity of interspike interval distributions and information gain in a stationary neuronal firing. *Biol Cybern*, **94**, 157–167. (IF: 1.398)
- [IV] Kostal, L. & Lansky, P. (2006c) Variability and randomness in stationary neuronal activity. *Biosystems*. doi:10.1016/j.biosystems.2006.05.010. (IF: 1.144)
- [V] Kostal, L., Lansky, P. & Rospars, J.-P. (2006d) Encoding of pheromone intensity by dynamic activation of pheromone receptors. *Neurocomputing*. doi:10.1016/j.neucom.2006.10.110. (IF: 0.790)
- [VI] Kostal, L., Lansky, P. & Zucca, C. (2007) Randomness and variability of the neuronal activity described by the ornstein-uhlenbeck model. *Network: Comp Neur Sys*, in print. (IF: 2.055)

Active participation in meetings with abstracts

- [i] Kostal L, Lansky P (2004) *The comparison of model and experimental ISI distributions using Kullback-Leibler distance*, (poster), Computational Systems Biology of the Neuronal Cell, SISSA-ICTP, Trieste, Italy
- [ii] Kostal L, Lansky P (2005) *A method for analyzing stationary neuronal activity from the information-theoretic point of view*, (poster), ECMTB 2005 Dresden, Germany
- [iii] Kostal L (2005) Lansky P, *Classification of stationary neuronal activity according to its information rate*, (contributed talk), Neural Coding 2005, Marburg, Germany
- [iv] Kostal L, Lansky P (2005) *Variability and randomness in stationary neuronal activity*, (poster), Fifth Conference of the Czech Neuroscience Society, Prague, Czech Republic
- [v] Kostal L, Lansky P (2006) *Comparison of variability and randomness in stationary neuronal firing*, (contributed talk), Information Theory, Neurobiology and Cognition, MPI MIS Leipzig, Germany
- [vi] Kostal L, Lansky P, Rospars J-P (2006) *Pheromone plumes provide optimal signal for the olfactory sensory neuron*, (poster), CNS 2006, Edinburgh, UK
- [vii] Kostal L, Lansky P (2006) *Characterization of stationary neuronal activity by its variability and randomness*, (poster), Prague Stochastics 2006, Prague, Czech Republic
- [viii] Kostal L, Lansky P (2007) *Variability and randomness in stationary neuronal activity*, (contributed talk), Stochastic analysis and its applications III, MFF UK, Prague, Czech Republic

