

Opponent Report

Title: Methods for enforcing non-negativity of solution in Krylov regularization

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Summary of the work's content

The Thesis deals with Krylov subspace methods with nonnegative constraints for inverse problems. It gives a good introduction to inverse problems and related difficulties. The role of Krylov subspace methods is discussed, explaining their regularization effect under certain assumptions. Several strategies to impose nonnegativity on the approximations are discussed, with an original algorithm developed by the author (MPRI, Algorithm 5). The discussed Krylov subspace methods with nonnegative constraints are then tested on several examples. Unfortunately, the example chapter is not entirely satisfying since some information is missing, and several data could have been presented better. Moreover, at places, the writing is confusing and the English too informal.

Overall, the thesis structure is well organized, the topics are well presented with clarifying examples, the proposed algorithm is interesting, and the methods were appropriately tested.

Evaluation

- **Thesis topic.** The topic difficulty is adequate for a master's thesis. The author satisfactorily fulfilled the assignment.
- **Author contribution.** The author summarized the basic idea of Krylov subspace methods for inverse problems focusing on variants for nonnegative solutions. A new variant of the Projected Restarted Iteration method (PRI), the Modified Projected Restarted Iteration method (MPRI), was developed and presented. Several Krylov subspace methods were also implemented by the candidate when needed. Finally, the candidate tested the presented methods on several benchmark problems.
- **Mathematics level.** The mathematical background is well presented with appropriate examples that help the explanation. Unfortunately, at places, the mathematical presentation is too loose. For instance, on p. 9, the singular values are generally not always decaying to zero; they do so in the considered applications. On p. 11, "That means that the coefficients $|u^T, e|$ will more or less stay or slightly oscillate around one level", it is not clear what "one level" means. On pp. 18-19, the concept of "being dominated by a vector" is not explained.
- **Literature and sources.** All the used sources are correctly cited. However, there are a few typos in the bibliography (e.g., dots are missing from the names' initials).
- **English and writing.** The English writing is overall good, but it still needs to be improved. It is often informal (verbs appear in the contracted form, e.g., p. 5, "won't", "can't", "don't"). Moreover, several sentences are unclear and have punctuation problems. For instance, on p. 21, "In reality that is unknown, however, it is available in

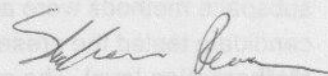
these artificial experiments”; which experiments? Also, there should be a comma after “reality”.

Questions and remarks

1. MCGLS contribution.
 - a. On p. 33, the Modified CGLS method (MCGLS) is presented. The author wrote that they “suggest” the method, but a “similar idea was studied in [9]”. Is MCGLS an original contribution? How is MCGLS related to the work in [9]?
 - b. On p. 34, “The issue with MCGLS is that we have no idea how the modification on line 5 in Algorithm 2 influences the convergence”. I have a curiosity; in your work with MCGLS, have you seen any example in which the method was not converging?
2. Lacks in the experiment presentation.
 - a. In Section 5.1, the results of the example “foxgood + baart” and “blur” are only commented, without presenting the numerical data. The following description is not enough for a scientific work: “Hence, it is not surprising that both PRI and RSPRI did really well and their results were identical. They found an accurate solution after 2 outer iterations and in total less than 5 overall iterations.” (p. 44). How accurate was the approximation?
 - b. In Section 5.2.1, a table such as the one in Figure 5.1 should complement the visual representation of the approximated solutions (Figure 5.2). What are the residual and error norm values?
 - c. Data presentation in the tables from figures 5.1, 5.7, 5.9 is not optimal. Most of the digits reported in the “relative residuum” and “relative error” columns are not significant.

Conclusion

I recommend accepting the presented thesis as a Master Thesis.



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