

In this work we develop algorithms for the k -SUPPLIER WITH OUTLIERS problem. In a network, we are given a set of *suppliers* and a set of *clients*. The goal is to choose k suppliers so that the distance between every served client and its nearest supplier is minimized. Clients that are not served are called *outliers* and the number of allowed outliers is given on input.

As k -SUPPLIER WITH OUTLIERS has numerous applications in logistics, we focus on parameters which are suitable for *transportation networks*. We study graphs with low *highway dimension*, which was proposed by Abraham et al. [SODA 2010], and low *doubling dimension*.

It is known that unless $P = NP$, k -SUPPLIER WITH OUTLIERS does not admit a $(3 - \varepsilon)$ -approximation algorithm for any constant $\varepsilon > 0$. The k -SUPPLIER WITH OUTLIERS problem is $W[1]$ -hard on graphs of constant doubling dimension for parameters k and highway dimension. We overcome both of these barriers through the paradigm of *parameterized approximation* algorithms.

In the case of highway dimension, we develop a $(1 + \varepsilon)$ -approximation algorithm for any $\varepsilon > 0$ with running time $f(k, p, h, \varepsilon) \cdot n^{O(1)}$ where p is the number of allowed outliers, h is the highway dimension of the input graph, and f is some computable function. In the case of doubling dimension, we develop a $(1 + \varepsilon)$ -approximation algorithm for any $\varepsilon > 0$ with running time $(k + p)^k \cdot \varepsilon^{-O(kd)} \cdot n^{O(1)}$ where p is the number of allowed outliers, and d is the doubling dimension of the input graph. In fact, the latter algorithm can be extended to a more general problem called CAPACITATED k -SUPPLIER WITH OUTLIERS.

Additionally, we consider a generalization of k -SUPPLIER WITH OUTLIERS called NON-UNIFORM k -SUPPLIER. It was shown that NON-UNIFORM k -SUPPLIER does not admit constant-approximation algorithms with a polynomial running time, unless $P = NP$. We extend this hardness result to the setting where the highway dimension and the doubling dimension are constant.