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Referee report on the Doctoral Thesis

"Abstract Models of Optimization Problems"

by Petr Škovroň

Linear programming is probably the most frequently solved algorithmic task in practice. Consequently a large amount of research is centered around devising fast algorithms for it. There exist algorithms working fast in practice and there exist algorithms with polynomial running time guarantees. Hence the story of LP may seem a great success, yet deeper inspection reveals that our understanding of its difficulties is not as satisfactory as the fundamental nature of the problem would desire. For all known polynomial algorithms, the running time analysis depends not only on the dimension of the problem and the number of constraints, but also on the complexity of the description of the input. In other words, we only know weakly polynomial algorithms, as opposed to strongly polynomial ones. Interpreting linear programming as a geometric optimization problem, such a dependence on the input complexity might seem superfluous as one feels that with a little a "jiggling" of the hyperplanes (constraints) the description can be shortened, while the combinatorial structure of the underlying polytope remains the same. Hence one would expect that a solution of the instance obtained by a combinatorial algorithm will not be effected by an infinitesimally small perturbation. Such a heuristic could lead to the hope that some of the succesful combinatorial algorithms should be polynomial just in the dimension and the number of constraints. A similarly small, technical-looking change "to avoid degeneracy of the problem" also plays an important role in the thesis of Petr Škovroň. Often it is assumed that such small technical changes can easily be made. One of the most interesting messages of the work of Petr is that occasionally this "technical" change **must** increase the dimension of the problem.

To formalize combinatorial algorithms, various abstract models for linear programming were developed during the last two decades. These models have two advantages. On the one hand the simplified abstract framework, stripped from the full complexity of the geometric environment, is often more transparent to perform a good running time analysis of the algorithm on. On the other hand, several completely different looking optimization problems were found to fit the abstract model and thus any algorithm developed for the abstract model would handle *all* these problems fitting in the model.

One of the most important such abstract models is the one of *LP-type problems* defined by Sharir and Welzl, and applied successfully by Matoušek, Sharir and Welzl (and Kalai in a dual context) for solving linear programming and numerous other geometric optimization problems. Besides LP-type problems (called *abstract LP-type problems* in the thesis to distinguish them from concrete LP-type problems) the dissertation deals with more recent abstract frameworks including *concrete LP-type problems* and *violator spaces*. The thesis on the one hand clarifies the relationships of all these models, on the other hand provides new insights about their power. I find the model of violator spaces particularly elegant in its simplicity considering the remarkable consequences it carries.

In the motivating example problem of linear programming the unboundedness of the instance is a strong possibility. Abstract models often ignore this outcome. An important contribution of the thesis deals with the issue of unboundedness in abstract models. In Chapter 2 great care is taken to define proper generalizations of the various models which would allow the presence of $-\infty$. This is sometimes easier sometimes harder where not even the proper definition is clear.

Another common assumption of algorithms on abstract models of linear programming is some kind of "non-degeneracy" of the instance. A significant portion of the thesis considers the combinatorial implications of this issue and arrives to a somewhat discouraging conclusion. The question asked is that for what price can this, seemingly only technical, assumption about non-degeneracy be ensured. In Chapter 3 of the dissertation Petr finds a sequence of abstract LP-type problems where in order to remove degeneracy one needs to pay a possibly quite hefty price in terms of a dimension increase. This is maybe the most involved part of the thesis, which I find very impressive. Surprisingly, an *actual* linear program is invoked to solve this theoretical question *about* linear programming. To complete the surprise, Petr shows that the seemingly artificial abstract LP-type problem constructed can be realized by a relatively simple "real" linear program.

A possible approach to gain more insight into the strength and generality of various models goes through the determination of the number of different instances with fixed parameters. In Chapter 5 several bounds on the number of violator spaces with various properties (basis-regular non-degenerate, acyclic) are derived. These are quite sharp for small values of the dimension d.

Besides the numerous structural results mentioned above the thesis contains two algorithmic chapters. In Section 6, Clarkson's algorithm (originally developed for LP) is adapted for the model of violator spaces. In fact it is proved that in some sense (cyclic) violator spaces is **the** most general framework on which Clarkson's algorithm can work. This is then applied in in Section 7 to solve non-degenerate oriented matroid programs by establishing a violator space, where locating the optimum provides the optimum solution for the oriented matroid program. Here cycles of the violator space seem to be essential which I find a particularly nice feature of the deduction.

The text of the thesis is very enjoyable to read, it is clear from the beginning that great care was exercised in the presentation. There are only a couple of places (like the oriented matroid section or motivating cyclic violator spaces) where a more elaborate description would have been helpful.

In conclusion: the dissertation contains a well-written treatise of our knowledge of various abstract models of linear programming and significantly extends them in several directions, both structural and algorithmic. Based on the above I strongly recommend the acceptance of the thesis.