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Review of the PhD dissertation of Monika Balázsová "Numerical analysis of problems in time-dependent domains", Charles University of Prague

The PhD dissertation *Numerical analysis of problems in time-dependent domains* of M. Balázsová concerns the detailed and very rigorous theoretical analysis and subsequent practical application of the space-time discontinuous Galerkin finite element method (STDG) to the solution of time-dependent hyperbolic-parabolic partial differential equations on **moving domains** in multiple space dimensions using a total Arbitrary-Lagrangian-Eulerian (ALE) formulation of the problem, i.e. where the current configuration is mapped to a reference configuration. The theoretical analysis is carried out for a nonlinear scalar model problem in multiple dimensions, while the final applications concern the numerical solution of the full compressible Navier-Stokes equations in moving domains, coupled with the equations of nonlinear elasticity.

In addition to the introduction and conclusion sections, the thesis is divided into five main chapters. The first chapter briefly introduces the motivation and the governing partial differential equations used in the final applications of the method, namely the compressible Navier-Stokes equations on moving domains. Since these equations are too complex for a thorough theoretical analysis, which is the main focus of this thesis, the PDE are simplified and a scalar nonlinear model problem is subsequently introduced in the second chapter, together with the associated numerical discretization based on the space-time discontinuous Galerkin method. Particular care is taken concerning the definition and description of the underlying function spaces, the ALE mapping and the properties of the numerical fluxes for the discretization of the convective and the viscous terms, which are all essential ingredients of the studied STDG algorithm.
In chapter three a mathematically rigorous stability analysis of the numerical scheme is carried out. Starting from some auxiliary results first the properties of the discretization of the diffusive terms are analyzed, together with the necessary penalty terms. Next, the nonlinear convective terms are analyzed, as well as the ALE derivative. A main innovation of the present PhD thesis is the generalization of the concept of discrete characteristic function to general time-dependent domains. This concept is presented in detail in section 3.3 of the PhD thesis, where also the continuity of the discrete characteristic function is proven. Based on the previous results established in sections 3.1-3.3 the final stability result is proven in section 3.4. The proof is very technical and the final result is the rigorous proof of unconditional stability of the proposed space-time DG method. The rigorous proof of unconditional stability for such a complex method (note that the schemes under consideration are arbitrary high order accurate piecewise polynomial approximations of nonlinear partial differential equations in space and time on moving domains) can be seen as a **major step forward** and **real breakthrough** in the analysis of numerical schemes for the approximate solution of time-dependent partial differential equations.

In chapter four a detailed error estimation of the STDG method is provided. Similar to the procedure in chapter three, first the discretization of the viscous terms and the associated penalty terms is analyzed. Next, the nonlinear convective terms and the ALE derivative are studied. Furthermore, the use of piecewise linear and higher order polynomials in time is investigated in detail via abstract error estimates, including also the discrete characteristic function. The error analysis of the

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projection and interpolation operators is another fundamental ingredient to obtain an overall error estimate for the complete method. The main result is finally summarized in section 4.4.3. Again, obtaining mathematically rigorous error estimates for such a complex numerical algorithm for high order accurate piecewise polynomial approximations of time-dependent PDE on time-dependent domains is a **milestone** in numerical analysis and its importance for future developments in the field must be clearly **highlighted**.

The fifth and last of the main chapters is devoted to the application of the STDG method to a complex nonlinear fluid-structure interaction problem. The governing equations for the fluid are given by the compressible Navier-Stokes equations in a moving domain. The solid is modelled by the equations of nonlinear elasticity, employing either neo-Hookean materials or St. Venant-Kirchhoff material. First, the discretization of the fluid equations and of the nonlinear elasticity equations are presented separately. The determination of the ALE mapping is discussed, together with a presentation of the adopted FSI coupling method. A summary of the coupling algorithm is given in section 5.2.3. Some details about the practical solution of the resulting nonlinear and linear systems are given in section 5.3. Numerical benchmark problems are solved in section 5.4, one pure nonlinear elasticity benchmark problem and another full FSI problem concerning the flow in the human vocal folds and their resulting dynamic vibrations. The geometry of the FSI problem under consideration is quite complex, and so are also the governing equations and the employed numerical method.

I would like to emphasize that it is extremely rare to find a thorough and very rigorous **mathematical analysis** of complex numerical schemes for nonlinear time dependent PDE **together** with an **advanced implementation** of numerical methods on **multi-dimensional unstructured** meshes within one and the same PhD thesis. The present PhD dissertation of Monika Balázsová is one of these very rare examples.

Overall, the PhD thesis is very well written, the mathematical analysis is of utmost quality and rigor, the presented numerical methods are innovative and very complex and the obtained computational results are fully convincing. The high quality of the work is also confirmed by several publications of the PhD candidate in leading international peer-reviewed journals in the field of scientific computing and applied mathematics.

Overall **an excellent piece of work**, even according to the highest international standards. The PhD candidate is **definitely mature to defend her PhD thesis** in the final oral exam.

If there are any further questions, please do not hesitate to contact me.

With best regards,

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