Report of the Supervisor on the Doctoral Thesis
by Monika Balázsová

"Numerical analysis of problems in time-dependent domains"

for Obtaining the PhD Degree

The thesis is concerned with the analysis and applications of the discontinuous Galerkin method for the numerical solution of problems in time-dependent domains. This subject is very topical and important for science and technology, because many processes proceed in time-dependent domains. Unfortunately, theoretical analysis of many complicated processes is carried out under the assumption that the space domains are independent of time. Therefore, because of this fact, the subject of this thesis plays an important role from the point of view of practical problems. On the other hand, the development of methods allowing the solution of such processes and particularly of their theoretical analysis are extremally difficult. Typical is the simulation of flow in domains with deformable boundaries. This appeares in fluid-structure interaction, which is also a subject of this thesis.

The thesis consists of five chapters. Chapter 1 starts with the formulation of the compressible viscous flow in a time-dependent domain formed by the Navier-Stokes system equiped by thermodynamical relations, and initial and boundary conditions. Then the next sections - Chapters 2-4 are devoted to the theoretical analysis of a scalar model nonlinear nonstationary convection-diffusion initial-boundary value problem in a time-dependent domain. This problem can be considered as a simplified model of the Navier-Stokes system. In Chapter 2 this problem is reformulated with the aid of the arbitrary Lagrangian-Eulerian (ALE) method and discretized by space-time discontinuous Galerkin method (STDGM). After a new formulation of the ALE-STDGM, this method is theoretically analyzed. Chapter 3 is devoted to the proof of the unconditional stability of this method. In Chapter 4 the error estimation is analyzed. The theoretical part of the thesis formed by Chapters 2-4 represent new formulation of the ALE-STDGM method and its new deap theory.

The second part of the thesis formed by Chapter 5 is devoted to the application of the worked out numerical technique to the numerical solution of compressible viscous flow in time-dependent domains and the solution of dynamic nonlinear elasticity problems. The numerical experiments show the accuracy, efficiency and applicability to complicated problems including practical fluid-structure interaction (FSI) problems. The developed method was used for the simulation of flow induced vibrations of vocal folds. Among

other it was demonstrated that the application of nonlinear elasticity models gives more accurate results than the linear elasticity model.

The thesis contains a number of new theoretical as well as practical results in the extremally complicated problems. In spite the fact that the presented theoretical analysis is very difficult, the thesis is written in a careful and clear English. Monika Balázsová worked on the subject very intensively. She is a co-author of a number publications and presented her results with success at international conferences. The submitted work satisfies all demands laid on theses for obtaining the PhD degree. Therefore, I recommend to accept this work as a PhD thesis.

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