This work is concerned with the theoretical analysis of the space-time discontinuous Galerkin method applied to the numerical solution of nonstationary nonlinear convection-diffusion problem in a time-dependent domain. At first, the problem is reformulated by the use of the arbitrary Lagrangian-Eulerian (ALE) method, which replaces the classical partial time derivative by the so-called ALE derivative and an additional convection term. Then the problem is discretized with the use of the ALE space-time discontinuous Galerkin method. On the basis of a technical analysis we obtain an unconditional stability of this method. An important step in the analysis is the generalization of a discrete characteristic function associated with the approximate solution in a time-dependent domain and the derivation of its properties. Further we derive an a priori error estimate of the method in terms of the interpolation error, as well as in terms of h and tau. Finally, some practical applications of the ALE space-time discontinuos Galerkin method in a time-dependent domain are given. We are concerned with the numerical solution of a nonlinear elasticity benchmark problem and moreover with the interaction of compressible viscous flow with elastic structures. The main attention is paid to the modeling of flow induced vocal fold vibrations in a simplified human vocal tract.