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Review of the habilitation thesis "Selected exact solutions of Einstein equations and their properties" by Dr. Martin Zŏfka

The five sections of Dr. Zŏfka's thesis represent the five different areas of research on which the thesis is based. Clearly, Dr. Zŏfka's interests span a range of different parts of the general theme of the thesis which is aspects of exact solutions of Einstein's field equations as reflected in its title. It includes vacuum and Einstein-Maxwell solutions, some with sources and and some without.

The first section is about Einstein-Maxwell fields containing a purely magnetic Maxwell field in conjunction with a cosmological constant. Actually, the term "magnetic" in the Chapter title "Gravito-magnetic" here refers to a Maxwell magnetic field. This is in distinction to the more common usage where "gravito-magnetic" refers to the "magnetic" part of the Weyl tensor, a purely general relativistic object defined in analogy with the Maxwell magnetic field. In fact, the solutions studied in Sect. 1 are characterized by a Maxwell magnetic field and an electric Weyl field (the Weyl field was not discussed in the thesis and accompanying papers). As described in the thesis introduction, the main purpose of the study was to look for static solutions in which the energy density of a magnetic field is balanced by a cosmological constant. This was achieved in two ways, one with a homogeneous magnetic field and one with a non-homogeneous magnetic field. The homogeneous solution was actually a rediscovery of a known solution as acknowledged by the authors. This is something that happens from time to time in this field. The phenomenon of rediscovery is dependent on the famous equivalence problem in general relativity. To determine whether two metrics represent the same spacetime (locally) is a non-trivial problem since any given geometry can be expressed in terms of an infinite number coordinate systems. In the present case, the importance of the finding is determined by its physical interpretation which was apparently not known previously and which actually was the impetus for the discovery. The non-homogeneous solution is advertised as a new solution by Dr. Zŏfka and his co-author.

There are some issues in Sect. 1 that should be mentioned. The first one concerns the argument to show that the magnetic field is constant. Dr. Zofka's argument is that one of the two Maxwell invariants is constant. This is also true for a plane wave for example. Nevertheless, the conclusion is true. He also states that the solution "in general" has a deficit angle and that therefore the symmetry axis is singular. While a deficit angle is possible to achieve by "cutting" a spacetime and make appropriate identifications, it is clearly not necessary as shown by the metric form (1.32) which has no singularities. Maybe this is what Zŏfka is referring to by writing "in general"? Also, according to my calculations, the expression for the field tensor in (1.50) is not correct.

In Sect. 2, the thesis is concerned with solutions of the Einstein equations called "Spacetime crystals" by Dr. Zŏfka and a coauthor. This refers to static configurations of sources which are held in equilibrium by their mutual attraction and repulsion.

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The most well-known class of such solutions is known as the Majumdar-Papapetrou family. Such solutions are constructed by placing extremal (charge = mass) Reissner-Nordström solutions at arbitrary positions and strengths. The basic question posed by the authors is whether the spacetime of an extremally charged string can be thought of as a limit of extremal Reissner-Nordström sources with equidistant spacing along an axis of symmetry. The authors find that this is indeed the case. Some other solutions are also discussed in Sect. 2, including one obtained by dimensional reduction of a 5-dimensional Majumdar-Papapetrou solution.

I will just give a brief summary of the remaining three topics of the thesis:

Gravitational swimming
The focus here is to investigate the backreaction of the gravitational field on finite size objects which perform specified motions in a spacetime. The question is to what degree it is possible to travel on a non-geodesic trajectory by performing some suitable motions. This could then be described as an attempt to "swim" in the gravitational field. The results indicate that the simple models considered do lead to non-geodesic motion as expected, but do not allow for actual swimming in the sense of counteracting fully the gravitational field.

Higher dimensions
Some exact solutions of the Einstein equations in spacetimes with dimension higher than 4 are studied. Maybe the most interesting result in this section is the finding that dimension four is very special and allows for a greater variety of solutions and that consequently spacetimes of higher dimensions are more restricted in their properties.

Cylinders
In this section, Dr. Zŏfka gives an overview of published work on cylindrically symmetric solutions including some of his own. Among things mentioned are the possible physical conclusions that can be drawn from that class of exact solutions.

To conclude, I consider the work in Sect. 1 on Einstein-Maxwell equations to be the most interesting and significant part of the thesis. It gives important new insights into the relation between Maxwell theory and the cosmological constant. As for the other sections, they show, together with the first, that Dr. Zŏfka has made a substantial contribution to the field of general relativity.

As for the issues in Sect. 1 mentioned above I would like to say the following. In my view, the findings in Sect. 1 have a greater significance than what is realized by its authors with openings for new interesting applications. I am basing this conclusion on explicit calculations which I would be happy to share with Dr. Zŏfka. In my judgement, based on this assessment, I consider the issues in Sect. 1 as insignificant in comparison.

Going through the Turnitin report, I found no serious issues of plagiarism. In my judgement, the thesis and its accompanying papers represent original work by its authors.

In summary, I recommend that Dr. Zŏfka be granted the habilitation.

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