Opponent's Review

Subject: Habilitation Theses "Error contaminated linear approximation problems: Analysis and methods" by Dr. Iveta Hnetynkova

Opponent (external examiner): Jaroslav Kautsky

The Habilitation Theses presented by Dr. Hnetynkova consist of 11 papers which she co-authored during the last 10 years and a document summarizing her work.

The problems treated are also known as TLS (Total Least Squares); the word "total" emphasizing that the error contamination affects not only the observation data but also the data describing the system. The least square problems have been dealt with for a very long time; those of the total variety attracted attention some 40 years ago. Understanding, as well as finding reliable solutions, of these problems is of essential importance for maintaining the trust in the validity of solutions of mathematical and computational problem solving.

The typical aim is first to find a minimal change to the (usually linear) problem data so that a solution exists and, second and if not unique, find a solution of a minimal norm. The candidate's work concentrates on the situation, called multi-TLS here, where a TLS problems with the same system data is solved repeatedly with multiple observation data.

The main achievements of the candidate's work are:

- Complete characterization and classification of the existence and uniqueness of multi-TLS.
- Extension of the core technique to multi-TLS.
- Generalization of the Golub-Kahan bidiagonalization for treating multi-TLS.
- Formal development, using matrix and tensor algebra, of multi-TLS problems where the data have special structures, including its applications.
- Application of regularization techniques to the situations when multi-TLS methods deal with noise affected data.

The quality of the supporting papers is assured implicitly as they were published in peer reviewed top rank journals of long standing (LAA, BIT, relevant SIAM journals and others).

The range of results is wide and in an area which is hotly pursued. This means that new results are necessarily not only subtle and detailed but also need a deep insight in both formulation as well as resolving. The candidate's overview, although concise, is clear, provides sufficient definitions of the problems and links them well to the common topic of the theses.

Just for information I list some comments and questions.

(a) Using the notation of (1.7), does there exist a class $\mathcal{S}$ problem (that is with $\operatorname{rank}((V_{12} \quad V_{13})) < d$) such that $\operatorname{rank}(V_{13}) = d - e$? It would be good to

know as if no such problem exists then the $\text{rank}(V_{13}) = d - e$ would be a neat sufficient and necessary condition for the existence of the multi-TLS solution. On the other hand, demonstrating the existence of such a problem would re-enforce the subtlety of the situation: no easy results available.

(b) on page 12, between equations (1.9) and (1.10), the argument "where for the second one it is reasonable to put $x_2 = 0$" deserves some explanation how it effects the uniqueness of solutions.

(c) Terms "the problem is incompatible" and "the TLS solution does not exist" are used in parallel. I assume they mean the same; if yes may be it can be mentioned, if not it should be explained.

(d) some typos:

- page 7, line 13 down, "..the data, in particularly.." remove "in"
- page 11, line above (1.7), "$n \geq q \geq 0$" should be "$n \geq q > 0$"
- page 12 3 lines up, "and only the first one called the core problem has to be solved" could be "and only the first one, called the core problem, has to be solved"
- page 13, 2 lines down after (1.12), the $\mathbb{R}^m$ should be $\mathbb{R}^n$
- page 15, line 12 down, replace "geral" by "general"

Conclusion: My conclusion is that the presented theses fully satisfy the requirements of the degree they were submitted for.

(signed) [redacted] 21-01-2020

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