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Prof. Jan Hamhalter
Head of the Committee for habilitation of Dr. Marek Cuth
at Charles University, Prague

Dear Professor Hamhalter,

Please find below the requested

Review of the habilitation thesis of Mgr. Marek Cúth, Ph.D.

The habilitation thesis presented by Dr. Cúth falls inside the research area of Functional Analysis, concretely within the study of isometric and isomorphic properties of Banach spaces. It deals with the study of the geometry of the Lipchitz-free spaces and present a number of significative advances in this area.

The study of Lipschitz-free spaces goes back to the 1950's when it was introduced by Arens-Ells, and have been studied since them with different names as, Arens-Ells spaces, transportation cost spaces, free spaces... The name of Lipschitz-free spaces came from a 2003 paper by Kalton-Godefroy which gave a significative advance to the theory of non-linear geometry of Banach spaces. Even though this kind of spaces were used in their origin as a tool to attack different problems, as transportation problems or the Lipschitz classification of Banach spaces, nowadays there is a wide line of study of Banach space theoretical properties of Lipschitz-free spaces, with a number of mathematicians working in. The habilitation thesis of Dr. Cúth is related to this line of study. Lipschitz-free spaces allow to linearize Lipschitz maps from a metric space M to a Banach space Y with the cost of getting a more complicate domain, the Lipschitz-free space F(M) over M. An example of application of this technique is the proof that the bounded approximation property is a "Lipschitz" property, that is, a property which is preserved by Lipschitz isomorphisms. There are many other applications of Lipschitz-free spaces, some of them are briefly explained in this habilitation thesis.

The thesis consists in an introduction containing the motivation of the research and the main definitions, a chapter describing the scientific research of the thesis divided in three sections, and 8 appendices containing one publication of Dr. Cúth each. A sample of the results of Dr. Cúth's habilitation thesis includes, among many others, the following ones:

- The proof that the Lipschitz-free space over a separable infinite ultrametric space is isomorphic to l_1.
- The Lipschitz-free space over any infinite metric space M contains a complemented copy of l_1. Actually, there exists a closed subset N of M such that F(N) is isomorphic to l_1 and the canonical copy of F(N) in F(M) is complemented in F(M).
- A description of F(M) when M is a convex open subset of a finite-dimensional normed space E as a quotient of $L_1(M,E)$.
- $\mathrm{Lip}_0(\mathbb{R}^d)$ is isomorphic to $\mathrm{Lip}_0(\mathbb{Z}^d)$ for every positive integer d.
- A general theory of Lipschitz free p-spaces, as a generalization of the classical theory.

There is no doubt that this list constitutes a major contribution to the study of Banach space theoretical properties of Lipschitz-free spaces.

Dr. Cúth has nowadays 21 papers published, following MathSciNet, all of them published in very well-reputed journals. He presents 8 of these publications as appendices of the habilitation thesis: two published in the Journal of Functional Analysis (2019 and 2020), two in Israel Journal of Mathematics (one of 2019 and one to appear), two in the Proceedings of the American Mathematical Society (2016 and 2017), one in Mathematika of 2017, and one in Mediterranean Journal of Mathematics of 2016. Each of these papers contains original research and a non-trivial contribution to the study of the Banach space properties of Lipschitz-free spaces.

The percentage of plagiarism detected by the analysis given by Turnitin system is high, but it is not representative as mostly concern with references and definitions. In my opinion, there is absolutely no problem of plagiarism with this habilitation thesis and the material included is the original research of Dr. Cúth.

In conclusion, I think that the research of Dr. Cúth has a good International Recognition among specialists in Functional Analysis and that his habilitation thesis has enough quality to get the approval of the committee and, therefore, earn the habilitation.

Sincerely yours,

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Miguel Martín
Full Professor
University of Granada, Spain