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Report on the habilitation thesis
Structural Ramsey Theory and the Extension Property for Partial Isomorphisms
submitted by Jan Hubička

To whom it may concern

General remarks. I have for some years been aware of Jan Hubička's substantial contributions to a variety of fundamental topics in combinatorics and discrete mathematics, which also form the core of his comprehensive habilitation thesis. His thesis covers a considerable breadth of results in several areas that are being brought together in a new way and put into a common perspective. Connections with structural graph and hypergraph theory, fundamental investigations of an essentially combinatorial character in classical areas of model theory and universal algebra, and a far-reaching emphasis on structure theory account for the remarkable breadth and depth of the results that are being expounded in this habilitation thesis and the original research papers that underpin it. Only some of these topics are close to my own research area, which for the most part has been oriented more directly to logic-related questions. Especially with an emphasis on my own specialisation in finite model theory, however, mathematical logic shares a deep interest in combinatorial issues of a universal-algebraic nature in connection with the study of logical definability. As it turns out, some aspects of Jan Hubička's work, viz. the study of extension properties for partial automorphisms, do indeed touch on long-standing interests of my own. I have been happy to take this opportunity to take a closer look at the comprehensive presentation of his far-reaching contributions. This has been particularly rewarding in the wider context of his presentation in this habilitation thesis, which manages to embed the technical content matter into an overarching research programme that he has persistently pursued in fruitful collaboration with several

outstanding figures in his field. I should stress that, while I have occasionally met Jan Hubička at conferences and workshops, we have not had any direct collaborations or connections beyond the obvious awareness of some shared interests.

Content of the thesis. Jan Hubička's thesis on *Structural Ramsey Theory and the Extension Property for Partial Automorphisms* expounds recent achievements in two prominent research programmes devoted to very general investigations into model theory of a fundamentally combinatorial and universal-algebraic character. While Ramsey theory deals with far-reaching structural generalisations of the classical Ramsey theorem to important classes of finite structures, the study of extension properties for partial automorphisms deals with the relationship between local symmetries of finite structures and global symmetries in finite extensions. Classical universal-algebraic and model-theoretic results link natural classes of finite structures that arise in these contexts to fundamental closure properties of a universal-algebraic character (closure under substructure, joint embedding and amalgamation) which give rise to homogeneous countably infinite limit structures (Fraïssé limits) with particularly well-behaved logical theories and algebraic properties. The automorphism groups of structures obtained along this route and related constructions provide important examples for the fundamental algebraic analysis of symmetry groups, also in relation to questions of logical definability and classification. The main merit of Jan Hubičk'a exposition in this thesis lies on one hand in the combined treatment and considerable generalisation of perspective that arises from a more general setting for these fundamental problems and approaches. On the other hand it arises from considerable technical advances in the individual tracks that support striking new results in either field as well as powerful evidence for the intimate relationship between the two programmes. The conceptual generalisation of the very problem base concerns, for instance, a more general conception of the fundamental notion of *structures* and *symmetries* within and between structures. So the finite and countably infinite structures considered are not restricted to the format of relational structures, or even ordinary first-order structures with functions, but may comprise set-valued functions; while this turns out to be very natural from a universal-algebraic point of view it is quite an extension of the classical model-theoretic setting. With respect to local or global symmetries, consideration is fruitfully extended from the usual notion of partial or global automorphisms (permutations of the elements) to transformations that may simultaneously affect the underlying vocabulary (permutations of the names of relations or functions). The thesis manages to introduce these notions and concepts in an admirably self-contained manner. It also provides a concise outline of the background and directions for the two programmes concerned, and thereby manages to lay the foundations for the bulk of the technical material comprising 9 original publications and preprints. Those 9 papers are co-authored by Jan Hubička and eminent collaborators, most notably Jaroslav Nešetřil, and present an enormous wealth of technical

achievements in various separate directions.

The introductory chapter of some 40 pages is very well written as a survey and text-book style introduction that holds these pieces together. It manages to provide a wealth of background information, all the relevant basic definitions and emphasis on an overarching perspective. In all this Jan Hubička shows great clarity and skill in the exposition of technically very challenging material. So the 9 individual original contributions, each of which is highly significant on its own, are being put into the wider, combined context so that their synopsis supports the very ambitious and encompassing perspective that is being heralded in this thesis.

Among the combinatorial treasures in the individual papers that I consulted in more detail (especially (D) and (H), due to my own special interests) are substantial and bold generalisations of previous investigations and, for instance, also surprising simplifications of known proofs of extension properties. In my view, these results and their clear presentation show an enviable combination of combinatorial ingenuity, conceptual clarity and focus on the bigger picture of the greater mathematical context.

Research achievements and academic standing. In its scope and depth the material presented here would in my view by far exceed the usual requirements for a habitation thesis. The thesis manages to give a very mature and comprehensive view on rich areas of combinatorial studies that are notoriously hard. As is amply exemplified by the original contributions that form part of the thesis, Jan Hubička has contributed to very substantial advances in these areas. His direct contribution and involvement in several quite distinct strands of highly original and innovative technical developments makes it very clear that he is a leading force in the area with a great potential also for further research advances based on his expertise and tremendous immersion into this rich field.

Jan Hubička's publication record, as exemplified by the outstanding contributions collected into his habilitation thesis, is formidable in focus, breadth and depth. His collaboration with top-ranking senior researchers in several neighbouring fields further show his excellent standing in the research community.

In conclusion I would like to recommend unreservedly the habilitation of Jan Hubička at the prestigious Charles University. His dissertation gives ample evidence of his very considerable research achievements and excellent academic standing.

Sincerely,

Darmstadt, 3rd February 2020