School of Mathematics

University of Leeds
Leeds LS2 9JT UK

T +44 (0) 113 343 5166
F +44 (0) 113 343 5090
E h.d.macpherson@leeds.ac.uk

UNIVERSITY OF LEEDS

15 March 2020

To whom it may concern

Evaluation letter for the Habilitation of Dr. Jan Hubicka

I have known Dr. Hubicka for some 7 or 8 years, know the area of his work well, and have the highest regard for his creativity and his contributions.

Hubicka works on the boundary between combinatorics and model theory, and his work also has strong connections to topological dynamics, and to permutation group theory. Underpinning his work is Fraïssé's Amalgamation Property. This says that if L is a relational language and $\mathcal{K}$ is a collection of finite L-structures containing countably many members, closed under isomorphism and substructure, and with the Joint Embedding Property and Amalgamation Property, then there is a unique (up to isomorphism) countable L-structure M (the 'Fraïssé limit') such that the *age* of M is $\mathcal{K}$ (the age is the collection of finite L structures which embed in M), and which is *homogeneous* in the sense that any isomorphism between finite substructures of M extends to an automorphism; furthermore, for any such M, the class of finite L-structures which embed in M has the above properties.

As a starting point to Hubicka's work, we consider the following two properties which may or may not hold for a class $\mathcal{K}$ of finite first order structures over a finite relational language L. We shall always assume $\mathcal{K}$ is closed under isomorphism and substructure.

(1) The class $\mathcal{K}$ has the Extension Property for Partial Automorphisms (EPPA). This asserts that for every $A \in \mathcal{K}$ there is $B \in \mathcal{K}$ such that A is a substructure of B and every partial isomorphism between substructures of A extends to an automorphism of B. This condition was (I believe) first raised by Lascar around 1991 in connection with work on the 'small index property' for the automorphism group of the random graph. Hrushovski proved that EPPA holds for the class of all finite graphs, and this was extended in several directions (e.g. the class of all finite L-structures, or the class of all finite triangle-free

graphs) by Herwig, and a rather deep general result covering many classes $\mathcal{K}$ was obtained by Herwig and Lascar in work published in 2000. There remain tantalising cases which are open – for example, EPPA is I believe not known for the class of all finite tournaments. EPPA is an interesting combinatorial condition in its own right, and also has intriguing connections, noted by Herwig and Lascar, to questions on the residual topology on free groups. Its main motivation is that EPPA, in conjunction with the 'Amalgamation Property for Partial Automorphisms' (APA) guarantees the existence of ample generics for the automorphism group $\mathrm{Aut}(M)$ of the Fraïssé limit M (for every n, there is a comeagre orbit in the diagonal action of $\mathrm{Aut}(M)$ by conjugation on $\mathrm{Aut}(M)^n$). This latter condition has many powerful group-theoretic consequences.

(II) The class $\mathcal{K}$ has the *Ramsey Property* if the following holds: for every $A, B \in \mathcal{K}$ there is $C \in \mathcal{K}$ such that for every colouring of the embeddings $A \to C$ with two colours there is an embedding $f : B \to C$ such that all embeddings $A \to f(B)$ have the same colour. If $\mathcal{K}$ is the collection of finite total orders, this is just Ramsey's Theorem. In general it is a deep condition, with sophisticated proofs (though they tend to follow a pattern), stemming from work of Nesetril and Rödl in the 1970s. Typically, one considers this condition where the language L contains a binary symbol $\leq$ interpreted by a total order on members of $\mathcal{K}$. The Ramsey Property gained wider significance from a major 2005 piece of work by Kechris, Pestov, and Todorcevic on topological dynamics. Among other results, they showed that if G is a topologically closed subgroup of the symmetric group on a countably infinite set, then G is *extremely amenable* (every continuous G-action on a compact space has a fixed point) if and only if G is the automorphism group of the Fraïssé limit of a class of ordered structures with the Ramsey Property.

It has been clear for some time that (I) and (II) have similar flavour, and for example for many of the classes $\mathcal{K}$ for which EPPA is known, the Ramsey Property is also known once one adds an (independent) total order to the language. Trying to understand this connection has been a theme for some time – for example in an EPSRC-funded project of myself and Truss around 2009. I feel that the work of Hubicka and coauthors is the first which really starts to get a grip on this connection. I summarise below some of his main work, using the references in his Habilitation Thesis. I will not comment on the Introduction to his Habilitation Thesis, except to say that I found it an enlightening account showing a very clear and deep overview of the subject, and making clear the subtle connections between (I) and (II) above.

[HN15] This paper builds on an old theme, beyond Fraïssé amalgamation. If $\mathcal{K}$ is a class of finite structures over a relational language L, is there a countable L-structure with age $\mathcal{K}$ which is universal in the sense that it embeds every countable L-structure whose age is a subset of $\mathcal{K}$? And can M be chosen to be ω-categorical (that is, determined up to isomorphism by its cardinality and first order theory, or equivalently, admitting an 'oligomorphic' automorphism group)? This paper builds on work of Cherlin, Shelah and Shi, and shows that if $\mathcal{F}$ is a class of finite connected structures which is *regular* (has finitely many classes under a certain equivalence relation on its members) and $\mathcal{K}$ consists of finite L-structures not containing any homomorphic image of any member of $\mathcal{F}$, then there is indeed a countable universal structure with age $\mathcal{K}$, which can be chosen to be ω-categorical. This looks to me an important contribution to the subject, especially given a connection they make to constraint satisfaction.

[HN18A] This paper tackles the Ramsey property in connection with a very natural class of finite graphs, those that do not contain a 'bowtie' (two triangles amalgamated over a vertex). There is a connection to the previous paper, as there is a corresponding interesting universal graph. Answering an important earlier question, the authors show that this class has a 'Ramsey lift' – basically a way of expanding the graphs to a richer language so that the resulting class has the Ramsey property. The result is interesting as the first such involving a non-trivial algebraic closure operator, and some of the ideas initiated in this paper had important developments in the others below.

[HN19] This is a major paper on structural Ramsey theory, which puts many of the earlier results (the above one on Ramsey lifts for bowtie-free graphs, early results of Nesetril and Rödl, results on finite metric spaces) into a general framework, in which, for example, the language L is allowed to include function symbols.

[EHN17] This paper also works in languages with both relation and function symbols. There is a natural notion of 'free amalgamation', and, extending earlier work to the case including function symbols, the authors show that every free amalgamation class can be enriched to a Ramsey class by expanding by a total ordering in all possible ways. They also prove that a free amalgamation class has EPPA, provided the function symbols are assumed to be unary. This paper is an example of the very strong but still mysterious connection between EPPA and the Ramsey Property. There is a nice Ramsey Property application for ordered expansions of partial Steiner systems, and the methods also apply to bowtie-free graphs and other examples of Cherlin-Shelah-Shi flavour.

[EHN19] This is a really beautiful paper which includes an important counter-example to what was developing at the time as a hope. Using a small variant of a famous construction technique of Hrushovski, the authors give an example of an ω-categorical graph which has no ω-categorical expansion with extremely amenable automorphism group. Among several consequences, there is a closed oligomorphic permutation group on a countable set whose universal minimal flow is not metrisable. The paper mixes many ideas from model theory, permutation group theory, topological dynamics, and several aspects of combinatorics (Ramsey theory, sparse graphs).

[ABWH+17B] A countably infinite graph is 'metrically homogeneous' if it becomes homogeneous once the graph language is expanded by binary predicates P_d interpreted by the graph metric – $P_d(x, y)$ if the graph distance from x to y is d. Moss (around 1990) introduced this notion, and Cameron (around 2000) posed the challenge of classifying all metrically homogeneous graphs. Cherlin has produced a catalogue of examples, determined primarily (in the generic case) by forbidden triangle conditions, and conjectures that this catalogue is complete. Major progress has been made, and in particular it has been proved complete in the diameter 3 case (Amato, Cherlin, Macpherson). The authors tackle natural questions for these examples around EPPA and the existence of Ramsey expansions. They obtain very strong results, with just a few cases left unresolved. They also explore the Tent-Ziegler notion of 'stationary independence which has applications for proving simplicity of the automorphism group'. The results have many consequences for the automorphism groups of these structures. The work also sheds valuable new light on Cherlin's programme to prove that the catalogue is complete.

[HKN19A] This paper contains a very short and self-contained proof of an important result, proved earlier independently by Solecki and by Vershik, that the class of all finite metric spaces has EPPA.

[HKN19B] This paper develops very strongly the analogies between the Ramsey Property and EPPA, building on [HN19]. The authors develop a general framework for proving EPPA which generalises and strengthens many earlier results, in particular Herwig-Lascar, but also a nice approach of Hodkinson and Otto. I have not had a chance to study it closely, but I believe (partly from talks I have heard on the work) that it really gets to the heart of where obstacles to EPPA lie. It makes interesting links to much earlier ideas of Herwig around 'permorphisms' (automorphisms up to permuting symbols in the language), and indeed, the whole model-theoretic framework shows originality.

[EHKN19] A *two-graph* is a 3-uniform hypergraph such that any 4 vertices carry an even number of edges. This is a classical notion with strong connections to finite permutation group theory. The class of all finite two-graphs has the amalgamation property, and its Fraïssé limit is a very interesting reduct of the random graph, which acts as a counter-example to many over-optimistic thoughts. I had asked whether the class of finite two-graphs has EPPA. Using a beautiful argument building on ideas of Hodkinson and Otto, the authors show that it does. This is interesting result on a rather subtle example which tests the limits of the theory.

Conclusion. It is absolutely clear in my mind that the above constitutes an important and influential body of work, at the centre of much current activity around homogeneous structures and Ramsey theory. Dr. Hubicka is one of the leading players in the field and is also helping other young researchers to develop. I therefore recommend him very strongly indeed for Habilitation.

Your sincerely,

[redacted]

(Professor) H. Dugald Macpherson

Head of School of Mathematics,
University of Leeds