

Consider a domain $\Omega \subset \mathbb{R}^N$ with Lipschitz boundary and let $d(x) = \text{dist}(x, \partial\Omega)$. It is well known for $p \in (1, \infty)$ that $u \in W_0^{1,p}(\Omega)$ if and only if $u/d \in L^p(\Omega)$ and $\nabla u \in L^p(\Omega)$. Recently a new characterization appeared: it was proved that $u \in W_0^{1,p}(\Omega)$ if and only if $u/d \in L^1(\Omega)$ and $\nabla u \in L^p(\Omega)$. In the author's bachelor thesis the condition $u/d \in L^1(\Omega)$ was weakened to the condition $u/d \in L^{1,p}(\Omega)$, but only in the case $N = 1$. In this master thesis we prove that for $N \geq 1$, $p \in (1, \infty)$ and $q \in [1, \infty)$ we have $u \in W_0^{1,p}(\Omega)$ if and only if $u/d \in L^{1,q}(\Omega)$ and $\nabla u \in L^p(\Omega)$. Moreover, we present a counterexample to this equivalence in the case $q = \infty$.