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MASTER'S THESIS

Traditional Real Estate Portfolio Diversification and Risk Measures: Evidence from the Czech Republic and Slovakia

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Prague, January 5, 2021 Erik Müller

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Abstract

This thesis evaluates traditional real estate diversification strategy by region and by property type. Additionally, it provides common risk measures – reduction of total risk and tracking error. The main contribution is twofold. First, it extends the coverage of common real estate research to the area of the Czech Republic and Slovakia. To our knowledge, this is the first study of this kind on the local market. Second, this thesis accounts for non-divisibility of ownership. This is a specific attribute of real estate, which may deteriorate investors' efforts for optimal allocation. Researchers' methods depart from Capital Asset Pricing Model. Evaluation techniques include efficient and pseudo-efficient frontiers, quantiles of total risk and tracking error, both as a function of portfolio size and portfolio value. Main findings include: (i) there is no strictly superior strategy, but there is a difference for specific subcategories, (ii) impartible ownership decreases risk-adjusted performance, this might be partially overcome by leverage, (iii) diversification is costly and index tracking is hardly possible.

JEL Classification	C22, C61, G12, R33
Keywords	real estate diversification, direct investments, risk, ownership non-divisibility
Title	Traditional Real Estate Portfolio Diversification and Risk Measures: Evidence from the Czech Republic and Slovakia
Thesis length	151 730 characters, 23 150 words

Abstrakt

Tato práce vyhodnocuje účinnost tradičních nemovitostních diversifikačních strategií podle regionu a typu budovy. Dále publikuje běžné ukazatele rizika – redukce celkového rizika a sledování chyby. Hlavní přínos práce je dvojí. Zaprvé, práce rozšiřuje běžný nemovitostní výzkum na oblast České republiky a Slovenska. V rozsahu našich vědomostí, je práce první svého druhu pro lokální trh. Zadruhé, práce zohledňuje nedělitelnost vlastnictví. Jedná se o specifickou vlastnost nemovitostí, která může znehodnotit snahy investorů o dosažení optimální alokace. Vědecké metody mají počátek v modelu oceňování kapitálových aktiv. Použité techniky k vyhodnocování zahrnují: efektivní a pseudo-efektivní hranice, kvantily celkového rizika a sledování chyby, oba vyjádřeny jako funkce velikosti portfolia a hodnoty portfolia. Hlavní závěry zahrnují: (i) žádná z diversifikačních strategií není striktně dominující, avšak existují rozdíly pro dílčí kategorie, (ii) nedělitelnost vlastnictví snižuje rizikově upravenou výnosnost, toto může být částečně překonáno díky dluhové páce, (iii) diverzifikace je drahá a sledování indexu je velmi obtížné.

Klasifikace	C22, C61, G12, R33
Klíčová slova	nemovitostní diverzifikace, přímá investice, riziko, nedělitelnost vlastnictví
Název práce	Tradiční diverzifikace realitních portfolií a měření rizika: Zkušenosti z České republiky a Slovenska
Rozsah práce	151 730 znaků, 23 150 slov

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Acronyms

APAC Asia-Pacific

CAPM Capital Asset Pricing Model

CEE Central Eastern Europe

CNB Czech National Bank

ECB European Central Bank

EMEA Europe, the Middle East and Africa

IPD International Property Databank

Kč Czech Koruna

MAD Mean Absolute Deviation

MPT Modern Portfolio Theory

NPI National Council of Real Estate Investment Fiduciaries Property Index

NYSE New York Stock Exchange

OLS Ordinary Least Squares

PPS Percentage points

PSE Prague Stock Exchange

RECO Real Estate Consulting

REIT Real Estate Investment Trust

REOC Real Estate Operating Company

S&P Standard & Poor

SIM Single Index Model

SPV Single Purpose Vehicle

Master's Thesis Proposal

Author:	Bc. Erik Müller
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Defense Planned:	June 2020

Proposed Topic:
Shopping Centres as a Fundamental Component of Real Estate Portfolio Diversification in CEE

Motivation:
Allocating a part of a portfolio in real estate is a common way of portfolio diversification, especially for medium and large investors, such as mutual, pension and sovereign funds investment groups. Such a strategy is arguably effective both in short-term and long-term in case of direct investment and in long-term for pass through vehicles e.g. REITs. The reasoning is relatively lower volatility of returns compared to other instruments, typically shares and bonds and their lower correlation with them.

In the post-crisis period, searching for yields, excessive liquidity and cheap debt combined with above mentioned characteristics and long-term predictability of cash flow streams led to raising popularity of real estate asset class among other alternative investments. From 1996 to 2015, US allocation of pension funds into real estate has more than doubled from 3.3% to 7.0%. (Kräussl, R. et al., 2017) Similarly, increasing absolute and relative allocation has been seen around the World e.g. Europe, Gulf countries and various types of investors. Such a trend elevates requirements on real estate asset managers to effectively diversify their portfolios and to price the corresponding risk appropriately.

A common method is to diversify by region, property type and tenant mix. Marginal specific risk reduction in equally weighted portfolio tends to be low, starting from 20 to 40 properties (Brown and Matysiak, 2000), and 75% to 80% of attainable risk reduction may be achieved with only 10 -12 properties. However, standard portfolios are value weighted. Equal weighting is hard to achieve because of low marketability and divisibility of ownership. In such a case, 50-60 buildings, instead of 10-12, are necessary to achieve comparable results (Callender, M. et al., 2007) and up to 500 properties may be necessary to achieve 99% non-systematic risk elimination (Byrne, P. and Lee, S.L., 2001).

Higher tens of properties are available only for large funds and investment groups. Similarly, effective diversification by geographical area and building type is attainable only for the same investors because of corporate governance constraints. Medium sized asset managers have to therefore decide whether they prefer to diversify mainly within building type or within geographic area. Evidence from the US and UK shows that the choice of strategy matters. An allocation within region among property types mostly provides higher risk adjusted returns compared to other ways of diversification with one exception – retail. The efficient frontier of portfolio, composed from shopping centers, was close to all building portfolio efficient frontier and laid above all one-way diversified portfolios efficient frontiers (Eichholtz, P.M.A. et al., 1995).

Hypotheses:

1. Optimal portfolio allocation to shopping centers in CEE has the highest Sharpe ratio among all one-way diversification strategies.
2. Equally weighted portfolios composed from shopping centers in CEE have the lowest specific risk among all one-way diversification portfolios of the same size.
3. Shopping centers with the highest diversification potential have some common characteristics.

Methodology:

Evaluation of returns in financial and real estate markets differs in terms of data frequency – daily vs. quarterly or yearly and availability of total return structure – full vs. partial. Total return on property is composed from known income return and unknown capital growth (IPD 2014). Panel dataset, based on market index (e.g. IPD CEE annual index), region (city) and property type cap-rates, and observations on 3 property types – offices, warehouses, and retail, must be therefore extended by estimated capital growths from cap-rates respectively all risky yields depending on data availability. Since then, analysis of total returns follows common portfolio diversification theory.

Hypothesis #1 departs from Modern Portfolio Theory (Markowitz, 1952) and will follow the example of Eichholtz et al. (1995).

Hypothesis #2 uses the methodology framework of Capital Asset Pricing Model (Sharpe 1964) extended by Callender, M. et al. (2007) and Byrne, P. and Lee, S.L. (2001). The idea is to analyze variance of non-systematic risk components of randomly chosen equally weighted portfolios for every one-way diversification strategy s with m_s observations and for every portfolio size n, $1 \leq n \leq k$, where k is either the lowest or the highest number of observations m_s depending on chosen approach.

Hypothesis #3 aims to identify a common characteristic of the best performing shopping centers on the level of portfolio through correlation matrix and principal component analysis. Retail stores in the last quintile of abnormal returns adjusted per unit of specific risk may have similar properties e.g. composition of tenant mix in favor of higher tier fashion vendors, size range, availability of all sources of transportation and proximity to highway(s), significant capital expenditure in the last 5 years, etc. To the best of my knowledge, there has not been any study examining such a relation. Even though this area of research is interesting, it is anticipated endogeneity bias may arise, and that the majority of inspected variables will have statistically insignificant coefficients. Therefore, a proper framework must be established.

Expected Contribution:

The main purpose of this study is to evaluate whether shopping centers, in selected countries from CEE or in the entire CEE region, possess similar diversification properties as retail stores in the US and UK. Large and especially medium asset managers, who cannot attain the full diversification potential, may directly benefit from comparing key performance indicators of their portfolios with the best attainable diversification strategy given portfolio size. Assuming the validity of hypotheses 1 and 2, the most valuable outcome would be "a diversification manual" for medium investors. This could directly specify what kind of shopping centers should be considered for purchase and how to overcome difficulties of corporate governance and equity constraints associated with their acquisition.

Outline:

1. Introduction
2. Literature and theory review
3. Data description
4. Methodology
5. Results and robustness checks
6. Conclusion
7. A1. Diversification Manual

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1. Introduction

Investing in real estate is a common type of alternative investment. It has arguably become a popular strategy of portfolio diversification for many investors, ranging from sovereign and pension funds, to private equity investors. There are several potential reasons: real estate instruments have long-term predictable cash-flows, which are less volatile compared to shares, the magnitude of potential loss tends to be lower, and their returns have relatively low correlation with standard financial market instruments.

These properties, coupled with the search for yields after the financial crisis in 2008 (during which historical returns were pressed to its minimum and some national banks applied even negative interest rates), excessive stock of cash (especially for institutional investors), cheap and available debt financing, and elevated levels of risk aversion, led to a rise in absolute and relative allocation to real estate among other asset classes. According to Kräussl, Lehnert and Rinne (2017), between the years of 1996 and 2015, US pension funds increased relative allocation by more than 100% from 3.3% to 7.0%.

Persisting aggressivity of large core investors led to the drying of prime European and APAC markets and even the elevation of pressure on reduction of cap-rates. This implied that properties, especially in the case of offices, became more and more expensive relative to cash flows. They were also traded in a shorter time. Such a trend caused core plus and value add players to start seeking for new opportunities to invest and slowly shift towards other property types, such as residentials (apartments and single-family house) and alternative real estate (including student housing and flexible offices). Industry specialist from PWC in their annual report (2019, p. 3) noted: “With the price of core assets at record levels in many European cities, all investors face the challenge of how to deploy capital effectively and achieve the “sustainable cash-flows” they cherish. … Alternative real estate and residential – in all its forms – dominate the sector preferences of survey respondents, marking a remarkable shift in industry sentiment over the past few years.”

Similarly, the raising popularity of real estate among institutional investors in the last 5 years was seen in Europe, MENA, and APAC. Despite recent predictions that asset allocation in real estate peaked and steady decrease was expected, the recent Institutional Real Estate Allocations Monitor Survey Highlights by Hodes Weill & Associates (2018) indicated an increase by 30bps in comparison to the year 2017. Authors now expect that the maximum level has not yet been reached. For the following year, additional step up by 40bps for APAC and EMEA is expected. While in North and South America, the level is anticipated to stay constant. Another key finding was institutions in all regions, in comparison to target allocation, face actual underinvestment in properties by an average 90bps. A possible explanation for this is that, in sharp competition with other players, it is more difficult to find suitable assets for investing.

An increase of allocation in properties, and lack of new opportunities to invest, elevates quality requirements on diversification and risk assessment. Asset managers often use top-down strategy. They start with diversification among asset classes, among class types, and among specific investment opportunities. Arguably, allocating part of the total portfolio in real estate is effective in the long-term for listed property vehicles such as REITs (real estate investment trust). This is due to reasonably low to moderate covariance of returns with other financial market instruments. Direct investment into properties has even lower long-term covariance in comparison to pass-through vehicles, since it is relatively resistant to short-term market shocks and individual SPVs are non-listed. However, these favorable properties are not for free. Transfer of ownership is costly, and immediate acquisition, or disinvestment, is generally hard or impossible.

Compared to shares and bonds, real estate possesses a lower amount of mutual risk within the financial market. Yet, this does not mean that it is a risk-free asset. Nor, that the level of risk is lower. The source of risk is different. Therefore, investors deploy distinct diversification strategies to eliminate risk and construct a portfolio in accordance with their risk profile. Common strategies for direct investment include diversification by region, by property type, or by both. However, in comparison to their counterparts for other asset classes, these strategies tend to be less effective. Moreover, they are subject to specific constraints.

Poor divisibility of ownership almost excludes the possibility of creating an equally-weighted portfolio. The average correlation around 0.4 between real estate market index and individual property makes index tracking less attainable. In terms of non-systematic risk reduction, value-weighted portfolios are not that effective. Their effectiveness is further weakened by difficulty of index tracking. Jointly, these two constraints lead to the requirement of up to 50 properties, compared to 10-12 in case of equal-weighting, in order to attain 75%-80% of diversification potential (Callender, et al., 2007). Furthermore, up to 500 buildings are necessary to achieve 99% specific risk reduction (Byrne & Lee, 2001).

Small and medium investors often cannot afford to invest in higher tens of properties. This is due to limited financial resources and corporate government constraints. Therefore, they must decide whether to diversify mainly by property type or by region. Evidence from the United States and United Kingdom shows that the choice of strategy matters. Diversification within regions among property types mainly dominates the other way around (Eichholtz, et al., 1995). Research conducted by Jackson and White (2005), on an extended data set for the UK, found similar results.

Over the past years, a large amount of attention has been given to real estate in the US and UK. This is due to a great availability of high-quality data with full sample coverage. Part of this research is dedicated to direct property investment. However, the rest of the world has received substantially lower coverage, which includes Central-Eastern Europe. A potential cause of this may be poor data availability.

The objective of this thesis is to evaluate which diversification strategy performs better in terms of risk-adjusted performance. Furthermore, to provide additional risk measures – reduction in total risk and tracking error. This may provide answers to the questions: "What is the cost of diversification?" or "How possible is an index tracking?" The main contribution is twofold. First, we extend the coverage of previous research to the Czech Republic and Slovakia. This is only possible because of the valuable contribution of several industry professionals with construction of a unique dataset. Second, we account for value weighted portfolio structure and ownership non-divisibilty. This is particularly important in real estate, as these assumptions may substantially deteriorate investors' efforts for diversification. However, our empirical findings led to the formulation of an idea, which may partially overcome these adverse properties through leverage.

Focusing on the effectiveness of risk reduction and diversification strategies by region and by property type, our methodology departs from the concepts of Modern Portfolio Theory (Markowitz, 1952) and Capital Asset Pricing Model (Sharpe, 1964) (Lintner, 1965). We believe that these frameworks, with slight alternations, are still relevant today. Researchers' deployed methods include efficient and pseudo-efficient frontiers for diversification strategies and quantiles of total risk and tracking error for various portfolio sizes and target values.

This thesis is structured as follows: Chapter 2 provides classification of real estate diversification research and addresses each dimension with an individual literature overview. Chapter 3 is devoted to data and discusses specific requirements in real estate. Chapter 4 covers our applied methodology. Chapter 5 presents results, from which discussion follows. Chapter 6 concludes with the most important findings and outlines potential for future research. More detailed organization is provided in the Contents.

2. Literature review and theoretical concepts

The second chapter provides an introduction to portfolio diversification and its specific form for real estate. We present underneath segmentation of diversification into three dimensions. This is done in order to provide the reader with a better understanding of underlying theoretical concepts, different perception of diversification theory throughout history and classification of empirical papers. Subsequently, each dimension includes an independent literature review.

2.1. Three dimensions of real estate diversification

After conducting a literature overview, we unveiled that a large area of research is represented by a relatively specific term: “portfolio diversification in real estate investment”. Moreover, amongst various papers the degree of similarity ranges from high to literally none. Therefore, we present here our three-dimensional categorization and several concrete examples. This is an effort to classify literature regarding the above-mentioned specific term and categorize this thesis. The first dimension is where on a vertical level diversification occurs. The second dimension is what strategies (specific criteria) of diversification are used for the selection of instruments and final allocation of resources. The third dimension is how these strategies are evaluated in the context of underlying economic theory.

The First Dimension: Vertical level of diversification and selection of instruments

Asset managers mostly use a top-down iterative approach for vertical diversification. On the top level, there are decisions about how much to allocate to every asset class such as shares, bonds, and alternative instruments – commodities, real estate, private equity, and others. On the middle level, suitable instruments for investment are selected within each asset class. On the bottom level, given preferred strategy of diversification if applicable, final allocation among individual investments is chosen.

The middle level in real estate is composed of equity-based instruments. They may be classified as direct – single purpose vehicles (SPVs) and indirect – property funds, property stocks, real estate investment trusts (REITs), and fund-of-funds. In addition, we may include (commercial) debt instruments following a similar division. Direct ones mostly have loan structure of differing seniority and include debt-equity investment, such as mezzanine financing. Indirect ones are often marketable. They follow the structure of commercial mortgage-backed securities and their variations (Mercer LLC, 2019).

There are two kinds of asset managers in real estate whose role differs depending on the level which diversification occurs. The first of them manage multi-asset portfolios (typically pension funds). Mainly, they diversify on the top and middle level. The second of them manage real estate portfolios. These may or may not be part of multi-asset portfolios. They are on the top of asset, property, and facility management services. They focus mainly on middle and bottom level diversification. Both kinds of management may be performed either jointly or separately under different legal entities.

Our work is situated on the middle and bottom level. On the middle level we focus on direct investment through SPVs. On the bottom level, we conduct the entire analysis. Our findings may serve real estate asset managers with a direct comparison of their diversification strategies. Also, our contribution may aid multi-asset managers as an additional perspective for subsequent iterations.

The Second Dimension: Strategy of diversification for instruments Strategies of diversification are dependent on vertical level, type of instruments and followed objectives. Considered criteria on the top level between asset classes include regions, correlation of returns between asset classes, natural level of risk, currency, time horizon and others. Within asset class, as for instance shares, typical diversification would be by sector and by country or economic area. The parallel with shares in real estate is diversification by property type and by region. This is frequent within the scope of research, especially for direct investment through SPVs.

Often conducted to match a life cycle of investment, additional criteria for real estate might be inter- and intra-temporal diversification. McMahan (2006) adds three more – tenant mix, lease terms, and investment structure. Diversification by tenant mix takes place on the SPV and property fund level. It focuses on tenant’s credibility, its sector, and length of lease terms. Quality of tenant mix is more crucial in retail compared to other property types. It is one of the main determinants of shopping center positioning and revenues, due to synergies among tenants and shopping center specific rent surcharges (Anikeeff, 1996). As these criteria are important for the long-term profitability of a building, landlords mostly account for them. However, due to their semi-qualitative nature, deep quantitative analysis is barely possible. Therefore, they generally remain outside of a common researchers' interest.

For the same reasons, on the other side of popular research, stands diversification by property type and by region, also called "naïve" or traditional strategy. Diversification occurs on the level of real estate asset management. Property types include commercial buildings: offices, industrial (manufacturing and logistics), retail (shopping centers, retail warehouses, highstreets, urban retails), sometimes special purpose buildings (mainly hotels), and residential buildings (mainly apartments, single-family, and multi-family houses). Among various authors, diversification by region has alternations and its perception changed slightly over time.

Historically, diversification by region was understood to allocate resources in properties between different large geographical regions grouped on administrative level of states (US) or countries (UK). In mid-1980's, Hartzell, Hekman and Miles. (1986) proposed grouping of regions based on economic coherence and functionality, rather than administrative basis. They argued functional grouping better represents the nature of systematic risk. Later work of other authors, Mueller (1993), Eichholtz et al. (1995), Hamelink et al. (2000), Jackson and White (2005) suggest similar organization and also proposed more detailed disaggregation of functional areas.

Selected criteria for this research are diversification by property type and by region. Due to the nature of the data sample, the second is transformed into location in capital city and off-capital city. These criteria are then extended with diversification by building quality. The third measure might be viewed as a natural extension to the traditional approach with a set of different categorical variable. Its role is more advisory rather than decision-making. However, this does not limit that it provides additional perspective to possibly different performance of various property categories, which stands up to the present, outside of main researchers' interest.

The Third Dimension: Evaluation of strategies through economic theory

The majority of criteria effectiveness may be evaluated through techniques that measure risk as a degree of uncertainty of future cash-flows (returns). Often, the origins of quantitative analysis relate to one of two concepts: "non-differentiated" risk – analysis of total risk or its part which is not directly disaggregated to its market and individual components and "differentiated" risk – analysis of systematic and specific risk components apart. The first concept was introduced in Modern Portfolio Theory (MPT) (Markowitz, 1952) and is considered as square one for portfolio diversification and risk management. The second concept was developed by Sharpe (1964), and Lintner (1965) and is known as Capital Asset Pricing Model (CAPM).

Analytic and quantitative methods include correlation matrices of returns, selection of optimal portfolios, and modeling of efficient frontiers through mean-variance analysis (MPT), down-side risk, mean-absolute deviation, and ordinary least squares in Single index model (Sharpe, 1963). The breakdown of systematic and specific risk component further enables the entire concept of CAPM. This includes fundamental and technical analysis, returns distribution analysis of (randomly) chosen portfolios and its deviation from market index (known as tracking error). Additional tools include (marginal) specific and total risk reduction. Also, they comprise systematic variance to total variance ratio which is R^2 from regression equation (2.2). These techniques are then applied over different portfolio sizes and structures to evaluate effectiveness of selected criteria from the second dimension.

Our analysis relies on correlation of returns for index selection. Evaluation of diversification strategies is then conducted through modeling efficient and pseudo-efficient frontiers for various portfolio sizes, values, and structures and measuring total risk reduction and tracking error for various portfolio sizes and values.

2.2. Real estate as a part of multi-asset portfolios

Diversifying on the top level among various asset classes and considering real estate within the context of Modern Portfolio Theory started playing a role in the late 50's and early 60's. One of the earliest large research studies was by Ricks (1964). This focused on the decision-making process of big investors in allocating resources among various asset classes and pointed out the potential of increasing allocation to real estate. Furthermore, Webb and Rubens (1987) and Webb, Curcio and Rubens (1988), proposed similar research opinions. They found that real estate asset class, as a part of equity portfolios, is to some level of extent beyond the interest of researchers. Moreover, it does not receive as much attention from investors as it could and "should", which is a potentially missed opportunity for total risk reduction.

A more recent example is Lee and Stevenson (2006) who conducted their study on capital market, money market, and direct real estate instruments between the years 1977 and

2002. They found that inclusion of properties in multi-asset portfolios, with holding periods ranging from 5 to 25 years, yields mainly lower total portfolio variance and may increase portfolio return. Such an allocation tends to be consistent over time. This means that over different time periods of comparable length, allocation in real estate is always positive and does not vary significantly. Another finding was that the benefit of including real estate into portfolio increases in raising holding period length. In other words, portfolios with a longer duration tend to have a higher risk adjusted performance increase arising from real estate inclusion compared to portfolios with a shorter duration.

Barton, Tukker and Varanasi (2019) performed similar research to a previous study on data between January 1990 and May 2019 from European markets with one alteration. Rather than inspecting direct investment through SPVs, they focused on indirect investment through pass-through vehicles REITs (real estate investment trust) and joint-stocks companies REOCs (real estate operating companies). Key takeaways were the following: inclusion of listed property instruments into multi-asset portfolio mostly improves risk adjusted returns and its potential benefit increases in required risk adjusted returns, European investors “underinvest” into real estate in general, and allocation in vast majority of portfolios is positive and consistent over time besides one exception. After the global financial crisis of 2008, and only for low-risk portfolios, which are defined as having lower or equal variance than 20th percentile, the benefit of REITs and REOCs inclusion into portfolio is positive only with longer holding periods.

We consider the results of the last study potentially interesting for small investors. It provides a similar conclusion compared to previous mentioned research on direct investment (Lee & Stevenson, 2006). Main findings from Europe were in line with the outcome of another paper on indirect investment (Lee & Stevenson, 2005), which had comparable scope but focused on US market and only REITs between the years 1980 and 2002. According to our opinion, the key conclusion is that allocating part of portfolio in direct real estate investment through SPVs, and even indirect REITs and REOCs instruments, has a positive impact on risk-adjusted performance in the vast majority of portfolios. Mainly the second part of this statement is interesting since listed real estate instruments stand partially out of diversification seeking investors’ attention. This is due to their higher natural correlation with capital markets and therefore lower bearing diversification potential compared to relatively popular direct investment. A higher degree of systematic risk may be seen from correlations of annual returns published in Barton, Tukker and Varanasi (2019). Authors reported correlation between listed real estate instruments and equity market as 0.61, what is considered as high. Contrary, correlation between listed real estate and government bonds was substantially lower, 0.15. For comparison, Lee and Stevenson (2006) disclosed comparable correlations for data on a quarterly basis between direct investment, equity market and government bonds as 0.05 and -0.075.

Conover, Friday and Sirmans (2002) focused on diversification benefits from foreign real estate investments in their paper with the same name. The study extends the idea of the inclusion of domestic real estate stocks into portfolio in an international context. It provides evidence that inclusion of foreign real estate into U.S. and international stock portfolio yields better risk adjusted performance. Data between the years of 1986 to 1995, from the US, UK, Canada, France, and 3 more countries, show that besides 1 exception correlation between foreign listed real estate companies and U.S. stock market are lower to substantially lower compared to correlations between international exchange markets and U.S. stock market. The authors concluded (ibid., p. 25): ”Results suggest that the absence of foreign real estate reduces return and increases risk for a U.S. investor.”

To sumarize, not surprisingly, investment in real estate through SPVs or closed-end property funds bears higher diversification potential compared to investment through listed real estate instruments. Nonetheless, European and US real estate pass-through vehicles, and European and international joint-stock companies, performed in terms of multi-asset portfolio risk reduction sufficiently well. Their performance was consistent over time, and thus allowed investors to benefit from risk-reduction, especially in the case of higher risk-returns portfolios. In addition, assuming that holding period of REITs and REOCs is at least 5 years, which is a standardly recognized minimal holding period of these instruments for purpose of diversification (Lee & Stevenson, 2005), small investors in particular may prefer listed real estate instruments. This is due to their relatively high availability, liquidity, lower transactional costs, and very limited requirements on internal resources and capital constraints compared to direct investment and closed-end funds.

2.3. Traditional approach to real estate diversification

Origins of diversification by property and by type lays in the late 1970’s. The real estate industry began to implement new analytic and quantitative methods in risk assessment thanks to recent development in economic science and increasing availability of computers. An empirical study (Wiley, 1976) on life insurance companies, REITs and real estate corporation, unveiled that the majority of investors assessed risk mainly on investment-return basis. They deployed before or after-tax indicators, such as discounted cash flow model and internal rate of return, which were later not corrected or only corrected for risk mostly through "conservative" estimates and certainty-equivalent approach. However, already 27% of respondents used computers for investment analysis. In addition, 18 investors out of 150 already adopted computational power for regression analysis and forecasting respectively.

In the 1980's the role of real estate portfolio diversification became more important. Two surveys (Webb, 1984) (Webb & McIntosh, 1986) on investment acquisition rules for life insurance companies and pension funds respectively REITs pointed that the majority of investors adapted some kind of equity holding diversification strategy. This was either in terms of variation by property type, by geographical location or by limiting relative allocation to equity real estate holdings. The minority of investors did not adapt any systematic attempt – 43.8% for life insurance companies, 5.9% for pension managers and 29.8% for REITs. The most frequent way of diversifying was diversification by geographical location and followed by diversification by variation in property types. In addition, two more findings were discovered. Adaptation of computers increased and varied from 38.3% for REITs to 71.6% for pension funds, however REITs were still primarily focused on "manual" certainty-equivalent approach. Secondly, there was a change in investors' sentiment leading to a shift in major property type from apartment buildings to offices, industrial and retail buildings. Penny (1982) provided an explanation for low adaptation of Capital Asset Pricing Model in real estate which he found in missing commonly available real estate benchmark index. This is to some level an issue which persists for various regions today.

As previously mentioned, up until the 1980's, a region was more perceived as domestic or foreign geographical area was organized on the administrative level of states. Hartzell, Hekman and Miles (1986) proposed diversification by property type, metropolitan statistical area, and term length. They argued that diversification by geographical region is costly and ineffective especially due to low levels of systematic risk. Their paper extended the previous work of co-author (Miles & McCue, 1984), which was one of the first reassessment of region on functional rather than on geographical level. Attentive readers may question, why it is ineffective and costly to diversify when there is a low level of systematic risk whereas in theory the statement mostly holds when there is a high level of systematic risk. The reasoning is that within geographical regions the degree of similarity is lower than within regions of economic coherence. Therefore, without loss of generality, we may assume that systematic risk is lower within geographical region compared to region of economic coherence. However, as functionally grouped regions bear higher levels of systematic risk within them due to higher level of resemblance, they possess a lower level of systematic risk between them. This is not necessarily true for systematic risk between geographical regions. Therefore, improvement of risk-adjusted performance achieved through geographical diversification compared to functional grouping is limited which translates into low efficiency and effectiveness.

Following previous work, various authors Mueller (1993), Eichholtz et al. (1995), Hamelink et al. (2000), proposed similar organization based on functional and economic coherence on different levels of regional disaggregation. This is because of a higher degree of homogeneity within subject areas. A typical region size has 10 to 50 million inhabitants and share some common economic characteristics. In the case of the UK, it might be set as North, South and London (Eichholtz, et al., 1995).

Consecutive research on office and retail commercial buildings in the UK between years 1981 – 2000 by Jackson and White (2005) found that a degree of homogeneity among various vertical levels of regional disaggregation changed over time. Thus, explanatory power of property returns variance changed too. The paper suggested more detailed segmentation of functional areas compared to IPD classification, as it may improve visibility of patterns between regions. We find that authors also proposed different functional grouping for offices and retail interesting. This may change the perspective concerning what it means to diversify by economic coherent region. There would be no single classification of economic areas, but rather a set of classifications for every property type which even changes over time. Therefore, asset managers would have to periodically rebalance their portfolios given classification set. Even for large investors, this might be very hard to attain. For smaller investors, it would become nearly impossible due to limited internal resources and mainly equity constraints.

2.4. Evaluation of diversification through economic theories

In the previous chapter we mentioned that one of two key frameworks for evaluation of diversification strategies is Modern Portfolio Theory (Markowitz, 1952). Markowitz assumed that investors are risk averse with a quadratic utility function which is increasing in expected return and decreasing in its variance. He unveiled that portfolios composed from different assets have the same or lower variance than the sum of their variances. Moreover, for every attainable level of risk, there exists a specific combination of assets which maximizes expected return. The set of highest expected returns for attainable risk levels is called efficient frontier.

MPT implies that a rational investor always choses the one and only one combination of specific assets in a portfolio that maximize risk-return profile. Such a portfolio is called optimal and lays on efficient frontier. Tobin (1958) introduced the concept of borrowing and lending at risk-free rate. Under the assumption of availability of such an asset, the implication is even stronger. Each investor will hold the same portfolio, called a market portfolio, which is located on the intersection of efficient frontier and tangent line with risk free asset. Then, investor's risk awareness translates only in relative allocation between risk-free asset and market portfolio.

The measure of risk adjusted returns is called Sharpe ratio (Sharpe, 1966) which reflects risk premium to total amount of risk. The optimization problem of asset allocation is often solved through quantitative mean-variance analysis of historical returns, which is one of the biggest drawbacks for practical application especially in real estate. The reasoning is that it imposes strong requirements on length of time series which is restrictive. Let us assume that there is a set of n instruments for analysis. Then mean-variance portfolio requires n estimates of mean and $n \times (n+1) / 2$ estimates of variance-covariance so in total $n \times (n+3) / 2$ estimates. It implies that time series of historical returns need to be of length $(n+3) / 2$ what is hardly attainable as data are often available only for time horizon of 20 years or less on annual basis. Even if a data frequency is higher (quarterly or monthly), Byrne and Lee (2011) note that annual basis is preferred because higher frequency may lead to inconsistency, time lags, and seasonality.

There are other methods departing from "non-differentiated" risk, which overcome the above-mentioned weakness of mean-variance model analysis. Sing and Ong (2000) use downside risk framework to compare returns of 3 assets – real estate, stocks, and bonds in Singapore from 2Q/1983 to 2Q/1997. They concluded that assuming that returns are not normally and independently distributed, mean-variance model tend to over-estimate the risk. Therefore, they proposed to use downside risk framework, which would be preferred more by risk averse investors especially in the case of skewed returns.

Mean absolute deviation (MAD, absolute deviation of returns from expected return) is another considerable method. Early application was by Konno and Yamazaki (1991) who demonstrated through example of Japanese NIKKEI 225 market index that MAD is more effective compared to mean-variance model in terms of computational power and yields similar optimization as mean-variance model. Byrne and Lee (1998), and the revision of their previous work (2011), applied MAD method in real estate. In the second paper, authors compared various risk measures based on distribution of returns. The dataset is based on an example from the UK, with monthly data between years 1987 and 2002, for various property types and functional regions. Inspected techniques were mean-variance, semi-variance (return below the expected return), downside risk measure – lower partial moment of 2^{nd} order (return below specific target level, often 0), "minimax" rule (minimization of maximum loss in each period subject to constraint of minimum average return over all periods), and mean absolute deviation. Among all inspected techniques, MAD had the most similar asset allocation compared to mean variance model. On the contrary, methods of lowest partial moments and "minimax" had the least similar asset allocation with both previous approaches. Authors concluded that choice of the method depends more on investor's perception of risk rather than individual attributes of each model. Authors related to a paper by Cheng and Wolverton (2001) proposed downside risk framework (lower partial moments risk measure) as a different assessment of risk compared to MPT. However, as there is no universal way of measuring risk, it is not possible to determine which framework is superior. The final choice will depend on investors' utility curve. Investors preferring instruments with lower variance may deploy MPT whereas investors preferring instruments with negative skewness may involve mean absolute deviation.

The second concept of risk analysis was developed throughout the 60's, a decade after introduction of MPT. Single index model presented by Sharpe (1963) and establishment of Capital Asset Pricing Model (Sharpe, 1964) (Lintner, 1965) with parallel work of other authors (Treynor, 1962) (Mossin, 1966) created a new economic framework which is now frequently used. Instead of analyzing variance-covariance among all instruments as in MPT, analysis occurs between every instrument and suitable benchmark index. In the case of stocks, it is often NYSE composite, S&P 500, or other stock index related to shares of interest. The framework allows to brake-down risk into two components: systematic (or also called market risk) and idiosyncratic (or also called specific risk). The first component is common to all instruments, cannot be diversified away, and is rewarded through higher risk premium. The second one is specific to every instrument, is diversifiable and therefore does not bring any reward for possessing. In addition, this method liberates strict requirements on number of time periods for larger amount of analyzed assets, as the required number of estimates reduces to $3n + 2$.

The original version of the model known as Sharpe-Linter CAPM is specified by equation (2.1) where r_i stands for return of instrument i, r_M is return on market, r_f is a risk-free rate typically return on a government bond or a bill, $E(r_i)$- r_f is risk premium, $E(r_M)$- r_f is market premium, and β_i is sensitivity of risk premium to market premium.

$$E(r_i) = r_f + \beta_i\big(E(r_M) - r_f\big) \tag{2.1}$$

$$E(r_i) - r_f = \alpha_i + \beta_i\big(E(r_M) - r_f\big) \tag{2.2}$$

$$E\big(r_i - r_f\big) = \alpha_i + \beta_{iM}\big(E(r_M) - r_f\big) + \beta_{iS}SMB + \beta_{iH}HMB \tag{2.3}$$

Jensen (1968) extended the initial version with the inclusion of α_i in equation (2.2) which is an intercept from a time-series regression and stands for abnormal return – an amount by which a security i over- or under-performed a theoretical expected return. Strong assumptions of Sharpe-Linter CAPM assume that return on *i-th* asset is fully explained by risk-free rate and product of β_i and market premium. This directly implies that for every security i, α is expected to equal 0.

Blume and Friend (1970) expressed objections that assumption of equal risk-free rate for lending and borrowing is unfeasible. In (1972), Black introduced an extension to CAPM where he assumed that the condition of unlimited borrowing at risk-free rate does not hold, and thus ruled it out from his model. This version is known as Black CAPM or zero-beta. Later, Black, Jensen and Scholes (1972), Blume and Friend (1973), Fama and MacBeth (1973), conducted empirical tests and pointed out that the intercept from regressing portfolio mean-returns on portfolio beta exceeds risk-free rate. In other words, predicted returns on risk-free asset were unattainably high or α was on average greater than zero. Furthermore, they found that β was on average smaller than one. Whereas assumptions of Black CAPM were not violated in context of empirical evidence, Sharpe-Lintner CAPM was broadly criticized for inconsistency. This led to rejection of its original version.

Fama and French (1993), (1995), (1996) argued that β from equation (2.1) and (2.2) itself is unable to fully or at least sufficiently explain variance in returns of securities. Therefore, they proposed a framework known as Fama-French three-factor model specified by equation (2.3), where SMB stands for small minus big which is the difference of returns between two well diversified portfolios of small-caps and big-caps, and HML stands for high minus low which is the spread of returns between two well diversified portfolios composed from stocks with high market-to-book value and low market-to-book value. Recently, Fama and French (2015) presented an improved version of their previous model known as five-factor asset pricing model adding two more factors – RMW and CMA. The first stands for robust minus week which is the difference of returns between two well diversified portfolios of shares with robust and weak profitability. The second stands for conservative minus aggressive which is the spread between two well diversified portfolios of low and high investment stocks. Even though these models are commonly used in finance, their application in real estate, particularly in direct investments, is limited. Nonetheless, at least for listed property instruments, they may provide additional perspective for risk performance measurement.

One popular researchers' method in real estate which departs from CAPM and differentiated risk is measurement of total risk reduction or (marginal) specific risk reduction as a function of portfolio size. This concept originates from Archer and Evans (1968), where an analysis of 470 securities included in Standard & Poor's index were analyzed on a semi-annual basis between January 1958 and July 1967. The authors found that the majority of specific risk is diversified away with 8-10 securities and starting from 20 to 40 securities no significant risk reduction was possible. One of the examples of subsequent research in real estate for equally weighted portfolios is Brown (1988). Analysis of monthly data from 135 properties in the UK, between years 1979 and 1982, showed comparable results to the research of Archer and Evans (1968). In later work and popular textbook (Brown & Matysiak, 2000), previous analysis was revisited and similarities were found between the years of 1987 to 1997. Results showed that marginal specific risk reduction starts to be negligible from 20 to 40 properties for equally-weighted portfolios. Another finding was that individual properties have a higher specific risk in comparison to securities. Moreover, the correlation among real estate returns tends to be low. This have two implications: significant diversification is attainable with just a few properties, however, tracking a benchmark index would be difficult.

Morrell (1993) showed, through a data set composed of returns for 562 properties between the years 1984 and 1987, that an equally-weighted portfolio is generally unattainable due to non-divisibility of ownership. Morel concluded that in every case significantly more properties are necessary to attaint similar risk reduction in value-weighted portfolio compared to equally-weighted one. On the contrary, Brown and Schuck (1997, p. 184) opposed that even though value-weighting might have an impact on degree of systematic risk, "actual effect of value skewness will be a function of the number of properties, their individual total risks, the correlation structure between their returns and the forecasting skill of the investor."

To conclude, there are many forms of risk perception. Therefore, it is unrealistic to suppose that there would be a single unified way of risk evaluation. However, out of almost countless techniques, there is one concept which seems to us as the square-one for the majority of methods. Regardless of the fact that the Capital Asset Pricing Model in its original version was rejected, its extended versions remain popular among economists and investors. This is due to its simplicity, presumed effectivity, and possible risk differentiation compared to other frameworks.

3. Data

This third chapter contains information about data collection. At the beginning, we present source, structure, and transformation for indices and then for properties. Subsequently, data issues are discussed in a broader context. We reveal why current research mainly focuses on the US and UK markets. Finally, we provide descriptive statistics and figures on indices returns. We conclude with stylized facts.

3.1. Indices description

Data set for indices has a panel structure on a quarterly basis and is composed from an independent time series. Each time series is available from 3Q/1999 – 4Q/2009 to 4Q/2019 – 2Q/2020 allowing for the construction of a panel dataset of full rank for period 4Q2009-4Q/2019 when Q corresponds to the last day of subject quarter. Primary data are from European Central Bank (ECB), Czech National Bank (CNB), Prague Stock Exchange (PSE), and real estate consulting firm (RECO). We note here that data from RECO are unreferenced given request of the company, and terms real estate and property are in index names used interchangeably.

Primary data are represented by 4 published indices: Commercial Real Estate Price Index – CZ (RECO, 2020), Commercial Real Estate Price Index – European Monetary Union (ECB, 2020a), Residential Real Estate Price Index – CZ (ECB, 2020b), and Prague Stock Exchange Total Return Index (PSE, 2020). Additional 2 indices are constructed through weighting from publicly available data: Commercial Real Estate Price Index – CZ Value Weighted (RECO, 2020) to match the structure of building class (office, industrial, retail) of data set, and Risk-Free Rate Index (CNB, 2020) as an average of Czech government newly issued T-Bills yields weighted by their nominal value within the subject quarter. These 6 indices are then normalized at 100 for period 4Q/2010 by equation (3.1) to enable direct comparison among them and period *t* is reindexed to match 1 for base period 4Q/2010.

$$I_t^{base=4Q/2010} = \frac{I_t}{I_{t=4Q/2010}} \cdot 100 \tag{3.1}$$

Formal description of Commercial Real Estate Price Index – CZ Value Weighted follows. Let us suppose we have *N* properties, denoted by *n,* with complete observations for *U* time periods, denoted by *u*. Let *J* be the number of building classes and *j* represent building class. Let $x_{n,u}^{j}$ be a value in € for *n-th* property of class *j* in time *u* and X_n^j denotes average property value in € during *U* time periods. Then w_j described by equation (3.2) corresponds to relative time invariant share of building class *j* in portfolio of *N* properties. Let us suppose that we have *J* x *T* observations for index value, where *T* corresponds to time periods denoted by *t*, and $I_{j,t}^{CRE\_CZ}$ represents value of price index for property class *j* in time *t*. Then value of value weighted price index $I_t^{CRE\_CZ\_VW}$ in time *t* is determined by equation (3.3).

$$w_j = \frac{\sum_{n=1,k=j}^{N} X_n^k}{\sum_{n=1}^{N} X_n} \tag{3.2}$$

$$I_t^{CRE\_CZ\_VW} = \sum_{j=1}^{J} w_j \ . I_{j,t}^{CRE\_CZ} \tag{3.3}$$

We need to introduce a risk-free rate for purpose of modeling in Chapter 4. Bodie, Kane and Marcus (2014, p. 128) describe risk-free rate as the rate earned "by leaving money in risk-free assets such as T-bills, money market funds or the bank." In the context of Czech Republic, we decided to use government T-Bills which are commonly perceived as bearing no risk, at least for the domestic investor. Thus, disregarding foreign exchange risk and inflation. Formalization of risk-free rate and corresponding (return) index is the following.

Let us suppose that we have *N* government T-Bill emissions denoted by *n*. Let *T* stand for the number of time periods denoted by *t*. Let y_n^t represent emission average yield per annum and x_n^t represent emission nominal value both for emission *n* in time *t*. Then w_n^t, described by equation (3.4), corresponds to relative weight of emission *n* in time *t* to other emissions in time *t*. Let Y_t, described by equation (3.5), denote mean yield of government T-bills in time *t*. Then, index value I_t^{RF} is then determined by equation (3.6) and represents a value of one unit invested at time $t_0 = 0$ in time *t* assuming reinvestment.

$$w_n^t = \frac{x_n^t}{\sum_{n=1,v=t}^{N} x_n^v} \tag{3.4}$$

$$Y_t = \sum_{n=1,k=t}^{N} w_n^k \, . y_n^k \tag{3.5}$$

$$I_t^{RF} = \prod_{k=1}^{t} (1 + Y_k) \tag{3.6}$$

The above-mentioned indices might not be sufficient to conduct consecutive analysis because the results are only as good as their underlying assumptions. One of the key assumptions is that selected index serves sufficiently well as a benchmark.

S&P 500 index commonly serves for evaluating performance of American blue-chips. For direct real estate investment, researchers mostly use property indices from the National Council of Real Estate Investment Fiduciaries for US. In literature, this is commonly referred to as NPI (Viezer, 2000). Commonly used by the UK and rest of the world is International Property Databank (IPD) (Adair, et al., 2006). Given the scope of study, either price or total return index is selected. In our case, we are inspecting benefit of diversification, which is based on analysis of total returns, thus the second mentioned index is appropriate. However, it was not possible to directly obtain any similar index for the Czech Republic and Slovakia. Owner of IPD claimed that they do not have any collaboration with the local university. Additionally, several industry experts expressed an opinion that their indices for local market has limited relevance as its market coverage is very low.

The only directly obtained total return index is the one from Prague Stock Exchange. However, evidence shows that stock index is not suitable for explaining property returns, due to a relatively low degree of correlation and high volatility. (Elbaum & Hudson-Wilson, 1995) It is not difficult to deduce that stock index is not likely the best candidate, as it represents a benchmark for a completely different asset class. Therefore, we decided to model synthetic indices departing from IPD (2012) methodology.

Thanks to the valuable contribution of one real estate expert, we obtained a full panel dataset of prime yields and prime returns for each property type in Czech Republic and Slovakia. This enabled construction of 4 artificial indices: Synthetic Price Simple, Synthetic Price, Synthetic Rent and Synthetic Total Return. The second index serves as a proxy for capital growth with control of the first one. The third index approximates income return and the fourth total return.

IPD (2012, p. 13) describes capital growth, income return and total return as following: "Capital growth is calculated as the change in capital value, less any capital expenditure incurred, expressed as a percentage of capital employed over the period concerned. Income return is calculated as net income expressed as a percentage of capital employed over the period concerned. Total return is calculated as the change in capital value, less any capital expenditure incurred, plus net income, expressed as a percentage of capital employed over the period concerned."

In our methodology for indices, we disregard capital expenditure component for capital growth and total return as prime yield already includes requirement for capital expenditure. In addition, we will define income return in a different way to match it with reported (gross) initial yields for properties. The used definition is the following: "gross income expressed as a percentage of capital employed at the end of period concerned."

All synthetic indices are normalized at 100 on 4Q/2010 and value weighted by relative share of country specific building class in property data sample. Moreover, they are constructed in the form of price indices to ease comparison with previously introduced indices and to ease transformation to its returns for different reporting frequencies. Description of weighting is the same for all indices and analogous to weighting of Commercial Real Estate Price Index – CZ Value Weighted.

Let us suppose we have N properties, denoted by n, with complete observations for U time periods, denoted by u. Let J be the number of country specific building classes and j represents country specific building class. Let $x_{n,u}^{j}$ be a value in € for *n-th* property of class j in time u. We set X_n^j as an average property value in € during U time periods. Then w_j described by equation (3.7) corresponds to relative time invariant share of country specific building class j in portfolio of N properties.

$$w_j = \frac{\sum_{n=1,k=j}^{N} X_n^k}{\sum_{n=1}^{N} X_n} \tag{3.7}$$

Synthetic Price Simple Index is determined only by prime yield. Assuming full occupancy, constant rent and rack rented property, equation (3.8) corresponds to years necessary to repay the portfolio value. Synthetic Price Index differs by including capital gain arising from change in rent. This is clearly visible after substituting equation (3.10) into equation (3.11). It serves as an arguably effective proxy for capital growth component. Notation of both indices proceeds.

Let *J* represent the number of country specific building classes and *j* represent country specific building class. Let *T* stand for number of time periods denoted by *t*. We mark by $PY_{j,t}$ and $R_{j,t}$ prime yield respectively prime rent of *j*-th country specific building class in time *t*. Then the left-hand side of equation (3.8) is 1 over all property value weighted inversed prime yield in time *t* and I_t^{PS} from equation (3.9) is Synthetic Price Simple Index value in time *t*. Analogously, the left-hand side of equation (3.10) is value weighted term of all property prime rent divided by all property prime yield both in time *t* which is the value of 1 m^2 of synthetic property in time *t*. Then, the value of Synthetic Price Index in time *t* is expressed by I_t^P.

$$P_t^{PS} = \sum_{j=1}^{J} \frac{1}{PY_{j,t}} \,.\, w_j \tag{3.8}$$

$$I_t^{PS} = \frac{P_t^{PS}}{P_1^{PS}} \,.\, 100 \tag{3.9}$$

$$P_t^{P} = \sum_{j=1}^{J} \frac{R_{j,t}}{PY_{j,t}} \,.\, w_j \tag{3.10}$$

$$I_t^{P} = \frac{P_t^{P}}{P_1^{P}} \,.\, 100 \tag{3.11}$$

Synthetic Rent Index is the value weighted product of income returns. In long-run equilibrium, under assumption of zero vacation and rack rent, synthetic income return is equal to a fourth (because of 4 quarters) of value weighted prime yield. Value of its index I_t^R in time *t* from equation (3.13) then corresponds to value of 100 units invested on 4Q/2010 assuming a possibility of reinvestment. Synthetic Total Return Index in equation (3.14), or more precisely its return form, is a real estate parallel to actual rate of return on stock index containing capital growth and dividend component. It is simple product of Synthetic Price and Rent Index.

The return of every index in time *t* is obtained as a percentage change in index value between periods *t-1* and *t.* Expansion of return on Synthetic Total Return Index in time *t* is shown in equation (3.15). An attentive reader might have noticed that this definition implies compounding of capital growth by income return which is the third element on the right-hand side of the equation (3.15). Contrary to original IPD methodology, we include this element into total returns because it corresponds to the structure in which our property returns are reported. Let us use the same definitions of previously introduced variables for artificial indices in order to shorten the formalization. Let r_t^R in equation (3.12) denote a quartal income return of synthetic property in time t. Then value of Synthetic Return Index I_t^R is determined by equation (3.13) and value of Total Return Index I_t^{TR} is determined by equation (3.14) both in time *t*.

$$r_t^R = \frac{\sum_{j=1}^{J} PY_{j,t} \,.\, w_j}{4} \tag{3.12}$$

$$I_t^R = \frac{1}{(1+R_1)} \,.\, 100 \,.\, \prod_{t=1}^{T} (1+R_t) \tag{3.13}$$

$$I_t^{TR} = \frac{I_t^P \,.\, I_t^R}{100} \tag{3.14}$$

$$\%\Delta I_t^{TR} = \%\Delta I_t^P + \%\Delta I_t^R + \%\Delta I_t^P \,.\, \%\Delta I_t^R \tag{3.15}$$

3.2. Properties description

Dataset for properties is organized on a panel structure. Data was collected from 4 private investors in an anonymized form. Gross asset value of 66 properties from Czech Republic, Slovakia, and Poland is approximately €1.5 billion. For each property we have a set of descriptive categorical and quantitative variables. Covered building classes and types are:

- offices – grade A and B,
- retail – highstreet, shopping center, retail warehouse, and urban retail,
- industrial – logistics and manufacturing.

Other categorical variables are country, city, location within city, and building quality. Location within city attains following values:

- offices – central business district, mid-town, and edge of city,
- retail – city center, mid-town, and edge of city,
- industrial – highway, edge of city, and non-highway.

Binary variable is one – capital city attaining value 1 for city which is the capital city of a country and 0 elsewhere. Provided quantitative variables of interest are information on historical gross initial yields or gross income, equivalent yields, property values either in Kč or €, and often reversionary yields, and building size.

The availability of data and time structure during different years, between 2007 and 2019, is highly heterogeneous. Meaning that overall, there is more missing observations than realized ones. In addition, frequency of valuation for each property differs as always one of semi-annual, annual, biennial, or triennial. Valuation days differ too. Whereas most of the properties have the reference day on the last day of the 4^{th} quarter, some properties have their reference day on the last day of the 1^{st} and 3^{rd} quarter. An additional limitation is that most properties did not have reported capital expenditures – typically refurbishment and initial yields were reported in gross value including operating expense such as agent and legal fees, tenant incentives, and others. This would lead to inconsistency of capital gain and net operating income, respectively its return forms of capital growth and income return if not corrected for.

To obtain a homogeneous dataset we performed the following selection and transformation. All properties with a shorter holding period than 5 years or semi-annual, biennial, and triennial valuations were discarded. Whereas this excluded information would be valuable to, for example, business studies evaluating market performance, they are unsuitable for quantitative analysis. This is due to potentially introduced bias. Theoretically, the possible treatment to missing observations would be their estimation through interpolation. Whereas this might be a defensible practice for a large dataset with few missing observations, it is not justifiable for a small data sample. This is especially the case when estimating at least a half of observations. To justify this approach a brief test on properties with semi-annual valuation was conducted.

We inspected 3 sets of total returns for their mean correlations with selected index. The first set was in its original form on a semi-annual frequency on 1st and 3rd quarter. The second set was transformed to aggregated total returns on 3rd quarter on an annual basis. The third set was a linear interpolated second set with estimated total returns on the 1st quarter and reported returns on the 3rd quarter.

Observed correlations were 0.26, 0.37 and 0.35. The first two results are in line with our expectations and other findings (Brown & Matysiak, 2000). Capital growth component of total returns is mostly unobserved and therefore estimated. However, frequencies shorter than annual may lead to underestimation of correlation, since it is difficult to notice a change in property value. The third result on interpolated semi-annual data is higher by roughly 35%, or 9 % (pps.), in comparison to its original counterpart from the first dataset. A possible explanation is that in the long run returns on individual instruments tend to follow an average return on market index. However, missing unobserved disturbances on an individual instrument would on average exaggerate a degree of correlation between instrument and index.

Comparable results, in terms of difference between correlation of original returns and interpolated-aggregated ones, were found for annual and biennial valuation. A practical implication is that biennial and triennial appraised properties would have, on average, a higher estimated level of systematic risk on annual interpolated frequency than in reality. Therefore, they represent a source of bias, may deteriorate final results, and thus were disregarded.

Properties with semi-annual valuations on 1st and 3rd quarter were manually searched in the provided annual reports. This was done in order to prevent issues associated with the before mentioned interpolation. Reports included information on valuations and income returns on the 4th quarter. Properties that we were able to identify were extended by 4th quarter appraisal and included back into the remaining dataset. At this point, 52 properties remained with different valuation periods from 2008-2017 to 2013-2019. For each continuous period with at least 5 data points in time, degrees of freedom were then calculated. The period with the highest number – 266 was selected.

Reduced dataset with complete observations consists of 38 properties from the Czech Republic and Slovakia spanning from 2013 to 2019 collected on an annual basis. Portfolio average gross asset value during holding period is €593 million. 2 properties had a change in reporting currency from Kč to € over the holding period. To correct for the currency mismatch, market values in koruna were transferred into € on subject valuation days by a vector of exchange rates. (Yahoo Finance, 2020) The same transformation was applied on properties denoted in Kč to obtain a representative variable for indices value weighting. All properties had reported gross initial yield, or gross collected rent, instead of their net counterparts. Therefore, we introduced a previously mentioned altered definition of income return compared to IPD to match it with structure of data sample.

To correct for capital expenditure market value was adjusted if building size (lettable area) changed by more than 5%. This value was set because it is relatively common that lettable area may slightly increase in the long-run and may fluctuate a few percentages around its increasing trend even without need for a capital expenditure. The first occurs, for instance, when a tenant (typically a gastro) wants to lease or construct a terrace on part of the roof or ground floor. The second arises, for instance, from the change of use of technical area to lettable area and vice versa. Each time a building size changed by at least 5% market value was adjusted proportionally to change in lettable area. Afterwards, around 12 properties, with adjusted market values or capital growth exceeding 15% in any year, were inspected for consistency and omitted capital expenditure. Value of 15% was set as it is approximately the mean return increased by 1 standard deviation of Synthetic Price Index which is a benchmark for capital growth. Inspection technique is based on qualitative assessment of change in income return. Larger refurbishments take on average between 9 to 24 months during which tenants do not pay rent or at least they receive a significant rent reduction. Therefore, if there had been any major construction, it would have significantly reduced reported initial yields. This was found for 1 property which was excluded from further modeling.

Let us define an adjustment of market value in case of change of building size as it may seem nontrivial. Let us have *N* properties denoted by *n* with complete observations for *U* time periods denoted by *u*. Let $x_{n,u}$ be a value in Kč or € and $s_{n,u}$ be a building size in m^2 both for *n-th* property in time *u*. We set $f_{n,u}$ as an adjustment factor of market value for *n-th* property in time *u* described by equation (3.16). Then $\tilde{x}_{n,u}$ described by equation (3.17) corresponds to adjusted market value of *n-th* property in time *u*.

$$f_{n,u} = \begin{cases} 1\,, & \left|\dfrac{s_{n,u}}{s_{n,u-1}} - 1\right| < 0.05 \quad or \quad u = 1 \\ \dfrac{s_{n,u-1}}{s_{n,u}}\,, & \left|\dfrac{s_{n,u}}{s_{n,u-1}} - 1\right| \geq 0.05 \quad and \quad u \geq 2 \end{cases} \tag{3.16}$$

$$\tilde{x}_{n,u} = x_{n,u} \cdot \prod_{k=1}^{u} (f_{n,k})\,. \tag{3.17}$$

Roughly two thirds of properties had directly reported gross initial return defined as gross collected rent in year *t* over market value at the end of year *t*. The remaining third contained information about gross collected rent which was transformed to gross income return in line with previous definition. Capital growth was calculated as a percentage change of adjusted market value. Such a transformation leaves the final dataset with 228 observations. As for each property, 1 degree of freedom was lost. Total return is then the sum of income return and capital growth spanning from 2014 to 2019.

Following the lastly introduced variables, let $R_{n,u}$ be gross collected rent for *n-th* property in time *u*. Then, $iy_{n,u}$, determined by equation (3.18) is gross initial yield. Variable $cg_{n,u}$, determined by equation (3.19), is capital growth. Variable $tr_{n,u}$, determined by equation (3.20), is then the combination of both previously introduced variables and stands for total return all for *n-th* property in time *u*.

$$iy_{n,u} = \frac{R_{n,u}}{\tilde{x}_{n,u}} \tag{3.18}$$

$$cg_{n,u} = \frac{\tilde{x}_{n,u}}{\tilde{x}_{n,u-1}} - 1\,, \qquad u \geq 2 \tag{3.19}$$

$$tr_{n,u} = cg_{n,u} + iy_{n,u}\,, \qquad u \geq 2 \tag{3.20}$$

3.3. Real estate data problems

Data issues are a relatively common phenomenon to private data, especially from multiple independent resources. In our case, most of the issues may be related to the subsequent reasons. Each company has slightly different use of data and corresponding methodology. Thus, definitions of reported variables slightly differ as well as required frequencies. For many investors, especially in the case of cheaper properties, it is not necessary to observe a capital gain every year. The last year appraisal combined with market prediction is "a good enough" estimate.

Another cause is that the required holding period to obtain historical returns of 15 or 20 years is completely unrealistic. Even 10 years is hardly attainable. Furthermore, according to industry experts, the average holding period of commercial properties is below 5 years. Moreover, many A class properties were nonexistent 10 years ago. If we look at Prague's office market on flagship projects of White Star Real Estate and Penta Real Estate, we find that The Park was finished around 2009. Since then, it has changed its owner three times in the years 2009, 2013, and 2016 (ČT, 2013) (ČTK, 2016). Florentium received a permit of use in 2013 and was disposed 3 years later (Penta, 2016). Short holding periods lead to data unavailability respectively restricted availability but from multiple landlords. All of the above-mentioned could be, to some level of extent, overcome. For example, if there were to be at least one broadly accepted authority with a reasonable market coverage, such as International Property Databank, that would collect data from an entire market on granular form and then process them within unified methodology. However, this is not the case in the Czech Republic and Slovakia. Possible explanations might be that to construct such an index would be difficult, time consuming and costly. Another reason might be that there is no sufficient need for such a tool and investors are afraid of sharing confidential financial reports. Lastly, there might be no will. Whereas market intransparency leads to overall dead-way loss, some players are able to realize higher profit in short-term. We believe that the true cause is a combination of all, but "the unwillingness to democratize market information" plays an important role. Paraphrasing words of an industry expert: "To sustain our margins we have to understand the market better than others do. And this is what we are really good in."

3.4. Discussion on descriptive statistics

In this section, we provide a summary of the transformed dataset and comment on several important facts. Returns on indices are reported for an entire period and for a period which was selected for consecutive modeling both on a quarterly basis in Table 1 and on an annual basis in Table 2. Descriptive statistics for property returns and its components for every building class, are provided in Table 3. Set of classification and categorical variables are summarized in Table 4. Indices values are plotted in Figure 1 and 2, Indices quarter-over-quarter changes in Figure 3 and 4 and Indices year-over-year changes in Figure 5 and 6.

There are several comments that we may draw out of the provided statistics and overview. First, Prague Stock Exchange Total Return Index had substantially higher standard deviation of returns and substantially lower mean return during an entire period compared to Synthetic Total Return Index. Additionally, higher standard deviation and lower mean return on stock market index compared to all property return indicates that this benchmark is potentially unsuitable.

Second, Synthetic Total Return Index on an annual basis for period 4Q/2014 – 4/Q2019 had a higher mean return by around 65% and a lower standard deviation by around 30%, in comparison to a mean and standard deviation of property returns. Whereas relatively lower standard deviation of index does not attract attention, overperforming property returns by 6.1% (pps.) does. Therefore, opening a discussion as to whether the Synthetic Total Return is an appropriate proxy.

Simple comparison of descriptive statistics for different indices show that a potentially suitable candidate is Commercial Property Price Index – CZ Value Weighted which is a parallel to Synthetic Price Index. However, a closer look on individual components of property returns reveals that the source of relatively low total returns does not lay in initial yield, which on average overperforms return on Synthetic Rent Index. The source is in small capital growth of properties, where all property capital growth is on average only 2.15% compared to its synthetic index counterpart 9.15%. This rules out both Czech Commercial Property Price Indices as a single appropriate benchmark. In addition, Synthetic Price Index does not perform worse against capital growth compared to two previously mentioned indices and has more accurate methodology for its weighting mechanism. Therefore, it is potentially the best candidate for proxy to property capital growth out of all considered indices.

Third, relatively low capital growth compared to return on synthetic counterpart may impose a question whether capital growth is underestimated or return on Synthetic Price Index is overestimated. Overestimation seems to be relatively unlikely because the Czech Republic and Slovakia, especially during the years of 2014-2018, experienced a strong economic conjunction. After the financial crisis in 2008, the post-crisis elevated cap rates were pressed to their historical minimum. In addition, underlying data comes from a renowned real estate consulting company, are comparable with data from different companies, and constitute market average of several A class properties rather than a single estimate for each building class. Contrary underestimation of capital growth is possible. Lower returns on capital gain were found for every collection of properties from individual investors, especially in the case of retail and office market. Further comment is provided in the Section 4.1 on selecting an appropriate time lag for underlying benchmark.

Fourth, as previously mentioned, 1 property – retail with 66% capital growth in 1 year, was excluded because of concerning unreported capital expenditure. The majority of remaining properties with capital growth exceeding 15% in one year performed a pattern of sudden depreciation in value in the preceding year. This event was related mostly to cheaper properties, usually did not affect initial yield, and has not repeated. A possible explanation is that there was an increased probability of negative event occurrence which would affect market value that is discounted cash-flow of future revenue streams. As those properties are mostly single tenant or 1 dominant tenant buildings, a feasible cause is that there was an expiry of lease term with risk of increased vacancy and subsequent rent reduction. However, prolongation of covenant was successful. Moreover, landlords realized a rent surcharge, which then led to a value increase in the consecutive years.

Fifth, overview of categorical variables shows a potential limitation of this thesis because approximately 15% of properties account for 70% of portfolio value. Also, office and industrial building class consists of relatively few properties and the same holds for highstreet and shopping center building types for retail. To some level of extent, this may be overcome by bootstrapping what is discussed in the following methodology chapter.

To conclude, even though advance market data are generally unavailable in Central Eastern Europe, in particular for real estate, it is often possible to artificially substitute them. This enables a comparable analysis, as with the real data. In our case, the interlink between artificial and real data suggest that investors may underreport capital growth. We keep the possible answers to the questions: “When and why is capital growth underestimated?” for later discussion.

Table 1: Descriptive Statistics of Quarterly Indices Returns

Period	Index	Obs.	Min	Max	Median	Mean	Std. Dev.	Variance	Skewness	Exc. Kurt.
12/2014 - 12/2019	Synthetic Total Return	21	-0,92%	8,35%	3,20%	3,84%	2,15%	4,63E-04	0,19	-0,17
	Synthetic Price	21	-2,22%	6,83%	1,87%	2,44%	2,10%	4,41E-04	0,19	-0,17
	Synthetic Rent	21	1,24%	1,66%	1,36%	1,40%	0,13%	1,73E-06	0,75	-0,81
	Synthetic Price Simple	21	-0,18%	4,85%	1,18%	1,51%	1,35%	1,82E-04	0,54	-0,56
	Comercial Property Price - CZ W.	21	-0,44%	6,86%	1,65%	2,32%	2,10%	4,42E-04	1,05	-0,22
	Comercial Property Price - CZ	21	-0,48%	5,71%	2,24%	2,60%	1,74%	3,04E-04	0,23	-1,05
	Risk Free Rate	21	-0,30%	0,38%	-0,02%	0,00%	0,17%	3,00E-06	0,48	-0,34
	Prague Stock Exchange Total Return	21	-10,47%	9,22%	3,23%	1,97%	5,33%	2,84E-03	-0,52	-0,70
	Comercial Property Price - EMU	21	-0,95%	3,36%	1,00%	1,00%	1,11%	1,24E-04	0,15	-0,81
	Residential Property Price - CZ	21	0,72%	4,74%	2,04%	2,00%	0,92%	8,50E-05	0,98	1,27
12/2010 - 6/2020	Synthetic Total Return	39	-2,13%	8,35%	2,57%	2,71%	2,30%	5,27E-04	0,23	0,14
	Synthetic Price	39	-3,87%	6,83%	1,14%	1,17%	2,33%	5,43E-04	0,26	0,01
	Synthetic Rent	39	1,24%	1,77%	1,55%	1,53%	0,20%	4,08E-06	-0,13	-1,79
	Synthetic Price Simple	39	-1,47%	4,85%	0,39%	0,86%	1,27%	1,61E-04	1,01	0,77
	Comercial Property Price - CZ W.	39	-2,60%	6,86%	1,35%	1,45%	2,02%	4,08E-04	0,98	1,21
	Comercial Property Price - CZ	39	-1,60%	5,71%	1,56%	1,70%	1,83%	3,37E-04	0,41	-0,60
	Risk Free Rate	39	-0,30%	0,38%	0,02%	0,06%	0,16%	2,63E-06	0,07	-0,56
	Prague Stock Exchange Total Return	39	-29,16%	17,58%	2,31%	0,99%	8,48%	7,18E-03	-1,45	3,18
	Comercial Property Price - EMU	37	-0,95%	3,36%	0,29%	0,58%	1,08%	1,16E-04	0,66	-0,55
	Residential Property Price - CZ	37	-0,94%	4,74%	1,12%	1,18%	1,20%	1,45E-04	0,48	0,11

Table 2: Descriptive Statistics of Yearly Indices Returns

Period	Index	Obs.	Min	Max	Median	Mean	Std. Dev.	Variance	Skewness	Exc. Kurt.
12/2014 - 12/19	Synthetic Total Return	6	7,36%	25,51%	13,04%	15,44%	7,03%	4,94E-03	0,36	-1,81
	Synthetic Price	6	2,13%	18,90%	7,32%	9,15%	6,57%	4,32E-03	0,39	-1,78
	Synthetic Rent	6	5,14%	6,97%	5,66%	5,88%	0,71%	5,04E-05	0,43	-1,74
	Synthetic Price Simple	6	2,77%	10,96%	4,04%	5,73%	3,58%	1,28E-03	0,52	-1,87
	Comercial Property Price - CZ W.	6	4,75%	22,44%	5,97%	9,28%	6,94%	4,81E-03	1,02	-0,76
	Comercial Property Price - CZ	6	4,66%	20,43%	9,09%	10,21%	5,96%	3,56E-03	0,60	-1,34
	Risk Free Rate	6	-0,56%	0,79%	0,02%	0,00%	0,50%	2,51E-05	0,26	-1,55
	Prague Stock Exchange Total Return	6	-3,54%	23,58%	3,46%	7,67%	11,16%	1,25E-02	0,42	-1,88
	Comercial Property Price - EMU	6	1,97%	6,25%	3,68%	3,91%	1,64%	2,68E-04	0,18	-1,87
	Residential Property Price - CZ	6	3,73%	10,91%	8,68%	7,74%	2,93%	8,60E-04	-0,35	-1,90
12/2010 - 12/2019	Synthetic Total Return	10	0,33%	25,51%	11,94%	12,15%	7,42%	5,51E-03	0,37	-0,86
	Synthetic Price	10	-6,49%	18,90%	4,66%	5,49%	7,40%	5,47E-03	0,32	-0,86
	Synthetic Rent	10	5,14%	7,42%	6,73%	6,42%	0,88%	7,71E-05	-0,27	-1,86
	Synthetic Price Simple	10	-0,39%	10,96%	3,46%	4,43%	3,82%	1,46E-03	0,48	-1,38
	Comercial Property Price - CZ W.	10	-2,66%	22,44%	4,99%	6,53%	6,61%	4,36E-03	1,14	0,79
	Comercial Property Price - CZ	10	-1,96%	20,43%	5,79%	7,43%	6,09%	3,70E-03	0,65	-0,26
	Risk Free Rate	10	-0,56%	1,17%	0,19%	0,30%	0,61%	3,68E-05	0,02	-1,52
	Prague Stock Exchange Total Return	10	-21,29%	23,58%	3,46%	5,73%	13,57%	1,84E-02	-0,40	-0,90
	Comercial Property Price - EMU	10	-1,36%	6,25%	2,39%	2,45%	2,42%	5,85E-04	-0,06	-1,26
	Residential Property Price - CZ	10	-0,84%	10,91%	4,13%	4,49%	4,73%	2,24E-03	0,09	-1,90

Table 3: Descriptive Statistics of Yearly Property Total Return Components

Period	Building class	Obs.	Min	Max	Median	Mean	Std. Dev.	Variance	Skewness	Exc. Kurt.
12/2014 - 12/19	Initial Yield - All Property	228	-0,09%	12,52%	7,56%	7,15%	2,31%	5,34E-04	-1,13	1,79
	Capital Growth - All Property	228	-30,19%	66,17%	0,99%	2,15%	10,41%	1,08E-02	2,18	11,34
	Total Return - All Property	228	-23,21%	66,10%	8,32%	9,30%	10,33%	1,07E-02	1,66	8,31
	Initial Yield - Office	48	0,77%	9,96%	6,47%	6,28%	1,85%	3,42E-04	-0,67	1,36
	Capital Growth - Office	48	-20,88%	32,06%	1,08%	1,16%	7,50%	5,62E-03	0,64	5,87
	Total Return - Office	48	-14,13%	33,38%	7,39%	7,45%	7,52%	5,66E-03	-0,14	3,60
	Initial Yield - Retail	156	-0,09%	11,72%	7,82%	7,18%	2,38%	5,66E-04	-1,44	2,14
	Capital Gain - Retail	156	-30,19%	66,17%	0,47%	1,74%	11,33%	1,28E-02	2,40	11,40
	Total Return - Retail	156	-23,21%	66,10%	8,03%	8,92%	10,98%	1,21E-02	1,95	9,07
	Initial Yield - Industrial	24	4,41%	12,52%	8,53%	8,70%	1,85%	3,43E-04	-0,14	-0,48
	Capital Growth - Industrial	24	-6,85%	31,71%	5,69%	6,79%	7,97%	6,35E-03	1,08	1,79
	Total Return - Industrial	24	-0,35%	42,35%	14,59%	15,49%	8,71%	7,59E-03	0,88	1,79

Table 4: Overview of Property Categorical Variables

Joint Variables

Building Class	Count	€ mil.	Building Type	Count	€ mil.	Location	Count	€ mil.
Office	8	321	Grade A Office	4	258	CBD	0	0
			Grade B Office	4	63	Mid-town	7	316
						Edge of City	1	5
Retail	26	229	Highstreet	1	47	City Center	1	47
			Shopping Center	2	152	Mid-town	23	30
			Retail Warehouse	2	15	Edge of City	2	152
			Urban Retail	21	15			
Industrial	4	43	Logistics	3	28	Highway	3	28
			Manufacturing	1	15	Edge of City	0	0
						Non-highway	1	15
Total	38	593	Total	38	593	Total	38	593

Independent Variables

Building Quality	Count	€ mil.	Capital City	Count	€ mil.	Country	Count	€ mil.
A	9	522	Capital	28	496	CZ	33	257
B	29	72	Off-Capital	10	97	SK	5	337
Total	38	593	Total	38	593	Total	38	593

Note: € mil. corresponds to sum of average property value during holding period 2013-2019

Figure 1: Indices Value 1

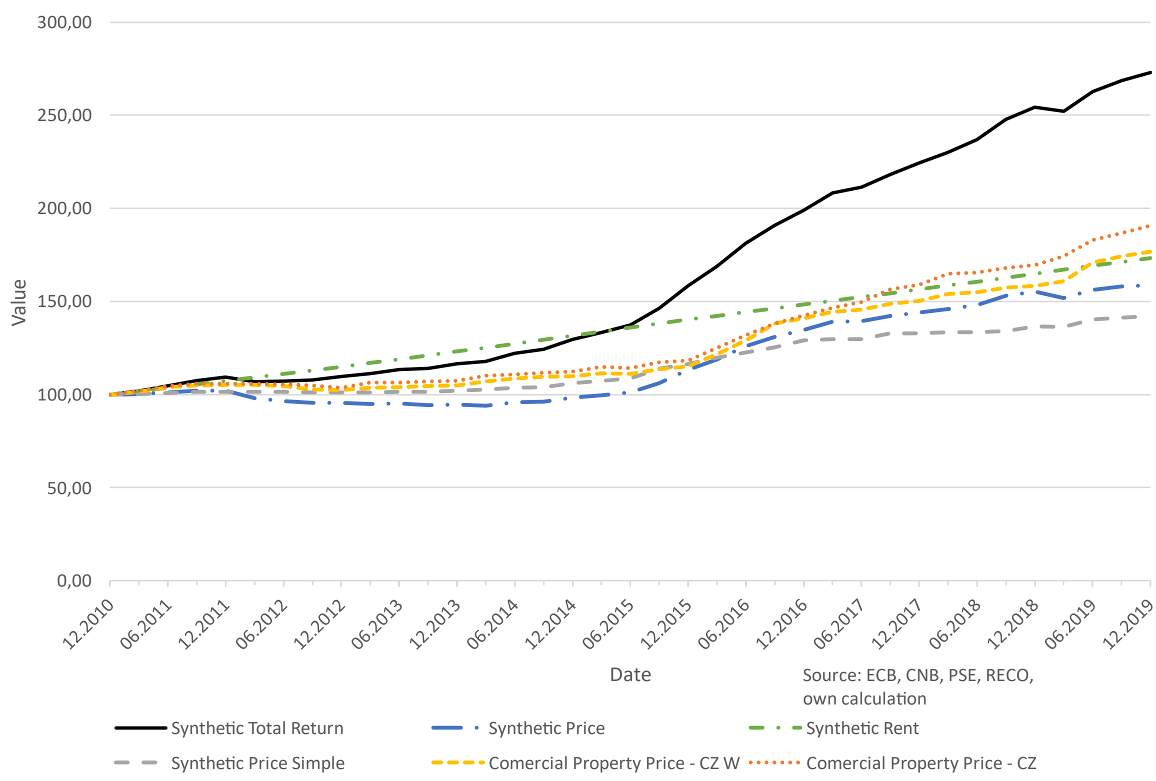


Figure 2: Indices Value 2

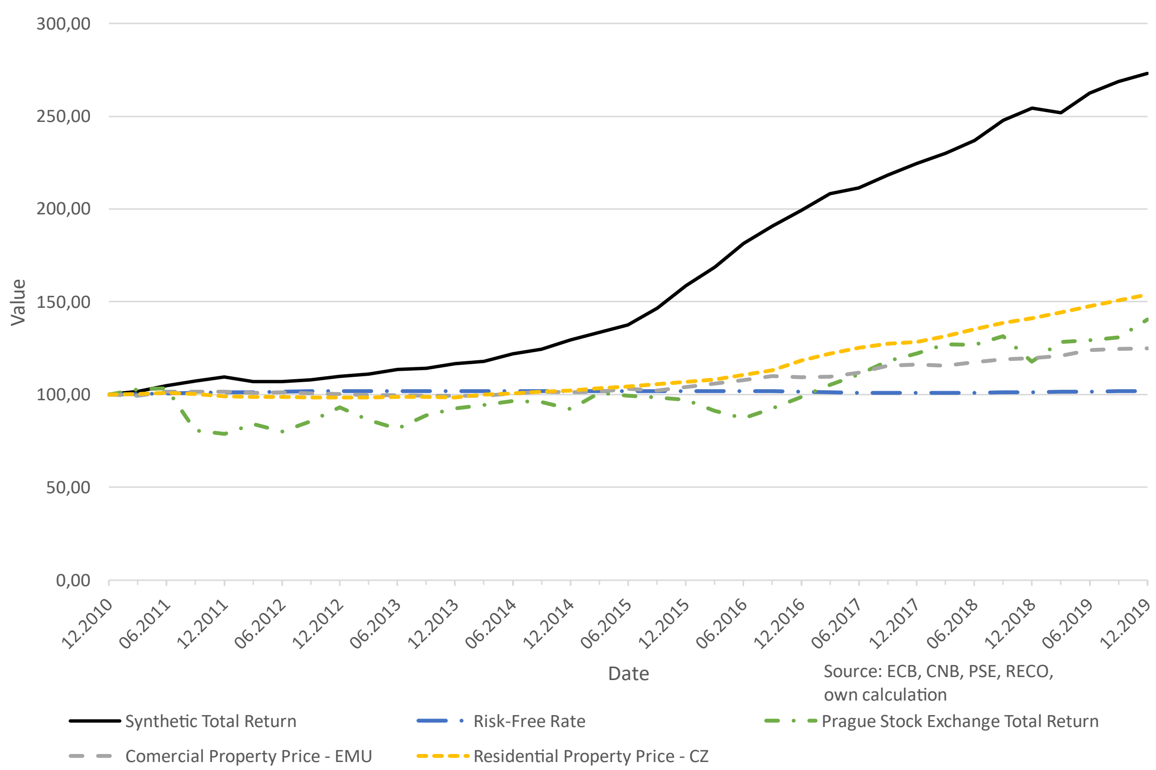


Figure 3: Indices Quarter-over-Quarter 1

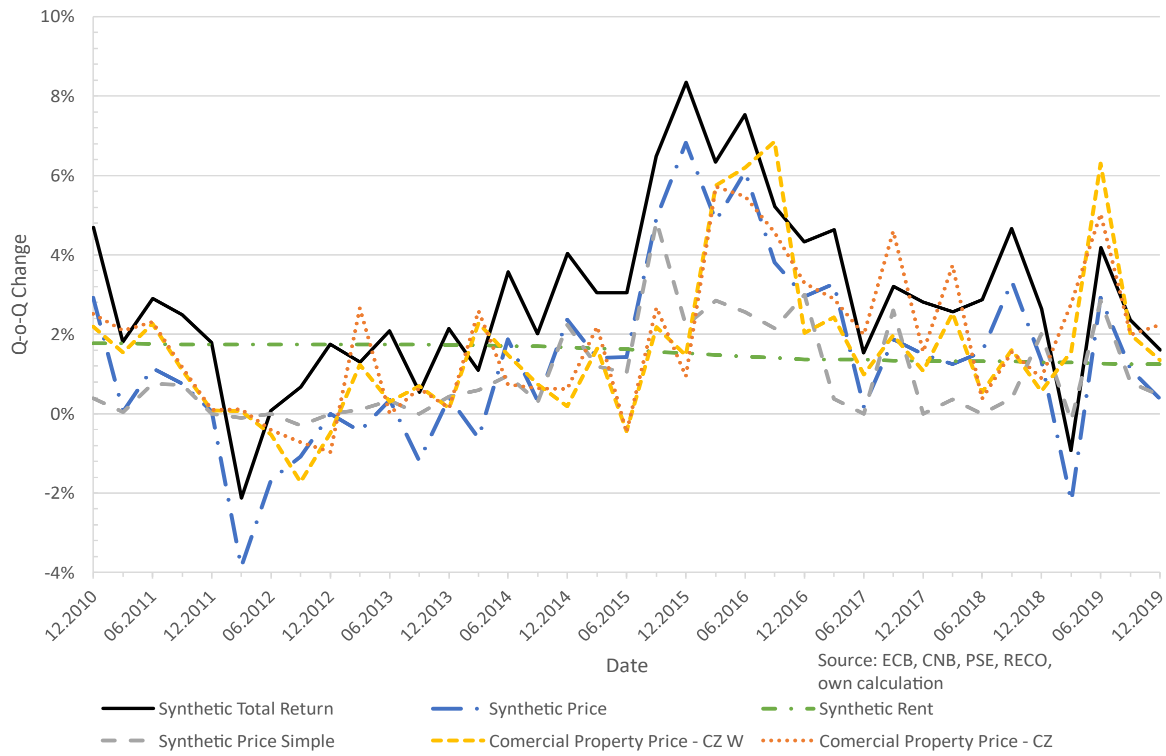


Figure 4: Indices Quarter-over-Quarter 2

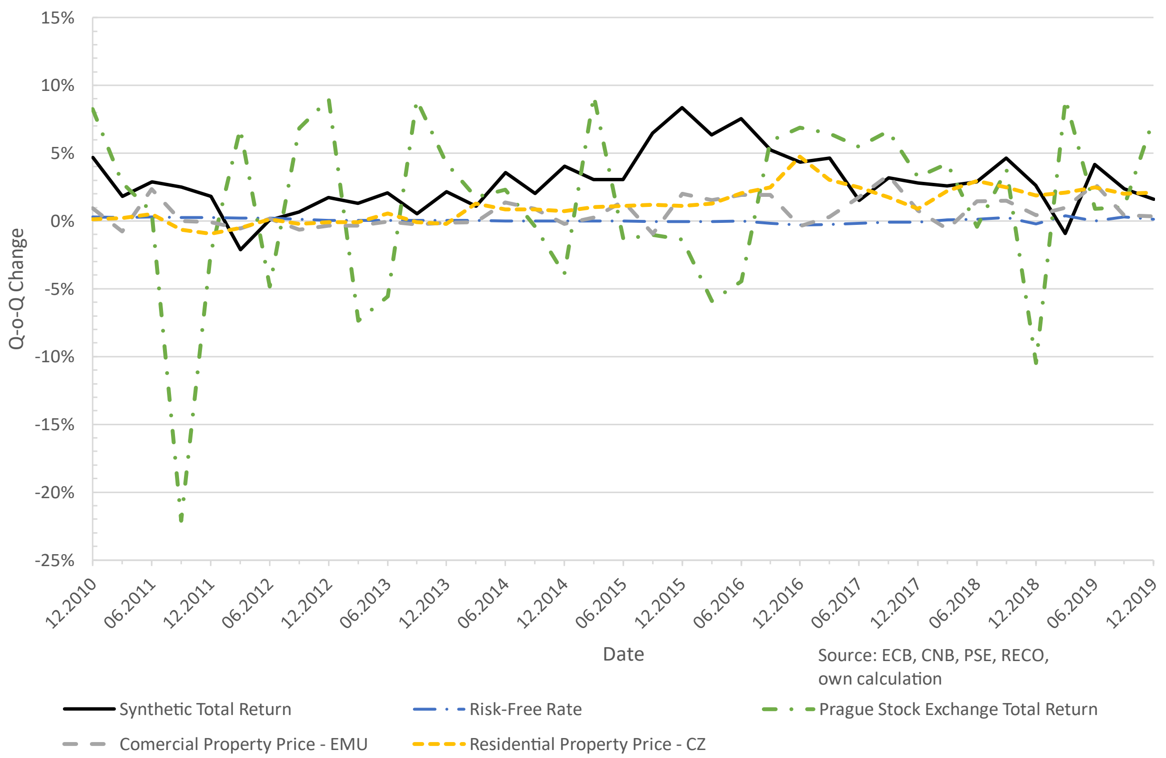


Figure 5: Indices Year-over-Year 1

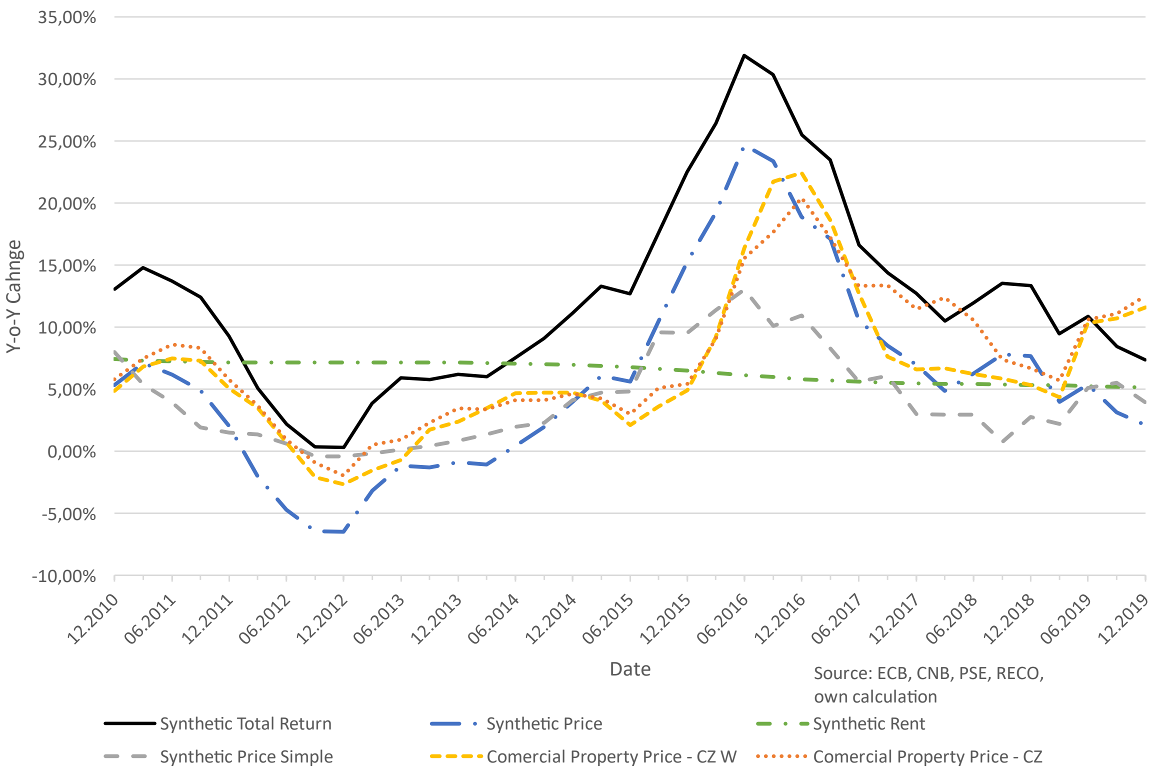
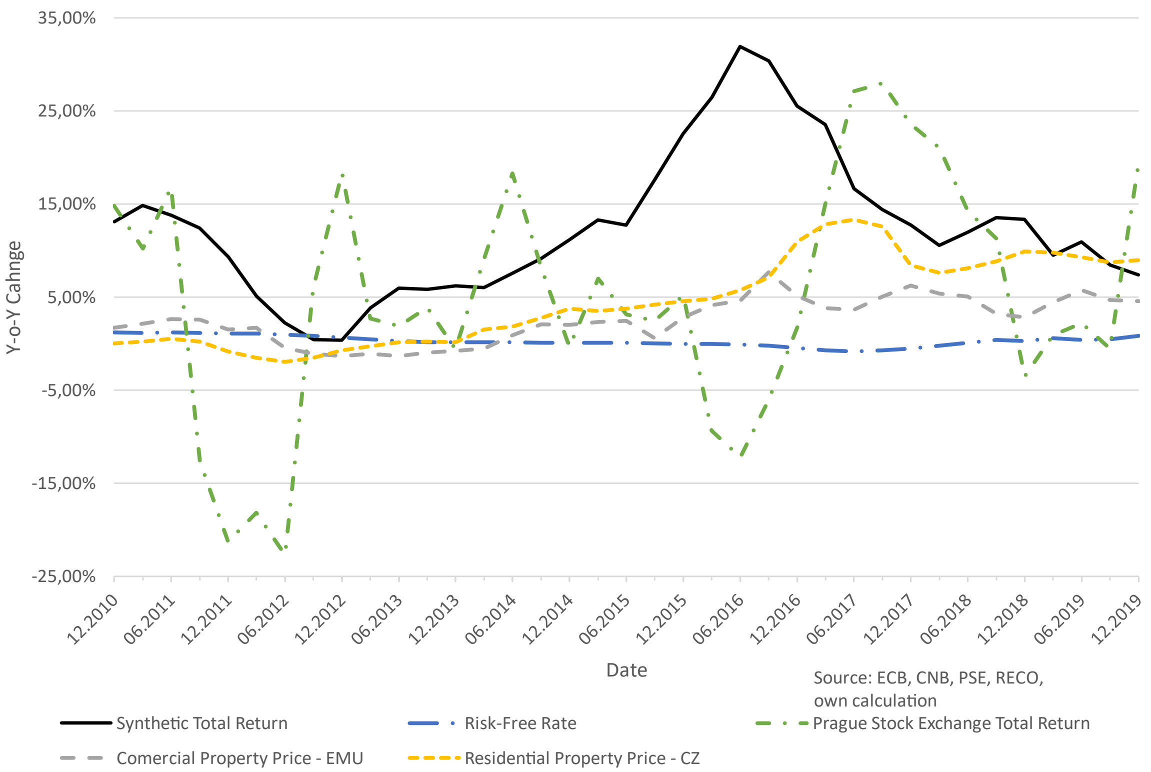


Figure 6: Indices Year-over-Year 2



4. Methodology

This fourth chapter discusses applied methodology. After obtaining a final dataset for indices and property return, we chose an appropriate time-lag of a suitable index. For each property we then estimate coefficients from CAPM and inspect for validity of estimators. Departing from differentiated systematic and specific risk, efficient frontiers are constructed for each diversification strategy. Furthermore, all property efficient frontier is then compared with attainable frontiers constrained by different portfolio values to inspect consistency. Following, quantiles of attainable risk reduction are estimated for equally and value weighted portfolios of different size, which are extended by quantiles of risk reduction for various portfolio values. Finally, applying the same concept used for risk reduction, we estimate tracking errors.

4.1. Selection of underlying index

To enable the entire concept of CAPM, a suitable benchmark and its time lag must be specified. Previously mentioned research relies on directly reported total return index based on known property returns where any transformation might be malicious. Contrary to this, we consider deployment of synthetical counterpart which may not be appropriate or might be time-lagged. The idea of period mismatch arose from the fact that the best potential index, Synthetic Total Return, had one of the lowest correlations with property returns. While the more likely worse candidate, Prague Stock Exchange Total Return, had the highest correlation. Evaluation is done through qualitative assessment of mean and value weighted correlation of all property excess total returns with lagged excess returns for each index and time lag. For completeness we add that use of total returns or excess returns for correlations do not change the results, under the assumption that we apply the same constant risk-free rate.

Let us assume we have *N* properties denoted by *n* with total returns on annual basis for *T* time periods denoted by *t*. Then $tr_{n,t}$, previously defined in equation (3.20), is total return on property *n* in time *t*. We obtain property excess total return $R_{n,t}$ as a difference between property total return $tr_{n,t}$ and mean annual return on Risk-Free Index during *T* time periods. Let us also assume we have *M* indices denoted by *m* with index value $I_{m,u}$ reported on quarterly basis for U time periods denoted by u. Let l be a time lag of index satisfying:

$l \in \mathbb{Z}$, $-4 \leq l \leq 4$, $L = \{ l_1, l_2, \dots \}$. Then, $I_{m,u,l}$ is lagged value of index m in time u lagged by l quarters.

We must obtain annual excess return on lagged indices. First, we subset indices value to obtain lagged values on annual frequency $I_{m,t,l}$ in time t synchronized with time periods of property excess total returns. For each index m in time t, we obtain lagged index excess returns $i_{m,t}^{l}$ as a difference between index return and lagged mean annual return on Risk-Free Index, when the lag of Risk-Free Index corresponds to lag of index. Second, we construct L x N x M matrices $\boldsymbol{A_{l,n,m}}$, $\boldsymbol{A_{l,n,m}} \in M(Tx2)$, when the first column of matrix $\boldsymbol{A_{l,n,m}}$ is vector of *n-th* property annual excess total returns, and the second column is vector of lagged *m-th* index annual excess returns lagged by l quarters. In other words, we create data frames from which we calculate correlations between property n and index m for each of its time-lag in the proceeding step.

Third, we obtain L correlation matrices $\boldsymbol{B_l}$, $\boldsymbol{B_l} \in M(NxM)$ where element $b_{n,m} \in \boldsymbol{B_l}$ is the correlation of the first column (property returns) and second column (lagged index return) in matrix $\boldsymbol{A_{l,n,m}}$. Fourth, we construct equal-weighted and value-weighted correlation matrices $\boldsymbol{C^{EW}}$ and $\boldsymbol{C^{VW}}$, $\boldsymbol{C^{()}} \in M(LxM)$. Let $\vec{\boldsymbol{w}}$ be a weighting vector described by equation (4.1) where $\tilde{X}_n$ is € denominated average of adjusted market values of *n-th* property during T holding periods from equation (3.17). In case of value-weighted correlations, vector $\vec{\boldsymbol{w}}$ stands for a relative share of *n-th* property value to portfolio value. We mark by vector $\overrightarrow{\boldsymbol{B_l^m}}$ *m-th* column in matrix $\boldsymbol{B_l}$. Then, element $c_{l,m}^{()}, c_{l,m}^{()} \in \boldsymbol{C^{()}}$, described by equation (4.2), represents mean respectively value weighted correlation of all property excess total returns and lagged excess returns of *m-th* index lagged by l quarters. In sum, we create equal- and value-weighted matrices of correlations in which column m is a vector of length L. This vector contains mean or value weighted correlations of excess returns between *m-th* index lags and properties.

$$\overrightarrow{\boldsymbol{w}} = \begin{cases} w_n = \dfrac{1}{N}\,, & for\ \boldsymbol{C^{EW}} \\ w_n = \dfrac{\widetilde{X}_n}{\sum_{n=1}^{N} \widetilde{X}_n}\,, & for\ \boldsymbol{C^{VW}} \end{cases}$$

(4.1)

$$c_{l,m}^{()} = \overrightarrow{\boldsymbol{B}_l^{\boldsymbol{m}}} \, . \, \overrightarrow{(\boldsymbol{w})^{\boldsymbol{T}}}$$

(4.2)

Fifth, visualized correlations between lagged indices and properties are inspected. The optimal lagged index should be relevant, possess comparable distribution, attain reasonably high degree of correlation in its lag l^* and should there be maximized.

4.2. Establishment of Capital Asset Pricing Model

We deploy a common framework of Jensen's extension (Jensen, 1968) of Sharpe-Lintner Capital Asset Pricing Model (Sharpe, 1964) (Lintner, 1965) specified by equation (4.3),

$$E(r_i) - r_f = \alpha_i + \beta_i\big(E(r_M) - r_f\big)$$

(4.3)

where r_i is return of instrument i, $E(r_i)$ is expected return of instrument i, and r_f is a risk-free rate. We mark the left-hand side of equation (4.3) as $E(R_i)$ which is a risk premium of *i-th* instrument. r_M is market return, $E(r_M)$ stands for expected market return, and $E(r_M) - r_f$ is market premium which is noted in collapsed form as $E(R_M)$. Parameter α_i is Jensen's alpha or so-called abnormal return for *i-th* instrument. Parameter β_i is sensitivity of risk premium to market premium for *i-th* instrument.

Equation (4.3) is estimated through Single Index Model (Sharpe, 1963). Equation (4.4) is the Single Index Model's expanded form with the same variables as in equation (4.3). Additional variable ε_i is error term on instrument i. Error term follows standard normal distribution $N(0, \sigma_{\epsilon,i}^2)$ where $\sigma_{\epsilon,i}^2$ is known as a specific or an idiosyncratic risk. Index t denotes period of return for instrument i and market, respectively the period for error term. The left-hand side of equation (4.4) is often called excess return on *i-th* instrument. On the right-hand side of the same equation, the second term in brackets is known as market excess return. To shorten the following notations, we rewrite equation (4.4) as equation (4.5). Taking an expected value of equation (4.5), we obtain expected risk premium relation. This is shown in equation (4.6) when E(ε_i) is 0.

$$r_{i,t} - r_f = \alpha_i + \beta_i\left(r_{M,t} - r_f\right) + \varepsilon_{i,t} \tag{4.4}$$

$$R_{i,t} = \alpha_i + \beta_i R_{M,t} + \varepsilon_{i,t} \tag{4.5}$$

$$E(R_i) = \alpha_i + \beta_i E(R_M) \tag{4.6}$$

Parameter β_i may be expressed as described in equation (4.7). Total risk of instrument *i* is variance of its risk premium, which equals to σ_i^2. One of the key advantages of CAPM, compared to Modern Portfolio Theory, is it allows for risk differentiation. Equation (4.8) shows breakdown of the total risk to systematic and specific risk components. Consider portfolio *P* composed from *N* instruments. Then, the calculation of risk premium R_p on portfolio *P* is shown in equation (4.9). Assuming zero correlation between specific risk components, the portfolio variance σ_p^2 is demonstrated in equation (4.10). Furthermore, considering equally weighted portfolio and normality of non-systematic risk, reducing variance of idiosyncratic component is visible from equation (4.11) for increasing *N*. Specific risk reduction is convex non-increasing function of portfolio size. Its 1st differentiation, marginal specific risk reduction, is concave non-decreasing function of portfolio size. Both functions have a limit in infinity equaling zero. This shows that non-systematic risk can be diversified away. Therefore, non-systematic risk does not bear any reward compared to market risk which is paid-off through higher risk premium. (Bodie, et al., 2014)

$$\beta_i = \frac{Cov(R_i, R_m)}{Var\ (R_m)} \tag{4.7}$$

$$\sigma_i^2 = \beta_i^2 . \sigma_M^2 + \sigma_{\epsilon,i}^2 \tag{4.8}$$

$$E(R_p) = \sum_{i=1}^{N} w_i.\alpha_i + E(R_M).\sum_{i=1}^{N} w_i.\beta_i + \overbrace{\sum_{i=1}^{N} w_i.\mathrm{E}(\varepsilon_i)}^{=0}, \quad \sum_{i=1}^{N} w_i = 1 \tag{4.9}$$

$$\sigma_p^2 = \sigma_m^2.\sum_{i=1}^{N} w_i^2.\beta_i^2 + \sum_{i=1}^{N} w_i^2.\sigma^2{}_{\varepsilon,i} \tag{4.10}$$

$$\sigma_{\epsilon,P}^2 = \sum_{i=1}^{N} \left(\frac{1}{N}\right)^2.\sigma_{\epsilon,i}^2 = \frac{1}{N}.\overline{\sigma}_\epsilon^2 \tag{4.11}$$

CAPM serves as square one for further modeling. We estimate equation (4.5) with the excess property total returns on the left-hand side. On the right-hand side of the equation, the second term in brackets is appropriately lagged excess market returns. Both excess returns are calculated as returns over risk free rate, which is an average of yearly returns on Risk-Free Rate Index between the years 2014 and 2019. For each property *n*, we save obtained coefficients. Simultaneously, we verify the first 3 conditions for unbiasedness of OLS from definition. Later in Chapter 5, we test for heteroskedasticity through the Breusch-Pagan Test, for no serial correlation of the 1^{st} order among residuals through the Breusch-Godfrey test, and for normality of residuals through the Shapiro-Wilk test to enable inference. (Wooldridge, 2013) (Shapiro & Wilk, 1965)

Our entire consecutive analysis relies only on regression coefficients – OLS estimators. Therefore, the only condition we need to satisfy for each property is unbiasedness of OLS: the OLS estimator $\hat{\boldsymbol{\beta}}$ is unbiased for $\boldsymbol{\beta}$. To comply with assumptions, 3 conditions must be met: linearity, zero conditional mean, and no perfect collinearity. The first condition is expected to be satisfied from definition when the model is supposed to be linear in parameter and panel of excess returns is supposed to be stationary because of its form in excess returns.

The second one is expected to be assured mainly because of exogeneity of explanatory variable and no omitted variable bias. In the case of any index, which is based on realized returns (IPD, NPI) and serves as an underlying benchmark, some level of endogeneity is always present by definition. The reasoning is total excess returns on properties affect excess returns on index in the subject period. However, the degree to which an individual property affects an index is so negligible, that without a loss of generality we may assume strict exogeneity.

In our case, the setting is different. Synthetic indices are based on averages of prime rents and yields consisting of approximately 10 A class properties. Under the assumption that any of these properties would be included in our transformed panel, the estimation with subject property would then suffer to some level from endogeneity. We were able to identify probably more half of these approximately 10 A class properties, out of which at least 3 are included in initial dataset. However, none of these 3 buildings are included in the transformed panel because of missing observations. The question is then what to do about unidentified ones. With a clean conscience, we are able to say nothing. There is no way to find this out without breaking confidentiality. Furthermore, this property may bias the estimated coefficient for itself, but it will not affect other property coefficients. Given that none of the potentially omitted A class properties in the transformed panel perform above expectations, consecutive modeling is believed to be unaffected.

Following on endogeneity, no omitted variable bias departs from assumption of full model specification in-line with Jensen's extension to CAPM. Due to discussed criticism of CAPM in Section 2.4 of literature review, such argumentation may sound buck-passing. However, other research, not limited to real estate, rely on the same extended CAPM. Furthermore, the length of series property returns left us with a very limited scope for more advanced techniques. The third condition is satisfied because there is only one independent variable, which in numeric and non-constant.

4.3. Construction of efficient and pseudo-efficient frontiers

One of the methods used for evaluating diversification strategy effectivity is through a comparison of its efficient frontiers. The efficient frontier is a curve connecting the highest attainable expected returns for the given level of risk from the feasibility set. The feasibility set is the result of mean-variance analysis of N instruments containing all attainable combinations of risk and return. By risk, we understand standard deviation of returns. The portfolio with the lowest variance, which is the origin of efficient frontier, is called minimum variance portfolio. This concept is a part of Modern Portfolio Theory and was originally introduced by Markowitz (1952). Even though the validity of MPT assumptions were broadly criticized as unfeasible, the idea of linear combination of multiple assets and its optimization in terms risk-adjusted performance was indeed revolutionary. However, to perform an original mean-variance analysis a long-time series of historical returns is necessary. This is one of its biggest drawbacks as was discussed in Section 2.4 of the literature review.

If we have a set of *N* properties for analysis. Then optimization problem requires *N* estimates of mean and *N x (N+1) / 2* estimates of variance-covariance. This totals to *N x (N+3) / 2* estimates and implies requirement on series length *(N +3) / 2* for historical returns. Our dataset is based on 37 properties after the exclusion of 1 possible outlier. In that case, we would need 20 observed historical returns. Therefore, we would need 21 consecutive annual valuation reports, as we lose 1 observation on calculating capital growth. As discussed in Section 3.3 on data issues, this is completely unrealistic due to short holding periods, which are estimated to be on average below 5 years. Therefore, we deploy previously estimated coefficients from a single index model, as described in equation (4.5). We explicitly mention that all efficient frontiers are reported in risk premiums (expected excess returns), rather than total returns to put it in line with other research. A direct benefit from this approach is setting the coordinates of the origin of risk-return space as risk-free rate.

Let us use previously introduced variables of risk premium of *i-th* property $E(R_i)$, market premium $E(R_M)$, risk premium of portfolio $E(R_P)$, portfolio variance σ_p^2, and relative share w_i of property i in portfolio P, all defined in equations (4.6, 4.9, 4.10). Furthermore, we introduce, target return R^* attaining values:

$\{ rU(R^{min}, q), \ldots, rU(R^{min}, q) + d.q, \ldots, rL(max(E(R_i)), q), d \in \mathbb{N}\}$, where R^{min} is return of minimal variance portfolio, $max(E(R_i))$ is maximum property risk premium, q is parameter of precision, and d is a multiplier of step-up precision difference. Function *rU(,)* rounds-up and function *rL (,)* rounds-down, both to the nearest integer multiple of precision parameter q. Parameter of precision q attains value 0.1% for all property efficient frontier and 0.2% elsewhere. Additionally, we define w^{MAX} as a maximal relative allocation in property *i.* Lastly, we extend all previously defined property related variables by an upper index ^{j}, which corresponds to *i-th* property subcategory in diversification strategy. Then, efficient frontier is a solution to the mean-variance optimization problem specified in equation (4.12), subject to constraints (1, 2, 3, 4), which define the specific optimization setting.

$$\min_{w_i} \sigma_p^2 = \sigma_m^2 \cdot \sum_{i=1}^{N} w_i^2 \cdot \beta_i^2 + \sum_{i=1}^{N} w_i^2 \cdot \sigma^2{}_{\varepsilon,i}$$
$$s.t. (1) \quad w_i \geq 0, \sum w_i = 1$$
$$s.t. (2) \quad E(R_p) = R^*$$
$$s.t. (3) \quad w^{MAX} \geq w_i$$
$$s.t. (4) \quad w_i^j = 0 \, for \, j \neq j^*$$

(4.12)

To properly specify the optimization problem, we classify all efficient frontiers into 3 categories: all property, all property with maximal relative allocation in property i with optional constraint (3), and subtype of diversification strategy with optional constraint (4). In the case of constraint (4), j* corresponds to inspected subtype. Moreover, conditions (1) and (2) must be satisfied for their extended versions with index ^{j}, $j = j$*. There are 3 diversification strategies – building class, building location, and capital city. Their subtypes are defined in Section 3.4, Table 4. The process is simple. The first step is to solve the problem for constraint (1) and an optional constraint ( ), if specified. The second step is to solve the problem for constraint (1), (2), and an optional constraint ( ), if specified. For each problem setting, we obtain a set of points that determine a path of efficient frontier. This path is then smoothed to approximate a curve of continuously differentiable function.

In the literature review Section 2.3, we discussed that a common problem in real estate is generally unattainable equal weighing due to non-divisibility of ownership. However, this might be an issue once comparing efficient frontiers. This is because their construction assumes the possibility of attaining any linear combination of asset allocation, within specified constraints. This might be perceived as an even stronger assumption than equal weighting. Therefore, we additionally construct pseudo-efficient frontiers, through Monte Carlo simulation, assuming only purchase of entire properties. There are 2 sets of these pseudo-efficient frontier – with replacement and without replacement, each for different portfolio values. Due to uneven distribution of data sample for different diversification strategies, we construct them only for the all property portfolio. These results allow for comparison between theoretically attainable diversification from mean-variance optimization and real attainable diversification. Arguably, the first is more relevant for a consulting company estimating the performance of a market, in particular larger markets such as the US and UK. The second is more representative for an asset manager comparing its risk adjusted performance with real attainable risk adjusted performance.

The process is as follows. At the beginning we define maximal portfolio size *N*, number of simulations *M* for each portfolio size in *N*, and vector of portfolio values $\vec{v}$. Portfolio value size *N* corresponds to number of properties *N*. Then for each portfolio size in *N*, we obtain *M* randomly selected portfolios – for which we obtain portfolio value, risk premium and portfolio variance in line with equations (4.9, 4.10). Portfolio value is a sum of mean adjusted building values defined previously for equation (4.1). If the portfolio value deviates by less than 10% from any element of vector $\vec{v}$, it is attributed to the feasible set of pseudo-efficient frontier of subject vector element. After finishing *N* x *M* simulations, the feasible set is evaluated in the same way as for efficient frontier. The set of highest attainable expected excess return for a given level of risk we then call a pseudo-efficient frontier.

4.4. Estimating risk reduction and tracking error

Total risk reduction is an additional measure to risk assessment. It is a relatively popular method among researchers and professionals. Mainly because of its simplicity, great market coverage for various instruments, and known theoretical values for different portfolio sizes. Its origins may be partially attributed to establishment of Modern Portfolio Theory and partially to Archer and Evans (1968). We previously showed, relating to equation (4.11), that specific risk is a non-increasing convex function of portfolio size. This statement holds under 2 assumptions – normality of systematic risk and equal weighting. Something similar may be shown for reduction of total risk. Let us say we have *N* properties for analysis, $\rho_{i,j}$ is correlation of returns between property *i* and *j*, σ_i is standard deviation of returns of property *i*, and w_i is relative share of property *i* in portfolio. Then, the function of total risk is expressed in equation (4.13). For an equally-weighted portfolio, function simplifies to equation (4.14). It is clear that function decreases in N.

$$\sigma_p^2 = \sum_{i=1}^{N} w_i^2 . \sigma_i^2 + \sum_{i=1}^{N} \sum_{j=1, i \neq j}^{N} w_i . w_j . \sigma_i . \sigma_j . \rho_{i,j}$$

(4.13)

$$\sigma_p^2 = \left(\frac{1}{N}\right)^2 \sum_{i=1}^{N} \sigma_i^2 + \left(\frac{1}{N}\right)^2 \sum_{i=1}^{N} \sum_{j=1, i \neq j}^{N} \overbrace{\sigma_i . \sigma_j . \rho_{i,j}}^{\sigma_{i,j}} =$$
$$= \left(\frac{1}{N}\right) \bar{\sigma}^2 + \left(\frac{N-1}{N}\right) \overbrace{\bar{\sigma}_{i,j}}^{\bar{\sigma}^2 . \bar{\rho}}$$
$$= \frac{\bar{\sigma}^2}{\mathrm{N}} + \left(\frac{N-1}{N}\right) \bar{\sigma}^2 . \bar{\rho} = \bar{\sigma}^2 \left(\bar{\rho} + \left(\frac{1-\bar{\rho}}{N}\right)\right)$$

(4.14)

However, for the same reason as in mean-variance analysis, we deploy the concept of Single Index Model. In that case, total variance of portfolio is shown in equation (4.10). Additionally, if we relax an assumption of equal-weighting, function of total risk mostly decreases in its size N.

Tracking error is a different tool, which is often associated with Capital Asset Pricing Model. It measures a standard deviation of dispersion between portfolio and index returns. The measure is commonly used in finance for evaluation of arguably diversified portfolios and their tracking to market benchmark. The lower the value, the higher the precision of index tracking. However, this does not imply that portfolios with very low tracking error have similar returns as index. The common problem with this measure is that it does not have an easy interpretation. To overcome this, let us introduce an active risk what is a mean dispersion of portfolio and index returns. Then, a realized active return in period *i* is expected to be with 95% probability within bounds (active risk ± 2 x tracking error). To formally define tracking error, let us use variables introduced in equation (4.5). Then, tracking error is defined in equation (4.14).

$$TE_{PM} = sd(R_P - \mathrm{R_M})$$

(4.14)

In this case, it does not matter whether the measure is obtained from returns or excess returns. It is a dispersion which is unaffected by possible subtraction of risk-free rate.

Following on both total risk and tracking error, we perform analysis of their quantiles distribution through Monte Carlo simulation with replacement. The process is similar to previously defined estimation of pseudo-efficient frontiers. First, we define maximal portfolio size *N*, number of simulations *M* for each portfolio size in *N*, vector of portfolio values $\vec{\boldsymbol{v}}$, and vector of quantiles $\vec{\boldsymbol{q}}$. By *n,* we will understand any portfolio size in *N*. Let *J* represent a set of variables of interest and *j* be a variable of interest. In our case, the ordered list of subject variables is: portfolio value, equally- and value-weighted total risk from equation (4.10), and equally- and value-weighted tracking error from equation (4.14). Portfolio value has the same definition as in previous section.

Let Q (.) represent a quantile function of parameter q. Then quantile function Q $(\vec{\boldsymbol{q}})$ with vector parameter $\vec{\boldsymbol{q}}$ applied over matrix Z ∈ M(MxN), returns matrix Z' $\in M(dim(\vec{\boldsymbol{q}})xN)$ where n-th column of matrix Z' is vector of length $dim(\vec{\boldsymbol{q}})$ containing values *y* from n-th column of matrix Z such that $F_Y(y) := Pr\ (Y \leq y) = \vec{\boldsymbol{q}}$, where function $F_Y\ (.)$ is a cumulative distribution function applied over *n-th* column in matrix *Z*. In other words, we defined a mechanism to obtain quantiles for every variable of interest which evolve either in portfolio size or portfolio value.

Second, let us have *J* matrices $A_j \in M(MxN)$ which serve as storage for obtained data. For each portfolio size *n* in *N*, we obtain *M* randomly selected portfolios. For each portfolio, we obtain *J* variables of interest. Now, we store these obtained variables for every *j, m, n* in element $a^j_{m,n}$,. $a^j_{m,n} \in A_{\mathrm{j}}$.

Third, we create storage for sorted data on joint variables. Let *k,* $k \in \mathbb{N}$, represent an index of column and matrix B_1, $B_1 \in M(\widetilde{D}xdim(\vec{\boldsymbol{q}}))$, is a matrix with variable dimension $\widetilde{D}$, $\widetilde{D} \in \mathbb{N}$. Now we sort joint variables value – value weighted total risk. For each *m*, and for each *n* we obtain portfolio value $a^1_{m,n}$. Let *k* satisfy condition: $\vec{\boldsymbol{v}}_{\boldsymbol{k-1}} < v_{i,j} \leq \vec{\boldsymbol{v}}_{\boldsymbol{k}}$, then we assign value-weighted total risk $a^3_{m,n}$ to element $b^1_{\sim,k}$, where ~ is the first empty row in column *k*. Analogously, we proceed for joint variables value – value weighted tracking error and matrix B_2, $B_2 \in M(\widetilde{D}xdim(\vec{\boldsymbol{q}}))$.

Fourth, we apply quantile function quantile function Q $(\vec{\boldsymbol{q}})$ with vector parameter $\vec{\boldsymbol{q}}$ over storage matrices, A_j , $_{j=\{2,\dots,J\}}$, B_1 , *and* B_2 . We obtain quantiles of interest and visualize them. Analogously we proceed for tracking error.

Equally- and value-weighted total risk as a function of portfolio size reveals how much of risk can be diversified away with increasing number of properties. Furthermore, we extend the common framework by expressing total risk as a function of different portfolio values. Dispersion in quantiles then shows how probable these reductions of risk are. This allows us to answer several common questions of investors: (1) What is the actual cost of diversification? (2) Is there potential for improvement? (3) If so, what would be its magnitude? Similarly, results on tracking error may serve to asset managers for fund analysis as a comparison to evaluate the attainability of index tracking. The key questions might be: (4) Is an index tracking even possible? (5) And if so, how costly would it be?

4.5. Seeming benefits of efficient frontiers modeling in Excel

In this section, we discuss the selection of used software and specific features of modeling efficient frontiers in Microsoft Excel. We evaluate the advantages and pitfalls of this technique, continue with unfeasibility for pseudo-efficient frontiers and conclude with a proposal for more convenient approach.

At square one of this research, we had several possible choices on text editor and data processor software. In consideration of an example of Knauff and Nejasmic (2014), about efficiency of writing in academic literature, we decided to use Microsoft Word. For data processing, we selected Microsoft Excel and R-Studio.

Various sources of data for entire modeling were pre-processed in spreadsheet processor up-to level of indices construction and calculation of returns on properties. Then, we transferred this data frame to R-Studio where all of modeling, besides one exception, was conducted. Deployed packages were zoo, xts, readxl, dplyr, and lmtest. The exception is modeling of classical efficient frontiers, as previous optimization problems for nonhomogeneous dataset might have potentially suffered from an error in a specification on a user side.

As discussed in Section 3.2 on properties description, we had a problem with collection of sufficient quality data for a long time. At that time, it appeared obtaining additional data with regular and continuous periods of returns on annual basis would be impossible. Therefore, we developed a spectrum of non-orthodox methods which might have been able to tackle specific encountered issues. This includes a construction of efficient frontiers through Microsoft Excel Solver, which is often not the most preferred method for medium and large data samples. Even though we currently evaluate these efforts to some level of extent as a search for phlogiston, at that time it seemed that the only other considerable choice would be completely respecifying the scope of the research. Therefore, we would like to share our experience and propose a more effective and efficient way of modeling.

In order to construct at least partially representative efficient frontiers, data sample should have the lowest minimum of 20 properties. However, we were able to exceed this level only with properties with interpolated returns when the data sample would have full rank maximum for 4 years. Such a period length is insufficient for any meaningful estimation. Therefore, properties with data for at least 7 years were accounted at the cost of allowing for a period mismatch. This means that a property might have observations for period 2012-2019, as well as well as for period 2007-2013. In terms of methodology, this might be a questionable approach. The reason is that returns on index differed between periods as well as the correlations with property returns. This implies different levels of systematic risk between periods. Moreover, obtained coefficients α, β were not easily comparable and this may raise questions about their consistency.

An additional complication is that commonly used packages in R-studio require a data frame of full rank for index and properties' returns. They do not allow for the construction of efficient frontiers from an estimated Single Index Model. Therefore, we would have to deploy linear programming end estimate efficient frontiers from equation (4.13) using matrix notation. However, this method is relatively vulnerable to any user error in code, particularly for constrained versions. Therefore, we decided to bring the search for phlogiston to a new level.

A potential candidate for the optimization problem was Microsoft Excel Solver, due to its simplicity and interface. Organization of optimization problem setting is well structured and output may be immediately compared with supposed solution. However, its standard configuration requires an exact specification for each optimization problem, making it relatively ineffective for construction of large efficient frontiers. This is often overcome for smaller data samples by minimizing a sum of portfolio variances for multiple target returns. This is possible since the sum of minimized portfolio variances would equal the minimized sum of portfolio variances. Then, minimizing a sum of portfolio variance is linearly difficult in increasing number of simultaneously analyzed target returns. Both approaches are exponentially difficult in increasing portfolio size.

However, our optimization problem, with at that time 25 buildings, exhibited different behavior. For one target return the computation time varied slightly around 30 seconds, for sum of two target returns already varied around 2 minutes. Starting from simultaneous optimization of 5 target returns model often choked and returned error, even though all constraints on computation time and number of iterations were relaxed, and 3 CPU cores at almost full load were deployed. A possible explanation is that Solver is to some level of extent "a black box", where the final processing mechanism is unknown.

In our case, step-by-step calculation of 10 efficient frontiers for 3 diversification strategies would require overall around 800 solutions. Using simple math, it would take around 7 hours of computation time. However, at least the same amount of time would be allocated to model respecification. Moreover, such a process again would suffer from potential user error. This was overcome by automating solver for each frontier through Visual Basic for Applications. Additionally, a significant reduction in computation time may be achieved through forwarding a solution for some target return to optimize the problem for the consecutive target return. The reasoning is that optimal allocations for sufficiently close target returns do not differ to a great extent. The shortcoming of this method is that it may produce efficient frontiers which are discontinuous.

This arises from the mechanism of how Solver evaluates potential solutions. The outcome of an iteration is set as an optimal solution, once its relative convergence in the consecutive 5 iterations drops below a user specified threshold and all constraints within specified precision are satisfied. For the common level of convergence parameter 0.01, a solution to preceding target return would be so similar to optimized target return that an improvement exceeding parameter of convergence is unattainable with following iterations.

In that case, we would receive a set of the same solutions for sufficiently close target returns when any solution besides the first one would be wrongly evaluated as presumably optimal. In particular, this holds for lower degrees of risk in a feasible set, when a relative change in risk for each increment in target return is lower compared to higher levels of risk. A practical implication is that close target returns would have the same allocation, which translates into spikes in image of efficient frontiers. These spikes would be then higher and more often at a lower level of risk. Moreover, the issue worsens in decreasing increment of target return, which may seem paradoxical. In other words, the more points we estimate in order to increase a precision of estimation, the more spikes would be present, even though they would be of lower height due to decreased target return increment.

To tackle this, we decreased the parameter by 2 orders, which improved the estimation but not in every case. Therefore, we adjusted the forwarded solution, so that the improvement over defined threshold would definitely be possible. However, the applied alternation was sufficiently small enough that an increase in difficulty and computation time would not raise to a level where the benefit from forwarding would deteriorate. We obtained the results of a previous optimization problem and saved them. Then, we divided them by factor 1.025, added a 0.002 increment, and forwarded them as a root to the next optimization problem. This led to reduction of computation time approximately by 85%.

With the same mechanism, we tried to estimate pseudo-efficient frontiers. However, this showed as unfeasible because these frontiers are by nature discontinuous. Under the assumption of non-divisibility of ownership, some levels of target returns do not have a solution, whereas higher levels may have one. The main issue lays in predicting whether some specific target return would have a solution and how long it will take to find it. We found that some solutions were identified in 20 seconds, others in 3 minutes, and some that existed may not have been identified at all. Therefore, we decided to model them through random resampling for a large population size as discussed in Section 4.3.

To conclude, a modeling efficient frontier in Excel is relatively popular for smaller data samples. Whereas analysis of medium and large portfolios may become cumbersome. Based on our experience, we believe that there are more effective, efficient and practical methods. A possible example is using packages quantmod or Fportfolio in R-Studio, both of which would have been easier ways to achieve the specified goals.

5. Results

In the fifth chapter we present the results of our research. First, we introduce a suitable market index and discuss possible reasons for its time lag. Second, we establish a modeling framework. Third, we evaluate different diversification strategies through efficient frontiers. Fourth, we question whether these efficient frontiers are attainable and whether the entire concept is suitable for application in real estate. Fifth, we present quantiles of total risk and tracking error for different portfolio sizes and values. A discussion on the actual cost of diversification follows.

5.1. Suitable market index and possible reasons for missing capital gains

In the previous sections 3.1 and 3.3, we discussed the absence of relevant market index for property returns in the Czech Republic and Slovakia. We proposed several potential proxies, suggested construction of synthetic benchmark departing from IPD (2012) methodology and questioned the relevance of stock exchange index for real estate. Additionally, we examined a degree of similarity between proposed indices. We found significant difference between index and property total returns, which was present for all building classes. The probable source was identified as absent capital gains.

In the previous Section 4.1, we showed that the synthetically constructed index had one of the lowest correlations with property returns. This is controversial, as it is supposedly the best candidate for market benchmark. Again, the potential reason relates to underestimated capital growth and a period mismatch. Additionally, we propose an evaluation mechanism for an index selection and a determination of its appropriate time lag. Now, we determine which index and its form has the highest relevance for explaining property returns. To do so, we present average correlation of property excess total returns with lagged indices in Figure 7 and its value weighted transformation in Figure 8.

Both figures show a development of average or value weighted correlations as a function of index time lag. Due to large variance in building values, we also construct value-weighted correlations to match the structure of sample portfolio. Assuming property returns in period 4Q/2015, Value -4 corresponds to correlation between index returns in period 4Q/2016 and property returns, value 0 means that periods are synchronized, and value 4 corresponds to correlation between index returns in period 4Q/2014 and property returns. The interpretation of the first time lag means that the index is behind the observed property returns. This may be caused by a delay in trend observation or its reporting. The possible interpretation of the third time lag is that index is in advance of observed property returns. This could be incorrectly evaluated as being deterministic to property returns, which would be a dispute. It is evident that the benchmark constructed to measure ex-post market performance cannot determine future market performance. The question is then, what are the other viable reasons? This will be discussed later. Meanwhile, we offer a space for the readers' own ideas.

The proposed evaluation mechanism is based on the theory that the most suitable index out of all considered should attain sufficiently high correlation in its optimal lag ***l****, in which should be maximized. Moreover, the optimal benchmark should be relevant and possess comparable distribution with the explained instruments. Relating to both Figures 7 and 8, the only index satisfying condition of relevance is Synthetic Total Return Index. Furthermore, it possesses relatively comparable distribution with property returns. This may be seen in Tables 2 and 3 from Section 3.4. Moreover, it attains reasonable levels of correlation in its lag, ***l* = 4***, for both correlations. Value-weighted correlation and average correlation are both maximized there. For correctness, we note that average correlation in lag = 5 is higher by approximately 0.02. This is followed by a sharp decrease in following lags.

However, if the benchmark is arguably non-deterministic, why is it in advance of its market? Moreover, why does it have a higher capital growth? We suppose that the correct form of the statement should be reformulated the other way around. What are the reasons that reported property returns are behind their benchmark and have lower capital growth? We believe the explanation may include the following: First, A class properties used for estimation of prime yields and prime rents receive the highest possible treatment. Their valuations are estimated regularly, often on quarterly frequency, and with the upmost precision. Therefore, any change in rent or market sentiment would be quickly reflected in their value, which is not the case for other properties. Different treatment or valuation methodology for these A class properties would then constitute a bias synthetic index.

Second, capital growth is, by nature, unobserved and therefore estimated. Moreover, it is not necessary to know exact capital gain, as the market value matters mostly once the transfer of ownership or collateralization is considered. This is in particular for private investors and cheaper properties. Therefore, the reasons may lay in estimation error and no need for more precise valuations.

Third, missing capital gain might be also unreported and there are various reasons for that. The conservatism principle in accounting implies that unsure future positive cash-flows are only recognized once it is more than probable that they will be realized. Additionally, some companies might be motivated to keep probable future gains undisclosed to create a buffer for worse times and to smooth valuation. Lastly, management may have an incentive to underestimate a value increase due to profit participation structure and non-overperfoming KPIs before general partner term maturity.

To conclude, we appointed a suitable index for further modeling and discussed potential reasons for absence of capital growth. We believe that the true rationale behind the missing capital growth component in sample portfolio is potentially the combination of all above mentioned reasons.

Figure 7: Average Correlation of Total Returns with Lagged Indices

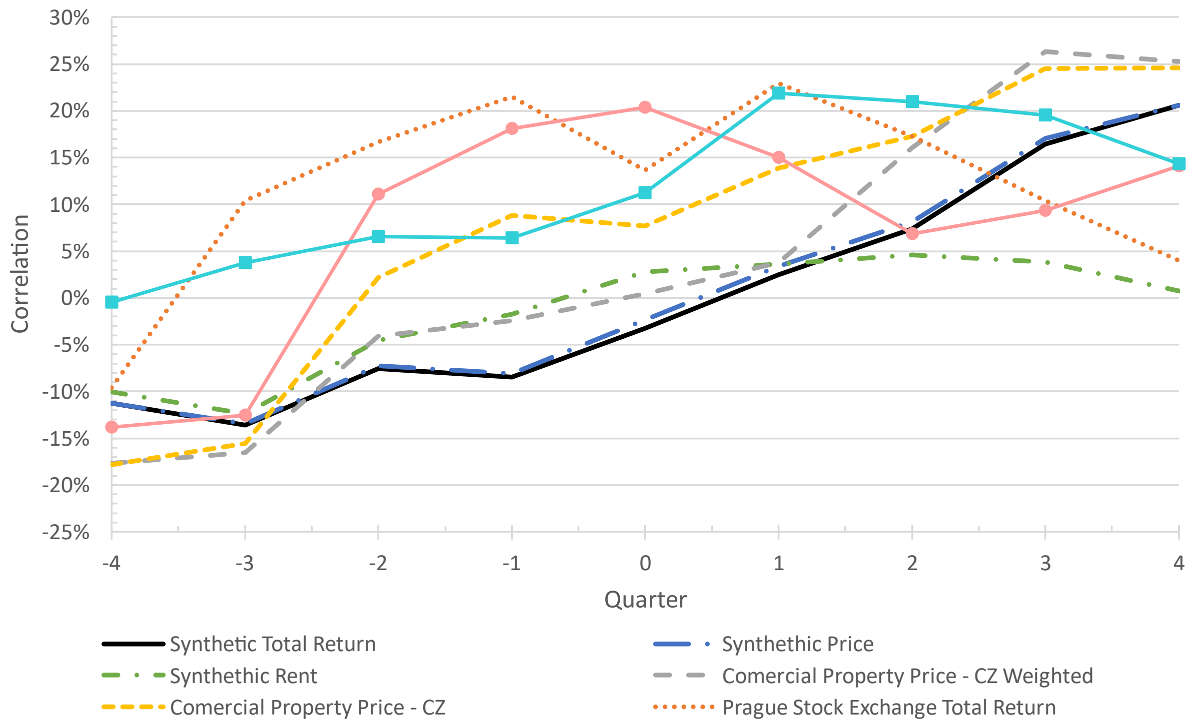
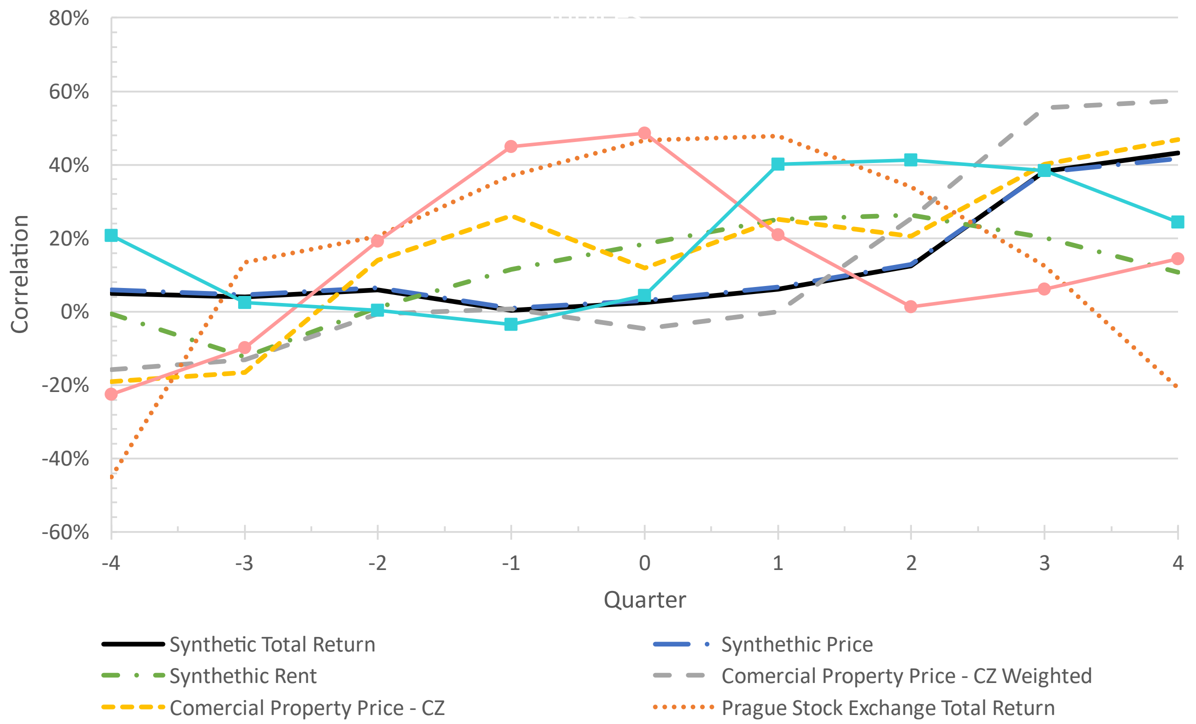


Figure 8: Value Weighted Correlation of Total Returns with Lagged Indices



5.2. Smaller portfolio and its limitations to modeling framework

In this section, we discuss validity and consistency of underlying model for all future results. In sections 4.2 and 4.3, we introduced Capital Asset Pricing Model and discussed limitations of mean-variance optimization based on Modern Portfolio Theory. Our results are based on Single-Index-Model in line with Jensen's extension to CAPM. We estimated equation (4.5) in Section 4.2 for 37 properties with excess returns between the years 2014 and 2019. But before we continue further, let us discuss the square one of modeling.

In the previous Section 5.1, we selected an underlying benchmark with an appropriate lag. However, even though we showed that Synthetic Total Return Index is the best out of all considered, we have not answered whether it is sufficient. Therefore, we searched for comparable correlations in literature to provide a broader context. Callender et. al. (2007, p. 15) published mean correlation of all property returns with IPD index as 0.41 with standard deviation in correlation as 0.3, both for period 1994-2004 and 1 728 properties. Our dataset has mean correlation 0.21 and standard deviation 0.43, both for period 2014-2019 and 37 properties. Simple statistics of total returns correlation is provided in Table 5.

Approximately 50% reduction in mean correlation and 33% raise in standard deviation may not seem as satisfactory. However, we found that authors might have wanted to publish mean correlation as 0.31. If their previous results are correct, we believe that the true result would be exactly around this number for the mean correlation. We base this on 3 techniques which yielded very similar results. For shortness we mention only the first one. We calculated a vector of relative share of each property category count in their results, which we multiplied by the transposed vector of property category correlations.

In context of this evidence and given 37 properties and shorter period, we find the mean correlation and its standard deviation for the Czech Republic and Slovakia as sufficient. This implies, that there is some level systematic risk and opens a way for subsequent modeling. Regression summary of estimated equation (4.5) is provided in Table 6.

Previously, in the same Section we discussed validity of the first 3 condition for classical linear model. We found linearity, zero conditional mean, and no perfect collinearity as valid from definition. At this point, we could end because necessary assumptions for Single-

Index-Model were satisfied. For correctness we shortly add that we inspected additional 3 conditions for classical linear model.

Homoskedasticity was rejected in favor of heteroskedasticity for 3 properties at 5% level of significance. As it is a problem only for minority of properties and heteroskedasticity robust standard errors did not vary substantially from their standard counterpart, we report them in natural form. None of the properties had Breusch-Godfrey test for 1st order autocorrelation of residuals with p-value smaller or equal to 0.05. Therefore, we have not found evidence for the 1st order serial correlation of errors. Shapiro-Wilk test for normality applied on residuals led to rejection of null hypothesis of normality at 5% of significance for 4 properties. This implies that estimators would be only asymptotically normally distributed. In terms of period length, the convergence of estimators to their true value would be at this point relatively weak. However, similarly as for heteroskedasticity, this was related only to minority of properties. Therefore, we find assumption of classical linear model satisfied for most of the properties and we proceed further.

The second part in this section examines validity of CAPM in general. In literature review we covered, that original version was broadly criticized for its inconsistency mainly for unfeasible returns on risk-free rate and lower level of systematic risk in portfolio compared to assumed level. Our model predicts the same, but the magnitude is even higher. Table 6 shows that mean α is 0.07 while mean β is 0.12. The probable cause arises from moderate degree of correlation and shorter data period availability. In other words, properties would tend to have on average higher abnormal returns, lower risk premiums, and lower market risk than in theory.

This represents a potential limit to outcome of this thesis as the predicted benefits from diversification might be overestimated. This is caused by potentially higher attainable risk reduction than in reality due to exaggerated idiosyncratic component and underrated systematic component of total risk. To partially overcome it as well as to account for smaller data sample, we deploy simulation-based techniques departing from Monte Carlo framework. The key advantage is that it allows to evaluate distribution of potential outcomes rather than single estimate.

To summarize, we evaluated sufficiency of selected benchmark index for modeling and discussed validity of assumptions for classical linear model. Later, we discussed theoretical properties of Capital Asset Pricing Model and compared them to results from our model.

Violation of these assumptions leads to potential overestimation of benefits from diversification. Therefore, we will deploy Monte Carlo simulation, which allows to some level of extent limiting exaggeration of risk reduction potential.

Table 5: Simple Statistics of Total Return Correlations between Properties and Selected Lagged Indices

	Lag ***l**** = 4		Synthetic Total Return		Synthetic Price		Synthetic Rent	
Period	Building class	Obs.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
2014 - 2019	All Property	38	0,21	0,43	0,21	0,43	0,01	0,29
	Office	8	0,23	0,35	0,22	0,34	0,00	0,41
	Retail	26	0,17	0,46	0,17	0,46	0,01	0,25
	Industrial	4	0,25	0,60	0,11	0,48	-0,19	0,42

Table 6: Regression Summary

No.	E(R)	α		β		No.	E(R)	α		β	
1	0,09	0,07	(0,06)	0,14	(0,34)	20	0,10	0,06	(0,05)	0,27	(0,28)
2	0,16	0,08	(0,05)	0,48	(0,29)	21	0,10	0,11	(0,03) *	-0,07	(0,15)
3	0,08	0,13	(0,04) *	-0,33	(0,26)	22	0,07	0,04	(0,02) *	0,21	(0,10) .
4	0,23	0,12	(0,11)	0,75	(0,64)	23	0,09	0,11	(0,04) *	-0,13	(0,23)
5	0,05	0,10	(0,10)	-0,33	(0,59)	24	0,11	0,00	(0,04)	0,76	(0,22) *
6	0,11	0,10	(0,05)	0,09	(0,29)	25	0,07	0,03	(0,00) **	0,24	(0,03) **
7	0,10	0,04	(0,03)	0,39	(0,17) .	26	0,09	-0,35	(0,22)	2,87	(1,30) .
8	0,02	0,02	(0,06)	0,02	(0,34)	27	-0,02	0,08	(0,14)	-0,66	(0,83)
9	0,12	0,17	(0,10)	-0,38	(0,57)	28	0,05	0,11	(0,02) **	-0,41	(0,11) *
10	0,14	0,14	(0,05) *	-0,03	(0,30)	29	0,10	0,03	(0,04)	0,44	(0,22) .
11	0,11	0,08	(0,08)	0,23	(0,48)	30	0,24	0,45	(0,16) *	-1,34	(0,94)
12	0,09	0,08	(0,08)	0,03	(0,47)	31	0,08	0,01	(0,06)	0,46	(0,33)
13	0,05	0,21	(0,16)	-1,03	(0,95)	32	0,00	0,00	(0,08)	-0,03	(0,50)
14	0,12	0,07	(0,06)	0,34	(0,35)	33	0,13	0,02	(0,06)	0,69	(0,35)
15	0,11	0,07	(0,03) *	0,27	(0,15)	34	0,14	0,19	(0,20)	-0,27	(1,16)
16	0,02	0,11	(0,10)	-0,60	(0,61)	35	0,09	0,08	(0,03) *	0,05	(0,18)
17	0,08	0,06	(0,01) *	0,07	(0,09)	36	0,07	0,00	(0,03)	0,46	(0,18) .
18	0,08	0,01	(0,02)	0,44	(0,14) *	37	0,09	0,05	(0,01) **	0,31	(0,05) **
19	0,02	-0,01	(0,08)	0,18	(0,45)						
	mean (E(R)) = 0.09			mean (α) = 0.07						mean (β) = 0.12	

Note: No. - property number, E(R) - risk premium, α, β - regression coefficients
standard errors in brackets, significance codes: 0.1 . 0.05 * 0.01 ** 0.001 ***

5.3. Industrial as a potential leader among diversification subcategories by property type

In Section 4.3 we provided a description of efficient frontiers, discussed motivation for modeling them and outlined potential drawbacks of application in real estate. One of which was that classical mean-variance optimization is based on the possibility to attain any allocation within given constraint. This is often problematic due to ownership non-divisibility

Next, we provide an answer to probably the most frequently asked question related to traditional real estate diversification: "Which strategy is better?" We present modeled frontiers for 2 traditional strategies – by region and by property type. Then we discuss their meaning. Naturally, we extend this approach by segmentation on building quality and provide efficient frontiers for different maximal relative allocation. Lastly, we discuss real attainability of risk adjusted returns.

We directly begin with the anticipated answer. Similarly, as Eichholtz et al. (1995), we find the results inconclusive. Based on evidence, none of the strategies strictly dominate the other. However, this does not mean that all segment strategies are performing equally well. In Figure 9 we present efficient frontiers for segmented diversification strategies by property type and by region. In Figure 10 we present frontiers for comparable strategies, segmented by building quality and upper locus of feasible set for different maximal relative allocations.

Let us introduce what each strategy means. Under diversification by property type, we understand to buy any single building class between all regions. Contrary, diversification by region stands for buying entire property mix within any single area. We adjusted the perspective of region to location within capital city and off-capital city. The reason is that our data structure does not allow to model meaningful diversification for regions, such as Bohemia and others.

To evaluate which strategy is better we compare location of efficient frontiers. The rule of thumb is the higher the efficient frontier is located the better. To conclude that any strategy is strictly dominant over the other, it must hold that all frontiers for its subcategory are above all subcategory frontiers to the other strategy. Moreover, this must hold for every single point located on the curve. We may see in Figure 9 that locations off-capital are at the same level, or above, compared to industrial building class. However, for most of its feasible set, industrial lays above capital. This implies that none of the strategies are strictly dominant.

Even though there is no superior strategy, the choice of subgroup still potentially matters. Our main findings are summarized below. First, for most of their feasible set, offices are inferior to any other subcategory. Moreover, with larger data sample they might be inferior even to Synthetic Index, which represents a single point of comparison and does not have any mean-variance optimization. This might be interesting for some investors, as it is relatively known that offices have on average lower cap rates. Thus, arguably lower expected returns, but at the same time they belong to a more liquid asset class with generally lower risk. This holds in particular for class A buildings. For these reasons, they are often selected by core and value-add investors, as a dominant property type in portfolio structure. However, our results show that their risk-adjusted performance is behind other property classes and mostly behind other diversification subcategories. Such a finding may beg the question whether strong preferences of larger risk-averse investors for class A offices is always the optimal decision.

Second, it is visible that industrial building class and location off-capital mostly dominates within their diversification strategies. Even though the dataset contains only 4 industrial properties, which accounts for approximately 7% of total portfolio value, we find its supposed building class dominance as plausible. Underlying prime yields and prime rents for synthetic index show that between the years 2014 and 2019, return solely on industrial exceeded on average both single returns on retail and on offices by at least 2% per year. On the contrary, off-capital dominance might be slightly spurious. Even-though, its superiority over capital region is assumed to hold at lower risk levels, at the higher levels its supremacy arises from large allocation into off-capital industrial. Therefore, to properly answer on the question, whether secondary regions in the Czech Republic and Slovakia exceed prime cities in terms of risk-adjusted performance, further research would be needed.

Third, in the thesis proposal we centered our hypotheses around the assumption that retail, as a building class, and in particular shopping centers as a building type, would be superior as a one-way diversification strategy. This was based on assumed dominance, which was found by Eichholtz, et al. (1995). However, our results show that retail attains only moderate performance. Additionally, it would be premature to make any conclusion about shopping centers, as there are observations only for 2 buildings. Therefore, in accordance with these findings, we appropriately adjusted our hypotheses to evaluate overall market performance. For these reasons, we will not proceed further on the creation of a diversification manual.

Forth, in Figure 10 we see that efficient frontier for Building Quality B is nearly identical to all property frontiers. We find this interesting because the set was generated for medium and higher risk premium levels, mostly as a linear combination of 4 specific properties. This might be partially visible from frontiers with constraint of maximal relative allocation (Max. allocation ≤ 20%) where 5 equally weighted properties may attain maximal portfolio risk premium of 18.1%. On average, these properties were substantially cheaper in comparison to mean building value in portfolio. They performed well due to reasonable total returns and low level of correlations with index. In other words, they had relatively high abnormal returns and relatively low levels of systematic risk. But why does this matter matters?

Fifth, the same properties constitute all property efficiency frontier which serves for evaluation of diversification strategies for entire market. Additionally, almost any A class property in portfolio costs more than double of their total value. This may question whether evaluating entire market performance through 4 small, but sufficiently well performing buildings, is relevant. Moreover, attainability of equal-weighting or even any portfolio allocation, as in mean-variance optimization, is more theoretical than real, due to impartible ownership. Therefore, use of efficient frontiers in real estate potentially provide theoretical ranking of diversification strategies and their subcategories. However, this does not answer what is the real maximal attainable risk premium given a certain level of risk. To address this, in the next section we will present pseudo-efficient frontiers, which also account for previously mentioned ownership non-divisibility.

To summarize, we showed there is no dominant traditional diversification strategy. It appears that, among diversification subcategories by property type, industrial is the plausible leader. For this reason, and lack of observations, we reformulated the original hypothesis from the thesis proposal to evaluate performance of the entire market. Subsequently, we discussed the relevance of mean-value optimization for direct real estate investment, due to unattainability of any portfolio allocation. In the context of diversification benefits, and especially of their evaluation, the key takeaway is that property value matters.

Figure 9: Efficient frontiers – by property type and by region

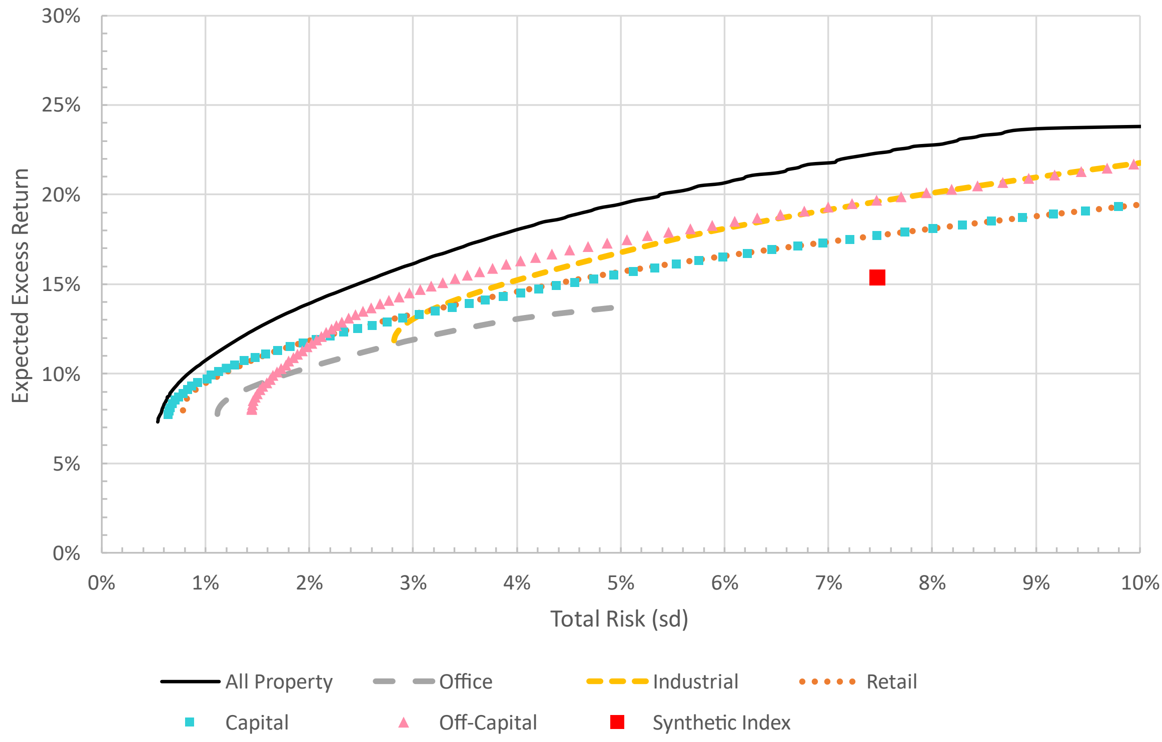
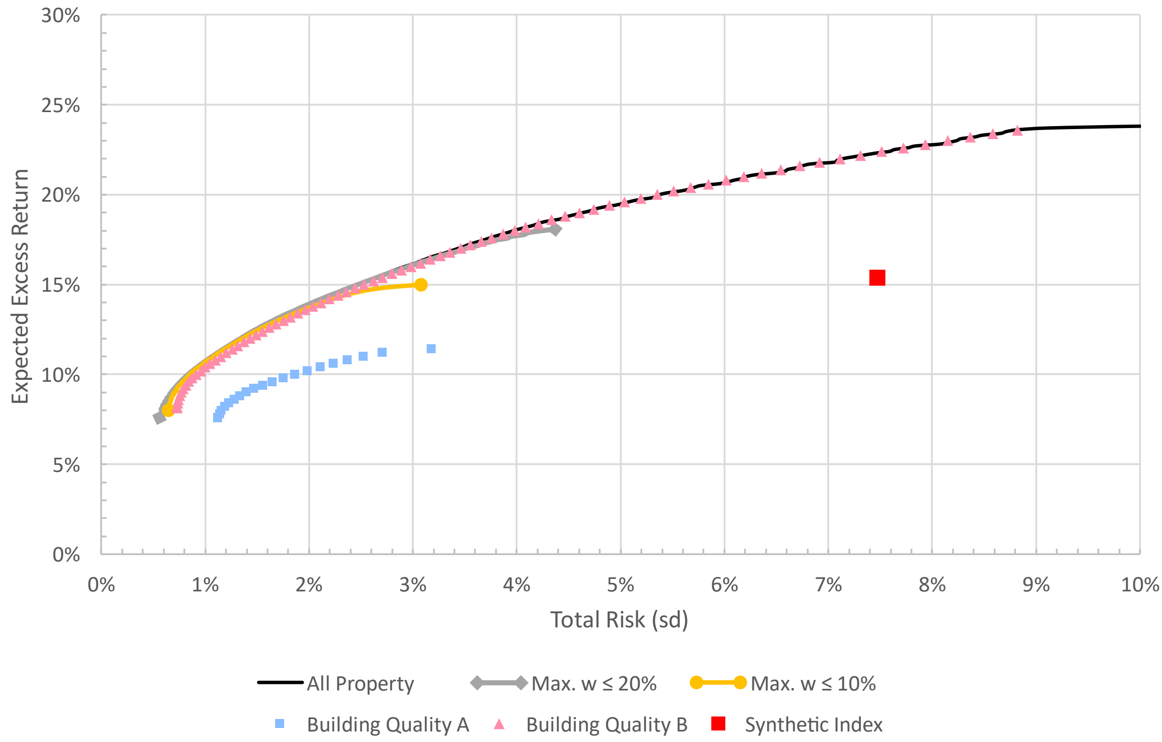


Figure 10: Efficient frontiers – by building quality and Max. % allocation



5.4. Unfeasibility of feasible set due to ownership non-divisibility and theoretical implications

In the previous section we discussed limits to the relevance of classical efficient frontiers within real estate. As a result, we model pseudo-efficient frontiers for different portfolio values, which also account for impartible ownership. This may have some practical implications, which we review in broader context at the end of this section. We estimate 2 sets of these frontiers – with property replacement and without property replacement, where each set is modeled for various target portfolio values. Complete methodology is specified in Section 4.3. We also present below a shortened version to provide readers with an idea of what we mean by pseudo-efficient frontier.

The process of estimation is the same for a set without replacement and with replacement. Set with replacement means that a hypothetical portfolio might include the same property multiple times. For each portfolio size up-to 37 properties, we bootstrap 1 million hypothetical portfolios, so in total 37 million trials. If the value of a hypothetical portfolio deviates by a maximum of 10% from any inspected portfolio value, for which we construct pseudo-efficient frontier, it is attributed to the feasible set of this portfolio. Then, pseudo-efficient frontier is the maximum attainable risk premium for a given level of risk in the subject feasible set. For completeness, we note that this method is based on a high number of trials to partially approximate the feasible set. Another specific feature is that it produces non-continuous frontiers. The reasoning is there may be no allocation for a given level of risk, while for a higher level of risk the allocation may exist.

Our data sample has a relatively uneven distribution of categories for different building classes and their types. Even though generally the same holds for any IPD index, the number of properties for this research is substantially lower. Therefore, we model pseudo-efficient frontiers only for all property frontier without differentiation for diversification strategies. However, to limit the impact of having a small data sample, we also model frontiers with replacement. This allows for multiple inclusions of the same property into a portfolio. This is particularly important for more valuable properties, which are less frequent in data sample, but have overall higher relevance and assumed higher precision of their reported total returns.

We present pseudo-efficient frontiers without replacement in Figure 11 and their counterparts with replacement in Figure 12, both with previously modeled efficient frontier of all property portfolio for comparison. Interpretation of both figures may include the following. First, a closer look at frontiers without replacement reveals that the feasible set contracted, particularly for higher risk levels. Moreover, almost any point of these frontiers lays below original all property efficient frontier. This implies that, in the real estate, the feasible set for original efficient frontiers is unattainable. In other words, mean-variance optimization overestimates maximal attainable risk premium for a given level of risk.

Second, it seems that cheaper portfolios with a target value around €5 mil. can attain higher risk premiums in general. At the same time, they are often riskier for the same risk premium than more expensive portfolios. Despite that our data show this finding is not analogous for higher portfolio values, we believe it is. This is further discussed at the end of this section. We attribute a possible dispute to limited sample size for more expensive properties. Change in risk-adjusted performance for different portfolio values may seem relatively evident, but we find it potentially interesting. We believe that it is against Modern Portfolio Theory and does not correspond to the world of finance. The reasoning is that as long as we disregard additional costs, there is arguably no difference in terms of risk and returns for a €5 mil. portfolio and €25 mil. portfolio composed from the same instruments, such as shares with the same relative allocation. However, as readers may assume, such a portfolio replication is mostly not possible for direct real estate investment. This may have additional implications, which we demonstrate in point 4.

Third, in Figure 12 we again may see that more expensive portfolios, up to €100 mil., tend to achieve higher risk adjusted returns. Similarly, as before, we believe that a violation of this concept, partly for €100 mil. and entirely for €250 mil. frontiers, is attributable to a limited data sample. Additionally, most of the portfolios were able to exceed all property frontier at least for some points. This is caused by multiple inclusion of the same property in the hypothetical portfolio, which we allowed to simulate for a greater number of properties in the market. A possible interpretation is that the maximum attainable risk premium, for a given level of risk on the Czech Republic and Slovakia real estate market, is assumed to be higher than our models show. However, if some investors are really able to attain these returns, what portfolio structure they would need to have is a different question.

Fourth, as previously mentioned we found a difference in the distribution of pseudo-efficient frontiers for different portfolio values. Based on this, we would like to propose the idea that due to ownership non-divisibility, rational investors will tend to deploy leverage to improve risk-adjusted performance. Larger investors will tend to have a higher risk-adjusted performance.

To justify this and provide a better understanding, let us introduce the Sharpe ratio and briefly revise Modern Portfolio Theory. The Sharpe ratio equals risk premium over total risk. It is a measure of risk-adjusted performance. Its maximized value corresponds to a slope of tangency line to (pseudo-) efficient frontier connecting optimal portfolio and risk-free rate, which equals in our case to coordinate [0,0] due to reporting in expected excess returns. Under full assumptions of Modern Portfolio Theory, all investors would hold the same market portfolio. Their risk awareness would only be reflected in allocation between risky and risk-free asset. To simplify the following idea, let us assume that unlimited borrowing and lending at risk-free rate is possible. Though assets are scarce, and no single market portfolio exists.

If, for instance, we have a rational investor who wants to invest €5 mil. in equity in line with our Figure 11. Then, the investor has two choices: (i) invest €5 mil. of pure equity or (ii) invest €25 mil. composed from €5 mil. equity and €20 mil. loan. The question is: "How does he decide?" Some readers might say that it would depend on his risk aversion and corresponding utility curve. This might be a correct answer. However, our empirical results show something different. We found that he would always choose option (ii), due to attainability of higher indifference curve. If an investor decides to make an allocation along tangency line with optimal €25 mil. portfolio, at the point where he would have 80% leverage or Debt-to-equity Ratio equal to 4, there he will receive a substantially higher risk premium compared to any attainable allocation with €5 mil. frontier. At the same time, he might face even lower risk. Such a decision might be attributed to attainability of more expensive portfolio with higher Sharpe ratio, due to leverage deployment. This might be freely paraphrased as: "leverage allowed to increase maximal attainable Sharpe ratio". In the real world an investor may make any choice, since assuming strict rationality is irrational. Additionally, such a high improvement in risk-adjusted performance would be probably unachievable. However, these do not limit the fact that there is a presumed existence of a leverage offsetting effect. Leverage deployment then shifts optimal allocation along tangency line with optimal holding portfolio, which also applies for MPT. However, it additionally increases the slope of the tangency line.

In theory, we often work under a framework where using leverage only increases risk. Let us suppose that there is a point on the tangency line with optimal portfolio, which corresponds to some level of leverage. Once borrowing becomes only possible at market interest rate, then to achieve the same level of risk premium as in the point there will be a higher risk. The magnitude of a risk increase for the same risk premium is higher for a higher interest rate. This is caused by a decrease in the slope of tangency line, with an optimal portfolio. However, non-divisibility of ownership in real estate also implies that risk decreases in portfolio value. This could be imagined as an opposite effect. The cause is that more valuable portfolios, which could be larger, possess higher diversification potential, or both would generally have a higher Sharpe ratio. Thus, resulting in a better risk-adjusted performance. Therefore, even-though leverage is overall increasing risk, it has a weak offsetting effect. This is due to a small increase in the maximal Sharpe ratio. This arises from potentially higher achievable portfolio value, which would have a higher efficient frontier. Investors would then tend to use leverage to improve risk-adjusted performance. They would receive a higher risk premium, but they would face only partially higher risk. This is due to the leverage offsetting effect. Therefore, such a decision could allow them to shift to a higher indifference curve.

As previously mentioned, under assumption of impartible ownership, higher portfolio value may generally induce higher position of efficient frontier. We would like to provide additional reasoning for decreasing risk in portfolio value and outline the underlying mechanism. As previously mentioned, increasing portfolio value may result in increasing portfolio size or increasing diversification potential of a specific building. The first arises from the possibility to acquire more properties into a portfolio, which generally implies reduction of idiosyncratic risk. The second relates to the general improvement of semi-qualitative indicators for more expensive buildings. Considering any specific building type, more valuable properties tend to have a larger size and arguably higher building quality. Either of these arguably leads to increased diversification potential of the building. A typical example is that a more expensive class A office building could have the same size, but better location and higher availability of on-site services. Therefore, it arguably would attract more solid and solvent tenants, which translates into a greater benefit from diversification by tenant mix. Similarly, larger net lettable area mostly implies a higher number of tenants. This again translates into improved diversification by tenant mix. Additionally, more tenants provide the possibility for more sophisticated diversification by lease terms. For more information on semi-qualitative diversifications see (McMahan, 2006).

These semi-qualitative diversification strategies contribute to specific risk reduction of property. However, it is often difficult to properly quantify them. Fortunately, this is not always necessary, since they directly impact cashflow and their stability. This, combined with market information on cap rates, constitutes market value and running initial yield of property. Based on these variables, the common evaluation of diversification strategies may depart. We find semi-qualitative diversification strategies as potentially interesting. However, they are beyond the scope of this research. Nonetheless, a highly motivated researcher, with strong support from private sector, might be able to explore these uncharted waters of real estate.

To conclude, simple techniques for risk assessment might be insufficient in real estate. We show that assumption of ownership non-divisibility implies different and less favorable results, in comparison to classical mean-variance optimization. Our empirical results allowed us to propose the idea that direct real estate investors would tend to use leverage to improve Sharpe ratio. Moreover, larger investors would tend to have higher risk-adjusted returns. We find this positive, as adverse effect of ownership non-divisibility might be partially reduced due to positive impact of leverage offsetting effect.

Figure 11: Pseudo-efficient frontiers without replacement

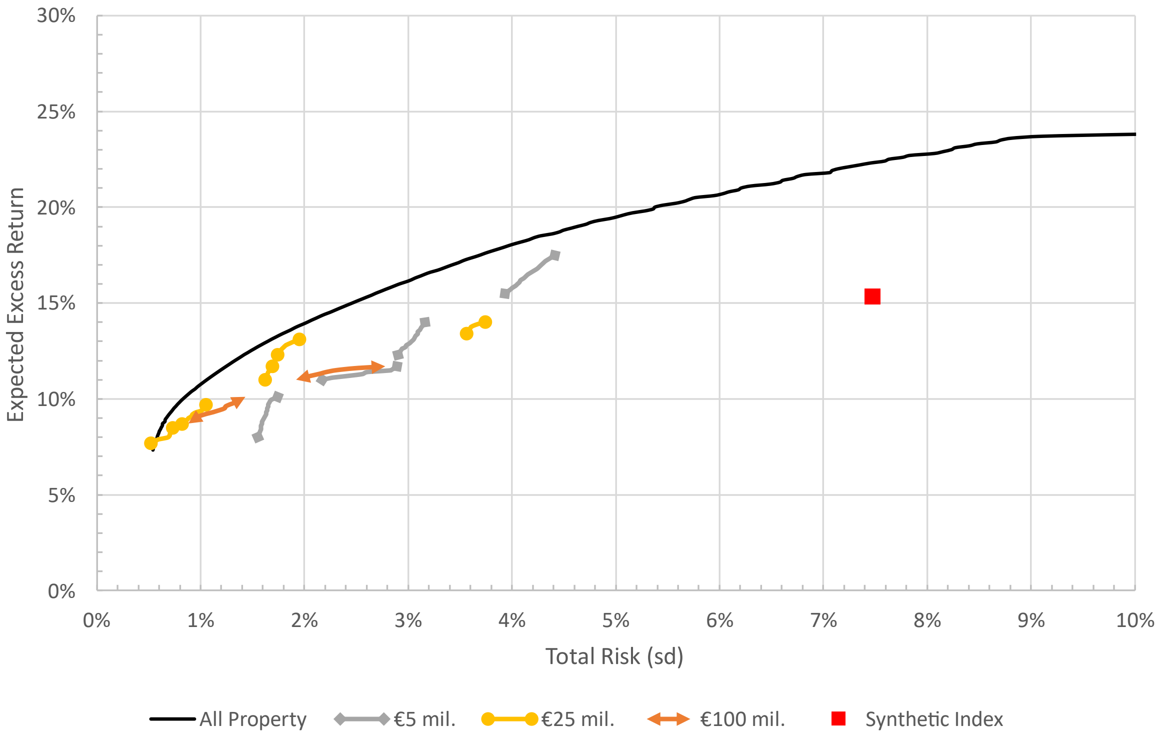
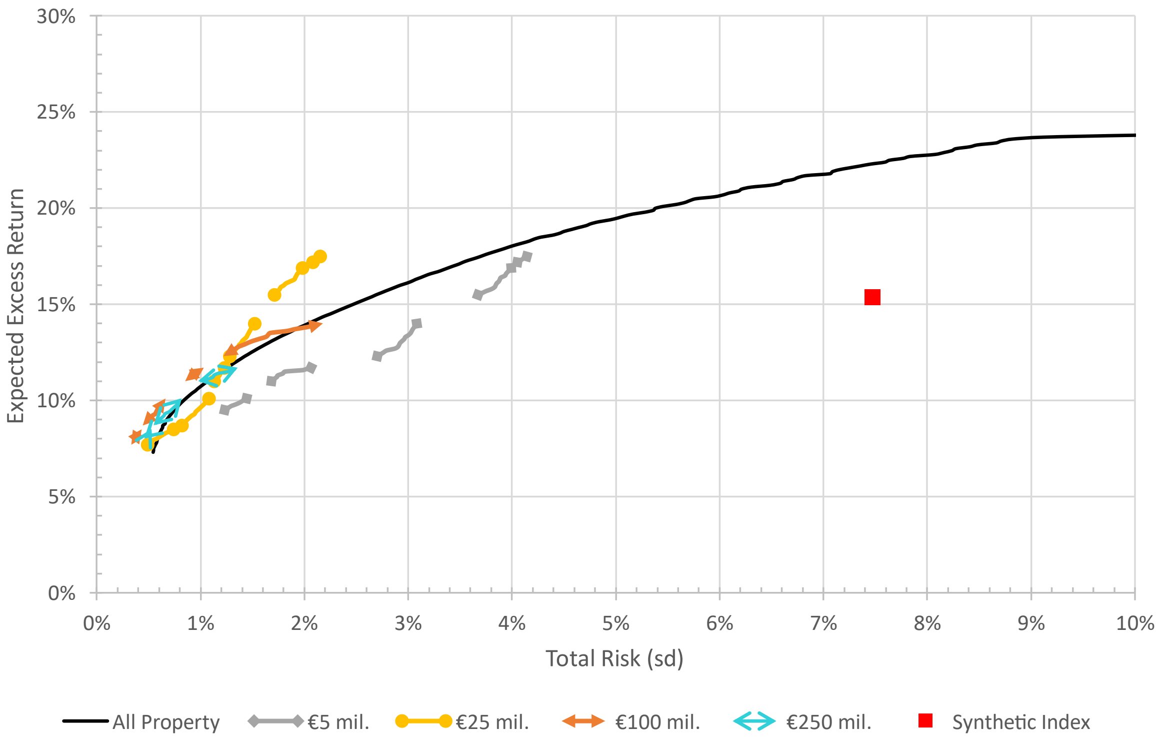


Figure 12: Pseudo-efficient frontiers with replacement



5.5. The cost of risk reduction and limits of index tracking

Previously in Chapter 5, we examined risk through efficient and pseudo-efficient frontiers. We showed that not all concepts from finance are easily transferable to real estate. The reasoning being that properties are to some level of extent a scarce asset, portfolios are value-weighted, and ownership is impartible. Therefore, the assumption of any allocation is unfeasible. In this section, we provide additional risk measures of risk reduction and tracking error, which are conventionally expressed as a function of portfolio size. To provide a broader perspective, we additionally express these in quantiles and as a function of portfolio value.

In Figures 13, 14 and 15 we present quantiles of total risk. In Figure 16, 17 and 18 we present quantiles of tracking error. Their methodology is provided in Section 4.4. We believe that readers are familiar with these measures and quantiles, therefore we proceed with only a short note on modeling. Each figure is constructed as an outcome of 10 000 simulations for each portfolio size up to 50. Results of value-weighted trials were then attributed to 1 out of 36 portfolio values. We plot X-axis for higher values as contracted to better orient readers. Interpretation of the figures may include:

First, Total Return Synthetic Index mostly has a higher level of total risk then all quantiles for all portfolio sizes or portfolio values. The majority of exceptions are attributable to the worst performing 0.95 quantile, which encompass especially at the lower values of the X-axis the worst performing properties. This might be a dispute. The reason relates to the previously mentioned very high abnormal returns and very low risk premium sensitivities to market premium, see Section 5.2. For the same reason, we may observe a small increase in total risk for the lowest reported quantile and the first 3 properties in portfolio. Possible improvements for these results are discussed later.

Second, besides 0.95 quantile in value weighted total risk, value weighted portfolios behave comparably as equally weighted portfolios. This means that in terms of portfolio size, there is no significant difference between both weighting mechanisms for risk reduction. Such a finding corresponds to (Brown & Schuck, 1997) and is opposed to (Morrell, 1993) as discussed in Section 2.4. However, we explicitly contend that this does not imply that value no longer matters. Nor, that different selected benchmark necessarily would produce comparable results.

Third, Callender et al. (2007) disclosed the lowest attainable risk from diversification for 0.50 quantile, which is also almost identical to IPD market risk, as approximately 5% starting from approximately 200 properties. This corresponds to approximate risk reduction by 3.5% (measured in returns, pps.) or relative risk reduction approximately by 40%. Our data show that the theoretical limit of risk for all quantiles lay at approximately 1.0% and 0.50 quantile would closely approach this level above 50 properties. This would correspond to reduction in total risk by 4% (pps.), or relative risk reduction approximately by 80%.

Fourth, in Figure 15 we see that trials, which resulted in total risk that was equal or higher than in the remaining 95% of trials, have spikes for portfolio values at €15 mil. and €40 mil. This is caused by multiple inclusions of the worst performing properties into portfolio. From the same figure, we may also identify the price of diversification. A portfolio with the median performance may attain 50% total risk reduction for approximately €125 mil., which we find relatively expensive. Comparable results for a value-weighted portfolio may be achieved with 6 properties. Almost full diversification potential will then be reached at €1 000 mil.

Fifth, from Figure 16 and 17 we may see that similar to total risk, besides the worst performing reported quantile, these results are comparable for both weighting mechanisms. As previously mentioned in Section 4.4, the interpretation of tracking error is relatively difficult. However, there is still one more conclusion to be drawn. Given our results, the tracking of synthetic index would be across all common levels of precision difficult to impossible. Whereas from a comparable study on the UK market (Baum & Struempell, 2006) we may conclude that on average 6 properties or approximately £60 mil. are necessary to achieve 5% tracking error, our data show that beside the best performing trials these numbers are unachievable for any portfolio size and value. For these reasons, we keep the potential interpretation of Figure 18 for the readers.

Sixth, potential improvement of results could be achieved through longer period of returns, more suitable underlying index, introduction of non-zero correlation to idiosyncratic risk between properties or their combination. Whereas the improvement through the first and second way seems to be unlikely, without a large contribution from the private sector, the third represents a potential field for additional research relying mostly on researchers' skills. Additionally, we propose to our readers a potential interpretation technique of total risk quantiles. This might indicate an upper limit of risk, which is assumed to be actually present on the market. If our readers are willing, they may elevate, in their imagination, plotted curves of quantiles by 6.5% (pps.) what corresponds to difference between synthetic index risk (7.5%) and limit of total risk reduction (1.0%). This is possible, because lower coefficient β would imply a higher degree of systematic risk and lower degree of specific risk. Then quantiles of total risk for this lower coefficient β would originate higher, but would converge faster, and their limit would be lower. These elevated curves would then constitute an upper limit for each quantile, below which the assumed market risk would be located.

To summarize, we provided an explanation for breaching the theoretical limit of systematic risk by total risk of simulated portfolios. We found similar evidence to other researchers for the relevance of both weighting mechanisms. Our results provide answers to the question in Section 4.4. They suggest that for a median portfolio 50% of total risk could be diversified away with 6 properties, which would cost around €125 mil. Also, there is evidence that index tracking within common levels of precision might be very difficult to attain in CZ and SK real estate market.

Figure 13: Quantiles of Total Risk in No. of Properties – Equally Weighted

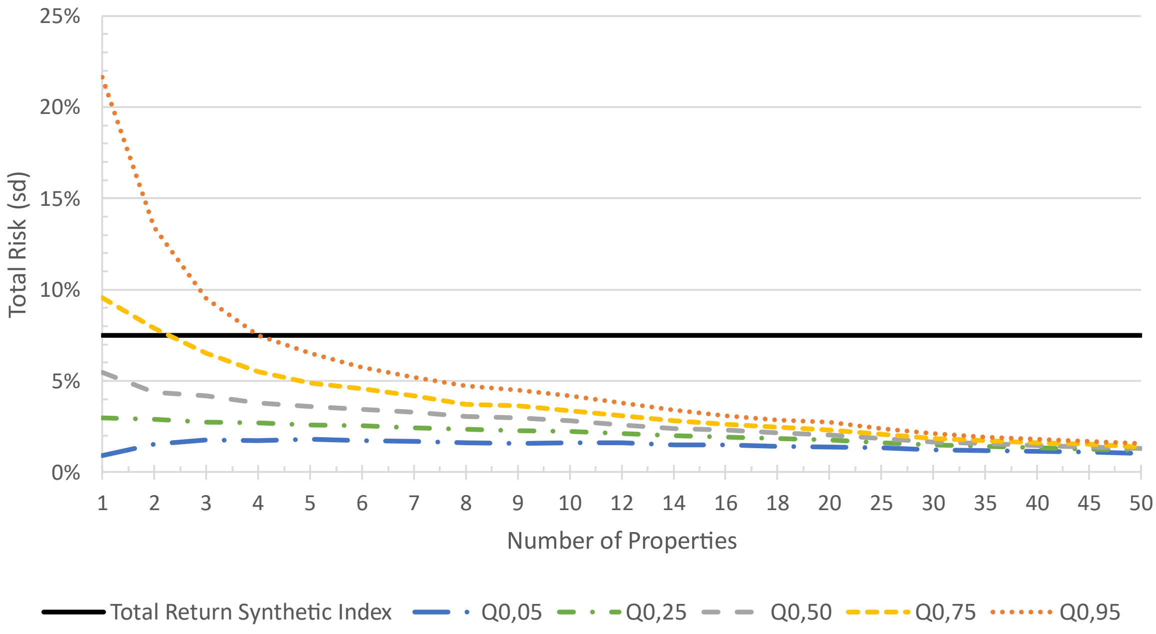


Figure 14: Quantiles of Total Risk in No. of Properties – Value Weighted

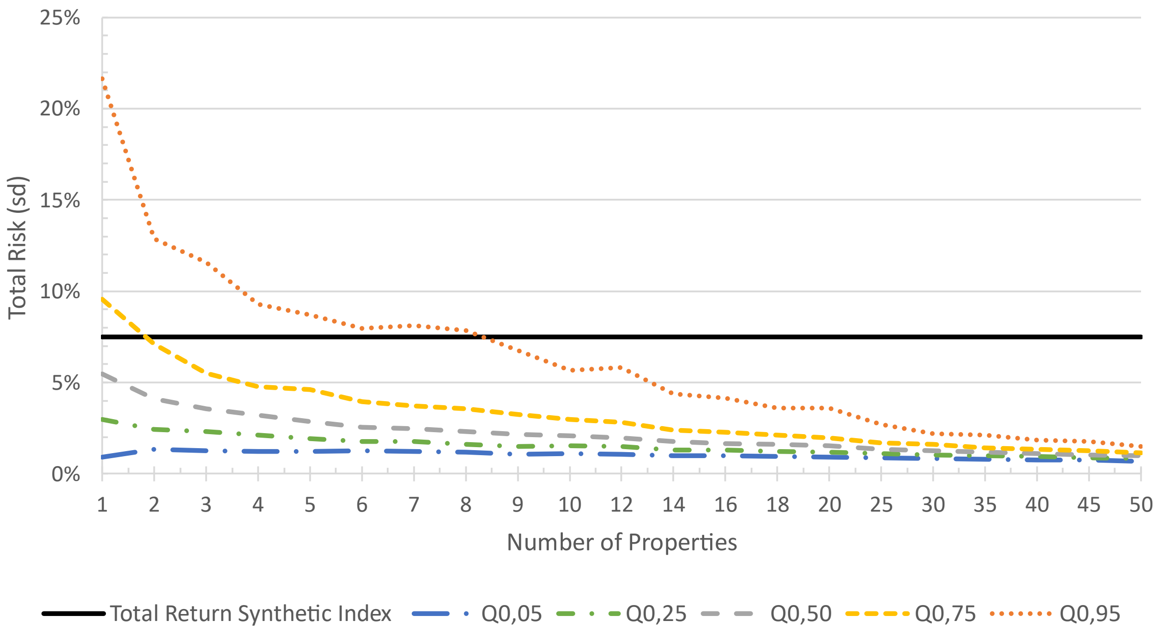


Figure 15: Quantiles of Total Risk in Portfolio Value – Value Weighted

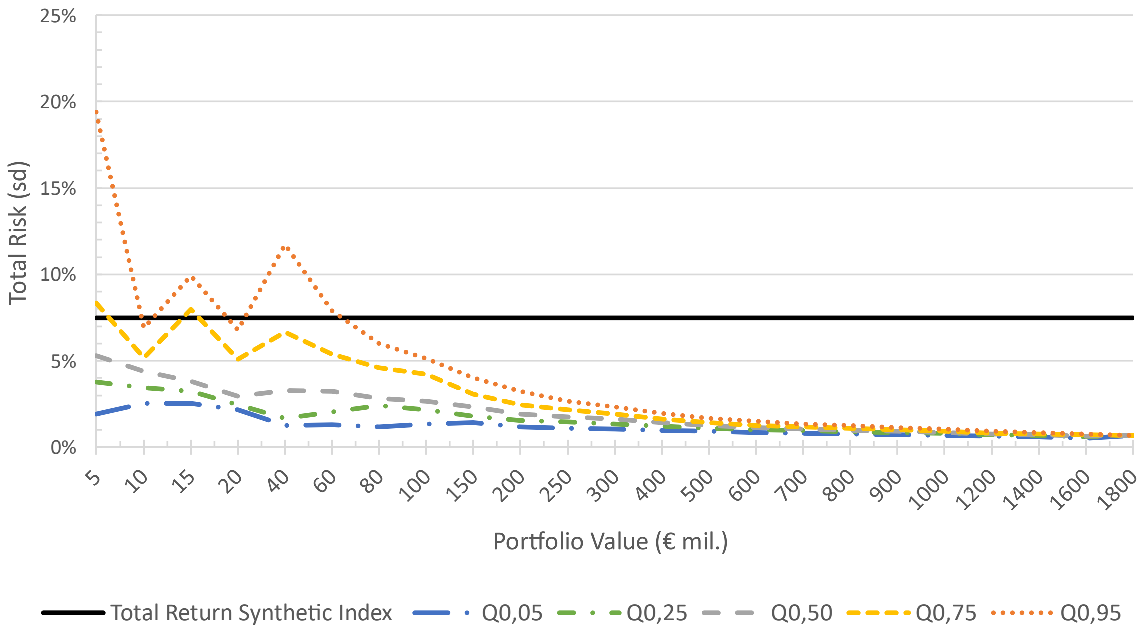


Figure 16: Quantiles of Tracking Error in No. of Properties – Equally Weighted

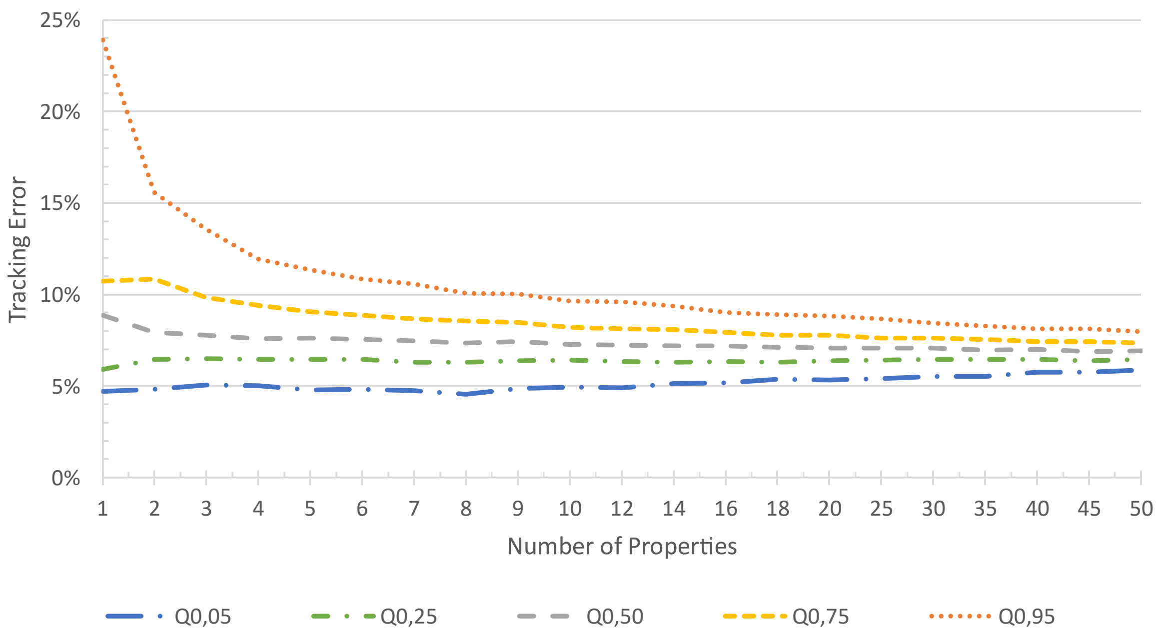


Figure 17: Quantiles of Tracking Error in No. of Properties – Value Weighted

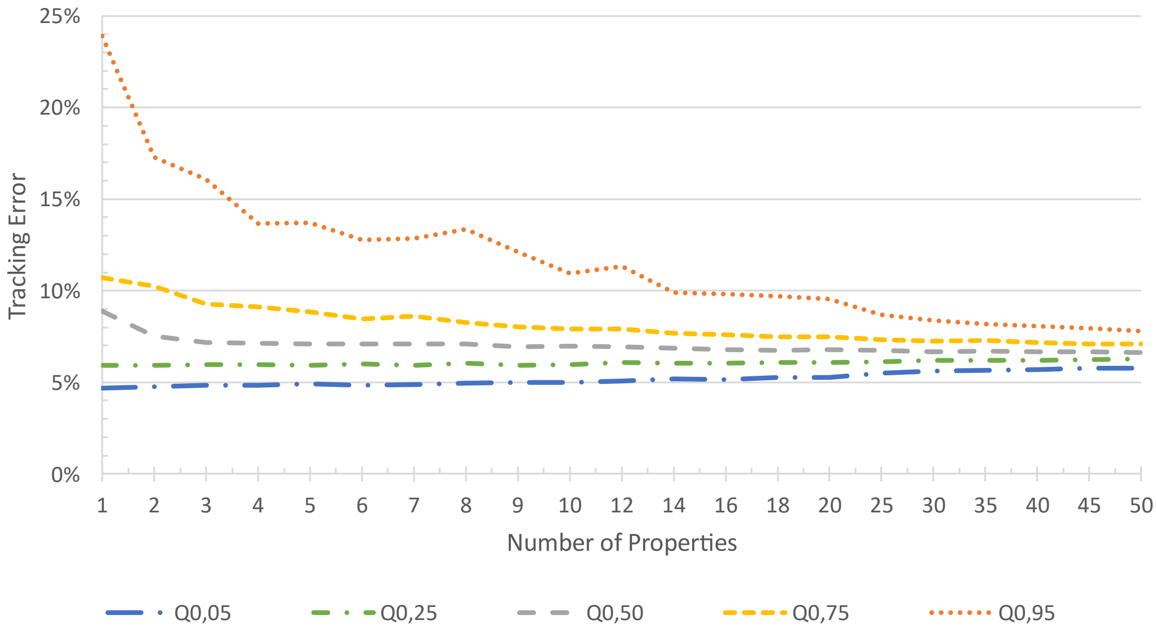
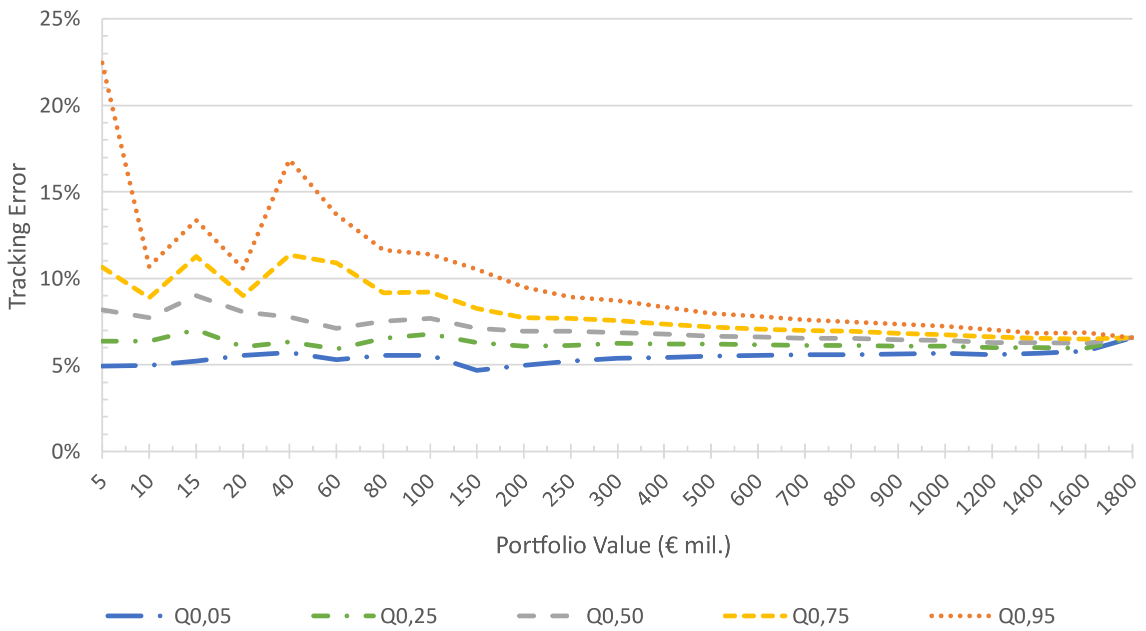


Figure 18: Quantiles of Tracking Error in Portfolio Value – Value Weighted



6. Conclusion

Direct real estate investment is arguably a popular diversification strategy for multi-asset portfolio managers, due to a positive impact on increased risk-adjusted performance. Higher diversification potential of real estate originates to different nature as an asset class and lower mutual risk with financial markets. However, this does not necessarily mean that real estate has lower risk in general. To address the risk and attain optimal holding structure, real estate investors often deploy common risk measures and traditional diversification strategy by region and by property type. Their efforts face specific constraints, which are unconceivable for listed instruments. Assumption of ownership non-divisibility makes equal-weighting difficult to attain. Moreover, it implies that only full investment or disinvestment is a considerable option. This results in decreased effectivity and increased cost of diversification compared to other equity holdings.

The purpose of this thesis is to evaluate unique data from the Czech Republic and Slovakia and determine which strategies are better. Additionally, we provide complementary risk measures and account for ownership non-divisibility. These results may then broaden the horizons of risk perception. At the same time, and within our knowledge, they provide the first kind of benchmarks to the local market.

Previous research in Central Eastern Europe has been substantially limited, due to the absence of at least one dominant institution, which would collect data of sufficient quality. This was to a moderate degree of extent overcome by high personal dedication of several real estate experts. Such a contribution enabled analysis of property total returns and construction of synthetic index, which serves as a proxy for market benchmark. The primary researcher technique is an estimation of Capital Asset Pricing Model, with Jensen's extension through Single Index Model. Estimated coefficients, risk premiums and various versions of risk serve as a base for further modeling.

Our main findings include: First, a relatively large difference between total returns on index and properties was identified in missing capital gains. Such a trend was generally present among all properties, regardless of their value, building type and owner. A possible explanation is attributed to (i) use of a different valuation approach to core properties, which establish synthetic index, (ii) unobserved nature of capital growth, (iii) and underreporting capital growth, for instance to follow the accounting conservatism principle.

Second, none of the diversification strategies strictly dominate the other. Among diversification subcategories, industrial is a plausible leader for differentiation by property type. However, we assume that core and value-add investors may have strong preference for a different building type – A class offices. This is arguably due to lower perceived risk, higher liquidity and larger deployable volume of capital. However, our results also suggest that such a preference might not always be optimal. Estimated risk-adjusted performance was almost always inferior to all other diversification subcategories.

Third, off-capital region may dominate regional division. However, further research on a larger data sample is needed to confirm its relative superiority. A larger data sample would also allow to inspect region through a traditional perception, as a district, or through a modern approach, as a functional and economic coherent area. A natural extension of traditional diversification strategies is segmentation by different categorical variables.

Fourth, our extension was based on differentiation by building quality. Models predicted that quality B would dominate over quality A. However, we suppose that such a conclusion would be premature. A large part of upper feasible set was constructed through allocation into 4 small buildings, which represent around 10% of properties, but less than 1% of total sample value. This reveals a potential vulnerability of mean-variance optimization in real estate and may question its relevance. Whereas we find this method appropriate for theoretical diversification strategy assessment, it is not able to outline real risk-adjusted performance as a consequence of impartible ownership.

Fifth, as previously mentioned, non-divisibility of ownership implies specific obstacles to real estate, for which investors should account for. The direct impact on a classical feasible set is that it becomes unfeasible. In other words, traditional mean variance optimization would almost always overestimate attainable risk premium for given level of risk. A possible substitute is pseudo-efficient frontier, which is constructed through Monte Carlo simulation for a large number of trials. Empirical findings allowed us to propose an idea that, under assumption of impartible ownership, total risk would be decreasing function in portfolio value. Therefore, investors would tend to deploy leverage to improve risk-adjusted returns and larger investors would tend to achieve higher risk-adjusted performance.

Sixth, complementary risk measures expressed in quantiles for various portfolio sizes and values, revealed that diversification may become pricy. Our model predicts that a portfolio with the median performance would attain approximately 50% of total risk reduction, with €125 mil. portfolio value or 6 properties. This corresponds to a decrease in total risk by 2.6% (pps.). Full diversification potential would be almost realized starting from €1 000 mil. Comparable model predicts that index tracking, within common levels of precision, would mostly be unattainable, even at 5% of tracking error. A potential improvement in results may be achieved through the introduction of correlation among idiosyncratic risk. However, the biggest improvement would be realized through an extended dataset and an introduction of relevant real estate market index, what we consider to be square one.

Last, we would like to express our belief that establishing cooperation between private sector and universities in the Czech Republic would enable a new dimension of real estate research. This may lead to overcoming the chronic problem of data unavailability. Additionally, it may lead to the evaluation of a large variety of strategies, which are only believed to be effective through evidence. Finally, it may bring a new age to real estate – the age of data.

To conclude, we presented our results for 3 different risk measures on examples of the Czech Republic and Slovakia. We found a choice of diversification subcategory may impact investors' performance. The plausible leader among diversification subcategory by property type is industrial. Additionally, an assumption of ownership impartation negatively impacts investors' efforts for optimal allocation. Based on empirical findings, we propose the idea that investors may partially mimic this adverse characteristic by deployment of leverage. Lastly, our results suggest that diversification may become costly and index tracking is hardly possible. As a final point, speaking of diversification for direct real estate investment, the key takeaway of this thesis is that value matters!

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