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FACULTY OF SOCIAL SCIENCES
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Using spin model to determine FTTx connectivity market potential in the Czech Republic

Master's thesis

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Declaration of Authorship

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Prague, January 5, 2021

Pavel Munduch

Abstract

The our thesis "Using spin model to determine FTTx connectivity market potential in the Czech Republic", we firstly map the current landscape of the Czech broadband technology market. Additionally, we present an overview of Ising model's interdisciplinary applications. Afterwards, we describe the dynamics of the Ising model and in particular we study the convergence tendencies of Ising model generated series as well as the spin positioning in the Ising model lattices based on the input parameters.

Consequently, we assume the spins in the model to represent the fiber technology and alternative technology and thus we link the Ising model, its parameters and outputs to the problem of fiber connectivity potential. Apart from the standard input parameters of the Ising model, we also introduce variability in terms of the distribution of the initial lattice and we define four archetypes to represent real market situations. Ultimately, we describe the sets of parameters for which the market appears to have the most potential of fiber deployment.

JEL Classification	A12, C6, C15
Keywords	Ising model, econophysics, fiber technology, broadband connection
Title	Using spin model to determine FTTx connectivity market potential in the Czech Republic

Abstrakt

V naší diplomové práci "Využití spinových modelů k určení potencialu FTTx připojení v České republice" prvně mapujeme současnou situaci na českém trhu širokopásmové připojení k internetu. Dále uvádíme přehled mezioborových využití Isingova modelu. Poté popisujeme závislost dynamiky modelu a jím vygenerovaných posloupností na vstupních parametrech. Nato spojíme teoretický model s realitou tak, že definujeme spinové pozice Isingova modelu jako případy optického a alternativního připojení. Kromě klasických vstupních parametrů Isingova modelu ješté uvádíme variabilitu závislou na stavu vstupní matice - zároveň tedy i definujeme čtyři archetypy vstupní matice. Nakonec popisujeme, pro které parametry modelu se trh zdá být nejvíce nakloněn rozestavení optických sítí.

Klasifikace JEL	A12, C6, C15
Klíčová slova	Isingův model, Ekonofyzika, Optické technologie, Širokopásmové připojení
Název práce	Využití spinových modelů k určení potenciálu FTTx připojení v České republice

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Acronyms

4G Fourth generation of broadband cellular network technology

5G Fifth generation of broadband cellular network technology

ESIF European Structural and Investment Funds

EU European Union

EU28 28 European Union members

ČTÚ Czech Telecommunications Office

FTTx Fiber to the x

FTTP Fiber to the premise

FTTH Fiber to the home

CETIN Czech Telecommunication Infrastructure

DESI The Digital Economy and Society Index

NGA New Generation Access

ICT Information and Communication Technology

xDSL Digital subscriber loop

ADSL Asymmetric digital subscriber line

VDSL Very-high-bit-rate digital subscriber line

LLU Local Loop Unbundling

CATV Cable television

WLL Wireless local loop

CDMA Code division multiple access

UMTS Universal Mobile Telecommunication System

LTE Long Term Evolution

MNO Mobile Network Operator

MVNO Mobile Virtual Network Operator

ISP Internet Service Provider

OECD Organisation for Economic Co-operation and Development

Master’s Thesis Proposal

Author	Bc. Pavel Munduch
Supervisor	prof. PhDr. Ladislav Krištoufek, Ph.D.
Proposed topic	Using spin model to determine FTTx connectivity market potential in the Czech Republic

Motivation Fast internet connectivity is one of the key enablers of globalization and has significant effect on various economic variables. Gradually, countries around the world improve their fixed internet infrastructure via using several different technologies. One that stands out a bit is the fiber cable technology (FTTx), which has seen some development in the Czech Republic. Primary motivation is to evaluate its place within Czech infrastructure compared to other available structures. The issue is even more pressing when acknowledging the ongoing trend of and pressure on digitalization of public sphere operations and administration. Moreover, to fulfill the requirements and aims established by the European Union, subsidies are being offered to enhance the development.

By proxy, the spread of fiber technology can be connected to general broadband infrastructure. Using panel data from OECD countries, Rohman & Bohlin (2012) show that there exists positive effect of broadband speed on economic growth. Before that, Czernich *et al.* (2011) also concluded a positive effect of broadband infrastructure penetration on annual per-capita growth.

Additionally, the level of broadband infrastructure might influence other country characteristics as well, for instance entrepreneurial activity as shown by Heger *et al.* (2011) for the case of Czech Republicâ€™s neighbor, Germany. Even though the effect may be overridden by regional characteristics in some cases, the effect is apparent for high-tech industries.

Hypotheses

Hypothesis #1: FTTx technology is more viable to be constructed compared to other technologies (e.g. fixed LTE, wireless, coaxial cables).

Hypothesis #2: FTTx is has more impact when implemented in specific geographic areas (e.g. medium sized cities).

Hypothesis #3: FTTx technology is economically viable to be deployed at all.

Methodology We will tackle the problem by utilizing spin model analysis. By parametrization of key levers, we will be setting up and describing the decision making process for key agents and stakeholders. (i.e. mainly fixed NGA connection providers). The parametrization will include different specifications concerning the geography, existing infrastrucutre and competition in place (unbundling and presence of previously state infrastructure), eligibility for subsidies and other economical and qualitative influences to build a probabilistic framework. In this framework, binary outcome (i.e. fiber will be implemented vs other or none technology will be implemented) will be simulated using the afore mentioned spin modelling. Hopefully, arising from micro level foundations, we will be succesful in revealing and determining rules that would apply and be evident at macro level as well.

To properly set up the parametrization process, utilization of Czech state authorities' data resources would be convinient. (e.g. public consulation of NGA coverage conducted by Ministry of Industry and Trade, Czech Statistical Office, or Czech Telecommunications Office).

In more detail, the spin model revolves around the defining the spins as positive and negative 1s. This method stems from statistical mechanic and its scrutiny of ferromagnetism. In our particular environment, these two values represent either deploying the FTTx infrastructure or not (therefore an alternative). Based on the aforementioned parametrization, a (k x k) lattice is occupied. Spins are chosen pseudorandomly, but the simulation will be repeated numerously. Spins in a lattice can interact with its neighbours, but more importantly overall patterns ("total magnetism") should emerge within the system. The primary analysis will be conducted withing the programming language R environment.

Expected Contribution The development of the infrastructure is closely watch by both the European Union and local policy makers and plays crucial role in current debate. The analysis should be as well beneficial for the individual players. The analysis should be of help when determining the desired composition of technology usage and more importanly its effectiveness.

As presented below, there are papers/workings that empirically study the distribution of different telecommunications technologies and its factors from different perspectives. Nevertheless, the utilization of spin models has never been conducted in area of telecommunications (main use cases are in finance, or marketing, for instace.)

Several sources discuss the deployment the of various technologies to provide omnipresent broadband connectivity. Townsend *et al.* (2013) advocate that FTTx is the prevailing technology for several use cases. One areas of interest are newly covered sites and sites with high concetration of businesses, deploying FTTH/FTTP specifically. Other potential use case is upgrading existing core accesses with FTTC. Crucial conclusion is that fiber technology cannot effectively cover the whole country given population density and other constraints. These findings, however, lack to be backed up by deeper quantitative analysis. European-Commission (2019b) provides a comprehensive high level overview of FTTx's thought advantages and disadvantages. As suspected, FTTx technology deployment very much depends on the trade-off between high investment costs and great connectivity enhancement. Cik *et al.* (2017) recall there are few technical-economic analyses currently taking place with cooperation with the European Union combining different approaches on higher geopgraphical level. Expected contribution of this thesis is to examine the system dynamics and interagent actions more in more depth.

Outline

1. Introduction
2. Current situation on Czech fixed connection market
3. EU stand in developing NGA access with its territory
4. Parameter definition, relationship, input parameteres, causalities between
5. Explanation of the spin model methodology
6. Spin model calculations
7. Outcomes
8. Hypothesis confirmation or rejection
9. Conclusion

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Chapter 1

Introduction

In the following paper, we will be carrying out an analysis of a topic belonging under the field of econophysics. As the name aptly suggests, econophysics is a field merging economics and physics. More specifically, aim of econophysics is to apply models from physics to describe and analyze problems in the economic sciences, be it financial economics or macroeconomic models. In our study, we will utilize the Ising (Bornholdt) model in order to describe the behavior on the broadband technology market in the Czech Republic.
The objective of the thesis is to foremost delve into the dynamics of the Ising model, map its dynamics, and use the relevant characteristics to establish a model that helps to determine the market potential of fiber technology connection in the Czech Republic.
The thesis is structured as follows: Chapter 2 outlines the situation on the Czech fixed broadband technology market, Chapter 3 summarizes relevant literature review of fiber technology as well as it describes previous applications of the Ising model. Chapter 4 explains the dynamics of the Ising model and the technical background to its as well as intuition behind and connection to real life parameters. Chapter 5 presents the results of the Ising model generated series and theirs interpretation. Chapter 6 discusses the eligibility of the results, potential drawbacks of the Ising model and areas for further research. Lastly, Chapter 7 summarizes findings of our thesis.

Chapter 2

Czech broadband market overview

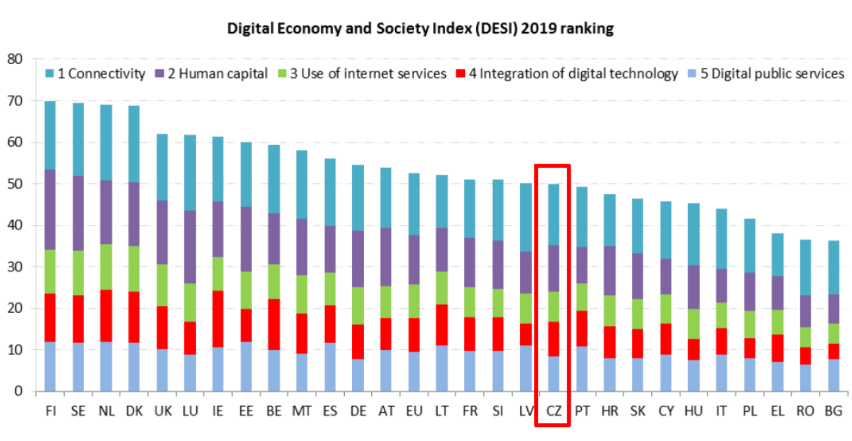
In Chapter 2, we will present an overview of the telecommunication market (broadband connection) market in the Czech Republic and its latest development.

2.1 Czech internet infrastructure and demand on a European level

The Czech Republic stands as rather an average country in terms of its digital and internet advancement. As can be seen in Figure 2.1, each year, on a European scale, European Union conducts a comparison of various metrics among its members and aggregates it into a single index, called DESI as presented by European-Commission (2019c). It comprises of five key components: Connectivity, Human capital, Use of internet services, Integration of digital technology and Digital public services. From perspective of this thesis, for our context the most important components are connectivity and use of internet services as we will examine them more closely.

In the connectivity, that takes into account mostly infrastructure measures including fixed broadband coverage and take-up in general as well as same measures for 5G, NGA (connections with over 30 Mb per second) or ultrafast broadband (connections with over 100 Mb per second). See Figure 2.2 for more detail. In this particular component, the Czech Republic ranks on the brink of average as 14th country out of the 28 EU member states with index of 59.2 out of 100 in 2018 (compared to EU average of 59.3). The Czech Republic mostly outranks EU average in the fixed infrastructure, nevertheless when it comes to mobile infrastructure, it is distinctive at 3rd place within EU28 with its 99%

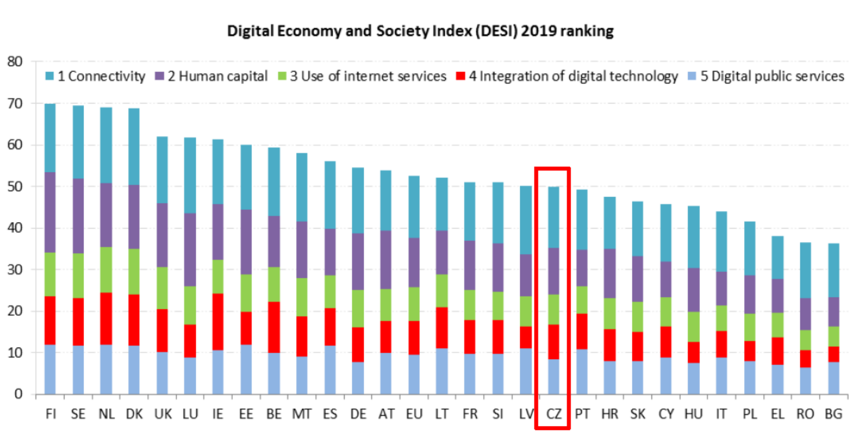
Figure 2.1: DESI index 2019



Source: European-Commission (2019c), European Commission.

coverage of 4G (EU average at 94%) and 11th place within EU28 with 17% 5G readiness (EU average at 14%).

Figure 2.2: DESI, connectivity component



Source: European-Commission (2019c), European Commission.

In the use of internet services, the Czech Republic ranks as the 19th country in the EU (see Figure 2.3) with index of 47.9 compared to EU average of 53.4 in 2018. Even though the Czech Republic is below average, we observe a growing trend in the general use of internet. Share of Czech individuals that never used internet dropped from 13% to 10% between 2016 and 2018. Moreover, the share of internet users (i.e. people who use the internet at least once a week) grew from 79% to 84% in the same time period.

The index also zooms in on different purposes of using internet. Interest-

Figure 2.3: DESI, use of internet services component

	Czechia				EU
	DESI 2017	DESI 2018	DESI 2019		DESI 2019
	value	value	value	rank	value
3a1 People who never used the internet % individuals	13% 2016	11% 2017	10% 2018	*11*	11% 2018
3a2 Internet users % individuals	79% 2016	81% 2017	84% 2018	*12*	83% 2018
3b1 News % internet users	NA 2016	91% 2017	91% 2017	*3*	72% 2017
3b2 Music, videos and games % internet users	72% 2016	72% 2016	70% 2018	*25*	81% 2018
3b3 Video on demand % internet users	4% 2016	4% 2016	5% 2018	*28*	31% 2018
3b4 Video calls % internet users	40% 2016	42% 2017	49% 2018	*18*	49% 2018
3b5 Social networks % internet users	55% 2016	57% 2017	64% 2018	*23*	65% 2018
3b6 Professional social networks % internet users	5% 2015	5% 2017	5% 2017	*26*	15% 2017
3b7 Doing an online course % internet users	3% 2016	4% 2017	4% 2017	*26*	9% 2017
3b8 Online consultations and voting % internet users	6% 2015	3% 2017	3% 2017	*27*	10% 2017
3c1 Banking % internet users	63% 2016	67% 2017	72% 2018	*10*	64% 2018
3c2 Shopping % internet users	57% 2016	65% 2017	67% 2018	*14*	69% 2018
3c3 Selling online % internet users	15% 2016	13% 2017	16% 2018	*16*	23% 2018

Source: European-Commission (2019c), European Commission.

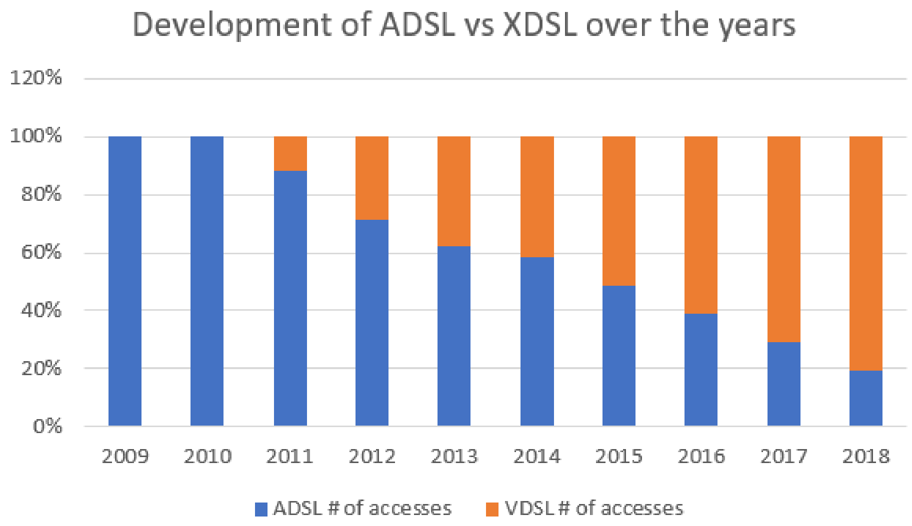
ingly enough, we see some discrepancies along the variables. While Czechs seem to lag behind when looking at more leisure focused activities (music, games, video, social networks), they leap over the EU average when it comes to more professional or serious activities such as shopping and selling online, internet banking or reading news (actually, Czechs rank 3rd in reading news within the EU28). Overall, we again observe a growing trend among almost all specific categories included in the use of internet component. This all may suggest that advancing broadband infrastructure can be increasingly more important in the future for Czech citizens.

2.2 State of Czech internet infrastructure, its key stakeholders and current development

2.2.1 Technology distribution

First, let us present an overview of different broadband technology on the Czech market, brief explanation of the technologies, comparison and Czech market shares closely followed by European-Commission (2019b). The main technologies employed in the broadband infrastructure are Digital Subscriber Lines (xDSL), Local Loop Unbundling (LLU), Cable TV (CATV), Fiber (FTTx), Wireless Local Loop (WLL) and Mobile. xDSL represents the backbone technology in the Czech Republic as it builds on copper wires that were historically established for fixed voice services (i.e. fixed telephone lines). Its main positive for future uses is their comparative advantage in required investment as the base infrastructure is already in place. At the end of July 2017, 92.9% of households in the Czech Republic were covered by xDSL. Nevertheless, xDSL can be highly asymmetrical in download and upload speeds, thus complicating activities that require balance in those two (e.g. video and cloud activities where mutually balanced communication is crucial). Initially, ADSL was the prevailing subcategory, lately being upgraded to VDSL that enables higher speeds. In Figure 2.4, we can see that the share of ADSL accesses has dropped from 100% to 19% in the last 10 years.

Figure 2.4: ADSL vs VDSL development



Source: Odbor-Ekonomické-RegulaceČTÚ (2019), ČTÚ

Local Loop Unbundling (LLU) is an advancement to the backbone infrastructure. LLU lines are those that are by regulation enabled to competitors of the backbone infrastructure owner (in Czech case it is CETIN and the lines are based on xDSL lines). As well as xDSL, Cable TV (CATV) lines are also metallic. Unlike xDSL though, CATV coaxial wires are also shielded in metallic/copper protection which allows them to reach substantially higher speeds. On the other hand, they are a bit problematic as the transfer is usually shared among multiple users and therefore may lag in peak hours. The coverage by

CATV is significantly lower than with xDSL with 41.6% of households covered at the end of July 2017.

Last typically fixed/wired solution are fiber (optic) cables - FTTx. Among all the technologies, fiber has the highest potential speed, whereby is usually deemed as the most long-sighted solution for broadband infrastructure. Moreover, compared to other technologies, it exhibits high rates of efficiency at various distances. Its main disadvantage however is the initial investment required. Unlike with other solutions, building fiber infrastructure usually requires starting construction from scratch with excessive land interventions.

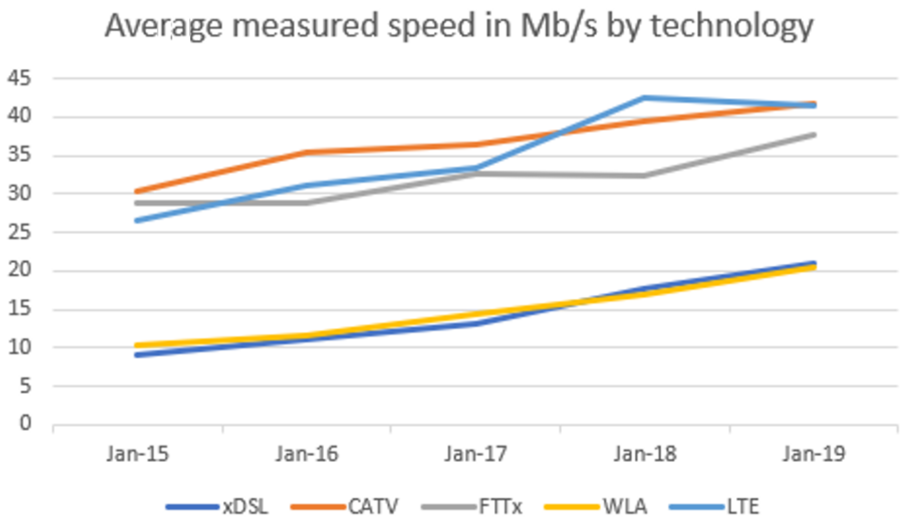
Moving on to more mobile solutions, we focus on Wireless local loop (WLL). In Czech Republic, this represents a set of antennas spread all over the country focusing on broadcasting internet signal in localized areas. Advantage of this technology is its low initial investment, therefore market with WLL providers is the most fragmented with 1600 unique local and national providers as counted by Internet-Pro-Všechny (2019). However, its main disadvantage would lie in users dependence on distance from the antennas and also susceptibility to weather and to topographical features.

Lastly, we need to mention mobile network connections. These are the standard cellular accesses using typical mobile networks, CDMA, UMTS or LTE (we may encounter a different terminology for different types of mobile connections, 3G, 4G, 5G; generally, 4G and 5G could fall in the LTE category, but the categories overlap). The set of advantages and disadvantages is quite similar to WLL, main difference to highlight is that for upgrading the features of mobile network, the current set of antennas used for voice services can be utilized.

In the Figure 2.5, we can observe data by Czech server DSL.cz (2019) that allows internet users to measure their download speed. Monthly, the server announces download averages for each technology. We see that for all technologies, the measured speed gradually increases over the last five years. Despite approximately doubled growth in speed, both xDSL and WLL lag behind its counterparts. At the top, we see LTE and CATV to compete for the first place with FTTx in close proximity.Even though these measurements may highly depend on current situation while they took place (based on weather, peak hours, technical issues, etc.), they give us a general comparison.

Annually, Czech Telecommunications Office publishes an overview of the absolute broadband accesses by technology. Firstly, in Figure 2.6 we provide a graph showing the development of market share among different fixed tech-

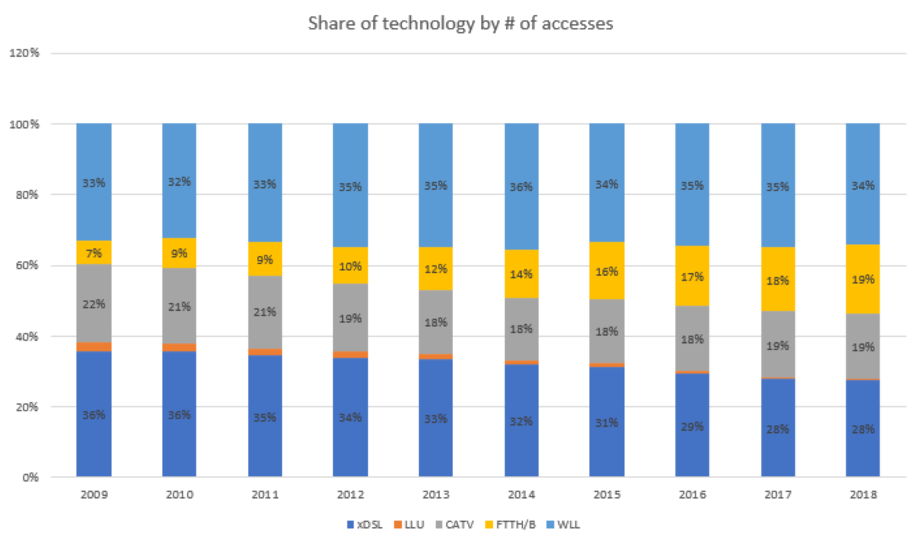
Figure 2.5: Evolution of connection speeds, Czech Republic



Source: DSL.cz (2019).

nologies. Unsurprisingly, the share of xDSL has been decreasing significantly despite the upgrade from ADSL to VDSL. On the other hand, the share of fiber connections is increasing considerably, in fact, it is the only technology whose share has increased in each year (with CAGR in absolute number of 9.1% between 2016 and 2018, whereas second highest growing CATV in absolute numbers reported CAGR of only 2.9%). Shares of other technologies either decline or rather stagnate.

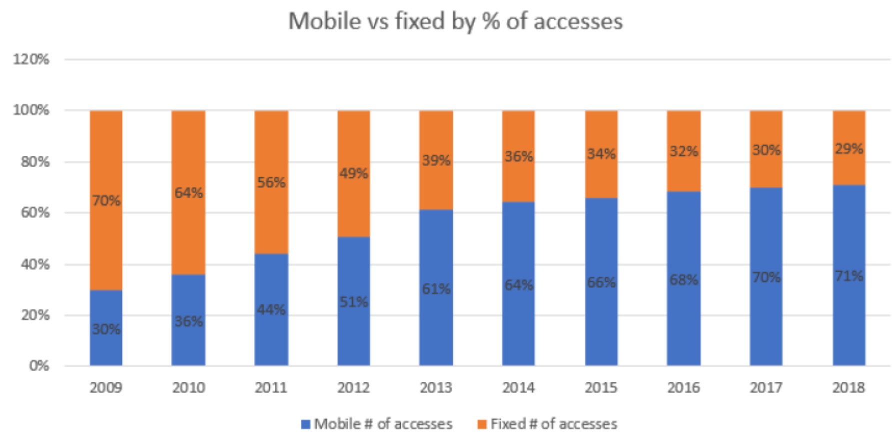
Figure 2.6: Share of accesses by technology



Source: Odbor-Ekonomické-RegulaceČTÚ (2019), ČTÚ

Secondly, in Figure 2.7 we observe the development of mobile versus fixed internet accesses. While fiber is not contested in terms of growth within fixed technologies, when put into perspective to mobile connections, fixed connections are losing their share. When zooming in on CAGR of mobile connections, we compute the exactly same figure for mobile as for fiber (i.e. 9.1% between 2016 and 2018), however.

Figure 2.7: Share of mobile vs fixed connections



Source: Odbor-Ekonomické-RegulaceČTÚ (2019), ČTÚ

Kumar (2012) also touches upon the idea of comparison of different technologies in their comparison: Wireless Versus Wireline, Competing Broadband Access Technologies. They come up with very similar remarks, making one overarching statement, in an ideal distribution of technologies, different approaches should be combined and not set as standalone entities. Now let us look at the main players and stakeholders on the market.

The regulator, Czech Telecommunications Office (ČTÚ)

Czech telecommunications market is regulated by the state via its subsidiary - Czech Telecommunications Office (CTU), which is owned by Czech Ministry of Trade and Industry. Apart from regulation, its role is in gathering data and statistics regarding the Czech telecommunication market. For our context, its main competencies are in granting licenses for entrepreneurs in telecommunications, managing the process of the frequency spectrum distribution and overseeing fair competition.

Czech Ministry of Trade and Industry

Czech Ministry of Trade and Industry is governmental body responsible for molding and pushing Czech national broadband strategy on national level as well as aligning strategy targets with European Union's agenda. Among others, its role also lies in overseeing auction and tenders for frequency domains and redistribution of subsidies for broadband infrastructure enhancement. Backbone infrastructure player - CETIN Ceska telekomunikacni infrastruktura (CETIN) is the natural incumbent in the Czech broadband infrastructure environment. It was established in June 2015 by separating company O2 into two entities, O2 kept the retail side of the business, while CETIN kept the infrastructure. As an ascendant of former state infrastructure player it owns a network of mobile towers/antennas across the whole state along with vastly spread wires network (20 million kilometers of metallic wires and 44 thousand kilometers of fiber cables). Given its unique position on the market, it leases the infrastructure to other players, 3 mobile network operators (MNOs that also lease their leased network to mobile virtual network providers - MVNOs) and internet service providers. PPF group owns the majority of CETIN.

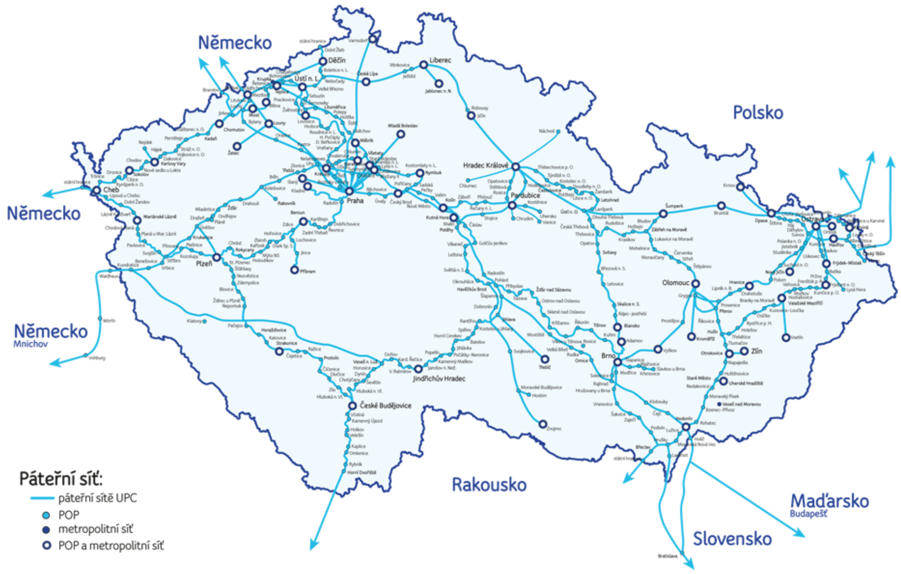
O2, T-Mobile and Vodafone

O2 Czech Republic is mobile and fixed telecommunication player connected to CETIN (O2 is as well in majority owned by PPF Group), O2 provides mobile services xDSL and fiber connections. T-Mobile and Vodafone are other market incumbents on the Czech telecommunication market. Basically, in terms of broadband services, they provide the same options as O2 via CETIN infrastructure. Nevertheless, both companies have expressed their interest in obtaining their own broadband infrastructure.

UPC

UPC was the key player owning CATV technology infrastructure (merged with Vodafone as of April 2020). It owned a nationwide network with reach to up to 1.4 million households (see Figure 2.8). In 2018, out of 596 thousand accesses via CATV in the Czech Republic, 507 thousand were operated by UPC (85%).

Figure 2.8: UPC network



Source: UPC (2019)

Poda and Wi-Fi players

PODA is the most visible players on the still quite fragmented FTTx market in the Czech Republic. Starting as a local player, it eventually built its fiber infrastructure all over the country (Ostrava in 2007, Brno in 2009, Prague in 2014). PODA prides itself as the company with most active fiber connections in the Czech Republic (in 2018, 63 thousand connections out of total 622 thousand - 10%) with continuous improvements and extensions. All together however, it enables the possibility of fiber connection to 200 thousand households.

As already mentioned, WLL market is very fragmented and dominated by local players. No players especially stand out among the 1600 providers.

2.2.2 Current development with recent and future investments undergone or planned

T-Mobile

At the beginning of 2018, T-Mobile has announced its plans to invest in its own fiber infrastructure. It dedicated between 20 and 30 mln EUR to reach coverage of 25%+ households in fiber (approximately one million of households). The investment in fiber should enable households to obtain ultra-fast broadband with speeds up to 1 Gb/s. Initially, the plan was not localized, however later that year, T-Mobile announced its main focus would be medium sized towns (up to 30 thousand of inhabitants). The story has been captured by Václavík (2018).

CETIN

After parting from O2 to become a separate infrastructure player, CETIN immediately announced its future investments into fiber connections. Ultimately, its goal is to connect up to one million households to fiber technology by 2026. As evidence of ongoing process, CETIN proclaims that average connection speed reached through their connections has increased by 25% (from 52 Mb/s to 67 Mb/s) throughout the year 2018. These plans were announced by the operator itself, CETIN (2019).

Vodafone

To enter the market with fixed infrastructure more deeply, Vodafone has decided to acquire UPC's business in the Czech Republic (as well as in Hungary, Romania and Germany) from Liberty Global. The transaction was approved by the European regulatory body and therefore the two companies are merged since August 2019, ultimately to be united under one brand Vodafone.

3600-3800 MHz auction (5G suitable frequencies)

In July 2017, Czech Telecommunication Office sold five 40 MHz frequency domains (3.6-3.8 GHz) suitable for 5G or IoT connection for total of CZK 1.015 bln, each for CZK 203 mln to O2, Vodafone, Nordic Telecom and Poda. In May 2017 it was announced that 6 companies will attend the auction for 5 frequency domains: O2, Vodafone, T-Mobile, Nordic Telecom, Poda and Suntel Net. Opening bid price was set to CZK 29 mln each, CZK 145 mln in total. The three current, well established mobile operators were entitled to buy up to one, newcomers up to two domains. In July 2017, winners of auctions were revealed, each O2, Vodafone and Poda will receive one domain, Nordic Telecom got the remaining two. Both T-Mobile and Suntel Net withdrew from the auction, T-Mobile later explained that from their point of view, prices were unrealistically high. Operators have 2 years to launch their network within the provided frequencies. In 3 years, 10% of cities with 5 to 10 thousand inhabitants must be covered, in 5 years it has to be 45%. On top of that, in 5 years also 40% coverage of cities with population 2-5 thousand must be attained. Nordic will construct its network with financial help of JT Private Equity Group, Poda has not revealed their financial means yet. Moreover, no collaboration on joint construction of the network was indicated.

Freeing frequencies for WLL

In December 2019, Czech Telecommunication Office has announced (Slížek (2019)) its intention to free up the 60 GHz radio spectrum (57-66 GHz). Their motion should be applied since January 2020. This is a substantial development for internet service providers employing WLL technologies. By enablement to use this spectrum, providers will be able to offer much higher connections speeds - without the need to devote extra energy to improve their current infrastructure (and immediately advancing to higher generations of WLL).

Broadband Europe plan

While all the investments may be viewed as initiatives carried out for economical reasons, there is a different relevant pressure to improve broadband infrastructure arising from European Union. European Union has expressed its plans multiple times; however, the latest targets presented are focused on the threshold of year 2025. The main goal of the strategy is to by then provide all households with access to internet with connection speed higher than 100 Mb/s. (European-Commission (2019a))

The Czech Republic is expected to get in line with these expectations. However, its current strategy only concerns target up to 2020. By then it is expected that (also) all households will have access only to 30 Mb/s and higher connection speeds. Concerning the 100 Mb/s target set by EU, Czech strategy involves plans to reach at least 50% of households having access to such bandwidth. Looking at latest statistics, the latter part of strategy has already been covered in July 2017, when the broadband penetration for 100 Mb/s and higher reached 58.2% of households. On the other hand, the former target had not been fulfilled then. At the end of July 2017, 89.8% of households had access (moreover, for the connection speed of only 2 Mb/s and above, 4% of households were still not covered). Generally, it is expected for the Czech Republic to converge its plans to the European strategy. However, Czech plans have encountered several hurdles.

Subsidy program and its execution in the Czech Republic

Delay in NGA infrastructure development in the Czech Republic is partially caused by the Czech Ministry of Industry and Trade's ambiguous execution of the subsidy application process. To support the plan to enhance connectivity (Strategy Europe 2020), EU/Czech Ministry of Industry and Trade designated 14 bln CZK for the development of the broadband infrastructure in the Czech Republic. In April 2017, the Ministry organized the application process for Czech entrepreneurs and public sector for the first 11.5 bln CZK (application deadline set to September 2017). By the time the deadline passed, only few companies had applied for subsidies in total worth only few tens of millions CZK. The lack of interest was allegedly caused by the fact that the application process and its conditions were vaguely defined and potentially in conflict with EU rules. CETIN CEO has stated the main problem was the risk of EU not reimbursing the infrastructure cost. Additionally, he claimed that the process was set to prefer certain technologies. In fact, CETIN filed a complaint regarding the process form to the European Commission in Summer 2017.

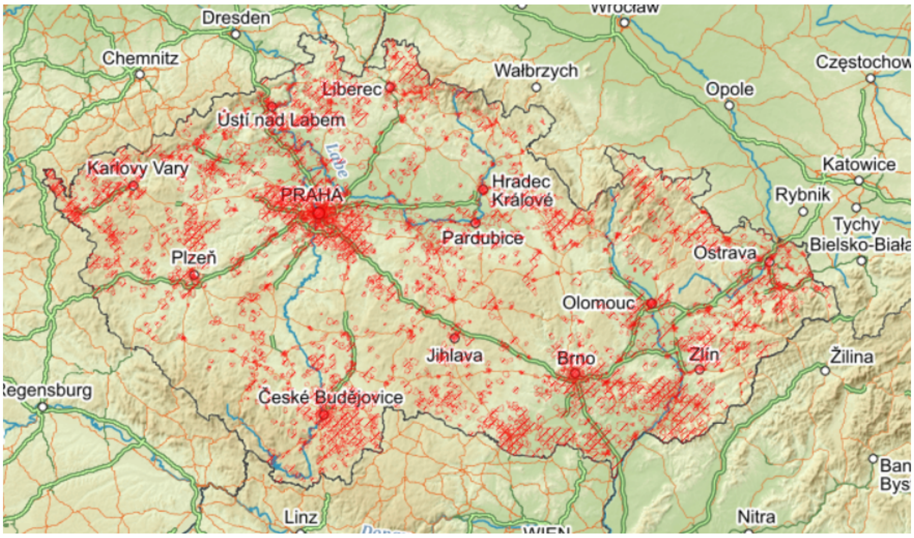
In 2018, the Ministry announced that it plans to initiate and design a new application process in Autumn 2018. In Summer 2018, after discussion with ESIF1, Ministress of Regional Development stated that the designated sum was excessive and announced that 5.2 bln CZK should be reallocated to programme devoted to revitalization of coal mining regions. Delay in NGA infrastructure development in the Czech Republic is partially caused by the Czech Ministry of Industry and Trade's ambiguous execution of the subsidy application process.

Another development of the subsidy programme is currently in question. In October 2019, Czech Ministry of Trade and Industry has commenced a public consultation (all internet providers are free to share their opinion on the development), mapping the current coverage of Czech Republic by different internet providers. This is a continuous effort (for instance see results from 31.12.2018 for fiber technologies in Figure 2.9) for the Ministry to better allocate its funds as well as to reveal blind spots in broadband coverage.

Action plan 2.0

In November 2019, Czech government along with the Czech Ministry of Trade and Industry has approved a new plan called Action plan 2.0, on how to contribute to construction of broad infrastructure (apart from the subsidy programmes). The plan stands on the two pillars. First one is the recognition of construction problems when crossing other line communication structures (for instance road, railways, etc.). To avoid the problem of cross-section, entities responsible for construction of other infrastructure should be soon required to adjust their work to be able to include broadband infrastructures as well. The second one is to redefine laws that relate to purchase of, lease of or access to land that could be used for infrastructure lines. In some cases, the financial cost of land rights settlement may constitute up to 50% of total investment required, which was deemed as an unnecessary burden for the infrastructure builders. (Czech-Ministry-Of-Trade-And-Industry (2019a)).

Figure 2.9: Fiber coverage in 2018



Source: Czech-Ministry-Of-Trade-And-Industry (2019b)

Chapter 3

Literature review

In Chapter 3, we will present an overview of available academic literature on our topic. Mainly we will focus on the economic relevance of the fiber deployment and on the application of the spin models in various industries.

3.1 Existing research on broadband infrastructure significance and different approaches

Broadband Infrastructure and Economic Growth

In their study, Czernich *et al.* (2011) examine the effect of improving and extending broadband infrastructure on macroeconomic outlook, namely they focus on annual GDP per capita growth. By looking at 25 OECD countries (the Czech Republic is included as well) and their statistics over one decade (1996-2007), they find a positive relation between broadband expansion and economic growth. Admittedly, there may be a reverse causality - economic growth may highly affect the wealth of citizens and therefore their ability to afford and overall demand for broadband services. Therefore, authors device an instrumental variable exploiting information about already existing infrastructure as they argue that already deployed legacy infrastructure used for fixed voice services and cable TV is a crucial factor. Ultimately, the authors conclude that extending broadband infrastructure leads to improved information sharing, followed by higher labor productivity, market competition and finally to higher GDP growth. In terms of absolute numbers, the authors reveal that based on their dataset, an increase of 10 p.p. in penetration of broadband results in 0.9-1.5 p.p. increase in annual GDP per capita growth.

Does Broadband Speed Really Matter for Driving Economic Growth? Investigating OECD Countries

Similarly, Rohman & Bohlin (2012) also investigate the impact of broadband enhancement on macroeconomic outlook, this time, the focus is broadband speed as independent variable and its impact on absolute GDP growth as dependent variable. Again, OECD countries are scrutinized using quarterly (more actual) dataset from 2008 to 2010. The positive and linear effect of broadband speed on economic growth is confirmed by the authors. Additionally, they observe the impact of increasing broadband speed even higher in countries whose economic growth was historically lower than other countries in the sample. The Czech Republic was again included in the analysis. The Effect of Broadband Infrastructure on Entrepreneurial Activities: The Case of Germany Positive effect of broadband infrastructure described in the literature is also accompanied by a case study by Heger *et al.* (2011). This study provides a closer look on the bridge between enhancing broadband infrastructure and the following economic growth; the bridge being entrepreneurial activities in the specific case of Germany. The paper summarizers the fact that infrastructure in general is crucial for entrepreneurship, but it extends the idea to broadband by creating an index that combines level of broadband infrastructure, general infrastructure (e.g. roads) and general physical knowledge. The authors find that for all industries as total, the expected positive effect may be overcome by regional differences (barriers to deploy the infrastructure, etc.). Nevertheless, the authors conclude that when it comes to high-tech based businesses, the positive effect of broadband infrastructure is present regardless of the regional aspects.

Structurally Independent Broadband Infrastructure Can Solve Perceived FTTH Coverage Issues

Felten & Langer (2016) try to compare two approaches to construction of universal FTTH infrastructure. They present two theoretical archetypical type of players with their business models. First type is a Vertically Integrated Operator, second type is Wholesale Network Operator (to put it in the perspective of the Czech Republic, O2 before separation would be an example of vertical integration, while CETIN after the separation in 2015 is an example of wholesale network operator). Following their business models based on the weighted average cost of capital, the authors frame a profitability model for each. In spite of several limitations, the main outcome of the case study is that the wholesale network operator is more efficient in deployment of universal fiber infrastructure. (For instance, at a certain level of weighted average cost of capital and keeping both types of players economically viable, the wholesale network operator's activities would result in 85% coverage compared to only 25% coverage for the vertically integrated players).

Transition from copper to fiber broadband: The role of connection speed and switching costs

Grzybowski *et al.* (2014) and Grzybowski *et al.* (2018) study the decisions of broadband internet consumers, the former study even focuses on Slovakian broadband market. The authors make the argument that their study can be extended to broader Central Europe - Czech Republic and Poland, specifically. In these studies, consumer preference is examined and it is revealed that the consumer is primarily affected by price, connection speed and the switching cost while using the technology. Futhermore, they conclude that among the fixed broadband technologies, fiber connection is the least elastic.

3.2 Applications of spin models in various fields

3.2.1 Social applications of two-dimensional Ising models

Stemming primarily from physics, Ising model (case of spin model) can be used in studies outside of the scope of this particular science. Stauffer (2008) introduces three cases of utilization of Ising model in social sciences - more precisely, three cases where agents involved in the problem behave binarily and affect each other (similar to ferromagnetism). First case is the case of German Ifo index that summarizes opinions on future outlook of economy as expressed by business relevant persons. These people give either a positive, neutral or negative opinion. Allegedly, these opinions are highly correlated to each other, i.e. opinions of other people strongly influence the perception of others. This phenomenon is also known as herding and can be studied by using Ising model. Second case is the study presented by Thomas Schelling who employed Ising model to describe the way racial segregation is formed based on preferences of United States cities' inhabitants. Ising model was also introduced into linguistics, where the replacement of one dominant language in a society by other languages was simulated. Example outcome of this study was that rate at which language is being replaced slows when the population of the entire society is higher.

3.2.2 Ising model and its ability to mimic financial series

Given the nature of financial markets, especially in the sense of the myriad of transactions happening within a short periods of time, Ising model seems to be well suited to describe financial series. Generally, it is supposed to well incorporate the herding behavior of traders (i.e. the influence of trades on consequent behavior - selling and buying) and subsequently simulate the most crucial outcome of any financial series - returns. This is done by observing the total magnetization of the system, however we will discuss this in more depth in Chapter 4.

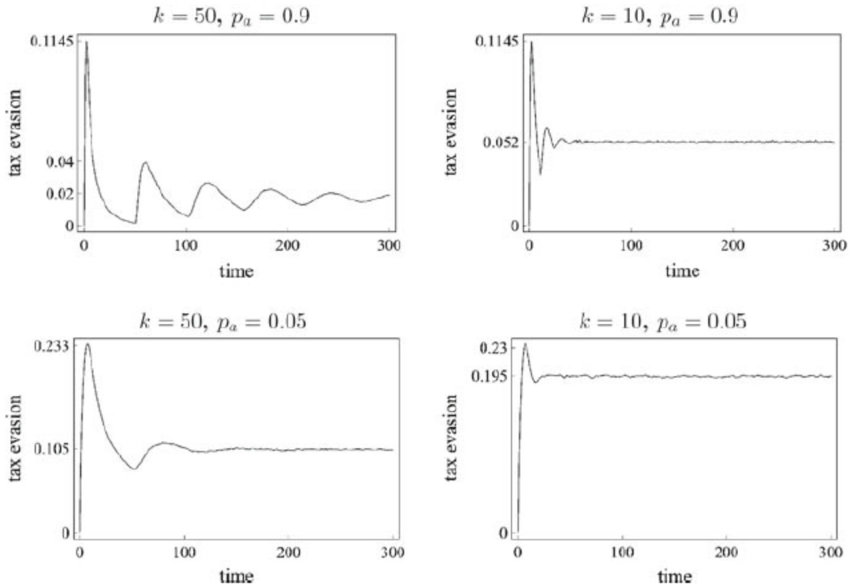
The connection between Ising (or its extensions) model and stylized facts of financial series has been studied already; giving a few examples: Takaishi (2015), Chrz (2014), Krause & Bornholdt (2011) or Dvořák (2012). Both Dvořák (2012) and Chrz (2014) perform an analysis of real financial data (S&P 500 index for former and S&P 500, gold and exchange rate for the latter). They conclude that some of the properties observable in financial series is apparent in all cases/simulations from the Ising model, e.g. autocorrelation of returns, deviation from normal distribution or the presence of long tails. Krause & Bornholdt (2011) accentuate these findings and thereby view the Ising model as potentially appropriate tool for financial modelling - especially for modelling expectional events, market bubbles and such.

3.2.3 Analyzing tax evasion using Ising model

All Zaklan *et al.* (2009), Seibold & Pickhardt (2013) and Seibold (2018) study the phenomena of tax evasion utilizing the Ising model. The binary outcome of the Ising model is represented by people who do not cheat on their taxes versus the people who evade paying full taxes. The use of the model is approapriate as they argue that the decision of evading taxes is dependent on the neighours as well as the mood of the whole system. In practice, we could view these as two separate influences: on local level, tax evader is more likely to cheat if they see that their neighbours do so and they can succumb to peer pressure or they can more tangibly see the benefits of the tax evasion for their neighbours; on a global level, the decision may be influenced by general opinion on tax evasion - either in mass media or in politics' rhetorics, etc.

This approach is fairly similar to other works utilizing the Ising model, however what is particularly intersting in this field is the introduction of invariability of spins. While in the basic Ising model, any spin (agent) can basically change its spin value at any moment, Zaklan *et al.* (2009) introduce some invariability in the form of tax audits and consequent punishment of tax evaders. At each time step, a fraction of players can be exposed to an audit with a certain probability. If audited and being revealed as a cheater, agent must remain honest in paying taxes for some level of subsequent periods.

Figure 3.1: Example of the outcome for Ising model in tax evasion



Fraction of tax evaders is on the y-axis, time step on the x-axis. k stands for the number of punishment periods, p_a for the probability of audit.
Source: Zaklan *et al.* (2009), Page 6.

In Figure 3.1, we can see the results of the simulation for a particular set of parameters of the Ising model and different audit and punishment parameters. The comparative dynamics presented in the paper are to be expected - with increasing probability of being audited and increasing punishment, the fraction of tax evaders eventually diminishes. On the other hand, the model presented has one serious advantage, i.e. it can describe a phenomena arising from microscopic interactions with a relatively simple model with a few parameters.

3.2.4 Distribution of labour in rural and urban sectors

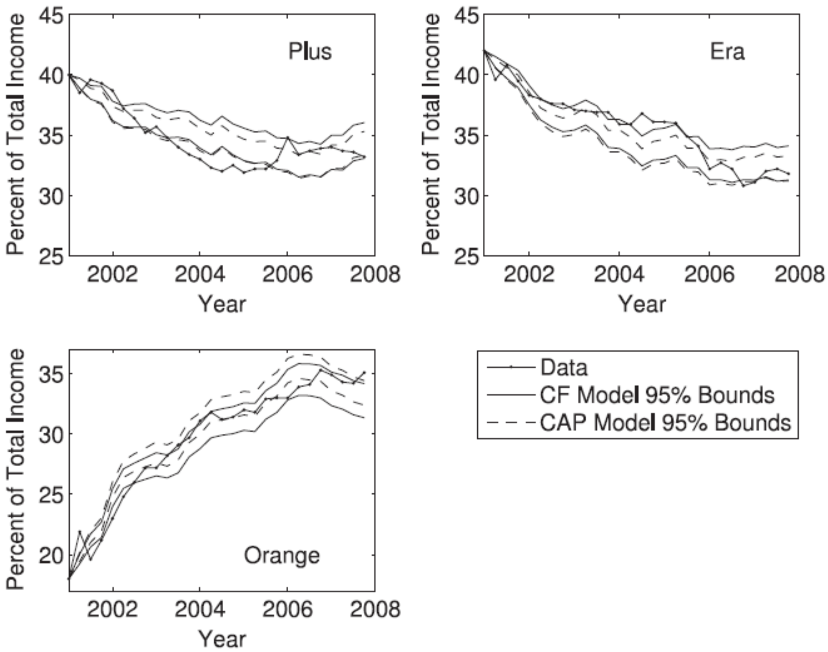
Silveira *et al.* (2006) apply the Ising model to investigate the distribution of labour between the rural and urban sector. This study stands out among others due to economic modification. Unlike many others, the authors develop an economic setting and they use it to transform the local field parameter. While in other models, the external field/influence is usually summarized by one single parameter, the authors introduce an utility framework based on the Cobb-Douglas production function for indiviual firms and they incorporate wage difference between sectors as one of the key parameters. The influence of immediate neighbours stays the same. The mechanism is then very similar to other works, however the parameters are then much more tangible and representable of the real system.

Incidentally, the authors introduce another parameter and a slight extension to the Ising model (though it is not paramount to their work): the fraction of agents that revise their decision (spin value) in each period, suggesting that changes in the system may be happening parallely. This is contrary to the original model, where only one spin is revised in each step. This is a way how to potentially introduce a new external force to the system.

3.2.5 Outflow dynamics in modeling oligopoly markets: the case of the mobile telecommunications market in Poland

Sznajd-Weron *et al.* (2008) study a problem in very similar setting as we do (they study the market shares in the Polish telecommunication market, i.e. consumers' choice of mobile operator). Whilst arguing that the consumer choice is largely affected by price and quality, they also acknowledge the importance of social interaction and advertisement. Again, the model's foundation is a $k \times k$ lattice. In this case, the values in the lattice stood for being a customer of one of the operators. Mainly based on marketing, they derive their own set of probability parameters and rules for a customer to switch from an operator to operator. Evenutally, they explore a whole set of parameters (share of social influence versus advertisement influence, initial market shares and relative advertisement strength of either mobile operator) to see the tendency of the market. Great emphasis is given on social interaction and two slightly different submodels are presented - "conformity first" (CF), a preference of social influence and "conformity and advertisement parallel" (CAP), where the decisions are influenced at the same time.

Figure 3.2: Market share results



Source: Sznajd-Weron *et al.* (2008), Page 18.

Finally, they are able to fit the model taking the parameters observable from the Polish market - level of advertisements spending by the players and market share are known. The level of social vs global advertisement is then chosen by the authors. The final results can be seen in Figure 3.2.

Chapter 4

Ising model

In this chapter, we will describe the basics of our model that is highly influenced by Ising (1925) and its origins. Furthermore, we will describe the simulation method and the economic interpretation of both Ising model's inputs and outputs related to our specific case of FTTx broadband technology distribution.

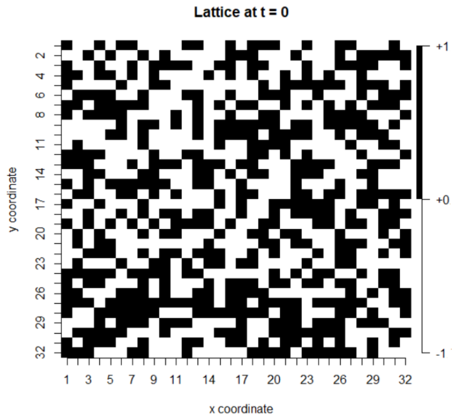
Brief origins of Ising (Bornholdt) model and its relation to econophysics

The origins of the Ising model are dated up to almost 100 years ago when Ising (1925) as part of his research introduced eponymous model simlifying the phenomena of ferromagnetism based on interaction of spins and its neighbours within the matter. Much further along the way, Krause & Bornholdt (2011) update the Ising model by not only including interaction with local neighbours but also allowing for a global effect. They apply the model to study financial markets as was already described in Subsection 3.2.2. We will be mostly working with their version of the Ising model.

4.1 General mechanism

Ising model allows to simulate behavior of some system in time represented on a $k \times k$ lattice. Each point on the lattice can take either of two values: -1 or +1. These values represent the opposite forces - be it the spins/charge of the particles as in the original physics model or any other binary decision (some applications were discussed in Section 3.2). A typical setting of the lattice can be seen in the Figure 4.1 where black and white squares represent the opposite states.

Figure 4.1: Typical configuration of a lattice $k \times k$.



Spin values are distributed at random with 50% probability for each.

According to the model, spins react to its neighbours and to the tendency of the system as a whole. The lattice is updated recurrently - in each time period, one random spin is chosen and either updated to the opposite value or remain unchanged according to the rules presented further.

Spins value in time t will be represented by $S_i(t)$ where i represents the position on the lattice and can take values of $i = \{1, 2, ..., k^2\}$. In Equation 4.1, we introduce quite simple concept of total magnetization M(t) that expresses the state of the lattice at time t. M(t) can take values between -1 and 1 and it measures the dominance of one spin. For instance, in our example lattice (Figure 4.1) the $M(t) \approx 0$ as we have roughly the same ratio of positive and negative values that cancel out.

$$M(t) = \frac{1}{k^2} \sum_{i=1}^{k^2} S_i(t) \quad \text{where } S_i(t) \in \{-1; +1\}. \tag{4.1}$$

Next, let us introduce a metric that expresses the microscopic decision of the spin and its consequent value.

In Equation 4.2, we see the decision process for each step - the random spin will adopt a certain value with certain probability in each step. The probability is influenced by two factors: β and function $h_i(t)$. $\beta > 0$ is a parameter and the function $h_i(t) \in \mathbb{R}$ will be discussed further on. First, let us present the dynamics of the probability in Table 4.1.

$$\begin{aligned} S_i(t+1) &= +1 \quad \text{with probability of } p_{pos} = \frac{1}{1+exp(-2\beta h_i(t))} \\ S_i(t+1) &= -1 \quad \text{with probability of } p_{neg} = 1 - p_{pos} \end{aligned} \tag{4.2}$$

Table 4.1: Dynamics of the Equation 4.2

Case (given $h_i(t) > 0$)	$exp(-2\beta h_i(t))$	p_{pos}
$h_i(t)$ or $\beta \uparrow$	$\downarrow$	$\uparrow$
$h_i(t)$ or $\beta \downarrow$	$\uparrow$	$\downarrow$
$h_i(t)$ or $\beta = 0$	0	1/2
$h_i(t)$ or $\beta \to \infty$	$\to 0$	$\to \infty$
$h_i(t)$ or $\beta \to -\infty$	$\to \infty$	$\to 0$

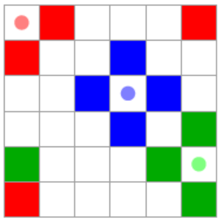
The results from Table 4.1 are quite trivial - with increasing value of parameters, the chance of the spin becoming positive is becoming more likely. We will keep that in mind for later on when we apply a more tangible interpretation of the parameters for our case. From the dynamics we can see that the dynamics of the probability assigning function is nearly perfect resemblence of the logistic function.

Finally, let us move on to the key factor in decision making - the function $h_i(t)$ described in Equation 4.3.

$$h_i(t) = \underbrace{\sum_{m=1}^{k^2} J_m S_m(t)}_{\text{local effect}} - \underbrace{\alpha S_i(t)|M(t)|}_{\text{global effect}} \tag{4.3}$$

As foreshadowed, the Equation 4.3 is comprised of two key components. First, we have the local effect, i.e. the effect of immediate neighbours of the spin considering the change of its value. In our case, we consider 4 neighbours in the lattice - 2 on the horizontal axis and 2 on the vertical axis. For these positions holds that $J = 1$, otherwise it is equal to zero. Effectively, the local effect is the sum of the spin values of the considered position's neighbours. Moreover, we incorporate mechanism called the periodic boundary condition. This means that if the considered spin is located on the boundary of the lattice, we complement the missing neighobours by taking the value from the opposite side of the lattice. This is well illustrated in the Figure 4.2. This way, our lattice transforms into a torus like structure (always considering the same amount of neighbours) and it allows to work with relatively smaller dimension of the lattice.

Figure 4.2: Periodic boundary conditions



Dot represents the spin considered, squares represent the respective neighbours within periodic boundary conditions.
Source: Dvořák (2012)

The global effect is dependent on the real parameter α, current value of the spin considered and total magnetization of the system. Based on the parameter α, the spin either submits to or defies the overall trend of the system. On top of that, the parameter β intensifies the probability outcome of local and global effects combined.

4.2 Choice of model inputs

In this section, we will briefly comment on the choice of parameters in our final simulations.

4.2.1 Numerical inputs

All in all, the initial parameters of the model are chosen quite arbitrarily. When it comes to one-dimensional input parameters, we need to (can) set up the following:

- Set of α parameters
- Set of β parameters
- Dimension of the lattice (k)
- Number of neighbours in consideration in local effect ($J_m \neq 0$)

- Strength of neighbour's impact (J)
- Number of steps simulated (length of sequence for one set of parameter)
- Number of simulations for one specific set of parameters
- Proportion of time steps stored for analysis

Following Krištoufek & Vošvrda (2016) or Chrz (2014), we choose our sets of α and β as intersection of both. Both parameters are defined as non-negative in the model setting and therefore we will focus on set of α between 0 and 10 with step increase of 1 and set of β between 0 and 3 with step increase of 0.25. Other studies do not include include the sensitivity of α and β as input parameter to such extent.

The choice of lattice dimension remains unaccounted for in literature and repeatedly arbitrary. For instance, the choice of k can rise up to 1000 (1 million spins in the lattice) in Zaklan *et al.* (2009). In our model, we choose the parameter to be 20, thus allowing for a system of total 400 spins. This way, one spin flip corresponds to $1/400 = 0.0025$ change in total magnetization or 0.25 p.p. in proportion to positive spin values in the whole system.

Our choice of the "t" parameter, i.e. number of potential spin changes would ideally be as big as possible. Unfortunately, this may largely affect the computational time of the simulations. Therefore, our choice is motivated by the actual ratio of simulation steps and number of spins' positions in the lattice. Ultimately, we choose the parameter to be $t = 10^5$, which, given the ratio of $10^5/2/400 = 125$, allows each spin to revise its value 125 times in total. In a model by Sznajd-Weron *et al.* (2008), which is loosely built on Ising model, they assign a exact period for one time step (in this case, $\delta t = 1$ month), however, the Ising level settings doesn't allow to make this connetion so simply.

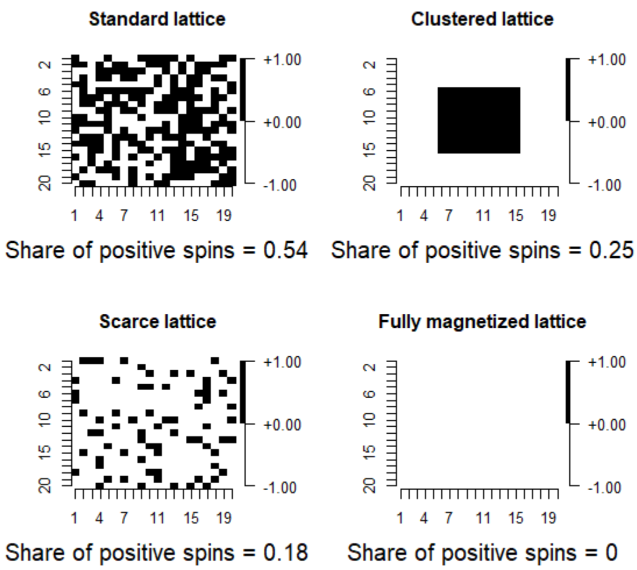
Another aspect to be put into consideration are the neighbours' specifications, i.e. the value of the J parameter (in standard model set to $J = 1$, visited by Jaroenjittichai & Laosiritaworn (2018), for instance) and the number and position of neighbours considered in the local effect (for instance, Krištoufek & Vošvrda (2016) include the spin itself among the neighbours, or Sznajd-Weron & Weron (2003) calculate with all 8 neighbours, i.e. including diagonal neighbours). After consideration, we decide not to include these parameters in our modelling. The local effect needs to be in the model to complement the global one. However, the effect on the individual spin function affecting probability of the spin flip can be substituted by variation in other parameters.

Lastly, we need to decide for how many times we will run the simulation when looking at a specific set of parameters, e.g. $\alpha = 1, \beta = 0.75$. In allignment with Central limit theorem, it are more likely that statistics of our generated series will approach normal distribution with increasing number of simulations. LaMorte (2016) states that sufficiently large sample starts with the number ≥ 30. This is the lowest threshold usually pronounced and as such should suffice, however we will be using the total number of simulations equal to 100. This way, we are more confident in better approaching the distribution and moreover, we may be able to analyze specific subsets with enough observations as well.

4.2.2 Initial state lattice distribution

There is another aspect that can be varied in the initial configuration of the model and that is shape of the lattice at the start of the simulation period. Majority of papers do not take this factor into account and operate with randomly distributed lattice with equal probability of each spin occuring. On top of the standard lattice distribution, we will introduce three more types of the inital lattices. An overview of all types considered can be seen in Figure 4.3.

Figure 4.3: Initial lattice states



Altogether, we distinguish four types of input lattices. Standard lattice represents a randomly distributed lattice with roughly the same proportion of spins. Clustered lattice stands for a lattice with a significant initial local magnetization. Moreover, we introduce a lattice with scarce proportion of positive spins and lastly, we will analyze a fully magnetized lattice. These four types represent situations that will be analyzed further on in this chapter.

4.3 Analyzed model outputs

4.3.1 Total magnetization of the system

One straightforward output of the simulation can be the total magnetization as described in Equation 4.1. For better further analysis, we will adjust the metric slightly in order to capture the share of spins in the system for which we use the following equation.

$$positive\,spin\,share\,(t) = \frac{1}{k^2} = Q(t)\sum_{i=1}^{k^2}\mathbb{I}_{s_i(t)=1}$$

This metric simply measures the fraction of positive spins in the whole lattice at time t.

4.3.2 Convergence

Another interesting output that we will analyze is the converegence of the simulated series. There are primarily three aspects of convergence that might yield interesting results. First of all, there is the binary value of whether the series converges at all. If it does converge, we can analyze the convergence value (the "limit of the sequence") and the speed of the convergence, i.e. how fast it takes for the series to reach the convergence interval around the convergence value. If the series does not converge, we can still generate some insights regarding the moments or range of the series, or similar.

Empirical approach

Given the fact that we are generating the time series one step by one step, we have to tailor the generic definitions of convergence for our purpose. In Equation 4.4, we see part of the mathematical definition for $a \in \mathbb{R}$ being a limit of the series. As we don't have the series defined by nth term, but it is rather recurrently defined stochastic process, we are unable to come up with analytical solution either.

$$\forall \epsilon \in \mathbb{R}, \epsilon > 0 \quad \exists n_0 \in \mathbb{N} \quad \forall n \in \mathbb{N}, n \geq n_0 : |a_n - a| < \epsilon \tag{4.4}$$

Therefore, we need to come up with conditions necesarry to be met to declare whether a series has converged or not. In words, we will declare that the series has converged, if for a fixed subset of the series' domain, the range of series remains within the ϵ-neighbourhood of final value of the series. Let us express the same in mathematical terms in the following equation.

Let a_n be series with $D(a_n) = [1,T] \in \mathbb{N}$.
Let us choose $\epsilon_0 \in \mathbb{R}$ and $n_0 \in \mathbb{N}$. If the following holds, then we say that the series converges to a_T.

$$\exists n \leq n_0 : \forall k \geq n, k \in \mathbb{N} : |a_k - a_T| < \epsilon_0 \tag{4.5}$$

Using this equation, we may also extend the concept to also analyze the speed of convergence. We will consider the number of steps that the series requires to irretrievably enter the ϵ-neighbourhood, i.e. $min\{n \leq n_0 : \forall k \in \mathbb{N} \geq n : |a_k - a_T| < \epsilon_0\}$.

Figure 4.4 summarizes the concept graphically. n_0 on the picture is set as the minimum steps required for the series to converge. That is, if the values of the series remain inside the red rectangle. Incidentally, the chosen n_0 corresponds to the lowest value to enter the ϵ-neighbourhood and therefore, in this case, it is also equal to the number of steps required for the series to converge.

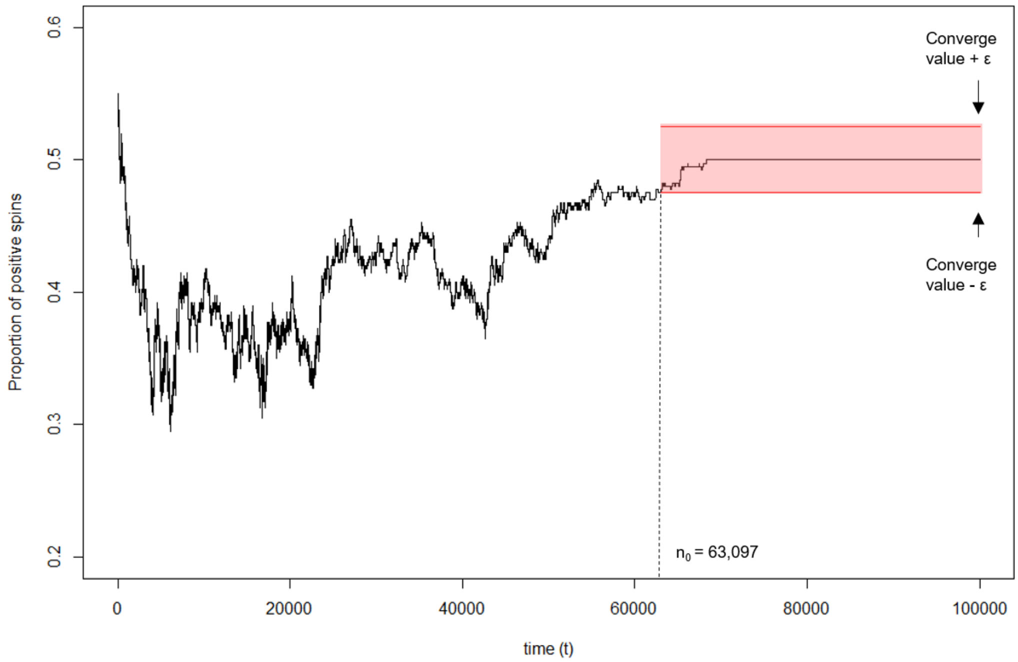
In our case, we will define n_0 as 1000 (this will represent potential 10 thousand spin flips). For the share of positive spins series, we will choose $\epsilon_0 = 0.025 = 2.5\%$. This will represent a total range of 5%.

4.3.3 Final state lattice distribution

Other outcome that can be analyzed is the final distribution of the lattice. More specifically, we may be interested in whether the spins will be placed seemingly randomly or whether they will form homogenous clusters.

We have considered a few potential metrics (Von-Neumann enthropy, Manhattan distance across the spins, summing all combinations of squares that are subsets of the lattice), but despite everything, the simplest method seems to be

Figure 4.4: Example of convergent series



the most appropriate and therefore in Equation 4.6 we define our own function $dist(t)$ with quite simple intuition behind.

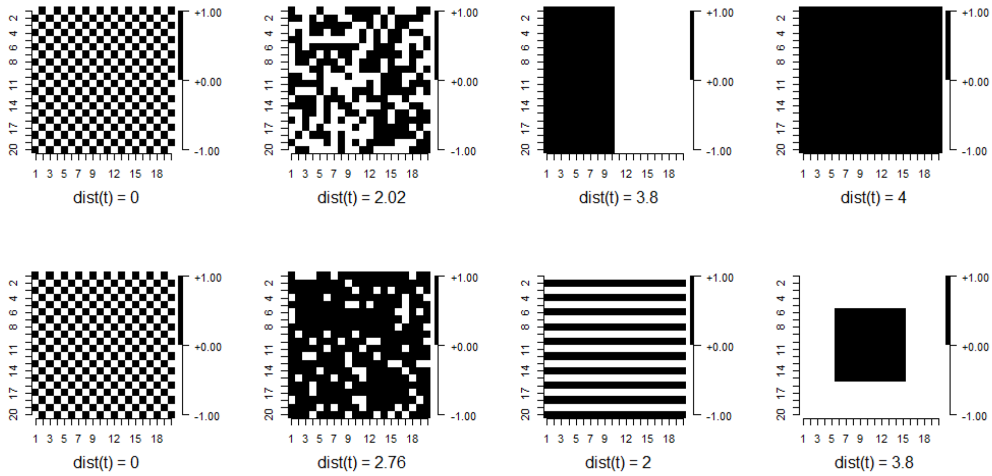
$$dist(t) = \frac{1}{k^2} \sum_{j=1}^{k^2} \sum_{i=1}^{4} \mathbb{I}_{s_j(t)=s_i(t)} \tag{4.6}$$

In Equation 4.6, the inner sum evaluates how many of the 4 neighbouring spins (same neighbours as in Figure 4.2) for a position have the same value. We obtain this sum for every position in the lattice and ultimately we take the average. Defined this way, the range of the function may include values from 0 to 4. We can understand the logic as imagining a fully positive spin having a value of 4. This number would be gradually decreasing with introducing borders between the binary spins. The metric is fairly similar to definition of Hamming distance (Hamming distance considers the count of different pairwise elements in two objects). In Figure 4.5, you may find eight illustrative lattices with their respective values of dist(t).

Notably, the issue we had with other considered metrics as well. For instance, though both graphs on the left in Figure 4.5 (chessboard like) exhibit some pattern, they are not considered to be clustered.

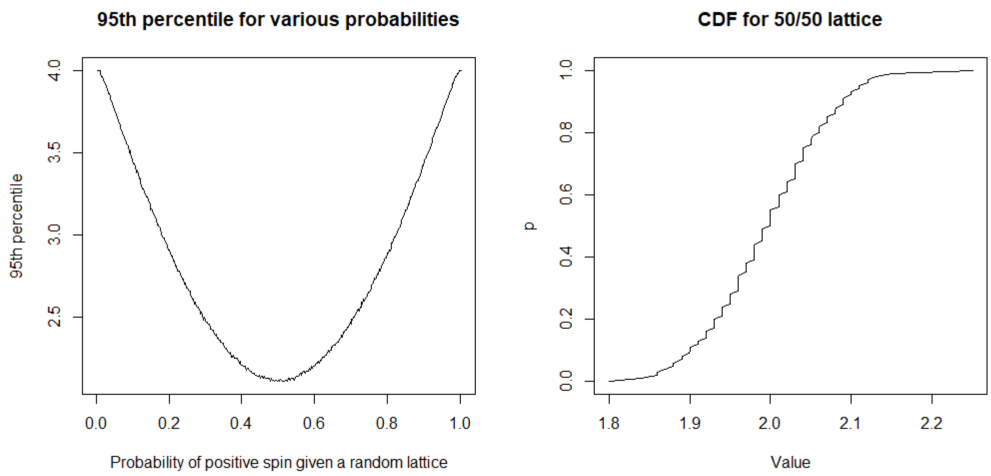
This metric is obviously dependent on the overall proportion of positive spins in the lattice. For instance, only lattices completely filled by either positive or negative spins can reach the value of 4.

Figure 4.5: Example cases of dist(t) function



To analyze whether a lattice generated by the Ising model is more clustered than a randomly generated lattice (lattice with randomly distributed spins with probability of one spin being positive is the same as the proportion of positive spins in Ising lattice), we are applying bootstrapping method and we generate 1000 lattices for each probability and we generate distribution statistics. In Figure 4.6, on the left part we see a relationship between 95th quantile for dist(t) applied on lattices simulated with given probability. On the right part, we see a cummulative distribution function for dist(t) applied on lattice with rough share of positive and negative spins is being 1:1.

Figure 4.6: Distribution of dist(t)



Ultimately, we will be collecting the data from the Ising generated series

and we will attempt to reject the null hypothesis of
H_0: Ising generated lattice is not different from a randomly generated lattice. On the other hand, the alternative hypothesis H_A states that the clustering differs from the randomly generated lattice.

4.3.4 Other outputs

Lastly, let us define a last metric that could be useful from our perspective. We will call it "risk cost eff" and we will define it as:

$$\text{risk cost eff}(t) = dist(t) \times Q(t) \times \Big(1 - (z_{90}(Q(t)) - z_{10}(Q(t)))\Big), \tag{4.7}$$

where z_x stand for the xth percentile.

Another potential output might be the difference in magnetization that is widely used in financial applications of the Ising model as already noted in Subsection 3.2.2. We refrain from tracking this output for two reasons. Firstly, the phenoma is already partially captured by measuring the convergence and its speed and secondly, in our interpretation, we are rather focusing on longer time dynamics.

4.4 Simulation process

In this section, let us walk through the process of simulating data stemming from the model. For all simulations and data handling, we will be using the R programming language. As discussed, one spin is updated in each time step. In order to see long term dynamics, we decide to use at least number of time steps $T = 10^5/2 = 50000$. Below we provide the reader with an extract from our script (R source script is provided as attachment to this thesis as well).

Function transforming the lattice to another state after one time period according to the rule in Equation 4.2:

```
tot_magnet <- function(lattice){
  sum(lattice)/(nrow(lattice)^2)
  }
one_step_spin_change <- function(lattice, alpha, beta){
  k <- dim(lattice)[1]
  x <- sample(1:k,1,replace = T)
  y <- sample(1:k,1,replace = T)

  J <- lattice * 0
  J_parameter <- 1

  # periodic boundary conditions
  J[ifelse((x-1)==(0),k,x-1),y] <- J_parameter #x_neg
  J[ifelse((x+1)==(k+1),1,x+1),y] <- J_parameter #x_pos
  J[x,ifelse((y-1)==(0),k,y-1)] <- J_parameter #y_neg
  J[x,ifelse((y+1)==(k+1),1,y+1)] <- J_parameter #y_pos

  h <- sum(J*lattice) - alpha*lattice[x,y]*abs(tot_magnet(lattice)) # h for indiviual
  prob_pos <- (1 + exp(-2*beta*h))^(-1) # p of becoming +1

  if(runif(1) < prob_pos){
    lattice[x,y] <- 1
  } else {
    lattice[x,y] <- -1
  }
  lattice
}
```

Unsurprisingly, the process can be computationally quite demanding - more so, if we attempt to collect information for different sets of parameters or initial lattices. To minimize time required for computation, we utilize the package 'parallel' by R Core Team (2020), which is an extension to R builtin apply functions that not only allows for parallel handling of operations but it also enables to employ all processor cores. We provide the reader with the code below. Plainly put, we construct a 3D array comprised of lattices with different input parameters and then we handle all the individual spin reversions simultaneously. Compared to brute force calculation - this may take 200 times less time (estimated time 50 hours versus 1000 hours given the simplest for loop algorithm).

Parallel handling of the lattices:

```
k <- 20 # width of the lattice
t <- 50000 # number of steps
for (m in 1:t) {
  our_array <- array(parSapply(clust,
                               1:dim(our_array)[3], # number of slices
                               function(x) one_step_spin_change(our_array[,,x],
                                                     set_pairs[x,2],
                                                     set_pairs[x,3])),
                     dim = c(k,k,nrow(set_pairs)) # rearrange into array again
  )
  clusterExport(clust,"our_array") # again export the array
}
```

During the process, we store the magnetization and distribution of spins information as well since they will be helpful in further analysis of the system.

4.5 Model interpretation and relation to the broadband infrastructure market

In this section, we will finally link Ising model application and its parameters and outputs with the fixed broadband internet technology market. Positive spins, i.e. $s_i(t) = 1$, are standing for fiber connection, whereas negative spins, i.e. $s_i(t) = 1$, imitate an alternative technology. Further on, we will maintain the point of view of a fiber deployment entity.

4.5.1 Inputs

Firstly, let us dive deeper into the inputs.

Main parameters - α and β

In general, the mechanics of the Ising model should be mainly influenced by choice of α and β parameters' configuration. While increasing β should have positive impact on the spread of fiber, α, on the other hand, should behave as the opposing force. In that sense, we may think about the problem in a way that β substitutes the comparative technology advantage compared to other technologies, be it connection speed (as seen in Figure 2.5), possibly more stable connection and reliability, or data transmission. Moreover, the perceived preference of fiber may be partially supported by empirical results, where (Grzybowski *et al.* (2014)) show that among fixed broaband technologies, fiber subscribers exhibit the smallest price elasticity.
On the other hand, α , as the opposite influence, might be a representative of

the market competition and innovation level of the other technologies. This can be any aspirations, ranging from updating the existing technology (as CETIN does with xDSL as depicted in Figure 2.4) to overall marketing efforts (as already used by Sznajd-Weron & Weron (2003)), e.g. bundling broadband internet with other telecommunication services offers.
Lastly, recalling Equation 4.3, we comment on the local effect part. With increasing neighbours with fiber, the probability of switching to fiber may increase. This can be explained simply by neighbourghly influences, however interpreting the local effect as the presence of higher maintenance and customer service (if locally more dominated by one technology) seems more fitting.

Initial state lattice distribution

In Figure 4.3, earlier in this chapter, we have introduced four archetypes of the initial lattice. These all should represent a slightly different market setting, or more precisely a geographical location.

1. Standard lattice (random)
2. Scarce lattice (village)
3. Clustered lattice (city)
4. Fully magnetized lattice (new location)

Firstly, the standard lattice responds to the most theoretical and unknown situation. Fiber connections are in the same ratio as alternative connections and randomly distributed across the market. We include this initial setting as prototype of an unmapped market. Furthermore, our analysis of this lattice is beneficial just from perspective of mapping the Ising model process.
Secondly, we present the scarce lattice. This prototype should generally respond to a village (or suburbs) setting. In this environment, we expect the units (spins) to be more scattered. Therefore, in the inital setting, we assign spins randomly across the environment. The share of fiber is assumed to be roughly 20%, which corresponds to the overall market share in the Czech Republic.
Thirdly, we present the counterpart to the village archetype - a city archetype. Unlike in the second option, spins are already stacked in order in the middle of the lattice. This may better reproduce a city structure where technology deployment can be more targeted.

Lastly, we introduce a lattice that is filled only with alternative technology connections. This archetype represents locations yet to be penetrated by fiber technology and thus can be interpreted as market entry settings.

4.5.2 Outputs

Finally, we will study the following outputs and their dependence on all parameters introduced in this chapter.

The ratio of all positive spins in the lattice can be interpreted as fiber technology market share. Further on, we examine the convergence. By that, we are able to analyze long term potential of the market. The market share value can either converge (the market will saturate) or oscillate. Furthermore, we investigate how the market clusters. More clustered markets may be more appealing from the perspective of deployment as focused deployment may reduce infrastructure costs significantly.

Ultimately, we cover the "risk cost eff" metric as defined in Equation 4.7. This is combined metric stemming from the generated series that comprises of three key terms. Firstly, we take the metric for measuring lattice distribution. This indicated the level of clustering and we assume that for a higher level of clustering, the cost of fiber deployment is more efficient. Secondly, we observe and multiply by the proportion of fiber individuals in the setting. The rationale behind is that while the clustering may be efficient, generally, the deployer may be more interested in deployment at larger scale. Lastly, we weigh the metric by bringing a proxy variable for volatility in the setting; by taking the range of market share values in the setting. If the market share values prevail in narrower range, the setting is more attractive in terms of risk. Finally, for the highest value of "risk cost eff", the deployment of fiber should be generally more attractive.

The core of analysis follows in the next chapter. Given that we simulate the Ising process for each set of parameters 100 times, we will be deriving comprehensive statistics for the above.

4.5.3 Expected outputs

Given the proposed interpretation and the model definition, we expect to observe general tendency towards clustering. Moreover, we suppose that lattices with lower initial share of positive spins would require higher strength of parameters to reach convergence.

Additionally, we expect the following:

Standard lattice

For the standard lattice, we expect an ideal behavior of the metrics. That is, for increasing fiber perception we expect higher share of convergent series with much faster pace. The opposite outcome goes for the competitiveness input. The clustering of the system is questionable but we would expect nearly perfect clustering arising.

City vs village

Given the initial position (the initial cluster) of the city lattice, we would assume the final share of fiber to reach higher values. On the other hand, we expect the village archetype to exhibit lower market share with higher clustering as the scarcity of initial composition might affect the series to demonstrate rather targeted deployment.

Market entry

For the market entry archetype, we foresee it series will require strong initial parameters to capture some market place. It will be interesting to see whether any combinations of parameters will eventually become feasible for fiber deployment when weighted by the series' volatility and the tendency to cluster. Given the dynamics of the model, it is also probable that without the initial base of fiber subscribers, the series depend upon extreme initial α and β parameters to gain stable market share.

Chapter 5

Results

In this chapter, we will take closer look on all situations described in Subsection 4.2.2 and their market representations. We will present the results for each of the archetypes from the perspective of the economic interpretation that was proposed up until this point. In the following Chapter 6, we will debate on the extent of how much these results align with out expectations.

Most outcomes will be represented in the form of heat maps (shading from red to orange, red being the minimum value). Black spots represent the not available ("NA") results - this may happen as for some cases no series will converge.

5.1 Random distribution with equal share

In this section, we will examine the development of a location ("standard lattice") that has the most theoretical representation. Above all, in Figure 5.11 we start by observing the proportion of series that have converged given the certain parameters. In general, it seems that the market is more likely to be saturated when the overall perception of fiber technology dominates the alternative technology (i.e. β is increasing). The effect is even more apparent when there is a little effort from competition to keep up with fiber deployment (i.e. lower values of α). Noticeably, the very left part of the graph (where β = 0) does not exhibit any sign of convergence. This was anticipated for all archetypes given the structure of the spin flip probability assignment function (Equation 4.2). In this particular case, the probability of a spin flip is always equal to one half.

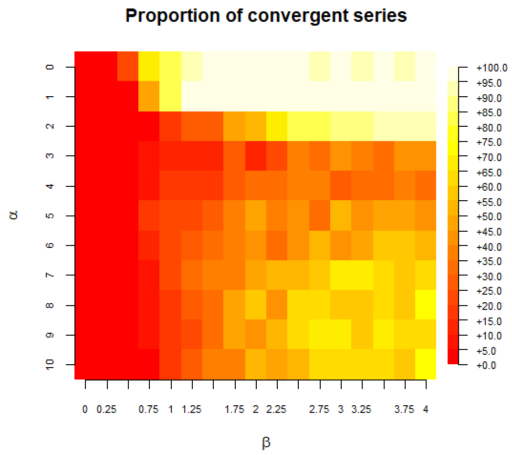


Figure 5.1: Standard: Number of convergent series

Convergent series

Further, let us more closely examine the subset of the convergent series. As we have seen in Figure 5.1, the main convergence parameter space is for low $\alpha < 3$ and higher $\beta > 0.5$. For other cases, the conclusion in this subsection might not be very apt as only a low share of series have converged. In Figure 5.2 and Figure 5.3, we confirm that not only the series converge when fiber perception prevails (accompanied by lower competition influences), but they also reach high levels of market shares. In fact, all series here converge to a dominant market share (i.e. at least 75%).

Moreover, we observe that with lower competitiveness, the market gets saturated at a faster pace. On the other hand, we are able to witness quite a large range of final market shares and even some non-negligible portion (around 10%) of series that converge below 5% market share. This suggests that even though the market might approach values of nearly 100% and may as such be recognized as attractive, it is counterbalanced by a relatively high risk and volatility.

Non-convergent series

Looking at the divergent series and its statistics in Figure 5.4, we are observing quite similar patterns in market shares' dependency on the initial parameters. Neglecting the sector that has been identified as converge prevalent, we again see decreasing market share with decreases fiber perception and higher competitiveness. Albeit, the market share level reach lower levels compared

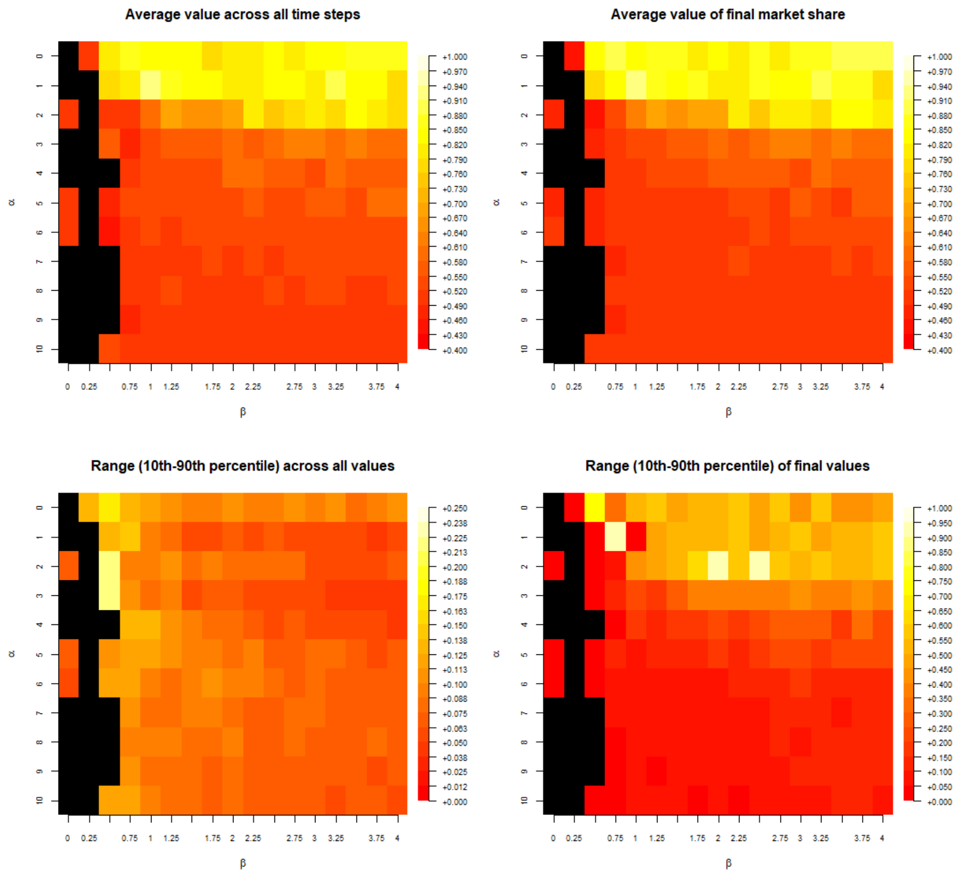


Figure 5.2: Standard: Convergent series statistics part A

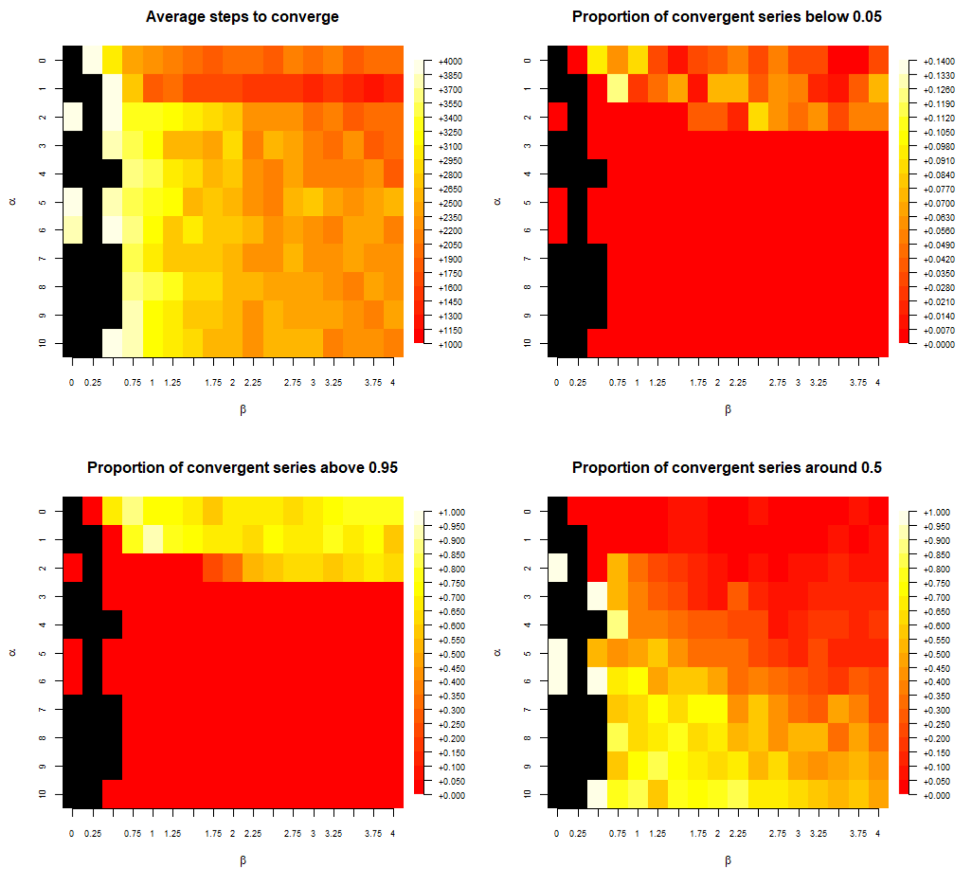


Figure 5.3: Standard: Convergent series statistics part B

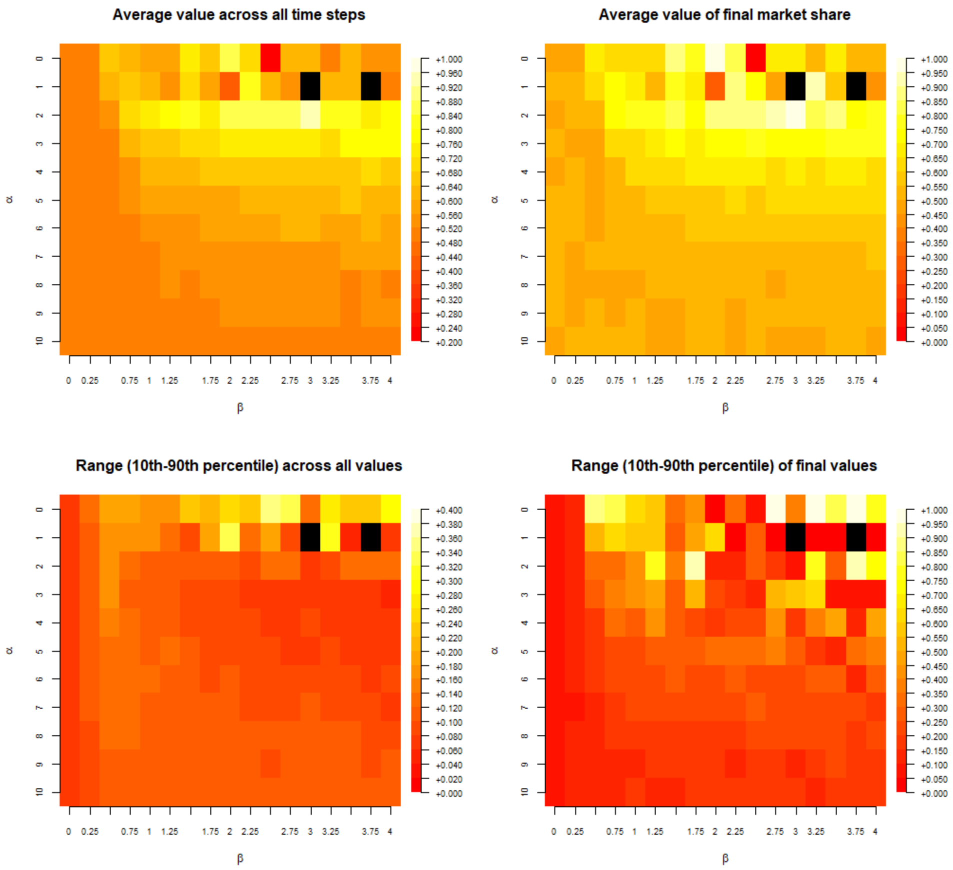


Figure 5.4: Standard: Divergent series statistics

to convergent series, they exhibit significantly lower range of final values and consequently lower volatility and lesser proneness to extremes.

Lattice distribution

In addition, let us comment on the behavior of the series in terms of clustering and its implications towards the convenience of fiber deployment. In Figure 5.5, we observe various depictions of the clustering. Firstly, we can see that on average, with better fiber perception and lower competition, the final clustering values tend to be bigger, even reaching the maximum values. This statement can be accompanied by the fact that overwhelming portion of series results in a clustering different than the one from randomly generated lattice with equal market share. Again, the one main exception is the borderline case of when β = 0, which is not surprising as for this parameter, the process acts like a toss of a coin. This phenomena also stands across the time steps, apart from the initial

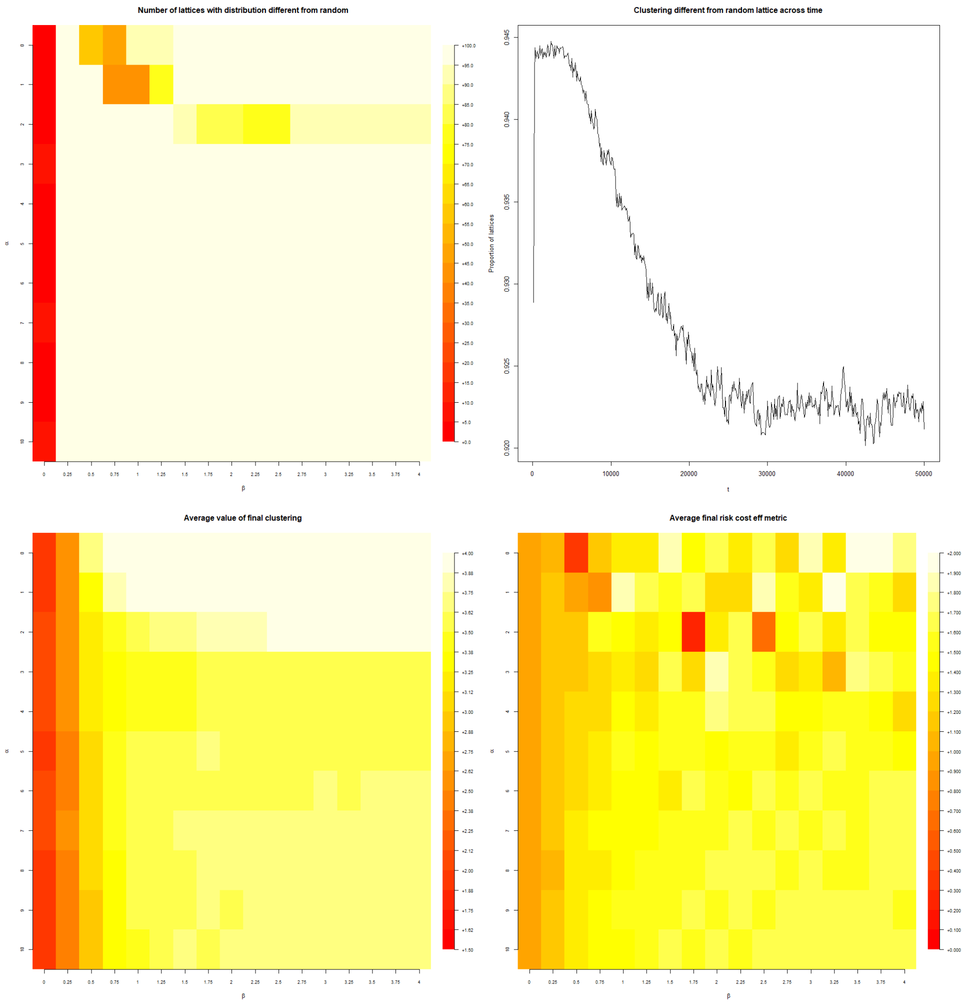


Figure 5.5: Standard: Lattice distribution statistics

"warming" period, when we see a quick switch between random clustering (as in the initial lattice) and ordered clustering.

Ultimately, let us merge the most relevant and inspect the "risk cost eff" that provides combination of the final market share, level of clustering and volatility. From Figure 5.5, we infer that this metric stays balanced over all set of parameters. We are not surprised to observe following the previous conclusions, i.e. we have seen that the larger market share was contrasted with a larger volatility.

5.2 Village entry

Furthermore, let us look at another market situation, which is a fiber deployment in village-like setting. In Figure 5.6, we identify the proportion of con-

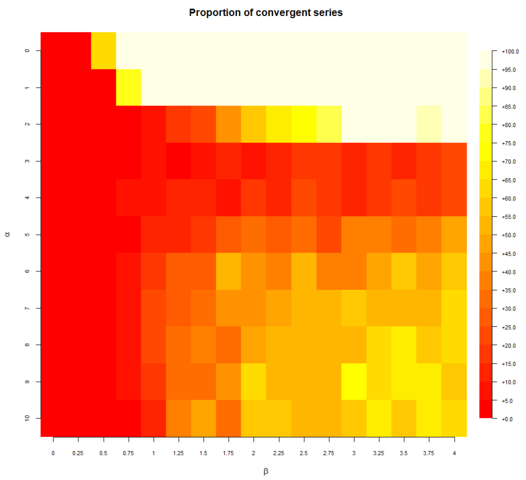


Figure 5.6: Village: Number of convergent series

vergent series. Similarly to the previous archetype, the market has a higher tendency to saturate with higher dominance of fiber traits over alternative and with lower competition from the alternative technologies.

Convergent series

Now let us once again examine the convergent series in slightly more depth. Though we see a great similarity to the standard lattice situation in terms of number of convergent series and even the speed of convergence, in Figure 5.7 and Figure 5.8, there are great dissimilarities in terms of the final market shares. In general, fiber deployment to a village with initially low market share appears not to result in significant market share gains. On the contrary, utter proportion of series converges to a market below 5%. There seems to be a notable portion of series that converge to roughly 50% market share with $\alpha > 5$, i.e. higher competition efforts. Nevertheless, we will refrain from making hasty conclusions here as the absolute count of the series is small.

Non-convergent series

Following with the divergent series, we again observe slighly different results than for the standard lattice (Figure 5.9). On top of that, it appears, rather unintuitively, that the fiber market share reaches higher average values (but also with higher volatility/range) with higher competition efforts. Even though this is not parallel with our intuition from the perspective of the fiber deployer, we may attempt to come up with potential rationale behind stemming from the

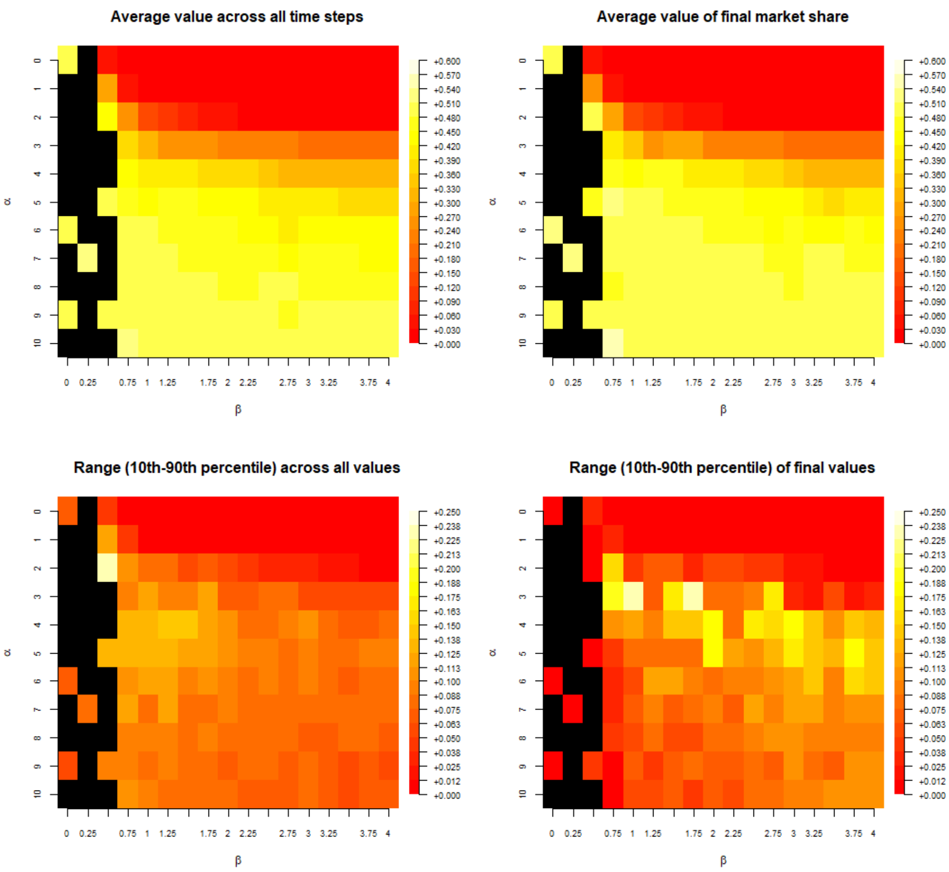


Figure 5.7: Village: Convergent series statistics part A

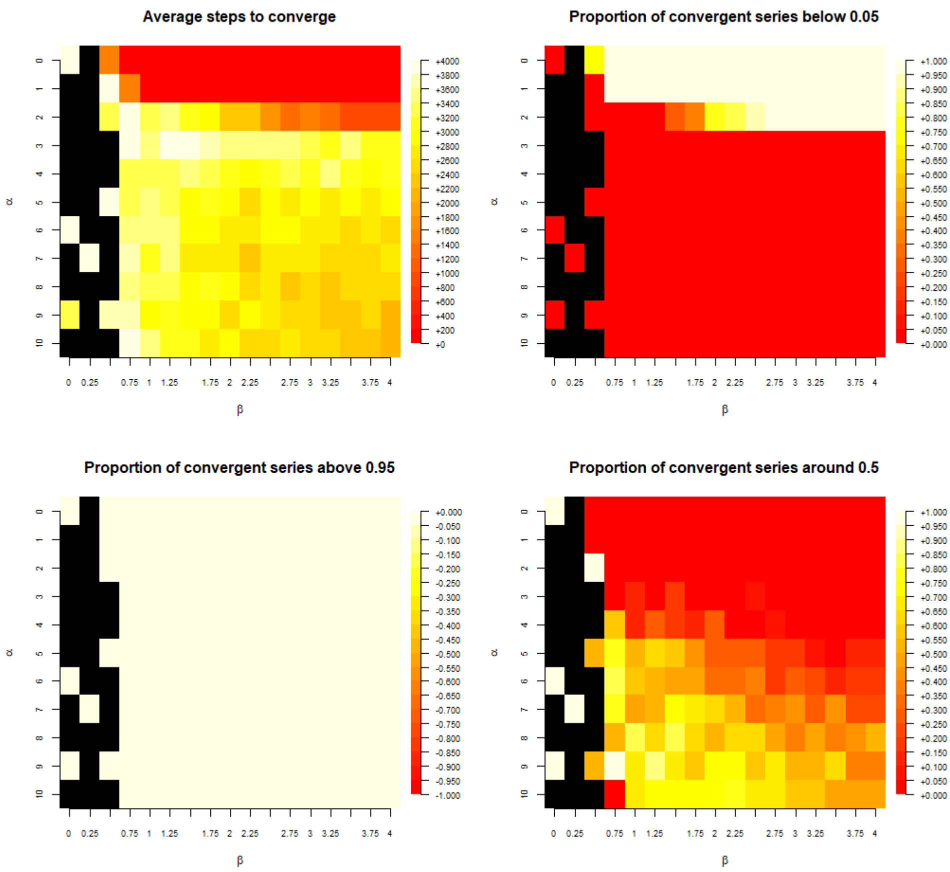


Figure 5.8: Village: Convergent series statistics part B

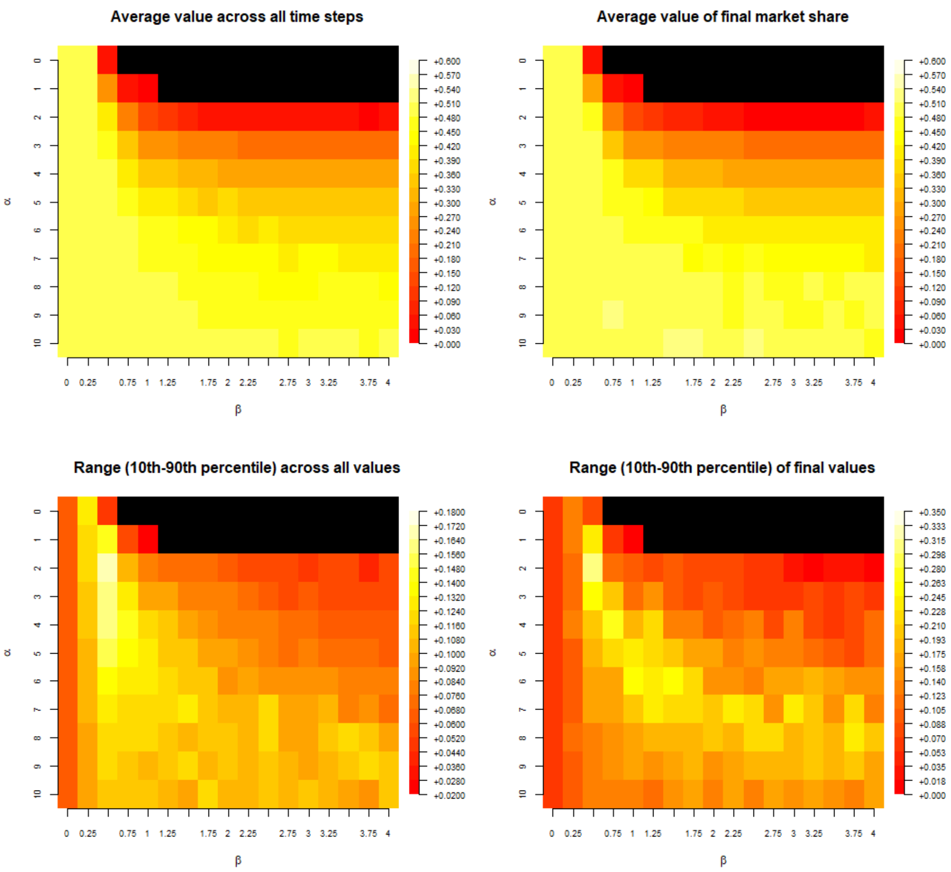


Figure 5.9: Village: Divergent series statistics

perspective of the market. Alternative explanation of these dynamics might be that with no competitive efforts around, fiber providers might be reluctant to this market (on account of superiority), eventually allocate their resources elsewhere and gradually abandon the market. When there is higher competition on the present at the initial moment, fiber companies may be inclined to engage in the competition themselves initially.

Lattice distribution

Last but not least, let us discuss the geographic technology distribution in the village environment. Clearly from Figure 5.10 and same as in previous setting, there is a strong tendency of cluster establishment arising. The tendency is apparent for the final values as well as for the development over time. The highest level of clustering appears for low parameters of α, which is in direction with the overall share evidence and thus not surprising (we have seen that for these parameters, there is a strong tendency to converge to minimal market share, i.e. one cluster of alternative technology is created).
Additionally, all of the findings suggest that in the village setting, the market potential of the fiber deployment coincides with a higher level of competition level of efforts.

5.3 City entry

To complement the village archetype, let us analyze the counterpart - the city setting. Interestingly enough, we see exactly the same pattern of convergence as in the previous two archetypes, i.e. for lower competition effort regardless fiber perception.

Convergent series

When it comes to the convergent series, unexpectedly, we witness a nearly identical behavior. Major proportion of series converges to a very low level of market share at similar pace, and again those series derived from higher α mostly converge to even market share. This leads us to believe that initial distibution of the lattice (in terms of initial positions, not initial market share) may not have significant impact on the final distribution.

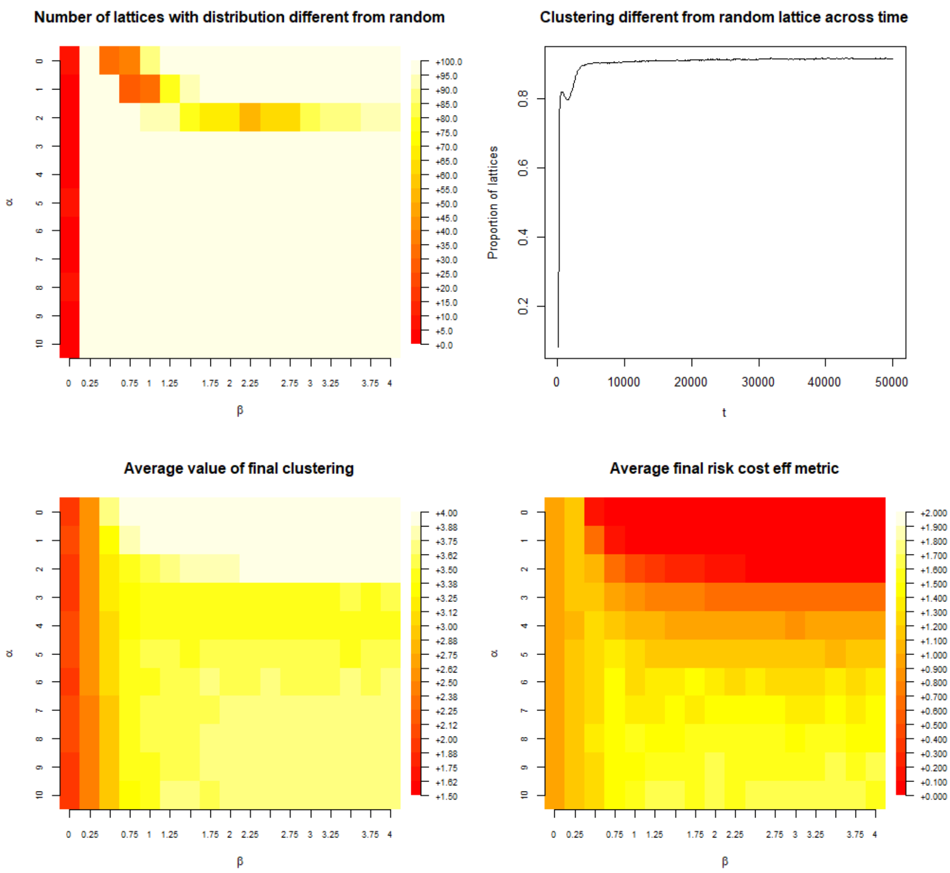


Figure 5.10: Village: Lattice distribution statistics

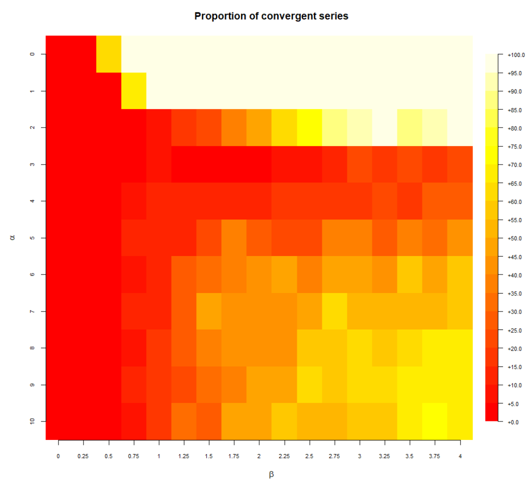


Figure 5.11: City: Number of convergent series

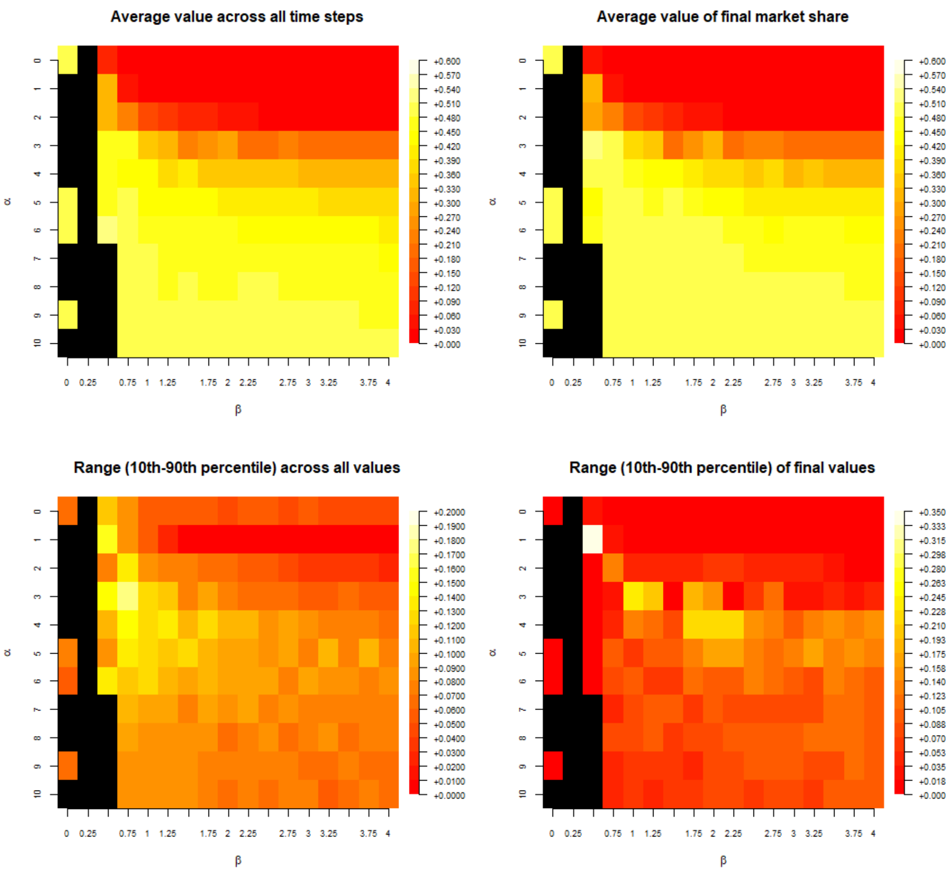


Figure 5.12: City: Convergent series statistics part A

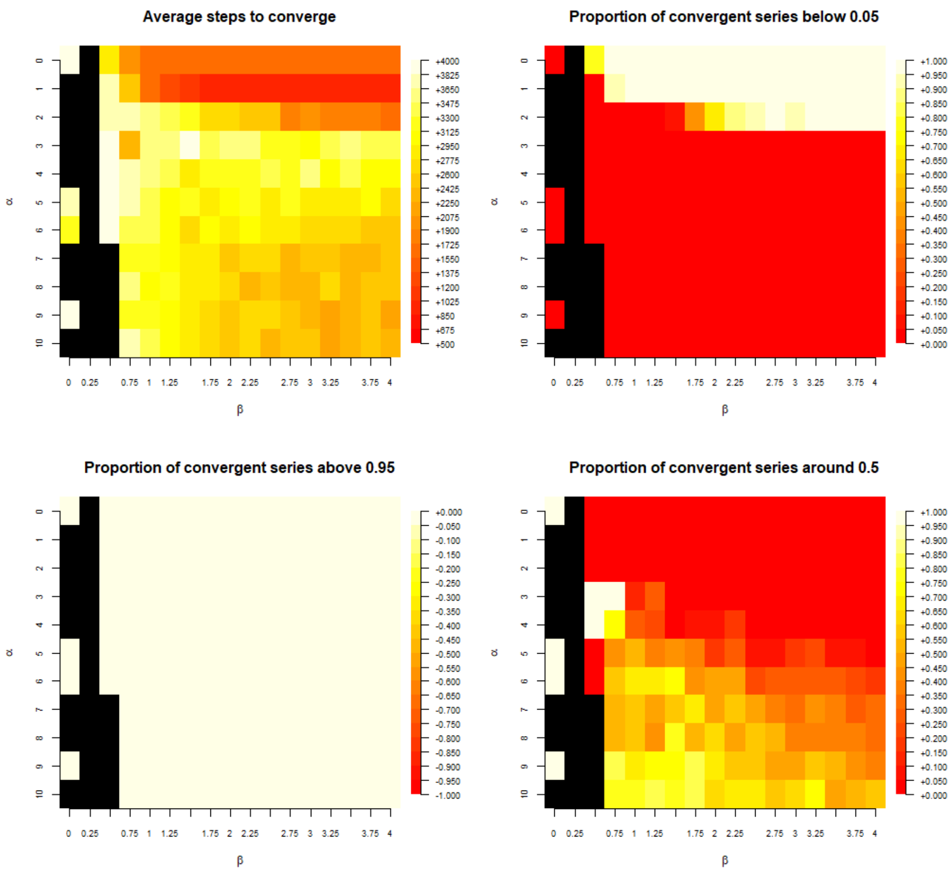


Figure 5.13: City: Convergent series statistics part B

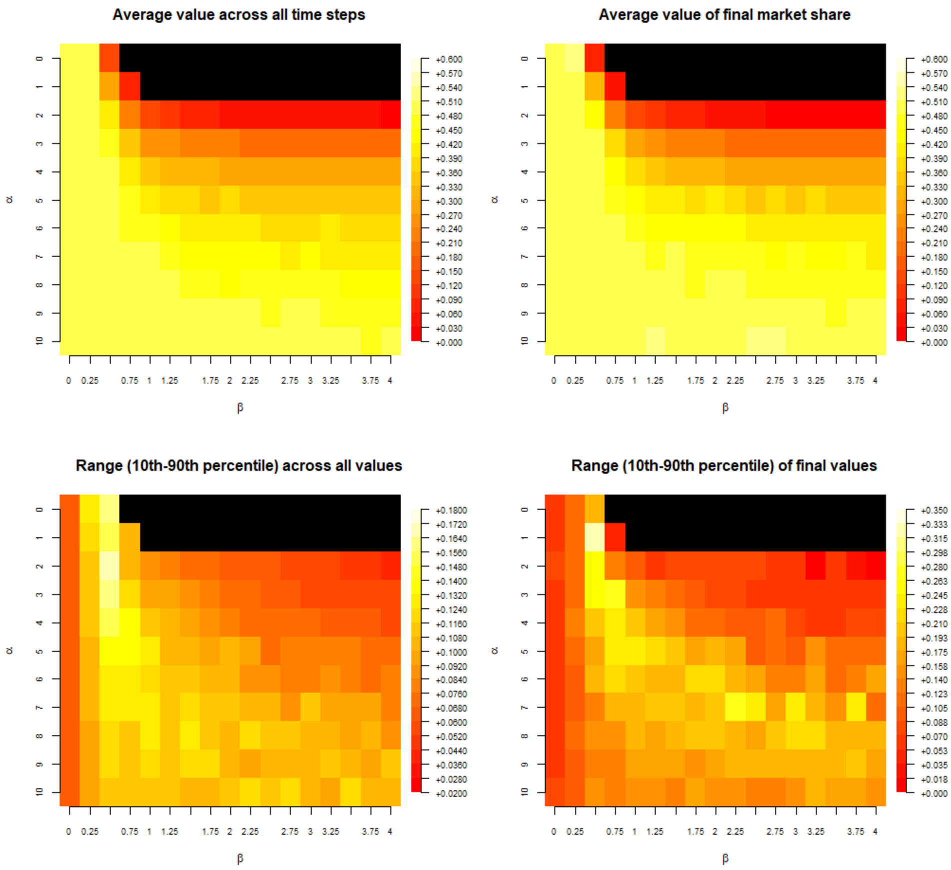


Figure 5.14: City: Divergent series statistics

Non-convergent series

For divergent series we have collected the same metric that again yields the same results as in the village archetype. Again, with higher competition effort, the market share of fiber rises, which is unintuitive. Let us debate once more the potential reasons behind this. One additional influence might be that the competition level of innovation efforts (α), might be associated with the general demand in the area. Given the overall higher competition, fiber providers are pushed to innovate as well and given the pressumed excellence of their product, they capture larger market share in the long run.

Lattice distribution

Lastly, let us look at the distribution of technologies. Again, it seems that our model has a tendency to cluster same technologies together. The Figure 5.15 shows development of number of simulations that have clustered differently

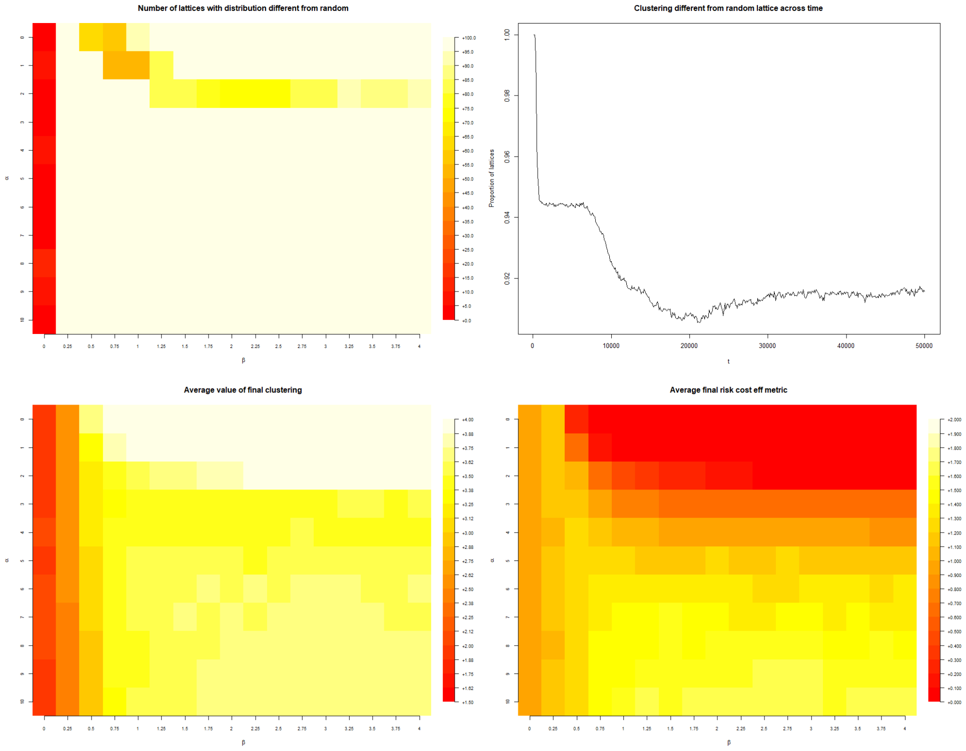


Figure 5.15: City: Lattice distribution statistics

than a random lattice. Slightly irregularly, there seems to be a large drop in the proportion of such simulations. Nevertheless, this was anticipated as the initial or a city archetype was artifically clustered (see Figure 4.3)). Finally, the "risk cost eff" scheme summarizes our findings, showing the parameter region with higher competitors' competency as more aspiring.

5.4 Situations wholly dominated by alternative technology - new market entries

Finally, let us examine the last defined archetype, which is a completely unpenetrated market. In this section, the primary aim is to identify the set of parameters that allow a successful entry to the market. After mapping all the previous cases, it is expected that for some subset of parameters, the fiber deployment will not happen with sufficient vigor and the fiber will remain dominated by the alternative technologies.

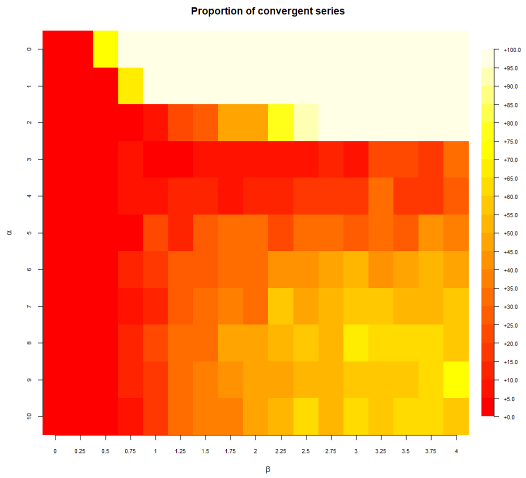


Figure 5.16: New market: Number of convergent series

Convergent series

For convergent series we witness the same emerging patterns in occurence of convergence and its speed, average convergence values or occurence of the special/extreme cases. Comparing Figure 5.17 or Figure 5.18 to their counterparts presented earlier, we suspect even more that the initial setting is irrelevant (or possibly highly relevant).

Non-convergent series

Unfortunately, when examining the divergent series, we do not see any unfamiliar patterns emerging. We have already presented two separate potential reasonings behind the dynamics, but let us supplement that a bit further. Perhaps, the level of competition's effort is determining the actual market share but serves as an indicator of quality (customers' propensity to spend, consumer's wealth) of the new market. Therefore higher effort of alternative technology providers signal higher attractivity of the market.

Lattice distribution

Commenting on the lattice distribution development of the new market entry is an encapsulation of all that has already been said. Repeatedly, the market has a tendency to cluster highly. Not counting the clustering of the simulations that have not been penetrated, we observe that the highest clustering levels occur for higher competition levels and higher level of fiber perception. One may

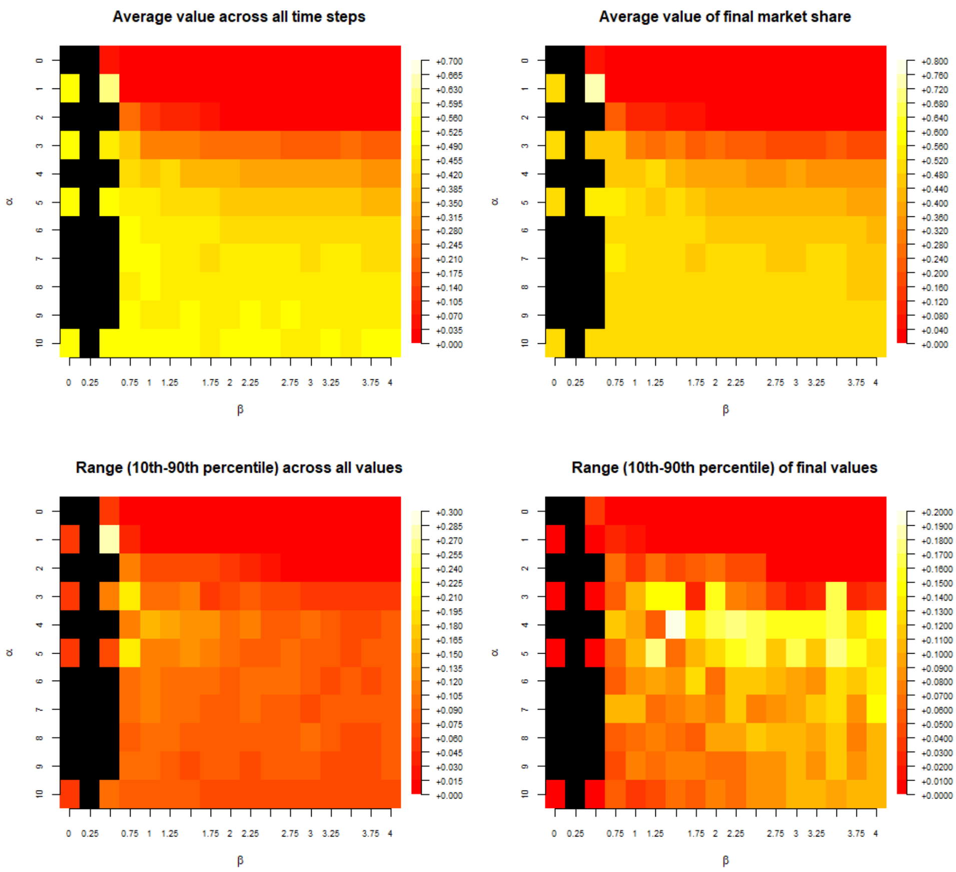


Figure 5.17: New market: Convergent series statistics part A

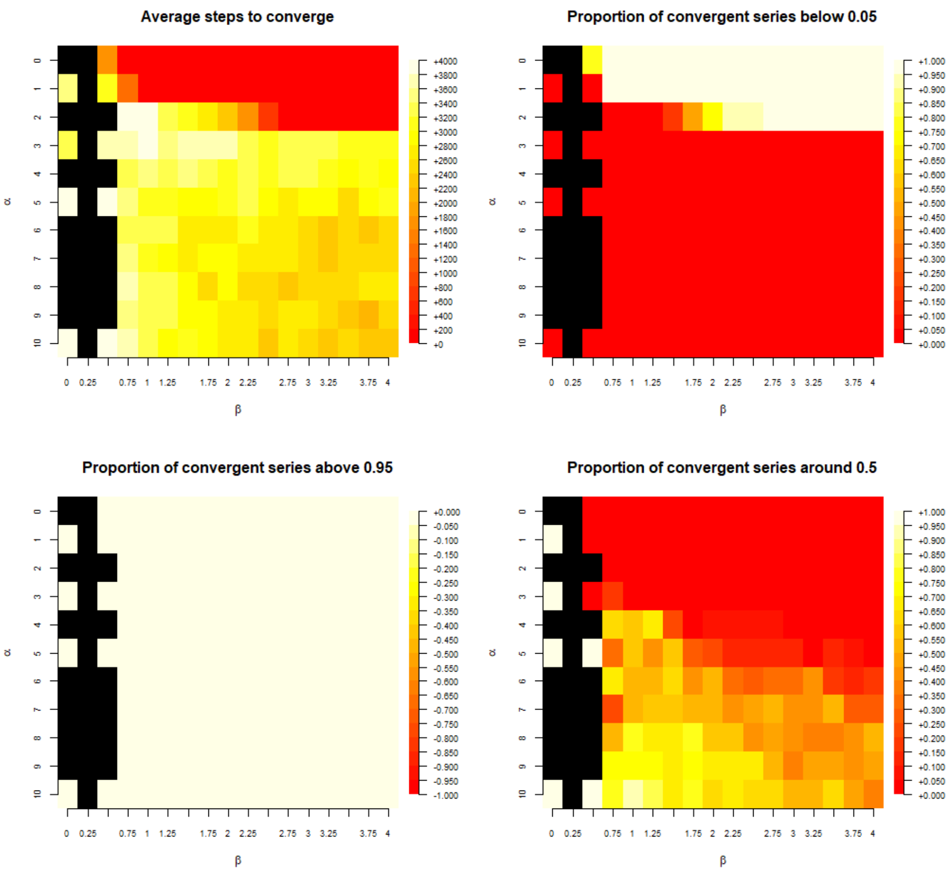


Figure 5.18: New market: Convergent series statistics part B

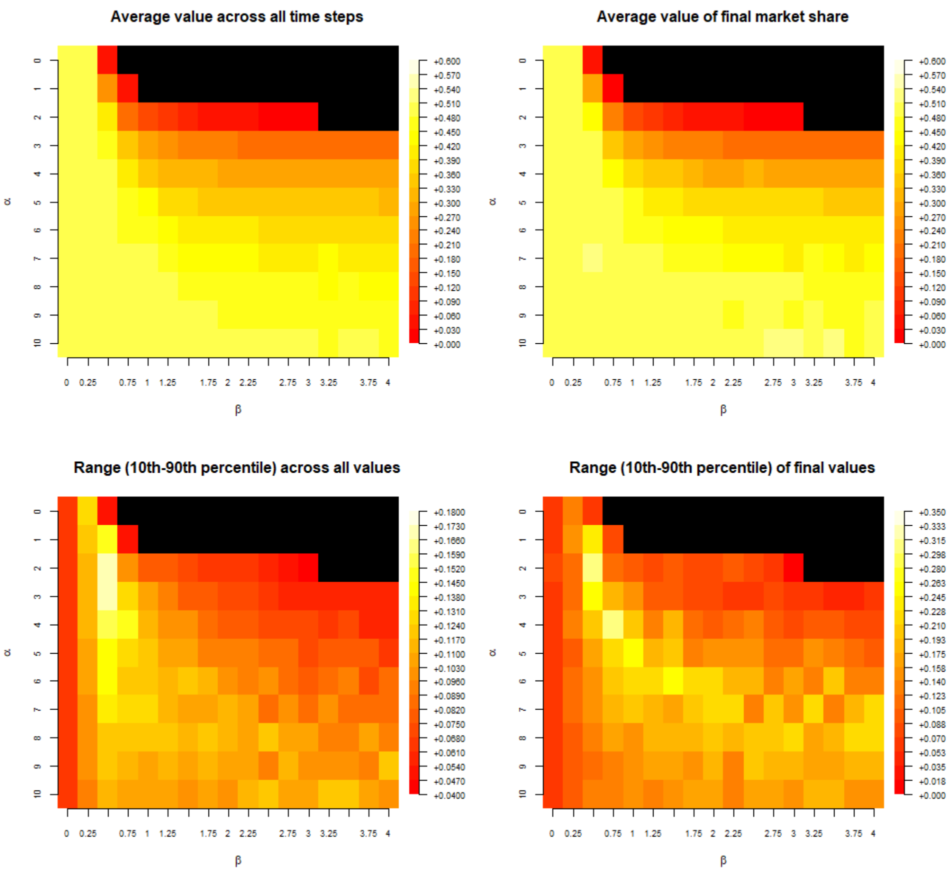


Figure 5.19: New market: Divergent series statistics

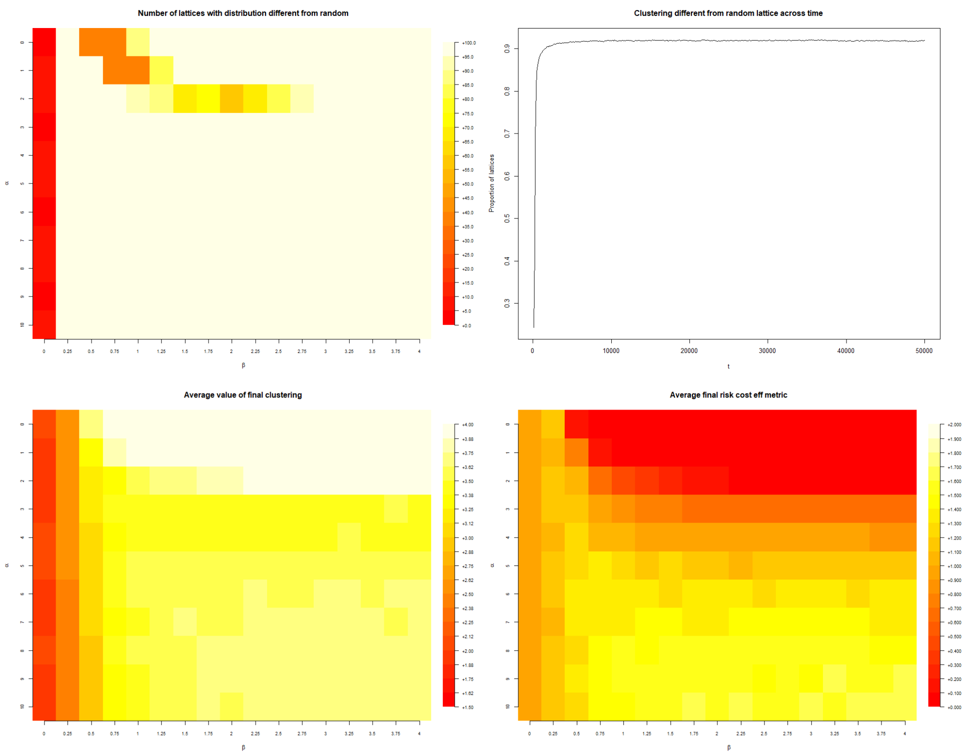


Figure 5.20: New market: Lattice distribution statistics

loosely infer that clustering (and thus decreasing costs) is closely intertwined with efforts. Lastly, putting all the pieces together, that the index combining market share, volatility and clustering levels indicate a market entry potential of fiber for $\alpha > 5$.

5.5 Overview

In summary, we have presented results of Ising generated series for four different archetypes. In terms of convergence and its speed, the time series behave very similarly in all four settings. The convergence is observable mainly for low valued $\alpha \in (0, 2)$ and $\beta \in (0.5, 4)$. The speed of convergence, if present, is accelerating with lower α and higher β.

In terms of market share, we see a huge discrepancy in behavior of the standard (random) lattice and three other (city, village, new entry). While for the standard lattice the results correspond with our intuition described in Section 4.5, for the other archetypes we see a completely opposite behavior (mainly the fact that higher competition efforts correspond to higher fiber market shares). We have already foreshadowed some alternative intuition behind that we will further summarize in the following chapter. However, it seems that this behavior has not been clearly grasped by our model.

For all archetypes, we are able to observe a clear propensity of the system to establish and preserve homogenous technology clusters. This behavior is apparent for all parameter setting (apart from the borderline case of $\beta = 0$). From this perspective, the distinction between village and city archetypes becomes negligible as their main distinguishing feature becomes smeared fastly.

Chapter 6

Discussion

In the following chapter, we will further discuss the results present in Chapter 5. Additionally, we discuss potential drawback of the whole model and lastly, we will contemplate aspects of model that could be further progressed.

Alternative intuition

As already implied - the behavior of market share series stemming from the three specific initial lattices within our model deviates significantly from the standard case. While we expected the same parameter dynamics (only shifted for different archetypes), the opposite is the case. Contrary to our intuition, according to our model, higher competitive efforts and innovation on the market in fact leads to higher levels of market shares of fiber technology. This might coordinate with another intuition that requires us to step out of the fiber deployment point of view and switch to market point of view. Likewise, it may require us to reverse the causality.

The unexpected results could be potentially explained if we assume that the α does not behave as determination of the market share but rather as indication of some theoretical market quality. Market quality could be embodied by topological features, customers' propensity to spend and wealth, or overall demand for broadband technology services. When we consider this relationship, the results become more tangible. Unfortunately though, this still does not explain the crucial difference.

Possibly, the issue might lie in the initial distribution of the lattice in terms of market share. It is not completely unrealistic that high initial market share would work as the indication of the market quality as itself and therefore the parameters would follow our intution as opposed to unforeseen situation in the three specific cases.

Potential drawbacks

Obviously, as for almost any other model, there exist potential caveats. Whilst being a great advantage of the model, its relative simplicity in input parameters might be a great obstacle in the way of drawing reasonable conclusions. For example, both α and β are not dynamic in the sense that they remain constant over time. This way, the possibility of (for instance) adjusting the level of competition based on market development remains unadressed.
Another potential inconvience is the fact that the lattice is occupied only in binary values. In our model we have presented the binary option as fiber technology opposed to any other fixed broadband technology, nonetheless, it is within reason, there may exist a third option that could disrupt the dynamics. Additionally, compared to other use cases, we have not found a meaningful derivation from the total magnetization of the system. This does not necessarily affect in a negative way, however the fact that we are not able to come up with similar interpretation as in financial series for instance (price and its returns) is unfortunate.

Further aspects for additional research

In any case, one of the directions of possible extensions to this work is to include more parameters or different mechanics and compare the results. Pitifully, each additional feature of the model results greatly influences the computational power required to generate the series that are to be further analyzed. This includes among others: including an assymetry between parameters (e.g. $B_{s_(t)=+1} > B_{s_(t)=+1}$, possibility of parallel switching, substantially increasing the number of steps (it may be possible that some of series have just not converged yet) or compare different neighbour dynamics. Also, as we've seen with difference between standard lattice setting and the rest, it might be the case that the generated series would behave differently based on the initial proportion of the spins.
Ultimately, the connection of the Ising model parameters and the real life situation is a very tough problem. Even though we may estimate the dynamics of the inputs, it is quite hard to calibrate and couple them with real life pairs. For instance, based on our market overview, we have been able to assume some

level of preference of fiber for customers (based on quality features), nevertheless it still does not give evidence on whether the β parameter is more close to being 2 or 4.

Chapter 7

Conclusion

At last, let us present the conclusions of our thesis.

Firstly, we have mapped and described the current Czech fixed broadband technology landscape, its expected development and its comparison to situation in the European Union. Furthermore, we have explored past research employing the Ising model and we have described their unique approaches. On top of our description of the Ising model, we contribute to the current research by newly introduced analysis of Ising generated series and lattices. Specifically, we conduct a focused analysis of Ising model inputs' (α, β, state of the initial lattice) effect on the model's outcomes - convergence of the series, development of proportion of the spins across the time and positioning of spins in the lattice (clustering).

Additionally, we link the technical results of the model with potential situation by defining real life equivalents to the model's parameters. Therefore, we assume the opposite spins in the model to represent fiber technology or alternative technology connections. Furthermore, we assume α input to be interpreted as competitiveness on the market, i.e. fiber competition's efforts to attract the customer. For β, we assume the representation to be a theoretical perception of fiber's relative dominance over other technologies (e.g. connection speed, higher reliability). Lastly (above the typical random lattice used in Ising model setting), we define three more archetypes of the inital lattices representing a different situations on the market - these are the "city", "village" and "new entry" situations. City and village archetypes represent different geographical settings, while new entry represents a situation of unpenetrated market segment.

We observe that for the standard typical situations, the market is more likely to saturate and to get dominated by fiber with lower competition's efforts and better perception of fiber. With the complementary parameter values, i.e. either higher competition or worse perception, we observe a tendency of the market to balance between the two alternatives.

For the other three archetypes, we observe an opposite trend. With higher competition's efforts, we observe higher fiber market shares. We contemplate the reasons behind this phenomena and we come up with imaginable hypothesis, which is that relationship is reversed and the α parameter does not determine the market share itself but rather it works as signalling for a attractive market situation.

Focusing on the convergence of the generated series, we conclude that series converge for specific set of parameters, regardless of the initial lattice setting. Initial lattice setting rigidity is also apparent for the homogenous clustering of system, i.e. the same technology has always a tendency to assemble into close proximity of itself. This effect works a represenation of a marginal cost effect. The clustering is apparent across all time steps.

Lastly, we have combined the market share, volatility of the market share and the level of clustering into an index that helps to identify attractive (high market share, low volatility and dense distribution) situations for fiber deployment. In all settings, this index appears to attract the fiber providers into places with higher level of competition.

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