

The thesis is divided into two parts. The first part deals with radiative corrections in muonic hydrogen. The effect of vacuum polarization is studied, and the simplified derivation of the Wichmann-Kroll potential is presented. The energy shift caused by vacuum polarization to the Lamb shift in muonic hydrogen is calculated and it agrees with results in literature. Further, the concept of the extended Bethe logarithm is introduced and its advantages are shown and used to calculate the combined self-energy vacuum polarization contribution to the Lamb shift in muonic hydrogen. The results given here are more accurate and somewhat different from others given in literature. In the second part, the ground-state energy splitting due to the tunneling in a two-dimensional double-well potential is calculated. A systematic WKB expansion of the energy splitting is given. An interplay between curvature of the classical tunneling path and quantum nature of motion is observed. A series is found that describes systems with strong coupling like the proton transfer in malonaldehyde. The results show a strong sensitivity of the splitting on slight variations of the parameters entering the Hamiltonian linearly. This indicates a presence of quantum chaos in this problem.