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Uncertainty In Macroeconomics: Making RBC path dependent

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Declaration:

Prohlašuji, že jsem diplomovou práci vypracoval samostatně a že jsem použil pouze uvedené prameny a literaturu.

Hereby I declare that I elaborated this thesis independently, using only the listed literature and resources.

21. 5. 2008

(signature)

I would like to thank to my advisor, prof. Sojka, for opening many windows and uncovering new ways how to think of economics. I thank also to participants of Economic Dynamics seminar for fruitful comments.

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Abstrakt

Práce zkoumá pohled na nejistotu, čas a očekávání v rámci neoklasické ekonomie a diskutuje možné alternativy a důsledky uvolnění některých předpokladů. Důležitost metodologie se projevuje mimo jiné, když jsou ekonomické teorie testovány na realných situacích. Předpoklady, na kterých jsou teorie postaveny, by tak měly být realistické.

Druhá část textu se snaží aplikovat myšlenky z předchozích metodologických úvah. Konkrétně je odvozen model, který je postaven na běžných neoklasických základech, je však navíc obohacen o realističtější zachycení nejistoty. Model je zkoumán pomocí odvozených impulzních odezev, simulací a rozptylu generovanych casových řad. V chovaní modifikovaného modelu můžeme nalézt známky tzv. závislosti na ceste. Vyšší míra persistence a nižší variabilita poukazuje na problematičnost použití technologických šoků jako nejdůležitějšího zdroje cyklického chovaní ekonomiky.

Abstrakt

Mainstream and alternative notions of uncertainty, time and expectations in economics are discussed. Then the consequences of relaxing some of the assumptions are examined. The relevance of the methodological issues is brought up, when economic theories are employed to deal with real economic problems. It is argued that the assumptions should be realistic.

In second part of this text, it is demonstrated how a mainstream neoclassical model can be enriched with more realistic assumptions in line with the previous argumentation. The model is then analyzed using impulse response functions, simulations and long run variance of generated time series. The model manifests interesting behavior, such as path dependency. The higher persistence and lower variance can be interpreted as evidence against a view that technological shocks are main sources of business cycles.

1 Introduction

For anyone trying to penetrate beyond the textbook interpretation, current state of economics is disturbing. Mainstream theory is formulating complex models with complicated formalisms, using profound results of mathematics.[1] Heterodox schools of thought put under question main building blocks of the orthodox paradigm, disputing all the results. The latter views provide interesting insights, often by dissolving borders between social sciences. However, they are unable to give quantitative answers. Practical questions are generally more appealing, as the methodology itself will not help us to deal with recessions. However, policy based on ill founded assumptions will hardly help us neither. Practical economics and fundamental methodology, both require profound study and it is impossible to understand fully both.

The 20th century witnessed many fierce debates in economics. After the triumph of the keynesian revolution, a clash between keynesians and monetarist took place. Then the pendulum swung back, with an impulse from the New Classical Economics. New Keynesians deserted their original keynesian positions and attacked new classics from behind, using their own weapons of general equilibrium models with rational expectations, implementing some of their old tools like price rigidities.

Now, the methodological questions seem to be abandoned.[2] Nevertheless, every economist should confront himself somehow with these themes.[3] Although the world does not need majority of economist working on methodology, every single economist should, at least briefly, study the methodology and enrich his view on the topic of his interest. Familiarity with methodological issues should then project itself into now ideas in practical topics.

In this text, the notion of uncertainty and time is scrutinised. As the methodology is defining for both orthodox and heterodox systems of thoughts, the selected notions have different implications in different frameworks. Because this text is not intended to be purely

[1] As an example, see macro textbook, Ljungqvist and Sargent (2004).

[2] Which is appraised by some as a success (Blanchard, 2000). On the other hand, Laidler (2001) shows that more broad perspective is useful.

[3] Laidler (2001) discusses importance of knowledge of history of thought for modern macroeconomics.

methodological, these issues are introduced only as an motivation to the modification of the model. Thus, this text served as an impulse for the author to organize his own thought on this issues.

Implications of the methodology should emanate in practical economics. The monetary policy is found to be such a field, which is both highly technical in terms of state-of-the-art models and practical at the same time. Therefore the implications of different attitudes toward the methodological issues manifest themselves in practice of central banks.

The basic structure of this text is as follows. Methodological issues are introduced in section 2.1. The mainstream and some alternative tractation is discussed to show that the current mainstream methodological paradigm is not completely unquestionable. Section 3, asks whether it is possible (and worth) to keep mainstream framework (and at least for the author the answer is affirmative) and explore some possible ways how to enrich and amend retained neoclassical paradigm. The rest of this text explores modified Real Business Cycle model (RBC) and tries to address discussed methodological issues.

The methodological part, first, the mainstream stance on particular topic is briefly described, then the implications of less restrictive assumptions are considered. Rather then to cover all the blocks of the mainstream economy methodology comprehensively, or to profoundly discuss theoretical consequences of few selected topics, the ambition is to briefly show that implication of non-mainstream methodology are logical and reasonable.

Second, connection between discussed issues and practical problems of monetary policy is examined. We try to find reflections of selected methodological issues in practical monetary policy conduct. This is intended to demonstrate, that heterodox inspirations can be generally appealing.

The most important and original part of the text consists of sections 4 and 5. Section 4 introduces one particular way how to amend the mainstream model. By making the standard RBC framework more realistic with regard to uncertainty, the model can account for interesting behavior. The uncertainty is crucial, as was identified in the methodological part of the text. The model thus logically follows from the previous methodological contemplations.

The impulse responses and simulations are shown and discussed in section 5. This effort is made in order to show, that even with only minor modifications, neoclassical model account for phenomena not often discussed in neoclassical economics.

The appendix contains step by step derivation of impulse response functions.

2 Few methodological contemplations

One could call current state of the economic debate as "phoney consensus". Currently, there is no great theoretical debate. The most of the economists are educated by using only a few textbooks. The waters seem to be calm.

Yet there are many competing explanations for many phenomena. Theories are often only falsified by empirical tests, not ruled out by superiority of a rival hypothesis. Even within the mainstream framework, there are views which are divergent. Moreover, the policy implications are completely opposite, for example while neokeynesian monetary policy framework can include the whole RBC theory. Yet the former leaves some space for welfare improving policies, the latter does not.

On the contrary, outside of the mainstream, schools which are completely opposite in their world views often share significant portion of insights and they accent the some topics, which are not fully covered by the orthodox schools. The optimistic point is, that no matter whether you are from orthodox or heterodox stream, if a model is applied critically and if the reasoning is done realistically, the outcomes need not to be distant.[4] In this section such points are going to be discussed.

Firstly, the uncertainty is going to be scrutinized. Then the implications of uncertainty in the time framework are going to be examined. Finally, the relation of rational expectations hypothesis with the issues of uncertainty and time is examined.

[4]For example, current neokeynesian monetary framework is near to post keynesians in respect to endogenity of money, even though the former is built primarily on general equilibrium model. Rochon (1999) provides comprehensive (rather critical) comparison of neo and post keynesian monetary economics. Another ideas can be found in Arestis and Sawyer (2004)

2.1 Uncertainty

Different people regard the nature of the world differently. Let's roughly categorise the possible views about the possible scope of uncertainty. Then we can discuss the implications of different views more easily. Because this text is not intended to be purely methodological, the following classification is only brief and by no means exhausting.

2.1.1 Two notions of uncertainty

The neoclassical mainstream assumes, that we know all states of the nature which can take place. That is, no outcome can be truly a surprise. It is assumed, that probabilities can be assigned to all the states, in another words, the distribution over all possible states of the nature is known. Moreover, it is, often silently, assumed, that this distribution is well behaving, that is, all the moments we need to know do exist and can be derived. In this setting, the uncertainty is reduced to risk. In this text, this notion is called *ergodicity.*

There is some *metastructure* of the world, which is *known.* The existence of metastructure denotes the existence of some fundamental laws. In the language of formalised models, there is some set of equations capable to describe the behavior of the economy.[5]

This metastructure can be described by some set of axioms. That corresponds to a *closed system of thought* (for description see Dow (1996, page 13)). From this set of axioms, more complex theorems are then derived in a deductive manner. This corresponds to Cartesian/Euclidean mode of thought. Initially, axioms were believed to be self evident so they were widely acceptable. While this seems plausible for mathematics (although the mainstream view in physics or mathematics does no longer supports this view), it is much more questionable for social sciences.

Note that, the assumption that the world around us is not changing has to be made, or that the change does not limit completeness of knowledge (in the risk sense). This premise

[5]The knowledge of the actual size of the parameters is second thing, it is not needed directly under the term metastructure. However, if the metastructure is known and stable, then people can learn from past realisations and form bayesian *a posterior* estimates which would converge to their true *a priory* values, because of the stability of the model.

is quite ambitious. These simplifications might be too strong. On the other hand, any model is only simplification of reality and modeling exercise is useful if the outcomes are taken with a grain of salt, being aware of the simplifications made.

If we do not assume the ergodic world, then we (*simply*) do not know. For example, the world around us is permanently evolving, and we do not know, what will be the outcomes and the consequences.[6]

Open system of thought is employed, if the assumption about existence of (closed) set of axioms is not made. Here, it can be distinguished basically between two variations. First option is, that we do not know, because the world is so complex, that we can *never* know. So, there is no observable and understandable metastructure. Second option is that, only our current knowledge is limited, and possibly in the future we can converge to some better understanding. So there could be some metastructure, but we do not know enough yet.[7] In the text we would call these views as *fundamental uncertainty.*

2.1.2 A critical appraisal

In the modern mainstream economics, the reality is modeled as ergodic. Despite its lower popularity, let's look at the fundamental uncertainty view in more detail. First of all, the economics seems not to be able to deliver fundamental answers. Even major economic turmoils are still unforecastable and the theory still grapples with many so called "puzzles".

Second, the uncertainty attitude, rather then the risk, is much more in line with more advanced methodology of science. Even in mathematics and physics, there is an evolution in what is seen as a self evident truth. The view that there is a closed set of axioms on which we could resolve any statement proved to be unsustainable[8] and any system based on such belief is untenable. This skepticism leads naturally to use of some form of critical

[6]For detailed treatment about different types of uncertainty, see fundamental contribution of Frank H. Knight: "Risk, Uncertainty and Profit" from 1921, or Rousseas (1986).

[7]Here, it depends on the qualitative judgment, what it means better understating. Let's take the ability to construct some decent model, what is decent is again some subjective qualitative judgment. The outcome for any practical problem is the same, and it is, that we do not know enough, for whatever reason.

[8]Moreover, axioms for contemporal math are not any more self evident and some are even quite counterintuitive, ie. axiom of choice.

realism.

The notion of existence of some metastructure is assumed in the famous *Lucas critique*. In his seminal paper, Lucas (1976), showed great insight, that pure econometrics often estimated only the reduced form coefficients. New policy based on reduced form estimates then can lead to unexpected results, because the estimated results were conditioned by environment at the time of estimation, particularly the existing policy setting. The new policy then changes the environment, so the agents re-examine their decisions. This is again assuming existence of stable metastructure. In modern language, there are the structural (or deep) parameters, which are the basis for micro agents optimisation problem.

2.2 Time

Let's extend the discussion over uncertainty to the notion of time. Saving for the future is fundamental for decisions made today. The extent of incertitude about the future development thus affects momentary decision as well. The time dimension is thus important feature of any economic discussion.

2.2.1 Mechanical vs. historical time

The term corresponding to the neoclassical ergodic view of the world is called *mechanical time*. This concept is used to model decision making, which takes place in within a standard economic model. In this sense, it is an artificial thing. You can simulate an experiment many times and you always obtain the same result. You can re-run the model with different parameters and then compare the outcomes and choose the best.

This is hardly possible in reality. More realistic notion of time is the *historic time*.[9] Here agents have to face fundamental uncertainty. In this world, each moment is unique and no decision can be taken back. Because people have only limited knowledge, the outcomes of the actions provide source of original information about functioning of the system. Thus we are changing the economy both by our action, but also by obtaining the

[9] Used by both post-keynesian and austrian economists.

knowledge how the system works. Even though we assume, that the metastructure is not completely observable, so we cannot observe what *it is* completely, we can at least observe what *it is not*, ie. build up some knowledge by falsification.[10]

The value of learning from past observations can differ.[11] If we assumed that the metastructure was stable, then one action should lead to the same outcome, whenever the conditions were the same. From this point of view, even this notion of time is a bit ahistorical. True historical time needs the metastructure to be changing, or impossible to be fully understandable. Assuming the fundamental uncertainty brings about another problem. Can we believe that any historical regularities will replicate themselves again in the future?

Moreover, what if the pace of development of the metastructure is not completely independent of our actions? Minsky (1957) describes how period of high interest rate in 50's provoked financial innovation, which eventually led to rise of money velocity. Induced reaction of economic system went directly against the policy conducted by the monetary authority.

2.2.2 Long run equilibrium

The neoclassical models are based in the ergodic framework. The structure of the model or economy is invariant in time. The equilibrium values are functions of the structure of the model so they are invariant as well. However, this is not the case in reality. Neither we do know the structure, nor we can expect that it stands still. What is then the equilibrium when we do not know the structures and we expect that it is even changing?

The change of metastructure can originate in learning by doing, as Kaldor (1972) points out:

> The gain in design by experience is even more important in making of plant and equipment; hence the *annual* gain of productivity due to "embodied technical

[10] Even this could be disputed, because we cannot know which variables we have omitted from our reasoning.

[11] See section 3.3 on learning and its problems.

> progress" will tend to be all grater then the number of plants constructed per year.

Kaldor proposed that increasing returns to scale are much more important then it was thought. With increasing returns, it is more profitable to invest even more in the area where you have already invested. So actual decision is conditioned by the primary decision where to invest, which could have taken place long time ago.

People can come up with hypotheses based on past observations. However, if we accept that the economy is permanently changing, then the value of observed historical regularities can be questioned. The historical regularities could not take place any more.

If economic agents are learning from actual behavior of the economy, then the actual knowledge is conditioned by historical path of the system. *Path dependency* then changes the properties of future equilibrium. In other words, in long run the agents can change their behavior, changing equilibrium. As an example we can look at unemployment. Higher rate of unemployment may decrease ability of people to work, making them more likely to be unemployed in the future. Moreover, the politicians can try to win these votes by increasing social spending, lowering the opportunity costs of being unemployed.

Economic modeling often makes exercises, where economy in equilibrium is exposed to a shock to some exogenous variable and then the path back to equilibrium is analyzed. When economic agents are learning from actual behavior of the economy, then the behavior itself is conditional on the events that took place in the past and any shock disturbing equilibrium then can have everlasting effect, changing attitude of the agents, resulting in different equilibria. In such a world, there is no unique equilibrium, which would prevail under all possible causes, questioning the whole concept of the term equilibrium.

Minsky's insight was that the decision making, particularly the risk aversion (Minsky, 1985), is conditioned by the historical record of the economy. His famous hypothesis of financial instability is simple and appealing. However, it is not in line with orthodox way of reasoning, where the rational agents have always the same optimising decision making procedure. Let's turn back to Lucas. He assumes stability of the structural parameters.

Obviously, there are no such stable parameters in Minsky's world.

Similar ways of reasoning in up to date research on inflation can be observed. Mishkin (2007) discusses recent developments in behavior of inflation. For Mishkin, the reason of great moderation is that the inflation expectations are now much more anchored then ever before. However, for last 20 years, there has not been a policy shift for example in FED policy making procedure. Therefore, it would be hard to explain this inflation dynamics by a structural model. This unprecedential stability tempts central bankers to make the inflation policy more expansionary. However, Mishkin clearly says, that any such temptation should be avoided, because the credibility could be easily lost. It seems, that Mishkin is more in line with Minsky's world views on uncertainty of existence of stable metastructure then with Lucas.

2.3 Role of expectations in economics

The role of expectations in behavior of economic systems is fundamental. Expectations are the way of assessing future situations and likelihood of its occurrence.

The cornerstone of modern economics are *Rational Expectations*. It logically constructs a manner of expectation formation, under which the agents cannot be systematically fooled by a deliberate policy regime, as was possible under adaptive expectations. This is entirely plausible. On the contrary, assuming that the agents actually know the metastructure, which we, as economists or econometricians, cannot say, is problematic. Let's quote Muth (1961), emphasis added:

> The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or more generally, the subjective probability distributions of outcomes) tend to be distributed, for the same information set, about the prediction of *the theory* (or the "objective" probability distribution).

Obviously, Muth assumes that there is a set of possible states and that the set is known to everybody. Moreover, according to him, the theory knows the objective distribution over the states of the nature. If we believe in ergodic view of the world, then the structure can be estimated from historical experience. If we know the equations describing the world, it is only a technical problem to calibrate the parameters in such a way that the errors have zero mean value.

Even if we assumed ergodic world for the moment, there would still be one big issue. Muth did not claim that entrepreneurs actually look into a textbook, compute marginal cost and then set their production: "*It does not assert that the scratch work of entrepreneurs resembles the system of equations in any way...*", Muth (1961). However, people are specialist in both production and consumption in reality. So the fraction of economy with which they have original experience is only small. Even though that people would somehow guess how to produce their own "optimal" quantities, about the rest of economy they have to make a guess about this particular quantity and price. And this guess is then based on *the theory*.

Note that *the theory* itself plays a crucial role in determining the actual result. The scientific paradigm is directly influencing the behavior of the object of the science itself. Let's recall Keynes's famous quote (Keynes, 1936, page 383):

> ..., the ideas of economists and political philosophers, both when they are right and when they are wrong, are more powerful then is commonly understood.

Let's turn back to Lucas once again. He, as well as Muth, assumed that the agents know the true model of the economy and are able to optimise. In a world, where the economic paradigm influences the actual behavior of the system, evidently, changes in the paradigm can influence the behavior of the people. Lucas critique then can be generalized, behavior of economic men is endogenous not only to the policy setting, but also to the current economic paradigm.

2.4 Rationality

Mainstream microeconomic literature[12] is build on preference relations. *Rational* preference relation is defined as preference relation which is complete and transitive. Following

[12] See Mas-Colell et al. (1995).

this stream, rational agents are then able to solve optimalisation problems, firms maximise profits and consumers maximise their utility by choosing optimal consumption bundle given vector of prices and some budget constraint.

However, in practical life, there are many examples, where people are not behaving in line with what would this "rationality" suggest.[13] Vanberg (2004) critically discusses ways how rationality hypothesis could be defended. The simplest neoclassical answer is that it's not the principle of rationality which is breached, only the utility function should include richer set of objectives. Then it's possible to talk about "rational" addiction to drugs etc (Becker, 1993). However, if every behavior is rational, then the term *rational* is useless, since there is nothing that is not *rational*.

The one way is to admit, that rationality is not full description of human behavior, nonetheless it's reasonable depiction of reality in presence of markets. And because markets are the playground in which economists are interested, thus rationality depicts economic behavior quite well. In some markets, the agents are confronted repeatedly with nearly identical problems. When dealing with identical problems, one can claim that agents will become rational, either because they can learn from their past actions or by elimination of suboptimal

However, the question is, whether markets that allow such learning prevail. In reality, many markets are permanently changing. Moreover, if agents do not share the same views about the likelihoods of different states of the nature, then some suboptimal behavior (suboptimal in global view) can be locally optimal (given mis-beliefs of some agents) and then such a system can be stable, ie. agents have no incentive to change their behavior eventhough it is not globally optimal.

3 Modifications and enrichments

The arguments presented above constitute serious critique of the neoclassical mainstream economics. Every economist should somehow confrontate himself with these issues and

[13] Nice examples can be found in Conslic (1996).

decide whether the critique is or is not relevant for him or not. There are basically three possible attitudes.

First, the neoclassical model is reasonable description of reality. Acknowledging that every model is only simplification of reality, some people believe that the simplifications dealt by neoclassical economists are not so strong that the model should be useless. Moreover, eventhough some stylized models are clearly very simplified, they provide interesting insights which can be applied later in the real situations.

Extreme case of this attitude is to resign the relation of the model to reality. In such a setting, economics gets next to the mathematics. Then outcomes are based only on assumptions, and economics is purely deductive system, without any direct relevance to reality.

For example, Lucas (1978) is building his model into completely unrealistic setting: the economy consists only of trees and its fruits. The elegance of the reasoning in unquestionable. The practical implications for any market are uncertain. However, research in mathematics has seldom direct applications yet its role in applications is obvious.

Second possible attitude is to accept the core of neoclassical economics and try to enrich the models with realistic features. The core means the models are based on rational micro optimising agents. The realistic features can account for various uncertainties in the models, various limits to the information set of the agents or any other limitation imposed on the agents. Typical example can be Stiglitz and Weiss (1981). This paper on credit rationing (and quantity not price adjustment) is one of the most important contributions is so Stiglitz's new economic paradigm based on information scarcity and assymetry (Stiglitz, 2000, 2002).

This view can be criticized. It can be argued that realism is added only as an *ad hoc* feature. Such models then can reasonably capture reality in one particular situation. However, those who find the core unsatisfactory question whether amendments can save the whole building when the basement are completely mistaken. Eventhough the optimisation is based on more realistic assumptions (cognitive limitations, incomplete information,. . .), for someone the mere optimisation is unacceptable, because it does not fairly represents the precess in human brain.

Another critique can be, that eventhough there are many obvious simplifications, the usual way how to construct a model is to choose just one amendment and see how this changes resulting behavior of the system. However, changing only one assumption often raises new questions. For example, to have the model analytically tractable, Stiglitz and Weiss (1981) impose new assumptions and behavior of bank, namely collateral requirements (or rather absence of collateral). Yet changes in collateral requirements would change the results significantly. Then the question is whether more realism was added by introducing assymetric information, or whether the realism was even decreased given simplified assumptions about the collateral requirements.

Next step in direction further away from neoclassical models is to add some features completely ad hoc, without dering them as a result of optimising agent behavior (which could be somehow constrained, as in the previous case). The typical example was introduction of price rigidities. This is the way how macroeconomic modeling was exercised in the 50's and 60's. The example can be famous Phillips curve, relation between inflation and unemployment (or output gap). This relation implies a possible trade-off, so policy-makers should choose such a combination to maximise social welfare. One of the interpretation of the rise of inflation in 70's is in fact trial to exploit the Phillips curve (Sargent, 2001).

The critique of this attitude is following. By introducing some feature to the model that is not derived from micro optimisation of the agents inside of the model, it is likely, that this new feature is not stable over different states of the other variables and thus Lucas critique applies. In another words, new features are introduced into the model as an exogenous thing (ie. not an internal result of the model itself). Then if this feature is not truly exogenous in the reality (ie. it is at least partly result of decisions within the model), then the new feature is not going to be valid under different circumstances. Assuming that agents will not change their behavior is the same as expect them not to adapt to the new environment and thus resing on assumption that they maxise their utility (Lucas and Sargent, 1979).

Under some circumstances, adaptive price setting can be reasonably accurate description of reality. However, if the predictions by adaptive expectations prove to be mistaken, people would likely completely change their price setting behavior, making adaptive assumption false (or at least invalidating estimated values of parameters). Assuming stability of such parameters, is in fact imposing some kind of rationality limitation onto the agents. This irrationality can be under some circumstances much less adequate model of human behavior then rational expectations.

This can be illustrated on a simple real life example. Suppose that my friend is always late and we have a meeting at 12 o'clock. I get stuck in traffic jam but I am not worried, because I know, I will arrive as the first anyway. Then once my friend realizes that she is expected to arrive with 15 minutes delay. Thus she will move her subjective time of meeting by 15 minutes, which will result in arriving 30 minutes later then original meeting time, etc...

The ultimate option is complete departure from neoclassical setting. It is difficult to find reasonable quantitatively tractable and empirically testable models outside of mainstream. One possible reason is that number of people working on development of alternative treatises is much lower and thus the development is slower.[14] One such a attempt is forming expectations in evolutionary manner (Vanberg, 2004).

Then there is a question, which option should be chosen. The new classical setting is without doubt the most elegant one. However, the realism is clearly lacking.

Viewed from classical keynesian perspective, new keynesian position is complete resignation on original Keynes' ideas. New keynesian policy models are still lagging behind, the realistic features can be incorporated only after a way, how to derive them from rational micro agent optimisation problem, is found. On many topics, new keynesians were able to derive similar results from micro optimisation as many (post) keynesians believe in (Rochon, 1999, chapter 7).

Classical keynesian position was attacked many time due to it's resilience to rational expectations. Yet it's used in policy models, where new keynesian alternative is not

[14] Which was however the case even for rational expectations before seminal contributions of Kydland, Prescott, Lucas and Sargent in 70's.

developed yet. For example, New Keynesian Phillips curve[15] is often depicted as

$$\pi_t = \gamma x_t + \beta \mathsf{E}_t \pi_{t+1} + u_t,$$

where x_t is output gap at time t, $\mathsf{E}_t \pi_{t+1}$ is expectation of inflation at time $t+1$ based on knowledge at time t and u_t is some inflationary shock. Thus this is forward looking equation. However, inflation inertia is well observed empirical fact and models should somehow reflect it. Thus for policy models include the Phillips curve which contain backward looking term[16]:

$$\pi_t = \gamma x_t + \beta \mathsf{E}_t \pi_{t+1} + \alpha \pi_{t-1} + u_t,$$

which captures inflation movements better. However, this model is not based on microfoundations and thus is unacceptable from new classical point of view (it cannot survive Lucas critique).

3.1 Example: price rigidities

Price rigidities are very interesting example. First of all, price rigidities are empirically observed fact (Dhyne et al., 2005).

Irving Fisher (Fisher, 1911, chapter 4) explains business cycles as a result of sluggish interest rate (interest rate as a price of money). The propagation mechanism of business cycle can be described as follows. First impulse is rise in price. Then the rate of interest rises, but with delay and not sufficiently. Thus real rate of interest declines. Lowered real rate of interest is incentive for firms to borrow even further and to expand production. The higher demand pushes the prices higher as long as the real interest rate is too low.

Keynes worked with wage rigidities as an observed fact, which need not to be explained further, because he believed that they are self evident.

[15]See section 3.1.

[16]So called Hybrid New Keynesian Phillips curve

When RBC theorists showed that business cycles can be explained without any rigidity, and thus left the rigidity assumption as not needed anymore. At this time major change in macroeconomics methodology occurred. The swing was toward micro based models with rational optimizing agents.

New classical economists won the battle about mainstream methodology in macroeconomics. The keynesian economists within the mainstream thus had to adapt and start using mainstream methodology. They still believed that price rigidities are important feature of real economy, thus they need to show that rigidity can be obtained as a result of optimisation of micro agents.

This was done in Fischer (1977) and Taylor (1980). Part of the macroeconomics debate at that time was about so called policy ineffectiveness which resulted in Lucas (1972). In Lucas' setting, rational agents would offset any systematic policy. Fischer demonstrated, that policy indeed affects the economy, if the government can act with higher frequency that is the frequency of wage bargaining and wage setting. Then Taylor showed that if there are wage contracts negotiated for more that 1 period and are not fully indexed against inflation, then policy is effective.

Later, Calvo (1983) showed simple way how rigidity can be modeled in macro models. He assumed that the firms face some probability that they will not be able to re-set their prices in the next period and from this setting he derived what is now called New keynesian Phillips curve. The fixed probability of not being able to re-set prices is result of menu costs etc.

Main advantage of so called *calvo pricing mechanism* is its practical tractability. Now the macro papers in leading journals starts with description of household utility function, tell that they are using calvo pricing mechanism and derive Euler equation and discuss the result.

The final acquisition of micro based macro framework by keynasian economists can be seen in Rotemberg and Woodford (1998). There central bank loss function, which was formerly critisied for being arbitrary, was derived from micro agent optimisation. The new keynesian paradigm is now used as the basis for monetary policy. Clarida et al. (1999)

provide good reference.

Moreover, now there are many different explanations of price and wage rigidities, ranging from menu costs to efficiency wages. Mankiw (1985) describes, how small, second order costs of changing prices (menu costs) leads to first order changes in aggregate variables. Interaction between nominal and real rigidities plays important role (Ball et al., 1989). For seminal contributions in this research project see for example Akerlof and Yellen (1985), Shapiro and Stiglitz (1984), Yellen (1984). The literature is summarized in by Mankiw and Romer (1991).

3.2 Internal and external consistency dilemma

Wren-Lewis (2006) shows the methodological dilemma using concept of *internal* and *external consistency*. By internal consistency he understands consistency of all elements of the model with its basic microeconomic assumptions, whereas external consistency is consistency between the model results and empirical data.

Of course, an ideal model should be both internally and externally consistent. Because any model is only simplification of reality, both are not possible. Obviously, there is a trade-off between internal and external consistency.

The neoclassical position is such that on internal consistency should not be compromised. Model which is internally inconsistent is simply false and should be rejected straightforwardly, while a model which is only externally inconsistent can be tolerated until a better model is found (Wren-Lewis, 2006, page 6).

Wren-Lewis gives example of *uncovered interest rate parity* (UIC). Imagine an investor considering whether he should invest domestically at rate r or abroad at rate r^*. UIC idea is simple, with no barriers, existence of possible arbitrage should make the exchange rate such that the yields are the same for both variants. However, the empirical evidence of UIC is at least mixed. Yet UIC is very common block of any open macro model.

Policy models in central banks still are using new keynesian phillips curve enriched with backward looking terms. It is because without it the models simply do not fit the data well. Thus, there is strong reason why do not insist on the internal consistency purity, or at least, when building policy models. However, this should provide strong impulse to work on ways how to incorporate backward looking terms into the model, or in another words, finding a way how to derive partly backward looking behavior from individual micro agent optimisation.

However, this leads to question whether this is right way of scientific work. Should not be the way of thinking going from reasonable assumptions to the result rather then finding assumptions from which the already known result can be derived? It seems that for virtually any result, there are some initial assumptions which allow the derivation. And this is the place, where critical realism must be applied. The degree of realism should be a measure with which competing initial assumptions should be judged in order to choose the best model.

If the objections of internal consistency purist were to be taken literally, this would limit the scope of possible question the research can address to only those in which reasonable microfoundations have been already developed. This would affect the policy related research (Wren-Lewis, 2006).

Mankiw (1989) argues that New classical economics will never be able to produce externally consistent models. On the other hand, according to Mankiw, new keynesians will eventually come up with explanations of micro foundations such that their externally consistent models will eventually become internally consistent as well. His concept of menu cost can be regarded as a step in that direction.

Interesting point is risen in Wren-Lewis (2006). By using Calvo pricing shortcut, we are not longer sure, that the model is in fat based on *true* microfundations. This simplification may be fine under some circumstances, but under another, the price setting behavior can just change. Thus models which are using Calvo pricing mechanisms are exposed to Lucas critique. On the other hand, the quest for true structural parameters seems to be neverending, there might be always another layer.

Wren-Lewis (2006) argues that Calvo pricing mechanism is not the only example of this. For example, the models which include money directly into the utility function (so called *money in utility* or Clower's *cash in advance* models) do the same thing.

3.2.1 VAR alternative

Unsatisfactory ability to mimic empirical data by optimisation based models led to development of new econometric techniques (Sims, 1996). One of the mostly used today is *vector autoregression* (VAR). Modern introduction to this method can be found in Stock and Watson (2001) or in Christiano et al. (1999). For practical example see King et al. (1991).

This method was pioneered by Sims (1972, 1980). For simplicity, consider bivariate system

$$\mathbf{X}_t = \mathbf{A}\mathbf{X}_{t-1} + \varepsilon_t,$$

$\mathbf{X}_t$ represents vector of variables in the VAR at time t, which is regressed on its historical past values. $\mathbf{A}$ is thus square matrix. Because the variables in the VAR are correlated, the shocks in ε_t vector are correlated as well. Thus the model need to impose additional restriction to identify the system. There are many ways how to impose the needed restrictions. The most straightforward way is to simply add residual from one equation to the next equation as a explanatory variable, this method leads to so called *recursive* VARs.

However, in this setting the result is dependent on the ordering of the equations. To avoid this, *structural* VARs impose restrictions on the vector of residuals to find true so called *innovations*, which are independent.

$$\varepsilon_t = \mathbf{B}\eta_t.$$

The matrix $\mathbf{B}$ is then set according to economic theory. One possible way how to do it is in Blanchard and Quah (1989). For example, let's have bivariate VAR with output and monetary policy stance as variables. Then one possible identification is to assume that innovation in monetary policy affects real economy only with some lag (so called *contemporaneous zero restriction*). Thus we have

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{xt} \\ \varepsilon_{mt} \end{pmatrix} = \begin{pmatrix} 1 & \phi \\ \varphi & 1 \end{pmatrix} \begin{pmatrix} \eta_{xt} \\ \eta_{mt} \end{pmatrix},$$

assuming contemporaneous zero restriction we set $\phi = 0$.

However there is plethora of papers which come up with different ways of identification. This leads to question whether the identification restrictions are really based on some consensual theory or whether it simply led to better results in particular paper. This point is strongly criticized, and VARs are depicted by some as atheoretical.

VARs have also practical shortcomings. The number of estimated parameters grows with square of the number of variables and often more lags are used, and big number of estimated parameters needs sufficiently long time period. It can be argued that economic systems evolve too fast. If there are structural brakes during the estimation period which changes values of the parameters, then the estimation over this period is not stable and leads to mistaken results. Rudebusch (1996) argues that this is the case when VARs are used for estimating monetary policy.[17]

3.3 Learning

To deal with critique that rational expectations are unrealistic, one way is to admit, that rationality is not full description of human behavior, nonetheless it's reasonable depiction of reality in presence of markets. In markets the effective behavior should eventually eliminate ineffective behavior.[18]

And because markets are the playground in which economists are interested, thus rationality depicts economic behavior quite well. In some markets, the agents are confronted repeatedly with nearly identical problems. When dealing with identical problems, one can claim that agents will become rational, either because they can learn from their past

[17] Which is exactly the area for which VARs were originally developed and for which they are frequently used.

[18] Critique of this assumption is explained later.

actions or by elimination of suboptimal

However, the question is, whether markets that allow such learning prevail. In reality, many markets are permanently changing. Moreover, if agents do not share the same views about the likelihoods of different states of the nature, then some suboptimal behavior (suboptimal in global view) can be locally optimal (given mis-beliefs of some agents) and then such a system can be stable, ie. agents have no incentive to change their behavior eventhough it'not globally optimal.

Learning is also suggested as an alternative way for notion of rationality (Vanberg, 2004). In mainstream current, basic introduction is provided in Honkapohja (1993), comprehensible review of the research in this areas is Evans and Honkapohja (1999), introduction with examples of application can be found in Sargent (1993). For more computational oriented comprehensive review see Brenner (2006).

The rational expectations are really strong in their assumptions. The agents in the models are assumed to have all the information about the economic system they are living in. Thus, they are much more clever then any economist. By introducing learning into the model, the agents are placed to the same position as an econometricians, they have to somehow arrive to their model of the economy they are living in. Let's summarize basic area of interest of learning modeling (following Evans and Honkapohja (1999)).

One set of questions which learning model address is still connected with rational expectations hypothesis. The question is, whether learning agents can converge to rational expectation equilibrium. And if it is so, under what circumstances?

Second field of interest is connected with models which contain more then one rational expectations equilibrium. The process of learning then can provide mechanism to select equlibria which are more likely to occur. Another question can be, what conditions are needed to converge to specific equilibrium.

Third, the trajectory toward new equilibrium itself is object of interest. Typically, actual trajectory after structural change would be different under learning and under rational expectations.

3.3.1 Least squares learning

Least squares is the most common econometric technique. It is used extensively in learning models. For example, consider Cobweb model, in which demand and supply is given by:

$$y_t^D = a - bp_t + \varepsilon_t^D \tag{1}$$

$$y_t^S = c + d\mathsf{E}_{t-1}p_t + \varepsilon_{t-1}^S, \tag{2}$$

where ε^D and ε^S are iid normally distributed shocks. Rational expectations equilibrium is then given by

$$p^* = \frac{a-c}{d+b}, \qquad y^* = \frac{ad+bc}{d+b}.$$

The question is, is it possible to converge to this allocation by gathering information about prices and quantities? Assume, that the firm knows its parameters, thus the only unknown are the parameters a and b. The firm can estimate following equation to grasp the knowledge about parameters a and b:

$$p_t = \beta_0 + \beta_1 y_t,$$

and then

$$a = -\frac{\beta_0}{\beta_1}, \qquad b = -\frac{1}{\beta_1}.$$

The equation is estimated by OLS, which, due to its general usage in economics and also in all other sciences, can be considered as a good approximation. The estimates by OLS obtained at time T (thus incorporating the information set prior to period T) are

$$\begin{pmatrix} \hat{\beta}_{0,T} \\ \hat{\beta}_{1,T} \end{pmatrix} = \left(\sum_{t=1}^{T} x_t' x_t \right)^{-1} \left(\sum_{t=1}^{T} x_t' p_t \right), \text{ where } x_t = (1 \quad y_t)$$

The numerical results are displayed on the figure 1. The actual trajectory is random, and depends on the realisation of the shocks We can observe, that learning can converge for long time. The calibration of the model is following: $a = 200$, $b = 1$, $c = 0$, $d = 0.2$, ε^D has standard deviation 0.33 and ε^S with standard deviation 2.

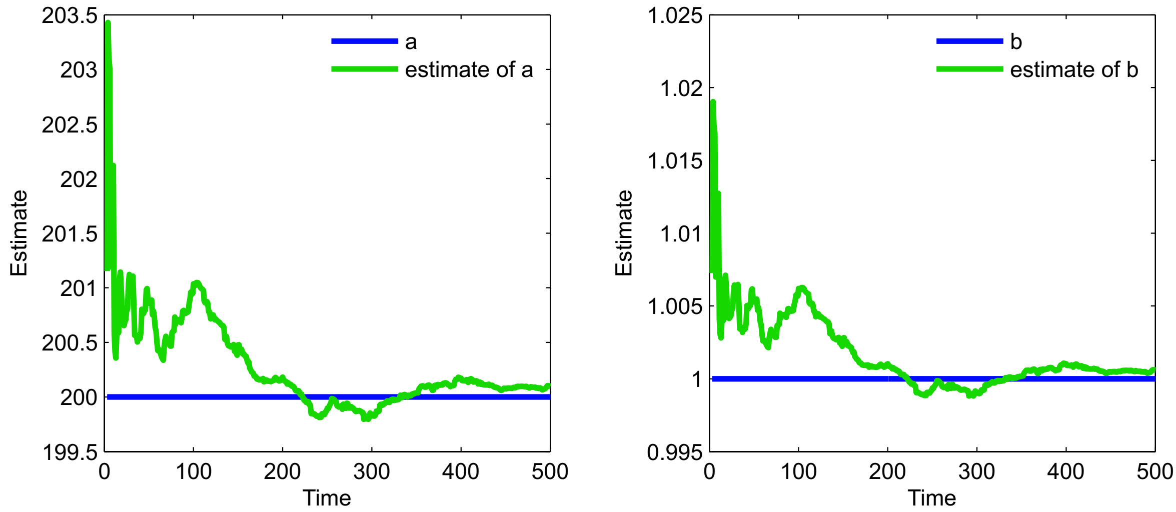

Figure 1: Learning dynamics of the parameters in the cobweb demand equation

Moreover, there is another very interesting issue. It happens that the learning mechanism converge much faster for equilibrial values of quantities and prices then for parameters in (here in demand) equation(s). This is not surprising, because for converge to the former does not imply knowledge of the latter, whereas knowledge of the parameters in demand and supply equation gives us automatically the equilibrial price and quantity.

Once the market has converged to it equilibrium position, there is no incentive to try to deviate from this position. This means, that from this point, the new observations will not be very different from equilibrial values (and this deviations exists only because of the presence of supply and demand shocks). This means, that the new observations will form near singular matrix. In another words, from this point, there is no incentive to deviate and thus there is no new information to learn from neither. Thus, the estimates of parameters in the demand and supply equations may converge surprisingly slowly.

This point has both practical and fundamental aspect. The practical one is, that in our particular calibration, the model converge extremely slowly without adding ε_S. To boost the speed of convergence, the variance of supply shock is much higher then the demand shock.[19] The practical speed of convergence is important for "the learning defense" of rational expectations. Eventhough if the least squares learning is assymptoticaly correct, the actual speed of convergence, due to the issues highlighted above, might be too slow to be of practical use.

The second point is more fundamental. Being very near to the equilibrium of the system does not imply that the agents have correct estimates of the parameters[20]. In one strand of economic though (Lucas, 1986; Sargent, 1993), rational expectations hypothesis is interpreted as situation, in which learning process has converged, rational expectations can be thus interpreted as equilibrium of the learning process of all the agents in the economy.[21] As was demonstrated above, the learning process is not straightforward all the time. But the rational expectations in fact assume that agents have the knowledge about situations which have never occur, because they were perceived as inferior to the actually chosen situation. Least squares learning cannot overcome this point.

Moreover, there is problem with identification of structural brakes. On the other hand, OLS is one of the most basic statistical method, thus one can imagine utilisation of more robust estimation technique.[22]

However, the research in learning is active and new ideas are being explored, for example Aragones et al. (2003); Gabaix et al. (2006); Gilboa et al. (2004).

One reason for adding learning to the models was to answer the critique of rational expectations and show that rational expectations equlibria can be reached by learning by agents. We showed, that even in the most simple case, the cobweb model, the dynamics is

[19]The demand shock affects only price in this simple setting. Note that the demand shock is not observable, without knowledge of true values of parameters a and b, while the supply shock is observable and affects the resulting price (in the next period).

[20]In this case true *structural* parameters.

[21]Lucas (1986):"*Technically, I think of economics as studying decision rules that are steady states of some adaptive process, decision rules that are found to work over a range of situations and hence are no longer revised appreciably as more experience accumulates.*"

[22]In the case of structural brakes, for example one can think of weighted least squares, with more wight added to more recent observations. In the extreme case, one could assign weight 0 to really old observations and do not take them into account at all.

not necessarily straightforward. Thus learning is interesting area for future research, but it cannot provide backing of rational expectations.

3.4 Central banks

Let's now look at reflections of this contemplation in practical economics. In the end, the quality of a tool should be measured by the quality of resulting piece. From this perspective of previous paragraphs, the monetary policy is suitable arena in which alternative economic theories can compete. First, it is trying to purposefully influence the state of the economy to achieve price stability.[23] Secondly, prominent economists always propose some policy measures so it is in the center of research.[24]

Moreover, recently many banks switched to inflation targeting regime. This particular regime, with its transparency, is very demanding in terms of forecasting ability.

3.4.1 Uncertainty reflected by central banks

Central bankers are well aware of the limits of their models.[25] They base their policies on data, which are often later subject to change. This data uncertainty means, that it is not clear, whether an economy is above or below its potential. One hypothesis claims that part of the 70's inflation was due to bad estimate of current output gap (or rather natural level of output) and the policy was too loose because the output was believed to be under potential, not above.

This problem can be called as *data uncertainty*. However, if we assume that the error term in the data has zero mean and symmetric distribution, than this kind of uncertainty should not change the optimal policy. The rationale is that we are aware of the possible errors, but since the probability that the true value is higher is the same as that it is lower, the optimal reaction is the same. This is called *certainty equivalence*.

[23] Often defined as some stable level of inflation.

[24] See Friedman (1970), Modigliani (1977) or Tobin (1980).

[25] See Smidkova (2003) for a recent discussion of uncertainty implications for central banks practicing inflation targeting strategy, or Svensson and Williams (2007).

However, the situation changes when the uncertainty moves from the error terms to the parameters describing the behavior of the system. Brainard (1967) showed that the certainty equivalence does not hold in this case. The optimal reaction, according to Brainard, is then lower than in case of certainty equivalence.

Policy persistence, or in other words interest rates smoothing, is now well established fact. There are many estimates of reaction function of many central banks and typical result is that this smoothing parameter[26] is as high as 0.8. This is sometimes criticised as inability of central banks to react promptly. However, in Brainard's logic, it is a clear way how to reduce consequences of inherent uncertainty.

3.4.2 Shocks

One of the most famous results of rational expectations concept is so called *dynamic inconsistency* (see Kydland and Prescott (1977)). The inflationary bias is the result of the agents knowing, in line with rational expectations, the true model of the economy. That comprises of the objective loss function of the central bank and the description of the economy, captured by Phillips curve. Rules are proposed as a way how to decrease this efficiency loss. Central bankers know, that if they broke their promise, the agents would switch to "discretionary regime", resulting again in high inflation bias and loss of welfare.

However, it seems quite difficult even for economists to asses quality of monetary policy conduct of different real central banks in developed market economy. There is a recent debate, comparing and discussing promptness and agresivity of European Central Bank and Federal Reserve during the period of slowdown in the first half current decade. From this first glance, the policy of ECB seems to be inferior and lagged behind the FED's policy measures. Recently, however, the discussion seems to have directly opposite momentum. Christiano et al. (2007) based on detailed estimation of various shocks affecting both economies claim that in fact the ECB had to deal with more severe conditions and coped with it soundly.

[26] ρ in $i_t = \rho i_{t-1} + (1-\rho) f(y, \pi, \ldots)$, where $f(y, \pi, \ldots)$ is some function of current or forecasted variables like output and inflation gap, etc.

The point is, that it is very difficult to analyze anything in the economy, even with state of the art model and even after some time. It seems impossible to asses precisely the shocks and their impact at the moment when they stroke. Still, standard business cycle models work with agents who realize the nature of the shock in the period when the shock appears, instantly re-optimising the future and current path of endogenous variables (or central bank re-asses its interest rate in case of neokeynesian framework with price rigidities).

3.4.3 Implications of degree of credibility

Higher credibility can make the policy more effective. The actual values of inflation and output obviously depend on the state of public expectations. This is a practical example of path dependency. Again, how precisely can the public tell the difference between a shock from policy (which was carried out deliberately by central bank to inflate the economy and expand the output) and a shock outside of the control of central bank? How the general public can asses the functioning of a central bank, even if the economists can not give simple answer? Still, Muth, Lucas, Kydland or Prescott in their models assume that the public knows.

If higher credibility makes the job for central bank easier and vice versa, then some exogenous shocks which made economic conditions harsh (for example supply shocks, which tend to lower output while increasing inflation) can for long periods decrease the central bank's credibility and make the trade off between inflation and output more expensive[27], resulting in higher inflation, eventually confirming public expectations. This is one of the most striking arguments, why transparency is said to by one of the key issues for modern central banks.

[27]If we take think in the neokeynesian monetary framework with the Phillips curve based on inflation expectations.

3.5 True structural parameters

To connect parts of the previous text, the true structural parameters are in fact assigning some values to the metastructure about which we had talked in the second section. The search of true baseline starts from current microeconomic description of economic agent as someone maximising his utility function.

One possible next step is enriching of the utility function of new arguments. Utility is no longer only function of consumption, but it includes working discomfort. A variable depiction costs of reasoning and deciding process can be included. The consumption need not to be in absolute values, it could be also in relative values to the consumption of others, taking into account social status issues.

One argument against mainstream rationality assumption is that humans are not able to maximise in the way neoclassical microeconomics wants them to. However, it could be argued, that maximisation principle should not be taken too literally. It could be argued that the arguments are more important then the maximisation principle. For example, inclusion of "decision making cost" would result in situation in which agents do not reoptimise frequently.

When applied for forecasting, the mainstream methodology can be described by two assumption.[28] The first is that the model is good representation of economy and at the same time, that the structure of the economy will remain relatively unchanged. In this setting, the predictions have favorable features. For example, it can be shown that the model which fits the data in the sample most accurately should be the best in predicting out of the sample.

However, the reality is different, so we should be aware that *models are simplified representations which are incorrect in many ways* and that *economies both evolve and suddenly shift* (Hendry and Clements, 2003, page 6). In this setting, there many reasons why model do not predict well. For example Hendry and Clements (2003, page 7) claim that the most of the forecasting error can be attributed to the shifts in the coefficients of

[28] See Hendry and Clements (2003).

deterministic terms.

3.6 Model modifications-middle way

Let's sum up the preceding considerations. The models based on general equilibrium with rational expectations are analytically tractable and internally consistent with simple assumption about micro decision making. These assumptions are clearly simplifying.

The opposite way is to try to build as realistic model as possible. Such models are however often difficult to deal with numerically and thus impractical for quantitative analysis. Such models are therefore of only limited use in policy making.

The middle way is to use core from micro optimisation and try to enrich the models with features which bring realism to particular area. In such a way, the model is still able to deliver quantitative results, yet is able to grasp realistically behavior of same part of the economic system.

Obviously, this middle position is questionable from both sides. From the neoclassical point of view, the new realistic features are often only ad hoc assumption. Because the internal consistency was compromised, the model is no longer logically sound construct.

The critique from the other side argues, that this kind of modeling is in fact not different from neoclassical mainstream and as such is based on completely unrealistic assumptions. And model with unrealistic assumptions cannot deliver realistic description of reality.

Both critiques are true in some degree. The middle way always inherit some negative aspects from its more theoretically extreme origins. It inhabits some positives too. This compromise makes it possible to work with at least partially internally consistent models which are trying to be realistic (at least in particular situation which is being modeled). The fact, that central banks use models based on this prism of reality could serve as a positive sign.

Thus, using neoclassical core and enriching it for realistic features seems to be the right way to pursue. The following sections contains a trial to follow this route. The model presented below is in core standard neoclassical RBC model in its simplest form.

The realism is added by denying the agents to know the true value of the shock hitting the economy. Not only the agent do not know the current realisation of shock to the technology, its value is not revealed at any time period later. By this construction, the agent never know the true state of the technology in the model and thus have to optimise under limiting informational constraint. The resulting behavior has some similar features to standard RBC models, yet some implications are different.

At the same time, one should be aware of the limitations. The chosen path does not lead (or at least in the short run) to complete universal model which would explain everything. The uncertainty in reality is fundamental and irreducible to risk, thus all the issues we talked about in sections about time and role of expectations implies that any quantitative model can deliver reasonable results only on short horizons.

It could be argued that this way of thinking is in line with open system of thought as depicted above.[29] It is because it is trying to build specific models to address specific question. Thus the model can be realistic in particular area which modeler wants to address.

[29]This is the near to the postkeynesian way of thinking, see Dow (1996, page 61,76).

4 Model

Let's now summarize the conclusions of the previous paragraphs. Uncertainty is found to be important in real economy, affectioning decision making of real agents. From this perspective, the simple neoclassical ergodic is simplistic and its assumptions a way too strong. Once we take the fundamental uncertainty into account, the economy starts to exhibit persistent behavior, because agents only slowly recognize the exact nature of the shocks. Because the true structure of the economic rules is uncertain, learning from actual results of actions is important. Once learning is important, the path dependency occurs, because the result today are influenced by the historical record of the economy.

The question is, whether a simple formal model can account for behavior described above. This paper advocates affirmative answer. The modern macro modeling, both RBC or neokeynesian framework, is based on Ramsey allocation problem. If we want to build a realistic model, we should start with realistic assumptions. The trial is therefore to modify the assumption of standard RBC model. The basic framework is taken from the basic RBC model, described for example in (Romer, 1999), which is based on original article (Kydland and Prescott, 1982). Note, that we do not need to work with more complicated versions at this stage, since it is most illustrative to describe the mechanics of the model with modification just in the simplest settings (one factor of production, etc.).

In canonical RBC, the uncertainty is modeled by stochastic disturbance to the technology in the production function. Moreover, this shock is immediately observed by the agents. In the text, the difficulties with estimation of the shock were discussed.

Therefore, let's modify the assumptions in following way. Let's include a new white noise shock to the output and make the shocks themselves unobservable. What is observable (and measurable in the reality) is level of output. So the agents will have to disentangle movements in output into two shock, classical RBC persistent shock to the technology and a new white noise shock to the total output. Because the endurance of the shocks differs, so would the reaction of the agents, if the shocks were observable. The question is, whether this modification can introduce to the model the notions discussed above.

4.1 RBC models in the literature

The whole RBC literature started by seminal paper by Kydland and Prescott (1982). Modern tractation of RBC methodology and literature is present in King and Rebelo (1999). RBC model is base for many modifications and extensions. RBC is also often criticized from various grounds (Summers, 1986; Mankiw, 1989). Yet modern dynamics stochastic general equilibrium models (DSGE) are using RBC core and are adding nominal rigidities and such models are used frequently in policy modeling in central banks.

Christiano (1988) explores what are the implication of not fully disclosing informations about technological shocks to agent. However, the information about the realisation of the shock is revealed after one period. Yet this setting can account for high inventory variability. The authors explore what are the implication of not fully disclosing informations about technological shocks to agent. However, the information about the realisation of the shock is revealed after one period. Yet this setting can account for high inventory variability.

RBC model with two different kinds of shocks is used for example in Aguiar and Gopinath (2004). This paper explains difference in business cycle patterns in developed and in developing countries. The new shock is in trend growth steady state, the interpretation is such that in developing countries, there are often structural reforms, big privatisations etc., which may affect the productivity growth in scale which is no longer possible in developed world. However, when the pendulum swings back, there can be on the other hand very big fall in productivity. Because this changes affect the permanent income of the population, the implication for consumption are much mere intense, because the agents do not smooth the consumption (because the shock are in growth level and thus are not transitory as are the shock in standard RBC). This setting can help to explain so called sudden stops, deep fall in output observed in developing countries crises, for example during tequila crisis in Mexico in 90's.

4.2 Definition

In this economy, the timeline is as follows. At the beginning of every period, the values of the shocks are drawn from their distributions. The values are not observed by the agents. Then the output is produced with the existing stock of capital and the volume of produced output is observed. With knowledge about distributions of the shocks (or its variance), stock of capital and previous belief about the level of technology, the agents guess the values of the shocks, based on the difference of the actual output and expectations based on the guess of technology from the previous period. From the inferred values of the shocks, the expectations of the next period level of technology is formed. Then the intertemporal optimisation takes place, the agents decide about the level of consumption and investment, resp. level of capital for the next period.

The production in period t is given by one factor Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} + \varepsilon_{Yt} \qquad \alpha \in (0,1), \tag{3}$$

where ε_{Yt} is shock to output at time t, K_t denotes total capital stock at time t. The total factor productivity at time t follows autoregresive process:

$$\log A_t = \rho \log A_{t-1} + \varepsilon_{At}, \tag{4}$$

where ε_{At} represents shock to the total factor productivity at time t.[30] The shocks are normally distributed

$$\forall t : \qquad \begin{bmatrix} \varepsilon_{Yt} \\ \varepsilon_{At} \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma \right), \qquad \Sigma = \begin{pmatrix} \sigma_Y^2 & 0 \\ 0 & \sigma_A^2 \end{pmatrix}. \tag{5}$$

The output can be either consumed or invested:

$$Y_t = C_t + I_t,$$

[30]This is a bit different from standard definition of technology, for the reasoning behind see section 4.3.

and the evolution of capital is ruled by

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where δ is the rate of depreciation. The budget constraint then looks like:

$$K_{t+1} = A_t K_t^{\alpha} + \varepsilon_{Yt} - C_t + (1-\delta)K_t. \tag{6}$$

The total utility of representative household is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t), \tag{7}$$

where β is the household subjective discount rate. For simplicity we assume $u(C_t) = \log(C_t)$.

4.3 Modification of Technology in the model

In the model the traditional Cobb-Douglas production function $Y_t = A_t K_t^{\alpha}$.

$$\log A_t = \rho \log A_{t-1} + \varepsilon_{At}.$$

We can plug this into the production function at time t to see the influence of the technology A_{t-1} and the shocks ε_{Yt} and ε_{At}. That is

$$\begin{aligned} A_t &= \exp(\rho \log A_{t-1} + \varepsilon_{At}) = A_{t-1}^{\rho} \exp(\varepsilon_{At}) \\ Y_t &= (A_{t-1}^{\rho} \exp(\varepsilon_{At}))K_t^{\alpha} + \varepsilon_{Yt}. \end{aligned} \tag{8}$$

Taylor polynomial for a function f at point a is

$$T_{f,a}(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

If we want to make a first order approximation of exponential function at $a = 0$ we get $f(x) \approx \frac{\exp'(a)}{0!}(x-a)^0 + \frac{\exp''(a)}{1!}(x-a) = 1 + x$. Therefore, we can simplify

$$\exp(\varepsilon_{At}) \approx 1 + \varepsilon_{At}.$$

We know, that A is fluctuating typically in some region near 1. So similar approximation can be done for A^ρ, where $a = 1$ and $f(x) = x^\rho$. So for A near 1, the result is

$$A_{t-1}^\rho \approx \rho A_{t-1},$$

let's define $\tilde{A} = A - 1$, so $\tilde{A} + 1 = A$ and we have

$$A_t \approx \rho(\tilde{A} + 1)(1 + \varepsilon_{At}) \approx 1 + \rho\tilde{A}_{t-1} + \varepsilon_{At}, \tag{9}$$

if we ignore the cross products. Obviously, when making simplifications, one has to be very cautious not to be diverted too far. In this case, the production function still have reasonable shape, with technology behaving as we would expect:

$$Y_t = (1 + \rho\tilde{A}_{t-1} + \varepsilon_{At})K_t^\alpha + \varepsilon_{Yt}. \tag{10}$$

We can see, that the technology can be interpreted as percentage deviation from some steady state. The output can never be negative in the classical RBC model. However, if we add an additive shock ε_{Yt}, it could happen, that the output would end up in negative values, given some extreme value of ε_{Yt}. If we make the linear approximation, again, we obtain negative output.

However, when we consider such a model, we always assume that the economy behaves reasonably. It seems realistic not to assume such extreme values for shocks, while keeping the formalism and normality for the sake of easier derivation. Moreover, such extreme shocks could be for example result of a total war, where standard economics does not work.

Simply, we have to keep in mind, that the result of the model for higher values of shocks are meaningless. Similar simplification is standard log-linearization procedure anyway. It would be possible to avoid to make this step, then in 4.6 we would need to work with one lognormal and one normal distribution, while with this step we can work with two normal distributions, which make the derivation much more easier.

At the end of the day, ε_Y could be implemented more directly by multiplying the cobb-douglas term of production function. Then we would that it is normally distributed and we would need the same discussion about plausible values of the shock. We could assume other distribution, for example uniformly distributed shock with narrow plausible interval. In that case, though, the derivation would not be as straightforward as in the case of normal or lognormal distribution.

4.4 Uncertainty in the model

The shocks, ε_{Yt} and ε_{At}, are not observable. Agents at time t observe only the last realization of total output Y_{t-1}. As the value of A_t is not observable, in order to form the expectations $E_t A_{t+1}$, the current level A_t has to be guessed by agents somehow. Assume, that there is some guessed level technology at time zero, $\hat{A}_0$. Agents compare current output with their expectations from previous period. Because $\mathrm{E}\varepsilon_{Yt} = \mathrm{E}\varepsilon_{At} = 0$, then from 4 we define the difference between actual and expected level of output as

$$\Delta_t := Y_t - E_{t-1} Y_t. \tag{11}$$

Agents have to guess the value of the shocks based on the observed difference of current output from its estimate. The best guess is the most likely one, ie. such that maximises the likelihood

$$\begin{pmatrix} \hat{\varepsilon}_{At} \\ \hat{\varepsilon}_{Yt} \end{pmatrix} = \operatorname{argmax} P(\varepsilon_{At}, \varepsilon_{Yt} | \Delta_t, K_{t-1}^{\alpha},) \tag{12}$$

$$\hat{\varepsilon}_{At} = f(\Delta_t, K_{t-1}^{\alpha}, \sigma_A^2, \sigma_Y^2),$$

$$\frac{\partial f}{\partial \Delta_t} > 0, \quad \frac{\partial f}{\partial \sigma_A^2} > 0, \quad \frac{\partial f}{\partial \sigma_Y^2} < 0, \quad \frac{\partial f}{\partial K_{t-1}} < 0.$$

4.5 Delta

Clearly Δ_t in equation (11) has to be the result of the shocks in the current period. If we use the approximation derived in 4.3, then

$$Y_t = A_t K_t^{\alpha} + \varepsilon_{Yt} = \left(1 + \rho \tilde{A}_{t-1} + \varepsilon_{At}\right) K_t^{\alpha} + \varepsilon_{Yt}.$$

Then we can derive

$$\begin{aligned} \Delta_t &= Y_t - E_{t-1} Y_t \\ &= (1 + \rho \tilde{A}_{t-1} + \varepsilon_{At}) K_t^{\alpha} + \varepsilon_{Yt} - (1 + \rho \hat{A}_{t-1}) K_t^{\alpha} \\ &= (\tilde{A}_{t-1} - \hat{A}_{t-1}) K_t^{\alpha} + \varepsilon_{At} K_t^{\alpha} + \varepsilon_{Yt}. \end{aligned} \tag{13}$$

Let's assume, that the agents at time t believe the previous estimate of A_{t-1}. Therefore $A_{t-1} - \hat{A}_{t-1} = 0$. The agent could optimise even more, re-estimating all the historical path of shocks. There is scope for future research, for example to develop some learning schemes. But for the moment we stick to the assumption, that $A_{t-1} - \hat{A}_{t-1} = 0$.

Then the expectations of technology in the next period is

$$E_t A_{t+1} = \rho \hat{A}_t = \rho(\rho \hat{A}_{t-1} + \hat{\varepsilon}_{At}). \tag{14}$$

4.6 Derivation of the most likely decomposition

Following (Andel, 2005), let's assume that X and Y are independent normally distributed variables with mean value equal to 0. Let's denote their distribution and variance by corresponding subscript. We observe their sum, and we want to find the most likely decomposition into guesses of x and y.

$$x + y = Z, \qquad y = Z - x.$$

Given Z, the likelihood is

$$L(x) = p_{x,y}(x, y|Z) = p_x(x)p_y(Z - x) = \varphi_x(x)\varphi_y(Z - x).$$

We want to find values of x and y (resp. x and $Z - x$) such that the probability above is maximised.

$$L(x) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{x^2}{2\sigma_x^2}\right) \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(-\frac{(Z - x)^2}{2\sigma_y^2}\right). \tag{15}$$

Logaritmization is increasing transformation, so it does not change the location of the extreme.

$$\log(L(x)) = l(x) = \log\left(\frac{1}{2\pi\sigma_x\sigma_y}\right) - \frac{x^2\sigma_y^2 + (Z - x)^2\sigma_x^2}{2\sigma_x^2\sigma_y^2},$$
$$l(x)' = 0 \quad \Rightarrow \quad x^* = \frac{\sigma_x^2 Z}{\sigma_x^2 + \sigma_y^2}. \tag{16}$$

To use this result in the model, we need to know, that if ε_A is normally distributed with variance σ_A^2, then $x = K^\alpha\varepsilon_A$ has variance $\sigma_x^2 = \sigma_A^2 K^{2\alpha}$.

Thus the most likely value of $\hat{\varepsilon}_{At}$ is[31]

$$\hat{\varepsilon}_{At} = \frac{K_{t-1}^{2\alpha}\sigma_A^2}{K_{t-1}^{2\alpha}\sigma_A^2 + \sigma_Y^2}\Delta_t, \tag{17}$$
$$\hat{A}_t = \rho\hat{A}_{t-1} + \hat{\varepsilon}_{At} = \rho\hat{A}_{t-1} + \frac{K_{t-1}^{2\alpha}\sigma_A^2}{K_{t-1}^{2\alpha}\sigma_A^2 + \sigma_Y^2}\Delta_t.$$

Equation (17) is intuitive result which occurs often in solution of so called *signal extraction problem*. The agents react to the signal and make their guess about true values of shocks in such a way to account for different variance of the shocks. For example, if σ_Y^2 is very high in comparison to σ_A^2, then agents will expect that the main source of signal is shock to the total output ε_Y and thus will expect the shock to be only transitory (because of the nonpersistence of ε_Y).

4.7 Path dependency

From 14 we can see, that

$$E_tA_{t+1} = \rho\hat{A}_t = \rho(\rho\hat{A}_{t-1} + \hat{\varepsilon}_{At}) = \rho^2(\rho\hat{A}_{t-2} + \hat{\varepsilon}_{At-1}) + \rho\hat{\varepsilon}_{At} = \dots$$
$$E_tA_{t+1} = \sum_{i=0}^{t-1}\rho^{i+1}\hat{\varepsilon}_{At-i} + \rho^{t+1}\hat{A}_0. \tag{18}$$

Therefore agents base their expectations of today productivity on the whole history. Because they only make a guess (although the best possible) about the relative importance of the both shocks, the true nature of the shock will move the economy away from some dynamic steady state.

Note that once the agents make mistake, this mistake is carried to the future and it is mitigated by ρ. The probability that the agents' guess is 100% correct is zero, the probability that the economy would be in equilibrium is zero as well.

Moreover, once one mistake is done and the estimate of technology is mistaken, then

[31]Because level of K doest not fluctuate substantially, in numerical simulation is actual level of capital substituted with steady state level of capital.

all future estimates will be biased. For example, imagine that $\Delta_t > 0$ is the result of $\varepsilon_{Yt} > 0$ only and $\varepsilon_{At} = 0$. Assume that $A_{t-1} = \hat{A}_{t-1}$. However, the agents' estimate would be such, that $\hat{\varepsilon}_{Yt} < \varepsilon_{Yt}$ and $\hat{\varepsilon}_{At} > 0 = \varepsilon_{At}$. Assume, that there are no future shocks, so $\varepsilon_{At+i} = \varepsilon_{Yt+i} = 0, \quad i = 1, 2, \ldots$ Because agents guessed $\hat{\varepsilon}_{At} > 0$, so the expectations of future technology is higher, then it actually is. When the output in the next period is realized, it is lower, then expected, because the true level of technology is lower then its guess. $\Delta_{t+1} < 0$ has to be explained by a guess of the shocks $\hat{\varepsilon}_{Yt+1}$ and $\hat{\varepsilon}_{Yt+1}$. The estimate would be that both $\hat{\varepsilon}_{Yt+1}, \hat{\varepsilon}_{At+1} < 0 = \varepsilon_{At+1} = \varepsilon_{Yt+1}$. The actual values of the guesses depend on the varinaces of shocks, σ_A^2 and σ_Y^2.

Every shock is bringing in some uncertainty, which affects all the future knowledge about the conditions in the system. The shock then affects the economy for much longer periods.

4.8 Solution

In this economy, there is neither information assymetry nor externalities, the competitive equilibrium is therefore the same as the solution to social planner problem, which is to maximise

$$\max U = \mathsf{E} \sum_{t=1}^{\infty} \beta^t u(C_t), \tag{19}$$

$$\begin{aligned} \text{s.t.} \quad K_{t+1} &= A_t K_t^\alpha + \varepsilon_{Yt} - C_t + (1-\delta) K_t, \\ \hat{A}_t &= \rho \hat{A}_{t-1} + \frac{K_{t-1}^{2\alpha} \sigma_A^2}{K_{t-1}^{2\alpha} \sigma_A^2 + \sigma_Y^2} \Delta_t, \\ A_t &= \rho A_{t-1} + \varepsilon_{At}, \\ &\hat{A}_0,\ K_1,\ \varepsilon_A \sim \mathrm{N}(0, \sigma_A^2),\ \varepsilon_Y \sim \mathrm{N}(0, \sigma_Y^2). \end{aligned}$$

When deciding about optimal consumption path, the problem is merely the same, as in standard RBC model (because $E\varepsilon_Y = 0$). The only difference is, that actual value of technology is only estimated, not observed. Therefore, the solution take the same Euler equation form

$$u'(C_t) = \beta E_t \left[(1 + r_{t+1}) u'(C_{t+1}) \right], \tag{20}$$

and the difference is that interest rate is more complex to derive. In standard RBC, it is the user cost of capital, ie. the marginal product of capital minus the depreciation rate. In standard RBC, r_t is decided simultaneously with output, whereas here, the interest rate is decided after the output is observed. Given $\hat{A}_t$, then rational expectation $E_t[A_{t+1}] = \rho \hat{A}_t$. Therefore, the expected marginal product of capital is $\rho \alpha \hat{A}_t K_{t+1}^{\alpha-1}$. The expected user price of capital (interest) is

$$\delta + E_t r_{t+1} = \rho \alpha \hat{A}_t K_{t+1}^{\alpha-1}, \tag{21}$$

which is corresponding to standard RBC model.

5 Results

To simulate the model, we have to firstly assign some values to the parameters of the model. With reference to King and Rebelo (1999), the parameters are set as follows:

α	β	δ	η	ρ	σ_A	σ_Y
0.36	0.95	0.05	1	0.9	0.005	0.002

Table 1: Calibration of the model

In the model, there are four possible sources of shocks, three real and one from agents' expectations. The real shocks are

1. initial shock to the level of capital,
2. ε_z, persistent shock to level of technology,
3. ε_y white noise shock to the level of output.

On top of these, there is another possible source of misalignment in the economy, the wrong expectations of the agents' about the state of the technology.

5.1 Impulse responses

The impulse responses are shown in figures 2 to 7. There are always two pictures in one figure. The left one is a standard impulse response graph. On the right hand side, the graph displays trajectories of true level of technology, the estimated level of technology (the level of technology which is expected by the agents) and the resulting mistake. Due to the definition, the mistake also depicts the resulting guess of the noise shock to the level of output.

Impulse response to the shock to the technology ε_A is depicted on the figure 2. We can observe that only part of the positive deviation of the output is explained by noise output shock ε_Y, thus the expected level of technology is lower then its true level. The result is that agents are not able to exploit completely positive period of higher technology and accumulate less capital then in standard RBC model settings.

Impulse response to the noise shock to the output ε_Y can be seen on the figure 3. Contrary to the previous case, now the agents believe that there was some positive technological shock eventhough the deviation of output was only due to the noise shock. We can observe interesting pattern on the mistake trajectory. In the first period, the mistake is positive, because the expected level of technology is higher then the true level. In the next period, however, the mistake is negative. It is because the agents still believe that the technology is still higher then its steady state (because the technological shock is persistent). But output is lower then it was supposed to be. Thus the explanation is that there occurred some negative output shock. The agents also lower their estimate of level of technology. There are two reasons, one because the positive technological shock is not permanent and secondly because if there was lower output then it was expected, in addition to the negative output shock there must have been also negative technological shock (which lowers expected level of technology a bit, but it remains above its steady state).

Lets assume the same noise output shock with the agents knowing the full information about the realisation of the shock. Then the response would look like portraited on 4. The mistake is present only at the beginning of the first period, when the realised output is higher then expected. However, the agents are told the true level of technology so they optimise as in standard RBC framework.

Response of the model to the deviation in the level of capital, which is not caused by shock is depicted on the figure 5. Note that because there was no shock, there is no mistake in disaggregation between technological and noise shock. The response is thus nearly similar to the previous case (the only exception is the mistake in the first period, which however does not have any implication for the shape of the dynamics of the model).

Suppose that there is a shock to the technology, which is correctly anticipated, is shown in figure 6. In fact, the impulse responses of the model are the same as in standard RBC model, because by giving the information about the true level of technological shock, the uncertainty is eliminated. Thus we can regard the model depicted in this text as a generalisation of standard RBC. Standard RBC model is thus a special case of the model with setting $\sigma_Y = 0$.

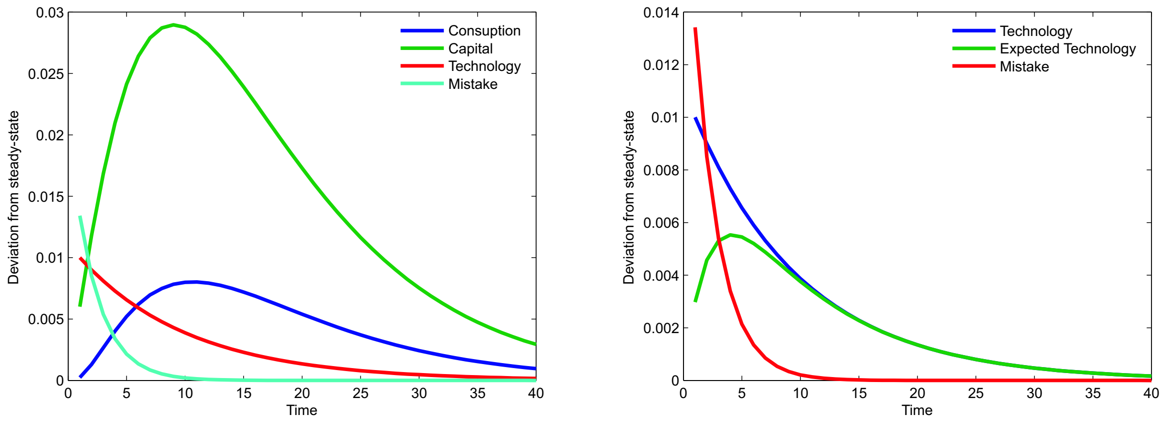

Figure 2: Shock to the level technology

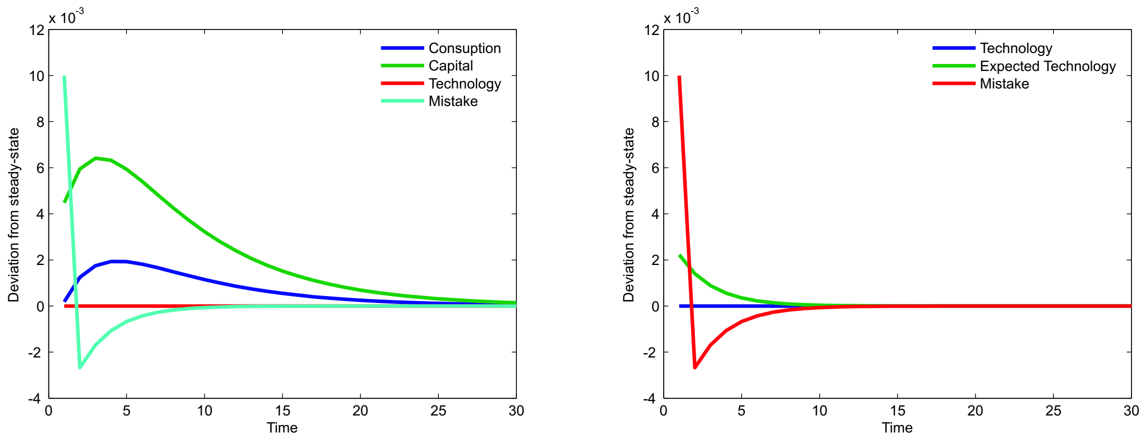

Figure 3: Shock to the level of output

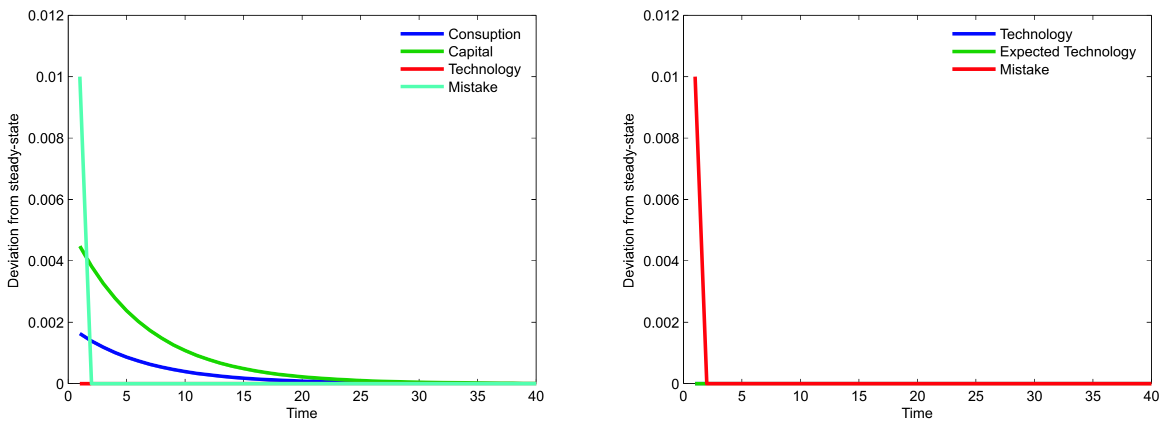

Figure 4: Shock to the level of output with full information

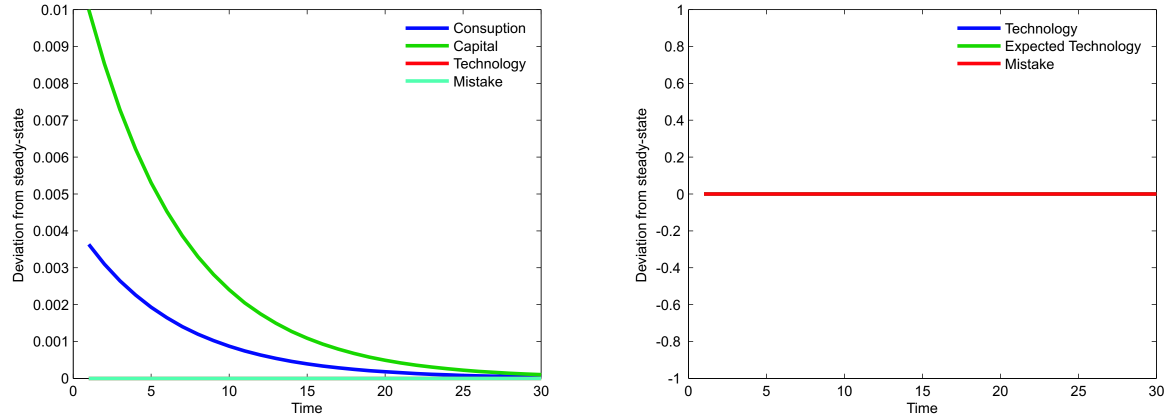

Figure 5: Shock to the level of capital

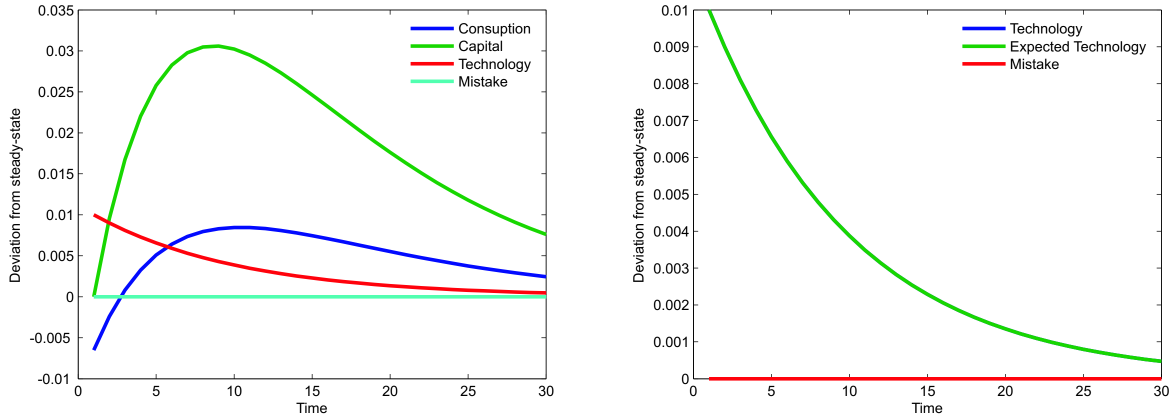

Figure 6: Shock to the level of technology with full information

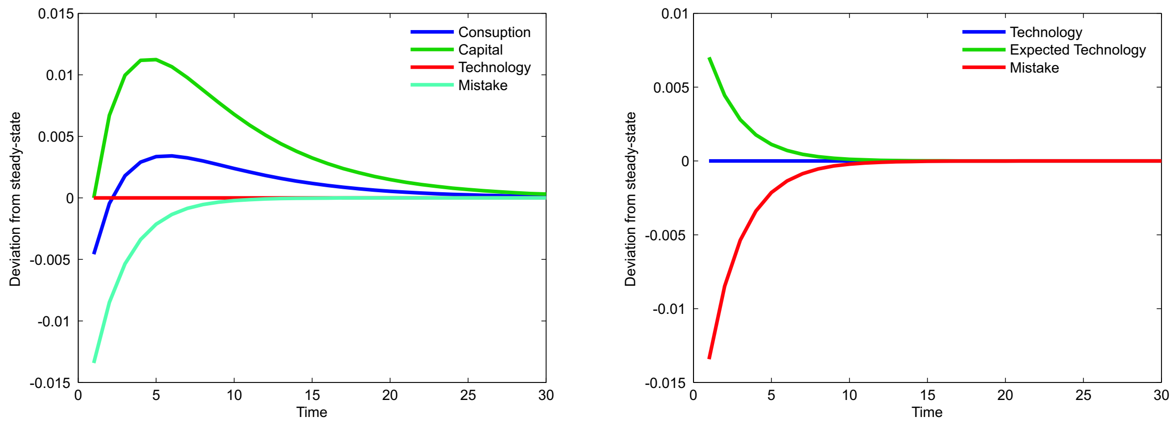

Figure 7: Shock to the expectations of the level of technology

The case, in which we believe that there was a shock to the level of technology, but there is no underlying technological shock that would justify the change in expectations is interesting. We can see, that mere expectations of higher productivity (which are not based on the real productivity), can produce trajectory similar to the situation of true technological shock, only of mitigated amplitude.

This is very interesting feature of the model, so let's discuss it a little further. This would not be possible in standard RBC. The reason why it is possible here, is the presence of noise output shocks. Thus people can believe that there is higher level of technology and that lower output is result only of a transitory of bad luck.

This can be interpreted that period of positive expectations about the state of the economy can result in similar shape of trajectory as actually higher level of technology. Keynesian economists relate to this effect, because original Keynes explanation of business cycle was build on entreprenial animal spirits and the psychology as the driving force behind investment.

The fact is that this cycle has lower amplitude than standard one. However, it is possible to come up with amplifying mechanisms. For example, imagine that the agent are divided to two groups and each group has different informational sets. The first one is behaving according to our model and has corresponding informational set. In the situation of higher expected productivity, they will behave according to the figure 7. The second group has different information set and do not form this optimistic behavior. However, it is only because of the reaction of the first group, the output is higher than its steady state. The second group can interpret this unexpected rise in output as an productivity shock and thus they can also start believing that the level of technology is above its steady state.

5.2 Simulations

Now let's simulate the model over longer time period[32] and compute the moments of the variables. Let's compare two cases, one is our benchmark case where agents do not have

[32]Results reported were calculated from simulated time series 500 periods long. Longer time span does not change the results.

information about the current state of the technology. The second is the case, where agents posses the complete information about the state of the technology and thus are surprised only by the noise (and they know it). Table 2 summarizes the resulting variances of deviations of consumption, capital and mistake in the expectations of realised output, viz section 4.5, equation (11).

	var(k)	var(c)	var(Δ)
benchmark	9.64e-005	8.28e-006	3.49e-005
alternative	1.37e-004	1.18e-005	2.78e-005
relative bench vs. alt	0.7	0.7	1.25

Table 2: Simulated variance

The absolute values are not that important because they result from the calibration of variance of the shocks. Thus more informative is the last row of the table 2, which reports relative values. As expected, the benchmark is more volatile in the Δ, but less volatile in capital or consumption. What we observe is that agents are unable to tell the persistent technological shock from unpersistent output shock. Therefore, they are less aggressive in their reactions, less exploiting possible gains when productivity is higher. On the other hand, they can also react to purely output shock, which rests only for 1 period by accumulating capital as it was a technological shock.

The figure 8 shows the simulation of the economy in the uncertainty case.

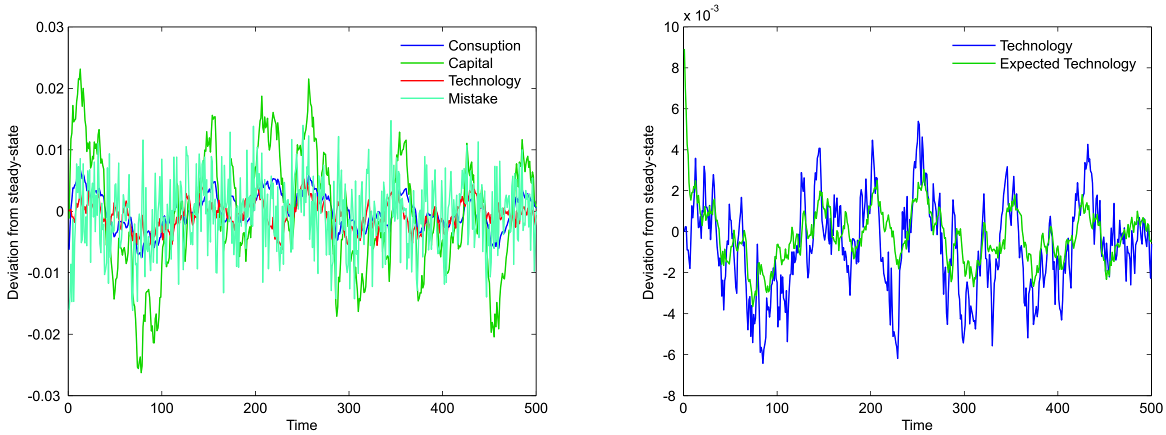

Figure 8: Simulation

The figure 9 shows the situation when agents observe correctly current technological level when optimising for next period.

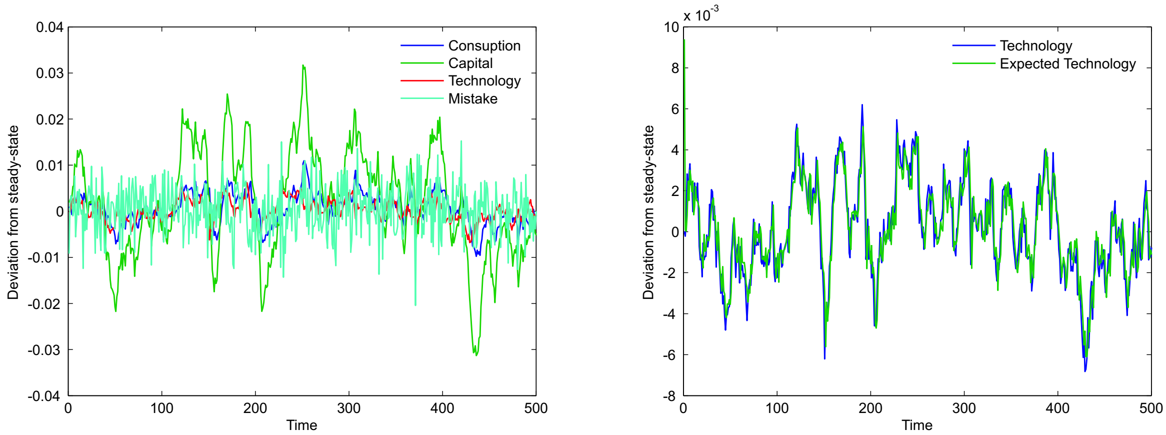

Figure 9: Simulation of benchmark RBC case

The difference between the figures 8 and 9 is striking. In the modified model, there occur long periods with significant mistake in the evaluation of the level of the technology.

However, we cannot compare these results with either empirical results (as for example in (Stock and Watson, 1999)) or developed RBC models. The reason is, that the model developed in this text is very simplified and works with one factor only. However, even this is enough to account for the features we wanted, ie. persistence and path dependency. This task is left for future research.

6 Conclusions

This text explored notion of uncertainty and proposed a modification of standard RBC model which addresses uncertainty more realistically. The impulse response functions and simulations were analyzed to show the differences to RBC benchmark.

Methodology We started by discussion of methodological issues and the reflections of these issues in mainstream neoclassical paradigm and postkeynesian alternative. It was observed that the way how uncertainty is perceived can be regarded as the most vivid distinction.

It was argued that in mainstream economics, the uncertainty is reduced to mere risk. In mainstream paradigm, the agents are given all the knowledge about the economic system they are living in. Contrary, alternative views stresses the importance of uncertainty in economic behavior. The agents do not know the true econonomic system. Moreover, the system itself is evolving and possibly suddenly shifting, thus knowledge of yesterday's behavior has only limited implications for the system today.

The micro based optimisation general equilibrium concept was yet regarded as useful unifying concept and practical core for modeling. However, the models should be judged by its realism of assumptions. This brings some tension into the modeling because the core assumptions about agents' rationality and optimisation are strongly simplified. The realism should be thus brought by enriching the core with certain realistic features which are helpful to solve particular question (or to model one selected market). By this construction, the model is not general, but it could address specific questions in the selected area and, at the same time, the model remains quantitatively tractable.

As one example how to make models more realistic, the text discusses so called learning mechanisms and gives a numerical example of learning dynamics in cobweb supply-demand model. Even in this extremely simple setting, the learning dynamics is found not to be completely straightforward and thus it is argued whether by learning the agents can converge to rational expectations knowledge[33] about the structure of the economy.

The rest of the methodological part of the text contemplates about implications of mainstream methodology, particularly the demand for models which are internally consistent. The internal consistency could constraint the research agenda to only such questions which can be addressed by micro optimisation derived models. However, it can be argued that this would retard policy related research. Hybrid New Keynesian Phillips Curve can be regarded as an example.

Model modifications The modification of standard RBC model presented in the text is adding more uncertainty into the model. The agents do not observe the true value of a technological shock. Another shock is introduced, this shock affects the actual output and is not persistent (where as the technological shock is persistent, as in the standard RBC setting). Neither the true value of this shock is known to the agents. Because the shocks have different persistence, they also have completely different implication for behavior of the agents in the model (consumption vs. capital accumulation).

The agents thus have to attribute movements in output (which is the observable) to the combination of the shocks. Because the agents do not know the true realisation of the shocks, they can form expectations about future technology which will not be met. The consumption and capital accumulation path are affected as well.

Persistence The presented model is more persistent then standard RBC. The reason is that part of the total shock to the output is ascribed to the output shock, which is white noise, therefore does not affect the productivity in the next period. Note that this is the case even if more complicated model of learning would be applied. Another source of persistence is due to the fact, that the agents are allowed to react only with one period delay, in comparison to the classical RBC. This source of persistence is only supplementary.

The persistence is sometimes implemented into standard RBC model ad hoc, to improve the fit on the actual data. The model presented in the text attains the persistence by result

[33] Rational expectations knowledge is the knowledge assumed by rational expectations, ie. the structural equations and the parameters of the economy.

of “rational” behavior of optimising agents. The implication of uncertainty is in a sense inspired by Brainard (1967), and the implication is the same, in the presence of uncertainty, it is “rational” to react less.

Moreover, because part of the shock is always believed to come from the noise output shock, thus the reaction is less aggressive. This also means, that in order to fit the data, one needs stronger source of persistent shocks then in standard setting. However, even the standard setting is criticised as it is difficult to find corresponding technological shocks in reality (Mankiw, 1989). Thus in this setting, this critique is even more pronounced. This modification of RBC model thus adds more arguments against using technological shocks as main business cycle driving force.

Path dependency Once the shocks are different then their estimations, mistake is built into the estimate of the technology and leads to uncorrect beliefs in the future. Therefore, negative or positive shocks have different implications of estimates of future technology, so the agents take different decisions then they would in the standard RBC setting. This behavior can be called as path dependency.

The actual behavior is closely linked to the learning mechanism the agents use. The model proposed in the paper is very simple in this aspect. However, one can come up with possible ways how to improve the estimates, using all the past estimates. This is an area for future work on this model. It may seem, that learning should allow the agents to converge quickly to the standard RBC model benchmark. Even if it is the case, there is another area for future work. To enhance the realism of the model, the assumption about stable know variance of the shocks should be relaxed. Then the behavior would be more complex. The ways how to make the model more realistic are huge. For example, if the technological shock would follow some markow switching process, then one can possibly simulate waves of optimism or pessimism, as if the model would be ruled by keynesian animal spirits.

Areas of future work The main future aspiration is of course to use this methodology in RBC model with labor market and try to fit actual data with the model. During calibration, we'll try to assign values of σ_Y and σ_A less arbitralilly.

Interesting question would be what would happen if learning is incorporated into the model. Because the periods of mistaken evaluation of the level of technology, the agents might be prevented from converging to the equilibrium rapidly.

Another interesting project could be incorporating cost to decision making (ie. re-estimating level of technology). In the standard RBC setting the question is simple, because the agent simply compare cost with benefits. However, in this modified setting the cost are not that straightforward, because there occur significant periods where estimates are mistaken. Thus the incentive not to optimise might be strong.

Path dependency could be more strengthen by eliminating decreasing returns to scale of the production function. One possible way is to introduce endogenized technology, as for example in AK growth models.

Lastly, future work could explore more implication of animal spirits for propagation of business cycle. It has been shown how initial belief of higher level of technology can lead to similar dynamics as productivity shock. Future system could focus on modeling this changes in market sentiments. One possible way could be to incorporate new industry, which would be deciding whether to increase or decrease its production only by observing the first RBC-like industry.

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A Deriving dynamics of the model

A.1 Defining model

This passage follows closely a derivation described in a text of Harald Uhlig "*A Toolkit for Analyzing Nonlinear Dynamic Stochastic Models Easily*", (Uhlig, 1995). The set up of the model is only a bit different.[34]

There are two ways how to solve dynamic problems. For *competitive equilibrium* we need to find the optimality conditions for the firms and for the households separately and then solve the system. Social planner solution is generally less demanding with regard to algebra. They are equivalent, if the welfare theorems holds (which is not the case for example in OLG models). So let's go through the Social planner problem path. The problem is to maximize

$$U = \max_{C_t} \mathsf{E} \sum_{t=0}^{\infty} \beta^t u(C_t),$$

such that

- budget constraint: $C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta)K_t$,
- technology: $A_t = Ae^{z_t}$,
- $z_t = \rho z_{t-1} + \varepsilon_t$, $\varepsilon_t \approx N(0, \sigma^2)$
- and K_{-1}, Z_0 are known.

There are many ways how to solve this problem. One is to form Bellman equation, we'll form the lagrangian

$$L = \max_{C_t} \mathsf{E} \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \lambda_t (C_t + K_{t+1} - A_t K_t^{\alpha} - (1-\delta)K_t) \right) \right]. \tag{22}$$

[34] Uhlig assumes differently time periods when capital is used for production.

We obtain following first order conditions:

$$\frac{\partial L}{\partial \lambda_t} = 0 \;\Rightarrow \qquad C_t + K_{t+1} = A_t K_t^{\alpha} + (1-\delta)K_t \tag{23}$$

$$\frac{\partial L}{\partial C_t} = 0 \;\Rightarrow \qquad u'(C_t) = \lambda_t \tag{24}$$

$$\frac{\partial L}{\partial K_{t+1}} = 0 \;\Rightarrow \qquad \lambda_t = \mathsf{E}_t\left[\beta\lambda_{t+1}(\alpha A_{t+1}K_{t+1}^{\alpha-1} + (1-\delta))\right] \tag{25}$$

Euler equation is obtained by combining equations (24) and (25):

$$u'(C_t) = \mathsf{E}_t\left[\beta u'(C_{t+1})(\alpha A_{t+1}K_{t+1}^{\alpha-1} + (1-\delta))\right]. \tag{26}$$

We can define interest rate R_t:

$$R_t = \alpha A_{t+1}K_{t+1}^{\alpha-1} + (1-\delta), \tag{27}$$

which is equal to marginal product of capital plus the costs of using capital, ie. the depreciation.

A.2 Equilibrium

Now we want to find the equilibrium values of the variables in the model. The equilibrium values of the variable X will be denoted by X^*. The only exemption is the level of the technology. Because $A_t = Ae^{z_t}$, then in equilibrium the value of $z_t = 0$, so the equilibrium value of technology is $A^* = A$ and we'll use A. Steady state values of the variables in the model. The equilibrium values of the variable X will be denoted by X^*. The only exemption is the level of the technology. Because $A_t = Ae^{z_t}$, then in equilibrium the value of $z_t = 0$, so the equilibrium value of technology is $A^* = A$ and we'll use A.

Equilibrium is such a situation, in which all the variables are not forced away from their values. Thus the budget constraint becomes

$$C^* + K^* = A(K^*)^\alpha + (1-\delta)K^*, \tag{28}$$

and the steady state interest rate

$$R^* = \alpha A(K^*)^{\alpha-1} + (1-\delta).$$

The Euler equation then becomes

$$\begin{aligned} u'(C^*) &= \mathsf{E}_t\left[\beta u'(C^*)(\alpha A(K^*)^{\alpha-1} + (1-\delta))\right], \\ 1 &= \beta R^*. \end{aligned} \tag{29}$$

To find the equilibrium level of capital, we have to solve for the capital the Euler equation in the equilibrium:

$$\begin{aligned} u'(C^*) &= \mathsf{E}_t\left[\beta u'(C^*)(\alpha A(K^*)_{t+1}^{\alpha-1} + (1-\delta))\right] \\ \frac{1}{\beta} &= \alpha A(K^*)^{\alpha-1} \\ (K^*)^{\alpha-1} &= \frac{1-\beta(1-\delta)}{\alpha\beta A} \\ K^* &= \left(\frac{1-\beta(1-\delta)}{\alpha\beta A}\right)^{\frac{1}{\alpha-1}} = \left(\frac{\alpha\beta A}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{30}$$

A.3 Linearization

A.3.1 Taylor polynomial approximation

Let's assume that the functions we are dealing with are smooth and there exists second derivation of utility function on some reasonable interval. Then the Taylor polynomial for function $f(x)$ at a point a is defined as

$$f(x) = f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \dots$$
$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(a)}{i!}(x-a)^i.$$

The linear problems are much easier to solve, thus we will use linear[35] approximation:

$$f(x) \approx f(a) + f'(a)(x-a)$$

or in two dimensions:

$$f(x,y) \approx f(a,b) + \frac{\partial f(a,b)}{\partial x}(x-a) + \frac{\partial f(a,b)}{\partial y}(y-b)$$

A.3.2 Deviations

Another useful step is to work with deviations from steady state values, not with the actual levels of variables. Thus we have

$$x_t = \frac{X_t - X^*}{X^*}.$$

Also, we have

$$\log(X_t) - \log(X^*) = \log\left(\frac{X_t}{X^*}\right) = \log\left(\frac{X_t - X^*}{X*} + 1\right) \approx x_t = \frac{X_t - X^*}{X^*}.$$

We want to use linear approximations of first order conditions and other equations. Namely, it was the budget constraint (23), interest rate (27) and the Euler equation (26)

[35]For quadratic approximation techniques, see for example Schmitt-Groh and Uribe (2001).

A.3.3 Budget constraint

The budget constraint is following:

$$C_t + K_{t+1} = A_t K_t^\alpha + (1-\delta)K_t.$$

The linearizations

- $C_t \approx C^* + (1)(C_t - C^*) = C^* + (C_t - C^*)\frac{C^*}{C^*} = C^* + c_t C^*$
- $K_t \approx K^* + k_t K^*$, thus $K_{t+1} \approx K^* + k_{t+1}K^*$
- $(1-\delta) \approx (1-\delta)(K^* + k_t)K^*$

A bit tricky is approximation of the term $A_t K_t^\alpha$. We have to use two dimensional approximation, total differential:

$$A_t K_t^\alpha \approx A(K^*)^\alpha + \alpha A(K^*)^{\alpha-1}(K_t - K^*) + A(K^*)^\alpha(z_t - z^*),$$

because $z^* = 0$ we have:

$$\begin{aligned} A_t K_t^\alpha &\approx A(K^*)^\alpha + \alpha A(K^*)^{\alpha-1}k_t K^* + A(K^*)^\alpha z_t \\ &= A(K^*)^\alpha + \alpha A(K^*)^\alpha k_t + A(K^*)^\alpha z_t \\ &= A(K^*)^\alpha + A(K^*)^\alpha(\alpha k_t + z_t) \end{aligned}$$

Now, we can put the approximations together and we obtain:

$$\begin{aligned} C_t + K_{t+1} &= A_t K_t^\alpha + (1-\delta)K_t \\ C^* + c_t C^* + K^* + k_{t+1}K^* &= A(K^*)^\alpha + A(K^*)^\alpha(\alpha k_t + z_t) + (1-\delta)(K^* + k_t)K^* \end{aligned}$$

if we notice that $C^* + K^* = A(K^*)^\alpha + (1-\delta)K^*$ always holds, because it's equilibrium condition (that's the way how the equilibrium was defined), we can simplify the linearized budget constraint and obtain:

$$c_t C^* + k_{t+1} K^* = A(K^*)^\alpha(\alpha k_t + z_t) + (1-\delta)k_t K^*) \tag{31}$$

A.3.4 Interest rate

The linearized equation for interest rate is

$$\begin{aligned}
R_t &= \alpha A_{t+1} K_{t+1}^{\alpha-1} + (1-\delta), \\
R^* + r_t R^* &\approx \alpha A(K^*)^{\alpha-1} + \alpha(\alpha-1)A(K^*)^{\alpha-2}(K_t - K^*) + \alpha A(K^*)^{\alpha-1} z_t + (1-\delta) \\
&= \alpha A(K^*)^{\alpha-1} + \alpha(\alpha-1)A(K^*)^{\alpha-1} k_t + \alpha A(K^*)^{\alpha-1} z_t + (1-\delta) \\
&= \alpha A(K^*)^{\alpha-1} + \alpha A(K^*)^{\alpha-1}((\alpha-1)k_t + z_t) + (1-\delta)
\end{aligned}$$

Because we already know that

$$\frac{1}{\beta} = R^* = \alpha A(K^*)^{\alpha-1} + (1-\delta), \tag{32}$$

we can simplify the equation for the interest into

$$\begin{aligned}
r_t R^* &= \alpha A(K^*)^{\alpha-1}((\alpha-1)k_t + z_t) \\
r_t &= \frac{\alpha A(K^*)^{\alpha-1}}{R^*}((\alpha-1)k_t + z_t) = \frac{\alpha A(K^*)^{\alpha-1} + (1-\delta) - (1-\delta)}{\alpha A(K^*)^{\alpha-1} + (1-\delta)}((\alpha-1)k_t + z_t) \\
r_t &= (1 - \beta(1-\delta))(z_t + (\alpha-1)k_t) \qquad (33)
\end{aligned}$$

A.3.5 Euler equation

Let's rewrite the Euler equation (26) using the interest rate:

$$u'(C_t) = \mathsf{E}\left[\beta u'(C_{t+1}) R_{t+1}\right].$$

So the linearized version looks like

$$u'(C^*) + u''(C^*)C^*c_t = \mathsf{E}\left[\beta(u'(C^*)R^* + u''(C^*)R^*C^*c_{t+1} + u'(C^*)R^*r_{t+1})\right].$$

Because we already know that $1 = \beta R^*$, we can simplify this equation and obtain

$$\begin{aligned} u'(C^*) + u''(C^*)C^*c_t &= \mathsf{E}_t\left[u'(C^*) + u''(C^*)C^*c_{t+1} + u'(C^*)r_{t+1}\right] \\ 0 &= \mathsf{E}_t\left[u''(C^*)C^*(c_{t+1} - c_t) + u'(C^*)r_{t+1}\right] \end{aligned} \tag{34}$$

A.4 Solving for the dynamics

The law of motion can be found for example by method of undetermined coefficients. The variables which are already given in the period t are called *state variables*. We want to find how to determine the other variables in the model, here k_{t+1}, r_t, c_t, given the state variables, k_t, z_t. We are looking for linear law of motion, which is described by

$$\begin{pmatrix} k_{t+1} \\ r_t \\ c_t \end{pmatrix} = \begin{pmatrix} \nu_{kk} & \nu_{kz} \\ \nu_{rk} & \nu_{rz} \\ \nu_{ck} & \nu_{cz} \end{pmatrix} \begin{pmatrix} k_t \\ z_t \end{pmatrix} \tag{35}$$

The coefficients ν can be understood as elasticities.

We have

$$\begin{aligned} c_tC^* + k_{t+1}K^* &= A(K^*)^\alpha(\alpha k_t + z_t) + (1-\delta)k_tK^* \\ c_tC^* &= A(K^*)^\alpha z_t + \left(A(K^*)^{\alpha-1} + (1-\delta)\right)k_tK^* - k_{t+1}K^* \\ c_tC^* &= Y^*z_t + R^*k_tK^* - k_{t+1}K^* \\ c_t &= \frac{Y^*}{C^*}z_t + \frac{1}{\beta}\frac{K^*}{C^*}k_tK^* - \frac{K^*}{C^*}k_{t+1} \end{aligned} \tag{36}$$

Now let's insert the law of motion for consumption:

$$\nu_{ck}k_t + \nu_{cz}z_t = \frac{Y^*}{C^*}z_t + \frac{K^*}{\beta C^*}k_t - \frac{K^*}{C^*}(\nu_{kk}k_t + \nu_{kz}z_t)$$
$$\nu_{ck}k_t + \nu_{cz}z_t = \frac{Y^*}{C^*}z_t + \left(\frac{K^*}{\beta C^*} - \frac{K^*}{C^*}\nu_{kk}\right)k_t - \frac{K^*}{C^*}\nu_{kz}z_t$$
$$\nu_{ck}k_t + \nu_{cz}z_t = \left(\frac{Y^*}{C^*} - \frac{K^*}{C^*}\nu_{kz}\right)z_t + \frac{K^*}{C^*}\left(\frac{1}{\beta} - \nu_{kk}\right)k_t.$$

Thus we obtain

$$\nu_{ck} = \frac{K^*}{C^*}\left(\frac{1}{\beta} - \nu_{kk}\right) \tag{37}$$
$$\nu_{cz} = \frac{Y^*}{C^*} - \frac{K^*}{C^*}\nu_{kz}. \tag{38}$$

Now turn to the interest rate. We already know that

$$r_t = (1-\beta(1-\delta))\,(z_t + (\alpha-1)k_t)$$
$$\nu_{rk}k_t + \nu_{rz}z_t = (1-\beta(1-\delta))(z_t + (\alpha-1)k_t)$$

thus we obtain

$$\nu_{rk} = (1-\beta(1-\delta))(\alpha-1), \tag{39}$$
$$\nu_{rz} = (1-\beta(1-\delta)). \tag{40}$$

Now turn back to the Euler equation (34) and rewrite it with the rule of motion:

$$0 = \mathsf{E}_t\left[u''(C^*)C^*(c_{t+1} - c_t) + u'(C^*)r_{t+1}\right]$$
$$0 = \mathsf{E}_t\left[u''(C^*)C^*((\nu_{ck}k_{t+1} + \nu_{cz}z_{t+1}) - (\nu_{ck}k_t + \nu_{cz}z_t)) + u'(C^*)(\nu_{rk}k_{t+1} + \nu_{rz}z_{t+1})\right]$$

Now, use the law of motion again for k_{t+1},

$$0 = \mathsf{E}_t \left[u''(C^*)C^*((\nu_{ck}(\nu_{kk}k_t + \nu_{kz}z_t) + \nu_{cz}z_{t+1}) - (\nu_{ck}k_t + \nu_{cz}z_t)) \right.$$
$$\left. + u'(C^*)(\nu_{rk}(\nu_{kk}k_t + \nu_{kz}z_t) + \nu_{rz}z_{t+1}) \right],$$

which after rearranging gives

$$\begin{aligned} 0 &= k_t(u''(C^*)C^*(\nu_{ck}\nu_{kk} - \nu_{ck}) + u'(C^*)\nu_{rk}\nu_{kk}) \\ &+ z_t(u''(C^*)C^*(\nu_{ck}\nu_{kz} - \nu_{cz}) + u'(C^*)\nu_{rk}\nu_{kz}) \\ &+ \mathsf{E}_t[z_{t+1}](u''(C^*)C^*\nu_{cz} + u'(C^*)\nu_{rz}) \end{aligned}$$

But we know that

$$\mathsf{E}_t[z_{t+1}] = \rho z_t$$

thus

$$\begin{aligned} 0 &= k_t(u''(C^*)C^*(\nu_{ck}\nu_{kk} - \nu_{ck}) + u'(C^*)\nu_{rk}\nu_{kk}) \\ &+ z_t(u''(C^*)C^*(\nu_{ck}\nu_{kz} - \nu_{cz}) + u'(C^*)\nu_{rk}\nu_{kz}) \\ &+ \rho z_t(u''(C^*)C^*\nu_{cz} + u'(C^*)\nu_{rz}) \end{aligned}$$

which can be simplified to

$$\begin{aligned} 0 &= k_t(\nu_{kk}(u''(C^*)C^*\nu_{ck} + u'(C^*)\nu_{rk}) - u''(C^*)C^*\nu_{ck}) \\ &+ z_t(u''(C^*)C^*(\nu_{ck}\nu_{kz} - (1-\rho)\nu_{cz}) + u'(C^*)(\nu_{rk}\nu_{kz} + \rho\nu_{rz}) \end{aligned}$$

So obtained equation for ν_{kk}:

$$0 = \nu_{kk}(u''(C^*)C^*\nu_{ck} + u'(C^*)\nu_{rk}) - u''(C^*)C^*\nu_{ck} \tag{41}$$

and another equation

$$0 = u''(C^*)C^*(\nu_{ck}\nu_{kz} - (1-\rho)\nu_{cz}) + u'(C^*)(\nu_{rk}\nu_{kz} + \rho\nu_{rz}). \tag{42}$$

Now, we can substitute to this equation previous results for ν_{ck} from (37) and ν_{rk} from (39)

$$\begin{aligned} 0 = \nu_{kk} &\left(u''(C^*)C^*\frac{K^*}{C^*}\left(\frac{1}{\beta} - \nu_{kk}\right) + u'(C^*)(1-\beta(1-\delta))(\alpha-1) \right) \\ &- u''(C^*)C^*\frac{K^*}{C^*}\left(\frac{1}{\beta} - \nu_{kk}\right) \end{aligned} \tag{43}$$

which can be further simplified

$$\begin{aligned} 0 = &-\nu_{kk}^2 u''(C^*)K^* \\ &+ \nu_{kk}\left(u''(C^*)K^*\left(\frac{1}{\beta}+1\right) u'(C^*)(1-\beta(1-\delta))(\alpha-1) \right) \\ &- u''(C^*)K^*\frac{1}{\beta} \end{aligned} \tag{44}$$

This is in fact a quadratic equation,

$$0 = \nu_{kk}^2 - \gamma\nu_{kk} + \frac{1}{\beta}, \tag{45}$$

where

$$\gamma = \frac{u''(C^*)K^*\left(\frac{1}{\beta}+1\right) u'(C^*)(1-\beta(1-\delta))(\alpha-1)}{u''(C^*)K^*} \tag{46}$$

Now we note that $\gamma > 0$, so we are looking for the smaller root so we are looking for a solution which is

$$\nu_{kk} = \frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} - \frac{1}{\beta}}. \tag{47}$$

Now, let's turn back to equation (42), substitute for ν_{cz} from (38) and solve it for ν_{kz}:

$$
\begin{aligned}
0 &= u''(C^*)C^*(\nu_{ck}\nu_{kz} - (1-\rho)\nu_{cz}) + u'(C^*)(\nu_{rk}\nu_{kz} + \rho\nu_{rz}) \\
0 &= u''(C^*)C^*\left(\nu_{ck}\nu_{kz} - (1-\rho)\left(\frac{Y^*}{C^*} - \frac{K^*}{C^*}\nu_{kz}\right)\right) + u'(C^*)(\nu_{rk}\nu_{kz} + \rho\nu_{rz})
\end{aligned}
$$

which after some manipulation gives

$$
\nu_{kz} = \frac{u''(C^*)C^*(1-\rho)\frac{K^*}{C^*} - u'(C^*)\rho\nu_{rz}}{u''(C^*)C^*\left(\nu_{ck} + (1-\rho)\frac{Y^*}{C^*}\right) + u'(C^*)\nu_{rk}} \tag{48}
$$

A.5 Summary

Let's summarize the result. First, we found the equilibrium values for capital and thus for output and consumption:

$$
\begin{aligned}
A^* &= A \\
K^* &= \left(\frac{\alpha\beta A}{1-\beta(1-\delta)}\right)^{\frac{1}{1-\alpha}} \\
Y^* &= A(K^*)^\alpha \\
C^* &= A(K^*)^\alpha - \delta K^* \\
R^* &= \alpha A(K^*)^{\alpha-1} + (1+\delta)
\end{aligned}
$$

Second, we linearize the model by using Taylor first order approximation. Then, using method of undetermined coefficients, we solved the system of difference equations

$$
\begin{pmatrix} k_{t+1} \\ r_t \\ c_t \end{pmatrix} = \begin{pmatrix} \nu_{kk} & \nu_{kz} \\ \nu_{rk} & \nu_{rz} \\ \nu_{ck} & \nu_{cz} \end{pmatrix} \begin{pmatrix} k_t \\ z_t \end{pmatrix}
$$

which describe the law of motion in our RBC model. The coefficients are

$$\nu_{rk} = (1-\beta(1-\delta))(\alpha-1), \qquad \nu_{rz} = (1-\beta(1-\delta)),$$

$$\nu_{kk} = \frac{\gamma}{2} - \sqrt{\frac{\gamma^2}{4} - \frac{1}{\beta}}, \qquad \nu_{kz} = \frac{u''(C^*)C^*(1-\rho)\frac{K^*}{C^*} - u'(C^*)\rho\nu_{rz}}{u''(C^*)C^*\left(\nu_{ck} + (1-\rho)\frac{Y^*}{C^*}\right) + u'(C^*)\nu_{rk}},$$

$$\nu_{ck} = \frac{K^*}{C^*}\left(\frac{1}{\beta} - \nu_{kk}\right), \qquad \nu_{cz} = \frac{Y^*}{C^*} - \frac{K^*}{C^*}\nu_{kz},$$

where

$$\gamma = \frac{u''(C^*)K^*\left(\frac{1}{\beta}+1\right)u'(C^*)(1-\beta(1-\delta))(\alpha-1)}{u''(C^*)K^*}.$$

B MATLAB codes

B.1 Learning

```
%%Learning
%demand:                y_d(t)=a-b*p(t)
%supply:                y_s(t)=c+d*Ep(t)
%market clearing:       p(t)=(a-y(t))/b

%number of periods
T=500;
%sdev of shock
sigma=0.3;

%Demand and Supply parameters
a=200;
b=1;
c=0;
d=0.2;

%steady states
p_star=(a-c)/(d+b);
y_star=a-b*p_star;

%initial settings
a_hat(1)=150;
b_hat(1)=2.5;

y(1)=(b_hat(1)*c+a_hat(1)*d)/(b_hat(1)+d)+36*normrnd(0,sigma);
p(1)=(a-y(1))/b+normrnd(0,sigma);

X=[1 y(1)];
P=[p(1)];
Odhad=[a_hat(1);b_hat(1)];

y(2)=y_star-10;
Ep(2)=(a_hat(1))/(1+b_hat(1));

for t=2:T;
    p(t)=(a-y(t))/b  +normrnd(0,sigma);
    P=[P;p(t)];

    X=[X;1 y(t)];
```

```
    B=((X'*X)^(-1))*(X'*P);


    a_hat(t)=-B(1,1)/B(2,1);
    b_hat(t)=-1/B(2,1);
    Odhad=[Odhad [a_hat(t);b_hat(t)]];

    y(t+1)=(b_hat(t)*c+a_hat(t)*d)/(b_hat(t)+d)+36*normrnd(0,sigma);

    Ep(t+1)=(a_hat(t)-c)/(d+b_hat(t));
end

p(T+1)=0;
i=1:1:T;
px=ones(1,T+1)*p_star;
al=ones(1,T+1)*a;
bl=ones(1,T+1)*b;
chyba=Ep-p;

[p;y];

a_chyba=a_hat-a;

%graphics
i=3:1:T;
figure(1);
plot1=plot(i,al(i),i,a_hat(i));
set(plot1,'LineWidth',2.5);
legend(plot1,'a','estimate of a');
legend('boxoff');
xlabel('Time');
ylabel('Estimate');

figure(2);
plot2=plot(i,bl(i),i,b_hat(i));
set(plot2,'LineWidth',2.5);
legend(plot2,'b','estimate of b');
legend('boxoff');
xlabel('Time');
ylabel('Estimate');
```

B.2 RBC

```
%%Model RBC - Impulse responses to shocks
%parameters
alpha=0.36;          %alpha..personal discount discount factor
delta=0.05;          %delta...capital depreciaton
A=1;                 %A...technology steady state
eta=0.9999999999;    %eta...utility parameter, propensity to consume
rho=0.9;             %rho...technological persistance
T=40;                %number of periods

sigma_Y=0.005;
sigma_A=0.002;


%initialisation
z=zeros(1,T);
k=zeros(1,T);
c=zeros(1,T);
eps_y=zeros(1,T);
eps_z=zeros(1,T);
Ez=zeros(1,T);
D=zeros(1,T);

%steady state levels
K_star=((alpha*beta*A)/(1-beta*(1-delta)))^(1/1-alpha);
Y_star=A*K_star^alpha;
C_star=Y_star-delta*K_star;

u2dCstar=-eta*C_star^(-2);  %u2dCstar... second derivation of utility in C_star
u1dCstar=C_star^(-1);       %u1dCstar... first derivation of utility in C_star

%shock decomposition
XXX=(K_star^(2*alpha)*sigma_A^2)/(K_star^(2*alpha)*sigma_A^2+sigma_Y^2);

%elasticities
gamma=(u2dCstar*K_star*(1/beta+1)+u1dCstar*(1-beta*(1-delta))*(alpha-1))/
           (u2dCstar*K_star);
v_rk=(1-beta*(1-delta))*(1-alpha);
v_rz=1-beta*(1-alpha);
v_kk=gamma/2-sqrt(gamma^2/4-1/beta);
v_ck=K_star/C_star*(1/beta-v_kk);
v_kz=(u2dCstar*C_star*(1-rho)*Y_star/C_star-u1dCstar*rho*v_rz)/
     (u2dCstar*C_star*(v_ck+(1-rho)*K_star/C_star)+u1dCstar*v_rk);
```

```
v_cz=Y_star/C_star-K_star/C_star*v_kz;

%initial shocks
z(1)=0.0;
eps_y(1)=0.01;
k(1)=0.00;

%initial expected technological shock
Ez(1)=0.00;



%trajectory
for i=1:T-1;
K(i)=K_star+K_star*k(i);
D(i)=A*exp(z(i))*(K(i))^alpha+eps_y(i)-A*exp(Ez(i))*(K(i))^alpha;
Ez(i)=z(i); %Full information available to the agents - standard RBC case
%Ez(i)=XXX*D(i)+Ez(i);   Alternative
k(i)=k(i)+D(i)/K_star;
k(i+1)=v_kk*k(i)+v_kz*z(i);
c(i)=v_ck*k(i)+v_cz*z(i);
z(i+1)=z(i)*rho;
Ez(i+1)=rho*z(i);
%Ez(i)=rho*Ez(i);   Alternative
end
c(T)=v_ck*k(T)+v_cz*Ez(T);%last period consumption (only technical thing)

%translation of shocks back to original levels
C=c*C_star+C_star;
K=k*K_star+K_star;

%graphics
i=1:1:T;

figure(1);
plot1=plot(i,C,i,K,i,A*exp(z),i,D);
set(plot1,'LineWidth',2.5);
legend(plot1,'Consuption','Capital','Technology','Mistake');
legend('boxoff');
xlabel('Time');
ylabel('Level');
title('RBC impulse')

figure(2);
plot2=plot(i,c,i,k,i,z,i,D);
```

```
set(plot2,'LineWidth',2.5);
legend(plot2,'Consuption','Capital','Technology','Mistake');
legend('boxoff');
xlabel('Time');
ylabel('Deviation from steady-state');
title('RBC')

figure(3);
plot3=plot(i,z,i,Ez,i,D);
set(plot3,'LineWidth',2.5);
legend(plot3,'Technology','Expected Technology','Mistake');
legend('boxoff');
xlabel('Time');
ylabel('Deviation from steady-state');
title('RBC impulse');
```

C Thesis proposal

Master Thesis Proposal

Author: Bc. Filip Rozsypal

Supervisor Prof. Ing. Milan Sojka, CSc.

Title Uncertainty in macroeconomic modelling

Motivation

The uncertainty is inherent in any economic system, yet different schools of thought view its scope and importance very differently. The mainstream approach is very demanding in its assumptions about agent's knowledge and rationality. The formalised models are able to give quantitative answers. However, the value of the results is questionable with unrealistic assumptions.

The thesis will examine the methodology behind selected assumptions. Mainstream methodology will be compared with its heterodox counterparts, focusing on issues related to macroeconomics, especially monetary economics. The thesis will try to address whether making the assumption more realistic while keeping the mainstream framework can reconcile the mainstream theory with its heterodox counterparts. The thesis will try to develop its own model in order to address the question asked above. New classical vs. Neokeynesian vs. Postkeynesian economics will be contrasted as three approaches with different accent on realism.

Methods

Review of existing literature, analysis of different assumption in a model by simulation, potentially estimation or calibration on real data

Theses

- Can a model be both based on realistic assumptions and deliver quantitative results at the same time?
- Can a unrealistic model be improved by systematically making one assumptions more realistic?

Expected structure

1. Discussion of methodological issues in mainstream and in heterodox schools
    - Uncertainty and its implication for the time and expectations formation
    - Comparison between different schools of thought
2. Relating theoretical debates to real problems in economic policy
    - Monetary policy and concepts of dynamic inconsistency
    - Assumption about the knowledge about the shocks in macro models
3. Discussion of the trade-off between quantitative tractability and degree of realism
    - Description of neoclassical position
    - Examples of heterodox criticism
    - Introducing the realism, ie. models of learning, bounded rationality, etc.
4. Model, probably based on standard intertemporal Ramsey allocation problem
    - Introducing uncertainty into RBC: two sources of shocks
    - Introducing aspects of learning
    - Indeterminacy and multiple equilibria, sunspots
    - Discussion of the implications for the dynamics of the model