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Report on Thesis "Noncommutative structures in quantum field theory"

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The thesis is devoted to operadic and properadic structures and related homotopy algebras. In this context, the noncommutativity refers to homotopy algebras with the (higher) operations being "noncommutative", i.e. not necessarily graded symmetric. This type of noncommutativity is dictated by a nontrivial action of the symmetric group on the underlying operad/properad and is in the spirit of Kontsevitsch's noncommutative symplectic geometry. In the thesis, the role of homological methods is central. The related algebraic structures are most naturally described within the framework of generalized Batain-Vilkovisky- (BV-) like structures and hence closely related to quantum field theory (QTF) and string field theory (STF).

This area of research is significant from both mathematics and physics point of views, leading to new interesting insights on higher algebraic and geometric structures relevant to QFT and STF and to a better understanding of the relation between homological algebra and quantum theory.

The thesis consists of two closely related parts, the part devoted to quantum homotopy algebras and their minimal models constructed using the homological perturbation lemma (HPL) and the part concerned with homotopy algebras generalizing the involutive Lie bialgebras. The interconnectedness is well reflected in the structure of the thesis, where the two aspects/parts, the (modular) operadic and the properadic ones, are described parallelly and the differences are commented.

The motivation for the first one comes from work of Markl and its generalization due to Barannikov, who observed that the structure of an algebra over the Feynman transform of an arbitrary modular operad can equivalently be encoded in a solution of the corresponding (generalized or noncommutative) BV (or master) equation, i.e. a BV-action. The commutative example is the quantum L-infinity, aka loop homotopy, algebra. A noncommutative one is, e.g. the quantum version of an A-infinity algebra. Barannikov's construction leads, starting from a

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modular operad P and an odd symplectic dg vector space V, to a BV-Laplacian and a BV-bracket on the space of invariants under the diagonal action of the symmetric group on the tensor product of P and V. Nevertheless, there is no obvious product compatible with the BV-operations as in case of an ordinary BV-algebra, where the structure naturally comprises the symmetric tensor algebra on V.

In order to remedy this, Lada introduces a notion of a modular operad with a connected sum. In the thesis, a biased definition as well as a monadic one is given along with relevant examples and their geometric interpretations. Barannikov's construction is extended to the case with the connected sum, leading to a full-fledged BV-algebra. Starting from this, the minimal model of the corresponding homotopy algebra is constructed and compared to the quantum field theory construction of an (Wilson-type) effective action using Feynman diagrams. As a byproduct, a homotopy between the original and effective action is constructed, generalizing the commutative case. Also, a possible notion of a morphim of quantum homotopy algebras is given.

The second one generalizes Barannikov's construction in a different direction. The aim is to describe algebras over the cobar construction of a general properad, again as solutions to an appropriate master equation. It was known that in case of an ordinary (or commutative) Frobenius properad, an algebra over its cobar construction is an IBL-infinity algebra (as introduced by Cieliebak, Fukaya and Latchev). Lada describes the general case leading to the notion of a homological differential operator encoding the corresponding homotopy structure in the general case. She introduces an associative analogue of the commutative Frobenius properad along with its geometric interpretation in terms of surfaces. The corresponding homotopy algebra could be seen as an associative cousin of a homotopy IBL algebra, an IBA-infinity algebra.

Both parts contain some original and non-trivial results based on collaborations.

Lada's contribution to these was essential. As examples, I would like to mention her observations leading to the monadic definition of the connected sum and the necessity of factorizing by the action of the "self-part" of the connected sum, both are related to a proper geometric understanding of operadic and properadic gradings, which play an important role in all constructions. Also, her contribution to the correct definition of the associative Frobenius operad, leading to a new and nontrivial example of the homotopy IBA algebra, was very important.

The techniques and methods used in the thesis are quite advanced and sophisticated. The reported results show that Lada gained a good understanding of these techniques and methods and applied them skilfully to the problems addressed in the thesis. Despite the numerous typos and inconsistencies (most of them could have easily be corrected during a careful second reading), the thesis is very good. There is no doubt that Lada can work successfully on her own prospective research projects in the future. I also believe that she will continue to produce relevant contributions to mathematics/mathematical physics.

Lada certainly deserves to be awarded the PhD title.

Branislav Jurčo Bonn, November 12, 2020