Report on the thesis:
Modèle d'écoulement biphasé en sciences de la Terre: fusion partielle, compactiion et différenciation

submitted by Mr. Ondřej Šrámek
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Mr. Šrámek's thesis is concerned with the mechanical and thermodynamical modelling of two-phase flow, a phenomenon of capital importance in the Earth's interior. The overall approach is to derive in rigorous fashion a set of equations describing the dynamics, and then to obtain analytical and/or numerical solutions for simple model problems to reveal the system's essential behavior.

Chapter 1 sets the stage by discussing the important role of two-phase flow in two main contexts in the Earth's interior: in partially molten regions of the mantle; and during the differentiation of the Earth's metallic core from the silicate mantle. The physical processes involved in both situations are clearly explained, and relevant previous work is abundantly cited. Mr. Šrámek then proceeds to discuss in general terms how two-phase flows can be modelled mathematically, with emphasis on how mean quantities are calculated on a scale much larger than the microstructure of the medium. The chapter closes with a clear and comprehensive survey of previous applications of two-phase flow models to geodynamical phenomena including mid-ocean ridges, mantle plumes, and solitary waves.

Chapter 2 introduces the main theoretical advance of the thesis: a generalization of the equations of Bercovici et al. (2001) to include the effect of phase changes (Šrámek et al 2007; henceforth "SRB07".) These new equations are derived in more detail in chapter 4, but are reproduced here in order to compare them with the older (and more familiar) two-phase theory of McKenzie (1984). The discussion nicely highlights the differences between the two theories, and clearly shows that the equations of SRB07 are more general and rigorous than those of McKenzie (1984).

Chapter 3 is an application of the author's equations to the problem of metal-silicate differentiation in the early Earth. For simplicity, Mr. Šrámek uses a reduced set of equations in which the melting/freezing of the metallic component is included in a simplified way via a temperature-dependent permeability coefficient. He then uses these equations to a study a series of models of phase separation of progressively increasing complexity: first the purely mechanical one-dimensional (1D) compaction of a layer comprising two phases; then purely radial compaction of a spherical self-gravitating body; and finally the differentiation of an intially homogeneous two-phase domain in two dimensions (2D), with thermal effects included. The solution of these problems requires fairly sophisticated numerical methods, which Mr. Šrámek discusses clearly and in detail. As the author himself notes, the solutions to the first two problems are qualititatively similar to solutions of 1D compaction problems obtained by several previous workers. More novel and interesting are the results of the 2D model, which show an initial stage of essentially 1D compaction followed by a catastrophic instability of Rayleigh-Taylor type that rapidly delivers a large metal-rich blob to the bottom of the "mantle". Unfortunately, only a single illustrative 2D solution is shown, without any more systematic study of how critical dynamical parameters (wavelength of the instability, timescale for blob descent, etc.) depend on the dimensionless governing parameters (nicely summarized by the author in § 3.3.) As a result, the chapter gives a somewhat "unfinished" impression - clearly much work remains to be done to understand the physics of this very complicated system and to draw geophysical conclusions from the numerical solutions.

Chapter 4 is devoted to a more detailed presentation of the SRB07 equations, with emphasis on the energy equation. The derivation, based on the principles of non-equilibrium thermodynamics, is impressively rigorous. The author's main new result is to identify two new terms that modify the usual (static) Clapeyron melting condition: a Gibbs-Thomson term representing the effect of surface tension, and a second term representing the dynamic pressure perturbation due to matrix dilatation/compaction. While the first of these is probably negligible, the second may be important, as discussed further in Chapter 6.

Chapter 5 discusses several simple model problems that illustrate the effects of coupled melting and matrix deformation. The basic "spherical compaction" model problem (originally proposed by McKenzie 1984) envisions two-phase material with uniform porosity inside a spherical permeable membrane that allows the melt phase to pass freely. The author's advance over previous studies of this system is to add the effects of melting, at a rate that is either uniform within the sphere or controlled by temperature and pressure according to the rate law of SRB07. The author's analytical solutions (valid at an initial instant in time) show that the presence of melting significantly affects the melt velocity and the rate of evolution of the porosity. The chapter ends somewhat abruptly after the presentation of these solutions, without any geophysical conclusions having been drawn. However, the author does address quantitatively the question of whether it might be possible to test the solutions against laboratory experiments. While the response turns out to be (probably) negative, the author is to be commended for trying to connect his abstract theoretical results to observable reality.

Chapter 6, which (like Chapter 4) represents published work, examines the problem of 1D steady-state pressure-release melting in upwellling mantle material. While models with this geometry have been studied by numerous workers in the past, none of them has considered properly the coupling between the mechanics and thermodynamics of melting. Mr. Šrámek's approach thus represents an important advance. By means of analytical solutions valid near the base of the melting zone, he shows that three distinct force balances are possible depending on the compaction length and the height above the base of the melting zone, and he confirms these predictions by means of numerical solutions of the full equations. The resulting phase diagram (fig. 6.2) is an elegant and pedagogical summary of the different dynamical regimes that can obtain within the melting zone. Another very interesting result is that the dynamic pressure perturbation first discussed in Chapter 4 can have a significant effect on the melting depth beneath ocean ridges, typically increasing it by a few km relative to the static Clapeyron prediction. The discussion throughout is clear and physically insightful.

Chapter 7 is a short discussion of the effects of surface tension, which were neglected in most of the models of the previous chapters. As far as I can tell, the discussion is largely an application of work by Hier-Majumder et al. (2006), and would probably be better as an Appendix rather than as a stand-alone chapter.

In conclusion, Mr. Šrámek's thesis effectively comprises two distinct parts. Chapters 4 and 6, which represent published work, are excellent in all respects and display the author's strengths: fine physical insight, mathematical and numerical finesse, and the ability to make significant advances over previous work. The other (unpublished) chapters are in a less finished state, and the reader feels to some extent a lack of systematic investigation, physical interpretation, and geophysically relevant conclusions. Even in these chapters, however, the work that is presented is excellent, and I have no doubt that high-quality publications will result from it. Overall, I judge that the thesis as it stands amply justifies the award of the degree of Doctor of Philosophy.

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