Referee's Comments on the Doctoral Thesis

Analysis of unsteady flows of incompressible heat-conducting rate-type viscoelastic fluids with stress-diffusion

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This excellent doctoral thesis is concerned with the proof of existence of global weak solutions to a class of viscoelastic fluids with stress diffusion. The results of the thesis represent a significant original contribution to the mathematical analysis of systems of nonlinear partial differential equations arising in models of non-Newtonian fluids, and demonstrate a clear ability by the candidate to conduct original and creative scientific work.

The thesis has vi+120 numbered pages, and includes a bibliography with 75 items. The thesis comprises of five chapters. Chapter 1 is of introductory nature: it presents a review of viscoelastic fluids, gives a brief description of the class of models under consideration, and formulates the objectives of the thesis. Chapter 2 is concerned with the derivation of the model from fundamental laws of continuum thermodynamics, the important feature of the class of models being consistency with the laws of thermodynamics.

Chapters 3 and 5 include the key contributions of the thesis. Chapter 3 in particular is devoted to the proof of the existence of large data global weak solutions to the nonisothermal model. Following the statement of the relevant function spaces, the systems of partial differential equations under consideration, and the assumptions on the material parameters/coefficients appearing in the model, the candidate states the definition of weak solution, and embarks on the proof of existence of suitable weak solution by a Galerkin approximation, and passes to the limit using a weak-compactness technique. The argument is highly nontrivial. It is the first result of this kind in the mathematical literature, and it is therefore all the more important.

Chapter 4 of the thesis is a survey of auxiliary technical tools used in the proofs, including sequential weak compactness results, results on intersections of Sobolev–Bochner spaces, and a collection of useful results from the calculus of positive definite matrices.

Chapter 5 includes the second substantial original contribution by the candidate. It contains the the proof of the existence of global weak solutions to a simpler isothermal model, and is based on the candidate's forthcoming journal article:

- M. Bathory, M. Bulíček, J. Málek, Large data existence theory for three-dimensional unsteady flows of rate-type viscoelastic fluids with stress diffusion. (Submitted); available from https://arxiv.org/abs/2002.11224v2, 2020.

The candidate has four other papers:

- M. Bathory, Joint weak type interpolation on Lorentz–Karamata spaces, Math. Inequal. Appl. 21 (2018), no. 2, 385–419.
- M. Bathory, Outflow Boundary Condition Leading to Minimal Energy Dissipation for an Incompressible Flow, WDS'17 Proceedings of Contributed Papers . Physics (Prague) (J. Šafránková, J. Pavlu, eds.), Prague, Matfyzpress, 2017, pp. 7–12.
- M. Bathory, M. Bulíček, Optimal outflow boundary condition for a stationary flow of an incompressible fluid, Preprint NCMM/2018/11, 2018.
- M. Bathory, M. Bulíček, O. Souček, Existence and qualitative theory for nonlinear elliptic systems with a nonlinear interface condition used in electrochemistry, Z. Angew. Math. Phys 71 (2020), 74.

Conclusions

The exposition in the thesis is clear and scholarly throughout. The work reported in the thesis represents a significant original scientific contribution to the mathematical analysis of the existence of large-data global-in-time weak solutions to models of incompressible non-Newtonian fluids. The results of Chapter 5, and Chapter 3 in particular, are of outstanding quality.