We prove a global-in-time and large-data existence of a suitable weak solution to a system of partial differential equations describing an unsteady flow of homogeneous incompressible viscoelastic rate-type fluid. The material parameters are continuous functions of temperature and, in particular, the dependence of the shear modulus is assumed to be linear. It is shown that studied models obey the fundamental laws of thermodynamics. The key step towards the existence proof is derivation of the balance of entropy. This inequality is paramount in the analysis and as its consequence, we obtain sufficient a priori estimates, positivity of temperature and also regularity of the elastic deformation. The second part of the thesis deals with the existence analysis for the isothermal case, however using a completely different method, which is of independent interest.