

## Review of “Counting extensions of imaginary quadratic fields”

The thesis of Alexandr Beneš is about the asymptotics for the number of quadratic extensions of a given discriminant. This is a rich area of research, revolving around the Cohen-Lenstra heuristics, with the most prominent recent advances made by Bhargava et al.

In his work, Alexandr extends the results to relative quadratic extensions over the imaginary quadratics. The first chapter of the thesis has preliminary results from commutative algebra, algebraic number theory and local fields. The second chapter introduces cohomology and various tools from class field theory. This involves introducing cohomology groups, functors, working with idèles, characters, Dedekind zeta functions and general L-functions. The main results are in Sections 2.4 and 2.5, in particular, Theorems 52 and 53, which describe the asymptotic for relative quadratic extension of imaginary quadratic number fields of the odd class number. The results presented seem to be original.

In the thesis, the author tries to communicate preliminaries overly concisely, which makes it hard to read. There are some stylistic issues, like introducing equivalences within the text and not using a new line (see Sections 1.1 and 1.3), or using text in indices of products (such as, in Theorem 40). Also, the author uses incorrect notation for the degree, which should be  $[L : K]$ , not  $[L/K]$ . There are some minor mistakes and typos which are unavoidable in larger texts.

The topics covered in the thesis are far more advanced than any baccalaureate curriculum. The later parts of the thesis, where the author provides his original work, are clear and well written. It demonstrates that not only did Alexandr manage to understand the material, but was also able to think critically and apply it. The results proven in the thesis are novel and could be considered for publication in a good reputable journal. Thus, I will recommend the top mark (výborně) for this thesis.

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