Two aspects of solving the incompressible Navier-Stokes equations are described in the thesis. The preconditioning of the algebraic systems arising from the Finite Element Method discretization of the Navier-Stokes equations is complex due to the saddle point structure of the resulting algebraic problems. The Pressure Convection Diffusion Reaction and the Least Squares Commutator preconditioners constitute two possible choices studied in the thesis. Solving the flow problems in time-dependent domains requires special numerical methods, such as the Fictitious Boundary method and the Arbitrary Lagrangian Eulerian formulation of Navier-Stokes equations which are used in the thesis. The problems examined in the thesis are simulations of experiments conducted in liquid Helium at low temperatures. These simulations can be used to establish a relationship between vorticity and new quantity pseudovorticity in an experiment-like setting.