David Janda: ***Simple categorization of mathematical objects: Examining students' decisions. PhD thesis*** **– opponent's review**

The thesis presents an elaborated research consisting of two pilot studies and the main study; the main study dealing with six research questions (pp. 47-48). I would like to concentrate on the main question of the main study. Allowing myself to paraphrase the author: What differences can we observe if the personal concept image of a category of mathematical objects is developed through examples and non-examples on the one side, and through a definition on the other? Mathematical objects being represented by functions; category by so-called Tall function – developed for this study (pp. 30–33). The definition of a Tall function was formulated in the following sentence: "Every function whose maximum is greater than the absolute value of its minimum will be called a Tall function." (p. 30) And I should add that in the inventive experimental design (p. 49) the author not only though two groups of students in above mentioned ways, tested them (asking to categorize a group of 20 graphs into examples and non-examples of Tall function – pp. 31–32) and in the following interview asked them to define the function and describe their decision processes during the test ("Describe how you were deciding during the test when categorizing presented figures – how the decision process ran." – p. 51). He, likewise, repeated the sequence providing each group with alternative instructional experience (the "definition group" with image activity; the "image group" with definition activity).

What differences did the author observe? Again, I would like to concentrate only on the main findings. Judging by the average percentage of correct answers (p. 60) the definition group outperformed the image group by 10% in the first trial. Both groups profited from the second instructional experience: the definition group by about 10%, the image group by about 15% – reaching about the same level of performance in the second trial (84% and 81% respectively).

The findings from interviews subsequently serve to explain the lower performance of the image group and the need of image activity in the definition group. Most of the definition group decided in most cases working from definition. Most of the image group based their decision only on property abstraction (expressing particular aspects of the presented graphs they considered for their decision) and some were not even able to describe concrete aspects, which determined their decision. As for the question of imperfection in performance of definition group, the interviews revealed that many of them do not follow working from definition in all cases. (p. 84)

In sum, the main findings seem to be psychologically plausible and educationally relevant, in line with e.g. Vygotsky's findings: teacher's verbalisation of know-about can serve as a scaffolding for pupil's development of know-how – correcting radical/simplified constructivist ideas.

I cannot judge whether the thesis makes original contribution to mathematics education as an expert on mathematics education. But judging by the sophisticated chapter Theoretical background and literature (pp. 11–27) and by appropriate Discussion and conclusions (pp. 83–90) it seems relevant and useful. And though the topic may be labelled as an evergreen rather than trendy or a breakthrough, authors approach gives the impression of being up to date. To acknowledge all the results, all the analytical work and all the scrupulous methodological work the author presents in his thesis would require many more pages of review. So, I can sum up, that I can recommend the thesis for defence with clear conscience.

For sake of discussion, I can offer one question, more preciously wish: Could the author think over possible advantages and disadvantages of integrating the questioning about decision criteria and testing: instead of asking for delayed introspection after the test asking students to argument the vote example vs. non-example after each graph?

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