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Price Level Targeting with Imperfect Rationality: A Heuristic Approach

Master's thesis

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Declaration of Authorship

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Abstract

The thesis compares price level targeting and inflation targeting regimes in a New Keynesian model without rational expectations hypothesis. Economic agents instead form their expectations using heuristics—they choose between a few simple rules based on their past forecasting performance. Two main specifications of the price level targeting model are examined—the agents form expectations either about price level or about inflation, which is *ex ante* not equivalent because of sequential nature of the model. In addition, several formulations of the forecasting rules are considered. According to the results, price level targeting is preferred in the case with expectations created about price level under the baseline calibration; but it is sensitive to some model parameters. Furthermore, when expectations are created about inflation, price level targeting over time loses credibility and leads to divergence of the economy. On the other hand, inflation targeting model functions stably. Therefore, while potential benefits of price level targeting have been confirmed under certain assumptions, the results suggest that inflation targeting constitutes significantly more robust choice for monetary policy.

JEL Classification	E31, E37, E52, E58, E70
Keywords	Price level targeting, Inflation targeting, Monetary policy, Bounded rationality, Heuristics
Title	Price Level Targeting with Imperfect Rationality: A Heuristic Approach

Abstrakt

Práce porovnává režimy cílování cenové hladiny a inflačního cílování v Novém Keynesiánském modelu bez hypotézy racionálních očekávání. Ekonomičtí agenti místo toho tvoří svá očekávání s využitím heuristiky—vybírají si mezi několika jednoduchými pravidly na základě přesnosti jejich minulých předpovědí. Dvě hlavní verze modelu cílování cenové hladiny jsou zkoumány—agenti tvoří očekávání buď o cenové hladině, nebo o inflaci, což je *ex ante* odlišné kvůli sekvenční podobě modelu. Kromě toho je porovnáno několik verzí pravidel pro předpovídání. Podle výsledků je cílování cenové hladiny preferováno v případě očekávání tvořených o cenové hladině a při základní kalibraci, ale zároveň je citlivé na změnu některých parametrů modelu. Navíc v případě, že jsou očekávání tvořena o inflaci, cílování cenové hladiny s časem ztrácí důvěryhodnost a vede k divergenci ekonomiky. Inflační cílování oproti tomu funguje stabilně. Zatímco potenciální výhody cílování cenové hladiny tedy byly za určitých předpokladů potvrzeny, inflační cílování dle výsledků představuje výrazně robustnější volbu pro měnovou politiku.

Klasifikace JEL	E31, E37, E52, E58, E70
Klíčová slova	Cílování cenové hladiny, Inflační cílování, Měnová politika, Omezená racionalita, Heuristika
Název práce	Cílování cenové hladiny s nedokonalou racionalitou: heuristický přístup

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Acronyms

PLT price level targeting

IT Inflation targeting

ECB European Central Bank

CNB Czech National Bank

DSGE Dynamic Stochastic General Equilibrium

ZLB zero lower bound

CPI consumer price index

RE rational expectations

GDP gross domestic product

VAR vector autoregression

SNB Swiss National Bank

Master's Thesis Proposal

Author	Bc. Vojtěch Molnár
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Proposed topic	Price Level Targeting with Imperfect Rationality: A Heuristic Approach

Motivation During recent decades, inflation targeting has become standard monetary policy regime across developed countries, but there has been ongoing debate regarding its comparison with price level targeting (which, unlike inflation targeting, attempts to compensate for past deviations from the target). As is discussed by Bohm et al. (2012), traditionally there was a perceived trade-off between lower uncertainty regarding future price level (offered by price level targeting) and between lower output and inflation variability (offered by inflation targeting). Nevertheless, Svensson (1999) suggests that under reasonable conditions, inflation variability could be actually lower under PLT than under IT. This result was also supported by subsequent literature (e.g. Clarida et al. (1999))—theoretical models generally suggest that price level targeting would lead to better results in terms of social welfare than inflation targeting. The compensation for past deviations helps to anchor inflation expectations closer to target, so it is easier to meet the target. On the other hand, no central bank applies price level targeting in practice. The problem is that results of theoretical models are largely driven by rational expectations of all economic agents. Policymakers consider this assumption as too restrictive and they are afraid that in reality, it would be much more difficult to communicate PLT to public, to establish its credibility and to anchor inflation expectations in the way the models suggest. Therefore, this thesis will attempt to address explicitly the role of rationality and of extent to which economic agents are forward-looking. It will be based on aforementioned standard macroeconomic models, but there will be a friction in rationality, so that inflation expectations will not be formed in perfectly rational way. This should offer new insight into comparison of inflation to price level targeting, which should be closer to reality than results suggested by models using rational expectations hypothesis.

Hypotheses

Hypothesis #1: In standard model with characteristics of the Czech Republic and with assumption of perfect rationality, price level targeting leads to better outcome in terms of economic stability and social welfare than inflation targeting.

Hypothesis #2: Abandoning rational expectations assumption decreases attractivity of price level targeting, where the higher the deviation from rational expectations hypothesis, the worse is performance of the price level targeting.

Hypothesis #3: When zero lower bound is considered, strength of price level targeting is higher than in normal conditions (even without fully rational expectations).

Methodology The core approach will be DSGE model, which is a standard tool in macroeconomic modelling. The model will be focused on the Czech Republic, although its results should be in general applicable also to other countries (at least to those with characteristics similar to the Czech Republic). The main deviation from classic DSGE models will be a friction in rationality, so that inflation expectations will no longer be formed with complete rationality—adaptive learning will be utilized instead (literature regarding this topic is summarized e.g. by Milani (2012)). Results of price level targeting in such model will then be compared with inflation targeting, where output and inflation variability will be the key measures of performance of the two monetary policy regimes. The comparison will be conducted under normal economic circumstances and at zero lower bound on interest rates.

Expected Contribution There is only small number of papers which investigates price level targeting with some imperfections in rationality of agents and credibility of the regime (Honkapohja and Mitra (2018) or Cateau and Shukayev (2018) are two examples), and to my best knowledge none of them focuses on the Czech Republic. Therefore the thesis should bring more realistic comparison of currently used inflation targeting and its alternative, price level targeting; and it should suggest whether price level targeting is a viable option for monetary policymaking in Czech conditions.

Outline

1. Introduction
2. Theoretical background, literature review (comparison of PLT and IT in the literature, role of rational expectations and of extent to which economic agents are forward-looking)

3. Methodology – DSGE model (description of basic model and approach to modelling imperfect rationality in formation of expectations)
4. Results (comparison of PLT and IT based on the model results)
5. Discussion, robustness assessment
6. Conclusion

Core bibliography

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Amendment: A few changes have been made in the final version of the thesis as compared with the proposal. Heuristic modelling has been applied, which represents more substantial deviation from rational expectations hypothesis than originally planned adaptive learning. Moreover, original plan to use model calibrated for the Czech Republic has been abandoned in favour of simple general model, which has enabled to focus mainly on process of expectations formation using heuristics. Nevertheless, the core of the original intention—examination of PLT in a model without perfect rationality—has remained the same.

Chapter 1

Introduction

Inflation targeting (IT) has become state-of-the-art in monetary policymaking during last three decades and it has been adopted by over 40 central banks (with the number gradually increasing). Nevertheless, there has been an ongoing debate and research of alternative policy frameworks, reinforced especially by the Great Recession and subsequent era of nominal interest rates close to their zero lower bound (ZLB)—problem, which has recently gained in topicality even more because of beginning economic recession induced by global pandemic. One significant stream of the research is focused on price level targeting (PLT), under which the targeted path of price level can still increase over time (consistently with small but positive inflation rate), but contrary to IT, the PLT regime compensates for past deviations of inflation from its steady state rate.

The difference seems small and in fact, in a hypothetical world where central bank was able to reach precisely its targeted inflation rate/price level under all circumstances (there are no shocks to prices and no regard for output stabilization), both regimes would be equivalent. In practice, however, the difference could have significant impact on conducting monetary policy. At first sight, price level targeting with its attempt to correct for past deviations might induce higher inflation and output volatility. Nevertheless, some literature contrarily suggests that it could actually decrease the volatility—when inflation is lower than targeted, economic agents know that the central bank will balance the deviation by higher inflation, which (if the regime is credible) increases inflation expectations, and that in turn helps the central bank to stabilize the economy and to return back to the targeted price level faster.

The crucial aspect here is that lower inflation now leads to higher inflation expectations for the future and *vice versa*. This works well in theoretical models with rational expectations (RE) and credible PLT regime—but such strong assumptions might not hold in reality. Moreover, there is very limited historical experience with PLT, so it is not possible to compare IT and PLT empirically. In fact, lack of belief in the strong assumptions among policymakers is probably the main reason why the discussion remains only theoretical and no central bank uses PLT in practice. Therefore, probably the only option is to try to relax the assumptions within the framework of the theoretical models and to compare IT and PLT in more realistic setting. This thesis represents one attempt to do so by replacing rational expectations hypothesis with heuristic approach as proposed for example by De Grauwe (2012). Economic agents form expectations by choosing between a few simple rules based on their past forecasting performance; in other aspects, the model corresponds to a standard textbook 3-equation New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. Analysis conducted within the thesis should contribute to the discussion of whether the theoretical benefits of price level targeting might hold also in reality, even without fully rational economic agents.

The thesis is structured as follows: Chapter 2 discusses price level targeting from theoretical perspective. Its advantages and drawbacks are summarized (where inflation targeting serves as a benchmark for comparison), issues with its practical implementation are discussed, literature examining its impact on social welfare and economic volatility is presented, and limited practical experience is briefly described. Chapter 3 discusses the methodology, i.e. macroeconomic model with expectations created using heuristics. Chapter 4 presents results of the analysis, while Chapter 5 assesses robustness of the results and their implications. Chapter 6 concludes. Appendices then contain some supplementary mathematical derivations and additional figures, respectively.

Chapter 2

Price Level Targeting

New Zealand was the first country to formally adopt IT framework in 1990. Number of inflation targeters has been gradually increasing since then and the framework has become state-of-the-art in monetary policy across the world. As of April 30, 2018, IT has been officially adopted by 41 countries (IMF 2019). In addition, other central banks including US Fed and European Central Bank (ECB) are using many aspects of IT despite not being formal inflation targeters.

IT has remained standard monetary policy regime across developed and middle income countries even during and after turbulent times of the Great Recession. Nevertheless, the crisis did challenge the framework to some extent. Many countries have been struggling with ZLB[1] on nominal interest rates and with a fear of deflation. Central banks have got by with unconventional monetary policy tools within the IT framework so far, but interest in alternative options has been renewed as well. Price level targeting represents one of the alternatives (and likely the most discussed one). Research dedicated to PLT goes back to 1990s, but it surged after the Great Recession.

Furthermore, as of half of 2020 it is clear that the world is coming into deep recession due to coronavirus pandemic. IMF (2020) in its *World Economic Outlook* forecasts global real gross domestic product (GDP) in 2020 to fall by 4.9%, and in case of advanced economies by 8%. CNB (2020) for the Czech Republic predicts output growth of -8% for 2020 in the baseline scenario. Moreover, all estimates are exceptionally uncertain with substantial further downside risks

[1] The term *zero lower bound* in whole thesis should be interpreted a bit loosely, in a sense of effective lower bound, which may differ from zero. For example, several central banks have applied slightly negative rates. Nevertheless, zero can serve as a good approximation of the effective lower bound in general discussions.

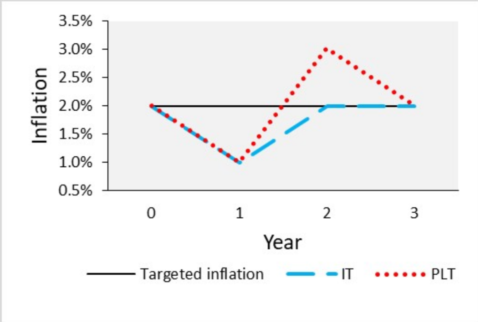
(e.g. forecast of the Czech National Bank (CNB) in a scenario with second wave of pandemic is -13.5%). And the recession comes in a situation with fairly limited space for monetary stimulus. Several central banks, such as ECB or Swiss National Bank (SNB), were stuck at the ZLB and were using unconventional policy tools even before the pandemic.[2] And those central banks which were actually able to increase their interest rates in the previous years and to get away from the ZLB, such as the Fed or the CNB, quickly lowered the rates after the pandemic broke out and have been back close to the lower bound since then. Such situation poses yet another challenge to the inflation targeting framework; and it strengthens even more the importance of researching further options, such as price level targeting.

Just as IT, PLT aims to attain price stability and it is consistent with a positive targeted inflation rate (targeted price level could easily increase every year by, say, 2%). The difference between the two regimes lies in a reaction to deviations from the targeted inflation. For example, let us assume that a central bank aims at 2% inflation rate and initial price level is 100. This is consistent with targeted price level 102 after one year and 104 after two years. When the actual inflation in the first year is due to a shock only 1%, so the price level increases to 101, central bank under inflation targeting will attempt to bring inflation back to 2% in the following year, which means price level 103 in the second year. Therefore, price level will be permanently affected by the shock. On the other hand, central bank under price level targeting will in the same situation aim at price level 104 in the second year, so the targeted inflation rate is 3% for one year, and only when the desired price level is reached, it stabilizes at 2%.[3] To put it simply, price level targeting compensates for past deviations of inflation from its target, while inflation targeting cares only about the future development and not about the past. Figures 2.1 and 2.2 depict the response on inflation being below the target in year 1 under the two regimes graphically. Now let us turn to implications of the difference between the two policy frameworks.

[2] ECB's deposit facility as well SNB's main policy rate have been negative and ECB has been still conducting its quantitative easing policy.

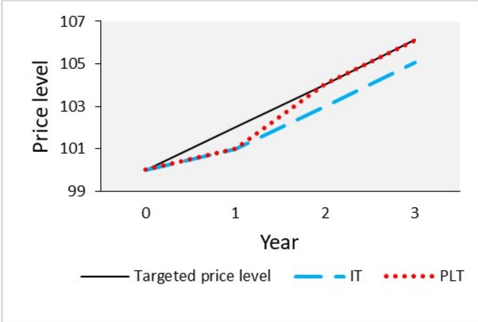
[3] For illustration purposes it is neglected that in reality, central bank under both regimes would likely try to smooth the economic development and would attempt to reach the desired inflation rate/price level in slightly longer time horizon. Moreover, given numbers are rounded (e.g. precise price level in the second year consistent with 2% inflation would be 104.04, which is, however, negligible difference.)

Figure 2.1: Inflation rate
Desired inflation rate when a shock causes inflation to be 1% in year 1.



Source: Author

Figure 2.2: Price level path
Desired price level path when a shock causes inflation to be 1% in year 1.



Source: Author

2.1 Advantages and drawbacks of PLT

This section presents theoretical benefits of PLT, as well as its shortcomings which could hinder realization of the benefits in practice. Inflation targeting serves as a benchmark for comparison. For detailed discussion of the points presented here see for example Cournede & Moccero (2011) or Popescu (2012).

Firstly, price level targeting regime can serve as an automatic stabilizer of the economy. For now let us assume that the regime is credible and well understood by public. Then any shock to prices in one direction leads to movement of inflation expectations in the opposite direction. A deflationary shock[4] under IT regime may cause inflation expectations to decrease and it can be a challenge for the central bank to return them back to the inflation target, because there is no reason to believe that inflation could actually exceed the target in the future. Therefore, as follows from the Fisher equation (with i being nominal interest rate, r *ex ante* real rate, and π^e future inflation expectations)

$$i = r + \pi^e \tag{2.1}$$

such situation requires sufficiently strong reduction of nominal interest rate by the central bank in order to decrease real interest rate and bring the economy back to the equilibrium. On the other hand, a deflationary shock under PLT leads to an *increase* in inflation expectations. Real interest rate then declines even for stable nominal interest rate. (The nominal rate can be of course adjusted as well, but the response does not need to be that strong). And this mechanism leads not only to less volatile nominal interest rate, but it can also

[4]The same logic in the opposite direction obviously applies for an inflationary shock.

anchor inflation closer to its long-term desired rate and decrease output volatility (see more detailed discussion about the volatility under PLT in Section 2.2).

The mechanism described above may play even larger role when there is a threat of falling into a liquidity trap. As there is lower need for nominal interest rate adjustments under PLT, there is also lower probability of hitting the ZLB on nominal interest rates. Price level targeting could potentially also help an economy with nominal rates already at the ZLB, as the only way to decrease real rate further with constant nominal rate is through higher inflation expectations, which could be achieved by the switch to PLT. However, as Cournede & Moccero (2011) argue, this benefit might not be as strong—firstly, switch of monetary policy regime under these exceptional circumstances could challenge long-term credibility of monetary policy[5], and secondly, elasticity of demand with respect to the real rate would be likely lower at the ZLB than normally. To get away from the liquidity trap more effectively, PLT could be (at least in case of an open economy) for example mixed with exchange rate peg in an unconventional temporary regime as proposed by Svensson (2000). Nevertheless, even though PLT itself might not be that efficient after the economy has already entered into the liquidity trap, the lower likelihood of the ZLB constitutes one of the key benefits of PLT, especially in current times of low nominal interest rates across the world.

Potentially lower short-term economic volatility under PLT has been considered mainly since the seminal paper of Svensson (1999b) (to be discussed below) and the benefits of the regime with respect to the ZLB has been emphasized especially since the Great Recession, but one traditional advantage of PLT discussed even before the Svensson's work is lower uncertainty about price level in the long-term. Price level under credible PLT is stationary around its targeted trend, as the central bank compensates for past inflation deviations and price level always returns back to its anticipated path. On the other hand, central bank targeting inflation does not correct for past deviations, so a temporary shock to inflation permanently affects price level. A sequence of shocks over time can then lead to a significant divergence of the actual price level from the price level consistent with targeted inflation rate. Technically, price level is no longer stationary and its variance increases up to the infinity in the very long term (Minford & Peel 2003). Such uncertainty about long-

[5]Similar conclusion has been drawn also by Bodenstein *et al.* (2019), who show in a model with learning that temporarily introducing PLT during deep recession would be ineffective, while adopting it permanently during calm times would be much more appropriate.

term prices may cause suboptimal intertemporal planning of economic agents, at least if indexation is costly or imperfect. Result is lower efficiency in the economy and lower social welfare. Conversely, lower uncertainty about future price level under PLT is closely related to lower inflation risk. As Cournede & Moccero (2011) argue, costs of long-term borrowing are reduced, which can lead to higher capital accumulation. Therefore, while the points discussed in the previous paragraphs deal rather with stabilization of the economy around the equilibrium, the argument described here suggests that PLT could actually lead to higher potential output.

Both Cournede & Moccero (2011) and Popescu (2012) also mention that PLT could decrease probability or severity of asset price bubbles. As discussed above, central bank after a deflationary shock needs to decrease nominal interest rate more aggressively under IT than under PLT. Nominal rates below the neutral level for a sustained period of time can then contribute to a creation of asset price bubbles.

The benefits of the PLT described above hinge on the anchoring expectations about future prices. The mechanism functions very well when all economic agents are forward looking, forming expectations in a rational way, and when the monetary policy regime is fully credible—the central bank is trusted to ensure that the targeted price level will be met. But the assumptions are hardly realistic, which poses significant obstacles into practical implementation of PLT. It is then a question of extent to which the assumptions hold at least partially in reality—whether PLT may still outperform IT.

One obvious practical challenge of PLT is communication. Central banks targeting inflation can simply use one number of the targeted rate to communicate its main goal. But explanation of the price level targeting regime to the public would be substantially more complicated. While the targeted inflation rate usually remains constant over time (at least when the regime is already well established), targeted price level changes over time.[6] For the long-term perspective, a central bank could communicate yearly rate of increase of targeted price level (i.e. steady state inflation consistent with the regime). But after a shock, inflation rate necessary to go back to the targeted price level would deviate from the long-term rate for some time. As the desired inflation rate consistent with PLT changes over time in the short-term (when there are

[6]Of course unless constant price level was targeted, which is however not likely to occur in practice. If a central bank adopted PLT in following years, it would very probably do so with targeted path of price level increasing over time by a constant rate.

shocks), its communication could be confusing. Central bank could instead of the inflation announce targeted level of consumer price index (CPI). But firstly, even this variable changes over time (although in a clear and predictable way, while the desired inflation rate changes unpredictably in response to shocks), and secondly, the CPI is much more abstract variable for general public than inflation rate—it is not difficult to imagine what 2% year-to-year inflation means, but CPI, say, 140 does not mean anything without putting it into context.

Effectiveness of price level targeting could be also dampened by imperfect credibility of central bank and of the regime. As described above, movement of inflation expectations in the opposite direction than deviation of current inflation is a crucial aspect of functioning of PLT. But when people are not persuaded that the central bank will really compensate for current shock and return back to the targeted price level path, they no longer form inflation expectations in the opposite direction to the shock and the key advantage of PLT stops working.

The truth is that credibility of central bank is important also in the IT regime, where it helps to anchor inflation expectations close to the target. Without the credibility, it is very difficult for the central bank to reach its objectives in both PLT and IT. So at the first sight there might be little difference between the regimes in this aspect. Moreover, central banks at least in developed countries usually have sufficient credibility among public. On the other hand, there are a few counterarguments to this reasoning. Firstly, inflation expectations play even higher role in PLT, so the requirements on the regime credibility may be even higher than in IT. Secondly, switch of the regime from IT to PLT could actually at least temporarily lower the credibility. Central banks adopting IT in some cases had to employ strong measures (significant adjustments of nominal interest rate) in initial phase of the regime to manifest their commitment to it. After the commitment had been shown, inflation expectations became better anchored and then it has been sufficient to adjust nominal interest rate less. But there is a risk that switch to a different regime could raise another need to show strong commitment to the new regime. To ensure that inflation expectations under PLT are really formed in the desired way, central bank might choose too short monetary policy horizon in the initial phase. It would aggressively react to all shocks to the price level in order to contain them and compensate for them as soon as possible. This would likely increase volatility of the economy. Effects of automatic stabilization of the economy described above as one of the benefits of PLT could then materialize only after this more volatile initial period. This drawback of the PLT adoption is even more pronounced when considering results of Smets (2003) which suggest (for a benchmark calibration, where loss function includes positive weights on both inflation and output gap variability and a slightly positive weight on interest rate volatility) that optimal policy horizon for PLT is about twice as long as optimal horizon for IT.

And thirdly, PLT is prone to dynamic inconsistency problem. As discussed by Bohm *et al.* (2012), a positive shock to prices today under the PLT framework leads to decrease in inflation expectations in the future, which should in turn help to lower the inflation rate. But once inflation is back at its long-term target, from short-term perspective it would be optimal for both the central bank and the whole economy to remain at the long-term rate instead of further decreasing inflation to meet targeted price level. The reason is that the further decrease of inflation required by the PLT regime depresses output. But if people are afraid that the central bank might actually abandon current targeted price level path and reset it to a new one, they will no longer decrease their inflation expectations in the first place and the whole mechanism of PLT breaks down. Therefore, high level of credibility and independence of the central bank is needed in order to avoid such situation—long-term perspective must prevail for the benefits of PLT to materialize. Originally, the fact that inflation targeting is not subject to the dynamic inconsistency has actually been one of its key theoretical underpinnings. This is another reason why the importance of central bank's credibility is even higher under PLT than under IT.

Moreover, central bank targeting inflation can under certain circumstances make use of escape clauses. For example, when a temporary deflationary external environment affects the economy (e.g. decrease of oil prices), the central bank can tolerate lower inflation for some time. Inflation below the target for this reason is more permissible than in case of sluggish domestic demand. In this case, the central bank can apply escape clause, does not respond aggressively to the shock, and only gradually aims at return to the target. But as IT takes into account only the future and not the past, and the long-term inflation target remains the same, the escape clause is consistent with whole policy regime. On the other hand, using the escape clause in the same situation under PLT and not reacting to the shock would mean change of the long-term targeted price level path, which would be challenging to communicate properly. Probably the only other option would be to react to the shock more aggressively, which would induce unnecessary volatility to the economy.

The drawbacks of price level targeting presented here show why the regime remains rather theoretical concept and no central bank uses it in practice. On the other hand, its potential benefits are quite strong as well, especially in times of low nominal interest rates and ubiquitous threat of the zero lower bound problem. Therefore, it remains a goal for economic research to compare both regimes in more detail under specific assumptions of various models.

2.2 Literature review

Much of the research regarding PLT in last two decades has been inspired by seminal paper of Svensson (1999b). This section is focused on this time period; see Bohm *et al.* (2012) for a brief overview of traditional debate (dealing with a perceived trade-off between lower uncertainty about long-term prices and higher inflation volatility under PLT) before Svensson.

Model presented in the Svensson's paper contains neoclassical Phillips curve and assumes that central bank has complete control over inflation rate and discretionary chooses the inflation in each period in order to minimize loss function. The loss function has two main specifications—one with deviation of inflation rate from its target (corresponding to IT) and one with deviation of price level from its targeted path (corresponding to PLT); both versions also contain deviation of output gap from its (non-negative) target. If output gap is at least moderately persistent, price level targeting brings lower inflation variability (lower unconditional variance of inflation) than inflation targeting. In particular, if the desired output gap is zero, level of output gap persistence above 0.5 is sufficient for PLT to bring lower inflation variability than IT. Moreover, even if society prefers inflation stabilization as opposed to price level stabilization (i.e. loss function of the society is specified in terms of inflation deviation from the target), assigning goal of price level stabilization to the central bank (i.e. central bank minimizes the other loss function with price level deviation) leads to results closer to optimal monetary policy than assigning inflation target.

While Svensson uses backward-looking model, Vestin (2006) confirms his key result in a standard forward-looking New Keynesian model. He assumes that central bank can operate only in discretionary environment, but optimal commitment solution can be replicated under PLT in scenario with no inflation persistence. This is a strong result, as optimal monetary policy under commitment generally leads to at least as high social welfare as under discretion (and usually to higher welfare). In addition, PLT leads to better inflation-output volatility trade-off than IT even with inflation persistence.

Some further support for price level targeting is provided e.g. by Cover & Pecorino (2005), Acuna-Roa & Parra-Polania (2016), or Covas & Zhang (2010). The first paper complements Svensson's analysis and shows that his results are valid under broader set of assumptions. In particular, PLT leads to lower volatility of the economy even without output gap persistence. The second paper applies New Keynesian model with inflation persistence and concludes that welfare loss from macroeconomic volatility can be reduced by 29% by adopting PLT in the baseline scenario. Furthermore, PLT becomes suboptimal only for level of indexation to lagged inflation exceeding at least 65%. Covas & Zhang (2010) then show dominance of PLT in a New Keynesian model with financial frictions in both debt and equity markets (difference between the regimes increases if the central bank also targets asset prices). Moreover, Woodford (2003) demonstrates that optimal monetary policy in forward-looking New Keynesian RE model contains certain level of history dependency, which is embedded in PLT, but not in IT.

Cateau (2017) analyses the problem from perspective of model uncertainty—economic agents form their expectations based on model different from one used by policymakers. On one hand, performance of PLT deteriorates faster than that of IT, which is more robust in this setting; on the other hand, for empirically reasonable models used by people, total gains from PLT still outweigh the decreasing performance of the regime.

Eggertsson & Woodford (2003) focus specifically on proper conduct of monetary policy constrained by the zero lower bound. They emphasis crucial role of correct management of inflation expectations. The authors use simple dynamic model and show that completely forward looking monetary policy leads to suboptimal outcomes at the ZLB. Conversely, a price level targeting rule is found to be an ideal solution to steer inflation expectations in the desired way. Zero lower bound is examined also by Nakov (2008), who shows that even though none simple monetary policy rule corresponds to optimal policy in presence of the ZLB, PLT leads to welfare loss exceeding the optimal policy loss only by 60%, while loss under IT with Taylor rule is twenty times higher as compared with the optimal policy. Coibion *et al.* (2012) then use New Keynesian model with non-zero steady state inflation and based on micro-founded welfare loss function, the authors demonstrate that optimal inflation target under IT is 2% at most. PLT then, firstly, leads to higher social welfare for any given inflation target; secondly, its optimal rate of (long-term) inflation is below 0.3%, which basically means actual price stability; and finally, it substantially reduces frequency of the zero lower bound episodes.

Impact of PLT as compared with IT on certain more specific economic aspects is examined e.g. by Azcona (2018) and Cole (2018). The first paper is focused on exchange rate volatility. The author uses a DSGE model of a small open economy with parameters estimated for Canada and concludes that replacement of IT by PLT could in case of supply shocks lower both nominal and real exchange rate volatility without increasing inflation or output volatility. Results for demand shocks are less clear—PLT delivers lower economic volatility only for shocks with low persistence. Simulation of multiple shocks hitting Canadian economy based on actual data indicates moderately lower real exchange rate volatility and significantly lower nominal exchange rate volatility under PLT. Reduced volatility of the exchange rate can be beneficial for international trade and may comprise one potential additional benefit of PLT. Cole (2018) then examines effectiveness of forward guidance. A DSGE model with two alternative policy rules (one for PLT and one for IT) either general or calibrated for the US economy shows that output and inflation react more strongly to forward guidance under PLT. When monetary policy rule contains interest rate inertia and when the central bank reacts aggressively to deviations of inflation from the target, difference between the policy regimes is lower, but PLT is still superior to IT quite robustly in terms of forward guidance effectiveness.

Quite different approach is applied by Berentsen & Waller (2011), who use a DSGE model with fully flexible prices, in which money is crucial for trade; the authors therefore adopt real business cycle approach rather than New Keynesian. Nevertheless, PLT improves social welfare even under these assumptions due to decreased short-term economic volatility. Additionally, the authors provide support for nominal interest rate smoothing. Ball *et al.* (2005) then apply inattention framework; they do not consider the impact of inflation expectations on aggregate demand, but firms incorporate new macroeconomic information into their pricing decisions only gradually. Flexible price level targeting is then found to be an optimal policy.

Many of the aforementioned studies deal with New Keynesian Phillips curve as given, but Minford & Peel (2003) consider indexation as endogenous—they show that PLT would result in lower degree of indexation in the economy, which would then bring substantial welfare gains from more flexible prices.

While support for PLT in rational expectations models is clearly substantial, there are several papers relaxing some of strong assumptions of such models. For example, Masson & Shukayev (2011) introduce an escape clause allowing for reset of the price level target after large shocks. It turns out that this possibility endangers credibility of the policy regime, it may lead to multiple equilibria (including low credibility equilibrium with high economic volatility and frequent changes of the target), and the result can be worse performance of PLT than of IT. These results are confirmed by Cateau & Shukayev (2018), who conduct similar analysis (the main difference between the studies is when and how the central bank resets the targeted price level) and who also conclude that the possibility to reevaluate targeted prices causes substantial deterioration in the effectiveness of PLT.

Honkapohja & Mitra (2019) replace rational expectations with adaptive learning (more on the methodology in section 3.2); full credibility of PLT is replaced by only partial credibility, which endogenously evolves over time. Their results are mixed—as long as there is at least some initial credibility of PLT, the regime outperforms IT during a liquidity trap; on the other hand, IT is superior when zero lower bound constraint is not binding. The authors also find that their model of PLT with learning fits well data from Swedish experiment with the regime in 1930s (more on this in section 2.3). Implications of imperfect knowledge and adaptive learning are also explored by Eusepi & Preston (2018) and Gaspar *et al.* (2007), who find that PLT can be appropriate policy regime even without rational expectations.

Transition from IT to PLT is examined by Cateau *et al.* (2009), who use large scale macroeconomic model used in Bank of Canada (ToTEM). They model transition between regimes as a Markov switching process with endogenous state of either low or high credibility of the policy regime. Low credibility corresponds to positive probability of switching back to IT, while high credibility represents rational expectations consistent with PLT. It turns out that potential welfare gain from the switch to PLT can be quite substantial, and it would become negative only if the economy was in the low credibility state for at least 13 years.

Yetman (2005) introduces consumers using rule-of-thumb forecasting into otherwise standard macroeconomic model with rational expectations and shows that even a small portion of rule-of-thumb consumers reverses optimality of price level targeting. While performance of both IT and PLT diminishes with increasing number of these consumers, PLT is less robust and deteriorates faster.

Such conclusion is actually in line with results of this thesis to be presented in chapter 4, even though particular details of both methodology and results differ between Yetman's paper and the thesis. In addition, PLT with economic agents making forecasts based on simple rules has been examined also by Ho *et al.* (2019), but as their paper is from methodological perspective closest to this thesis, it will be discussed in chapter 3.

There is also a question whether any practical difference between the two regimes actually exists. Ruge-Murcia (2014) empirically assesses five inflation targeting countries and finds that there is the difference for Australia, New Zealand, Sweden, and the United Kingdom. On the other hand, tests conducted by the author reject unit root in price level of Canada, suggesting that monetary development in this country has been similar to one which would be expected under PLT regime (but number of observations is not sufficiently high to make strong conclusions based on the analysis). Furthermore, Chadha & Nolan (2002) use simple New Keynesian model to show that while PLT outperforms IT, both regimes can in fact produce similar results in terms of economic volatility, as long as weight on inflation in IT Taylor rule is sufficiently high. In presence of costs of transition from IT to PLT, gains from the transition hence may be too low to outweigh the costs.

More detailed surveys of literature on price level targeting are provided by Ambler (2009), who in general confirms superiority of PLT in rational expectations models; and by Hatcher & Minford (2016), who focus on development since Ambler's survey and discuss papers on optimal monetary policy, ZLB, transition from IT to PLT, or financial frictions. They distinguish between central bank committing to simple rule of Taylor type versus minimizing loss function—PLT is dominant especially under the first approach. Overall, the authors favour PLT as well, although the conclusion is not unequivocal.

To summarize, models with rational expectations provide strong support for PLT in various model settings. There have been a few attempts to model expectations differently, which have led to mixed results—PLT is still superior to IT in some cases (but at least certain level of its credibility is crucial for such result), while it ceases to perform well without rational expectations according to other studies.

2.3 Practical experience

The only historical experience widely considered as price level targeting was Swedish monetary policy in 1930s. The most important points are briefly mentioned here; for a detailed discussion see e.g. Berg & Jonung (1999), Straumann & Woitek (2009), or Rathke *et al.* (2017).

Sweden suspended gold standard in 1931 and replaced it with a goal of maintaining price level stability. New regime was defined rather vaguely and it got more detailed outline only in 1932. Bohm *et al.* (2012) mention five key aspects of the new program—flexible exchange rate for now, but with foreseen return to gold standard or peg to British pound later on; resistance towards both deflation and inflation; allowing slight increase in prices, but not too large; focus on no particular price index; and an attempt to keep interest rates low unless it would be inconsistent with the other goals of the program. In 1933, exchange rate peg of Swedish krona to British pound was established, lasting until the beginning of WWII. The price level targeting regime effectively ended in 1937, when the mandate of the Swedish Riksbank has been broaden to include also other goals such as economic stability and employment.

Sweden achieved quite strong economic recovery in 1930s (when compared with other countries after the Great Depression), and abandoning of gold standard contributed to end of deflationary expectations. On the other hand, it is not possible to make strong conclusions about PLT based on this short experience. The regime definition differed from what is usually considered as price level targeting in the current discussions. The regime's goal was price stabilization, but no clear numerical target was set (as mentioned, the Riksbank did not target any particular price index). It did not include rise of the targeted price level—natural element of contemporaneous discussions about PLT. The regime was considered as temporary from the beginning (adopting PLT temporarily has been discussed in the recent literature mainly in relation to the zero lower bound on nominal interest rate, which was not the case of Sweden).

In addition, targeting price stability was accompanied by the exchange rate peg after 1933. Straumann & Woitek (2009) argue, based on their Bayesian vector autoregression (VAR) model and narrative evidence from archival sources, that actual actions of Riksbank did not correspond to its official goal of price stability—the main focus was on exchange rate stabilization (which was approached more as a goal in itself rather than just a measure to reach the price stability). Rathke *et al.* (2017) estimated DSGE model for small open economy and concluded that the crucial aspect for the economic recovery was undervaluation of krona rather than the price level targeting framework. The Swedish monetary policy in fact contained several aspects of "foolproof way of escaping from a liquidity trap" as described a couple decades later by Svensson (2000), although there was no rising price level target path. The authors moreover argue that the undervaluation was just a result of a need for increase in Riksbank's foreign exchange reserves, not an intentional policy aimed at economic stabilization—the recovery was just a pleasant side effect.

Bohm *et al.* (2012) mention also one other historical experience which could be considered as price level targeting—deflationary policy as a part of currency reform in Czechoslovakia after WWI. Its goal was to bring prices down to pre-war level. The policy was enforced mainly through appreciation of koruna induced by operations on foreign exchange market introduced in 1921. The result was successful decrease of prices, but at a cost of severe recession in 1922; nevertheless, the economy recovered in the following years and prices stabilized (although they were still more volatile than what is common in current inflation targeting countries). In 1929, gold standard was reintroduced. As the authors summarize, this experience does not say much about effectiveness of the PLT framework in general—the policy was adopted in unstable times after WWI, soon after separation of the koruna from the Austrian krone, when inflation rate exceeded 50% in several previous years. The key policy instrument were foreign exchange interventions rather than policy interest rates standard now, and just as in Sweden, the return to gold standard was foreseen from the beginning.

Apart from these two episodes, PLT is discussed only theoretically. Svensson (2000) recommends his already mentioned "foolproof way" specifically to Japan, which was struggling with low inflation, sluggish growth, and the ZLB in the 1990s, but Bank of Japan never employed this approach. The same paper served as a theoretical underpinning of exchange rate commitment in the Czech Republic in 2013-2017, which was adopted in time of low and decreasing inflation, and with a threat of deflation. As Franta *et al.* (2014) discuss, the CNB in fact foresaw that the commitment would lead to an increase in inflation above 2% target, followed by convergence to the target from above. This should compensate for previous period of low inflation and lead to average inflation close to the target; the policy therefore implicitly included price level targeting characteristics. On the other hand, the policy was based on nominal exchange rate peg without any commitment towards future price level and without intention to abandon inflation targeting framework. Such approach hence represents both the potential benefits of PLT as well as the reluctance of policymakers to adopt the regime fully due to perceived cognitive limitations of people.

During last decade, PLT has been discussed mostly in connection with Canada—it has been probably the only country seriously considering the policy. Intensive research was going on before 2011 trying to assess whether adoption of PLT would be beneficial for Canadian economy. Bank of Canada (2011) summarizes results—even though theoretical benefits of PLT were confirmed, policymakers were not convinced that the practical challenges of the regime would be outweighed by the theoretical benefits. Bank of Canada therefore continued in conducting inflation targeting—but it still claimed that the assessment of PLT might change in the future, especially given potentially higher probability of hitting the ZLB. Speech of the Bank's Senior Deputy Governor Wilkins (2018) is an example that the debate about PLT in Canada is still alive, although it is just one of more considered policy approaches.

Finally, with upcoming revision of Fed's monetary policy framework planned during 2020 together with pandemic-induced global recession (bringing the ZLB problem again to the foreground), topicality of PLT has recently increased even more. Article by Towning (2020) discusses how price level targeting could play a role in the new policy framework of the Fed, citing also former Fed's Chair Janet Yellen, who supports such regime. Way of actual implementation of PLT (if adopted) is yet to be determined—likely it would not be introduced in a pure sense, given challenges in communication and practical operation; rather just some of its elements might enrich current monetary policy regime. At the same time there is a concern whether it would be appropriate to introduce an important regime change during current time of unusually high economic uncertainty. Nevertheless, such debate confirms that PLT is at the frontier of current monetary policy discussions and that there is a need for its further research.

Chapter 3

Methodology

Now let us discuss macroeconomic modelling in context of bounded rationality. Firstly, several drawbacks of mainstream DSGE models are discussed, which provides rationale for different approaches. After that, various options of modelling departure from RE hypothesis are shortly discussed and model applied in this thesis is presented in detail. Lastly, general characteristics of the model are described to have a comparison with standard DSGE models before turning into own analysis in the next chapter.

3.1 Standard DSGE models

New Keynesian DSGE models have become crucial tools in macroeconomics during recent decades. Their key characteristics include, firstly, firms maximizing profits and consumers maximizing utility over time (i.e. the models are micro-founded and dynamic). Secondly, there is an assumption of price rigidity, where firms (operating in a monopolistic competition environment) are unable to immediately adjust all prices to a desired level. The rigidity is most often modelled by Calvo pricing (Calvo 1983), where in each period, a fraction θ of prices remains unchanged and only $1-\theta$ of prices can be adjusted. And most importantly in context of this thesis, all consumers and firms are assumed to have rational expectations, i.e. they know and utilize all available information, understand the model and whole economy, and are capable of complex computations in forecasting the future and optimizing their utilities and profits over time. The only existing uncertainty is from unknown realizations of future stochastic shocks—the agents, however, know at least statistical distribution of the shocks. For a textbook derivation of standard DSGE models see e.g. Gali

(2015); general discussion of the topic and description of the canonical model is provided, among others, by Clarida *et al.* (1999); and a typical example of actual application can be model of Smets & Wouters (2007).

Despite their widespread use, DSGE models suffer from several drawbacks. From behavioural and microeconomic perspective, the level of cognitive abilities they assign to economic agents is highly unrealistic and certainly not consistent with research of psychology and behavioural economics. Detailed overview of the field is beyond scope of this text, but to mention just a few contributions, Kahneman (1994) or Thaler (1980) discuss research questioning rationality assumption. Tversky & Kahneman (1974) describe use of heuristics in decision making (model with heuristics in a stylized way is to be applied in this thesis). And for more general overview of development of the gap between psychology and economics see e.g. Kahneman (2003).

Furthermore, significant stream of literature including Mankiw *et al.* (2003) or Andrade *et al.* (2016) shows significant and time-varying heterogeneity in inflation expectations (and in case of the latter paper also in expectations of other macroeconomic variables) across economic agents; Falck *et al.* (2019) build on this results to provide evidence on US data that level of the heterogeneity in the expectations can have strong influence on monetary policy transmission. This certainly contradicts the assumption of homogeneous rational expectations. De Grauwe (2012) also discusses that DSGE models have space only for risk (with known statistical distribution and hence quantifiable), while there is no room for fundamental unquantifiable uncertainty—which does not correspond to reality.

The truth is that each model is by definition a simplification and is therefore in some sense unrealistic. Nevertheless, the issues mentioned above have some undesirable consequences. Firstly, all economic fluctuations are explained by exogenous shocks and impossibility of economic agents to immediately adjust to them.[1] The model in a sense does not really explain business cycle, it just says a shock happened. Secondly and more empirically, De Grauwe (2012) contrasts normal distribution of output gaps simulated by DSGE models to non-normality in empirically observed US output (there is excess kurtosis, i.e. fat tails). Similar and more thorough analysis is provided by Ascari *et al.* (2015), who observe that US output growth rates are well approximated by Laplace

[1] Apart from Calvo pricing mechanism, elements such as autocorrelation in error terms or habit formation in consumption are usually added to the model in order to generate dynamics similar to real data.

distribution (which is characterized by fat tails). The authors use medium-scale DSGE model with either normal or Laplace distribution of shocks and show that simulated dynamics does not match real data well.[2] Similar results are achieved also for consumption, investment, inflation, employment, and real wage. The authors thus conclude that New Keynesian DSGE models are not able to approximate fat tails observed in real data sufficiently well.

This is certainly not to claim that DSGE models are useless. They still provide a coherent framework for thinking about the economy; they are immune to Lucas (1976) critique; they allow for various ways of potential expansion of the baseline model; and when used in practice, they are not applied mechanically, but they enable policymakers to insert expert judgment into them. But at the same time, the substantial imperfections discussed above motivate attempts to different modelling approaches, such as models with bounded rationality. While this holds for macroeconomic and business cycles modelling in general, it is even more important for situations where some of the assumptions of DSGE models are of the utmost importance. In particular, this is the case of price level targeting—as was discussed in previous chapter, these models generally favour PLT to IT, but policymakers are hesitant to apply it in practice, as they consider the assumption of rational expectations to be too restrictive in this context. That is why one of the models with bounded rationality is used to analyse PLT in this thesis.

3.2 Bounded rationality models

Modelling of rational expectations in standard DSGE models is more or less the same across various specifications of the models—there is basically just one way to be fully rational. On the other hand, there is plenty of ways how to model departure from the RE hypothesis, so there is also substantial variety of approaches among research papers. Complete survey of all the approaches is beyond scope of this text, see e.g. Woodford (2013) for more detailed overview. Just a few important streams of literature will be briefly discussed here.

One branch of bounded rationality models uses adaptive learning. Although details depend on a particular setup, the departure from the RE is in general still quite small, as economic agents are assumed to know full structure of the

[2] Normal shocks lead to normal distribution of output growth rate; Laplace shocks generate non-normality, but still not corresponding to actual data, as frictions present in DSGE models smooth the effects of the shocks.

economic model, they just do not know values of model parameters. They instead estimate them using simple econometric techniques, usually least squares. They re-estimate the parameters in each time period and learn about their true values over time. Such approach generally converges to rational expectations, but transition dynamics may have consequences for business cycle. This type of models is summarized in a comprehensive way by Evans & Honkapohja (2001); more recent surveys can be found in Milani (2012) and Eusepi & Preston (2018). Paper of Honkapohja & Mitra (2019) discussed in Section 2.2 applies this method in context of PLT.

Gabaix (Forthcoming) presents behavioural New Keynesian model, which introduces cognitive discounting parameter directly into IS curve and Phillips curve in otherwise standard version of a simple NK model. The method shows, besides other things, effectiveness of helicopter drops of money and fiscal stimulus at the ZLB, and non-optimality of price level targeting and nominal GDP targeting (which is caused by non-rationality of firms rather than non-rationality of households). However, further research is needed to examine model results in more detail and to assess their robustness.

Another class of models uses heuristic approach proposed by De Grauwe (2012)—economic agents do not have rational expectations, but they switch between several simple forecasting rules based on their past performance. Such model will be employed in this thesis, so the next section describes it in detail.

3.3 The Model

The model to be applied in this thesis consists of the same equations as the canonical version of standard small New Keynesian DSGE model, but it differs in the way in which expectations are formed. In particular, dynamic IS curve (or aggregate demand equation) based on utility maximization of individual households looks as follows:[3]

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \overline{r}) + \epsilon_t \tag{3.1}$$

where y_t is the output gap in period t, i_t is the nominal interest rate, $\overline{r}$

[3]Overall modelling approach follows De Grauwe (2012), but the particular version presented here differs from the original model by a few slight adjustments in the model equations. For example, De Grauwe assumes zero steady state real interest rate and zero inflation target, and therefore, he does not include these variables in the equations. He also does not consider PLT at all.

is the steady state real interest rate (assumed to be constant over time for simplicity), π_t is the inflation rate, and ϵ_t is a white noise disturbance term (no autocorrelation in the error term is used in the baseline model). The tilde above the expectations operator E captures the fact that expectations are not rational. Lagged output gap accounts for habit formation and while it is not part of the simplest version of textbook DSGE model, it is a standard element of almost all analyses, as it is needed to generate inertia corresponding to real data. De Grauwe (2011) actually shows how heuristic model to be used here does not need the habit formation to generate the inertia, but its inclusion, first, still improves fit of the model on real data; and second, it enables more direct comparison with rational expectations model. The parameter a_1 is inversely related to the degree of habit formation ($0 < a_1 < 1$) and $a_2 < 0$.

New Keynesian Phillips curve (aggregate supply equation) based on profit maximization of individual firms operating in monopolistic competition and Calvo price setting environment then has the form

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1} + b_2 y_t + \eta_t \tag{3.2}$$

where η_t is a white noise disturbance term. The equation includes also lagged inflation, which captures indexation of prices: while fraction $1-\theta$ of firms is able to optimize their prices in given time period freely, even remaining θ of firms adjust prices based on past inflation. Similarly to lagged output gap in the aggregate demand equation, this is not present in the simplest version of DSGE models, but it has become standard element of the models in order to generate more realistic dynamics. It is based on an idea that in reality, frequency of price adjustments by firms depends on state of the economy, which is inconsistent with the simple version of Calvo pricing scheme without indexation. Parameter b_1 then captures relative weight of forward and backward looking terms and it is a function of the underlying parameter θ ($0 < b_1 < 1$).

The model is closed by central bank interest rate rule. Two different rules will be analysed here—one corresponding to inflation targeting and one to price level targeting. In particular, central bank targeting inflation uses Taylor (1993) rule with interest rate smoothing:

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1^{IT}(\pi_t - \pi^*) + c_2 y_t\right] + (1 - c_3) i_{t-1} + u_t \tag{3.3}$$

and rule corresponding to PLT has the form

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1^{PLT}(p_t - \overline{p}_t) + c_2 y_t\right] + (1 - c_3)i_{t-1} + u_t \tag{3.4}$$

where p_t is log of price level in period t, while $\overline{p}_t$ is targeted price level in that period. This type of rule is called Wicksellian (see Woodford (2003) or Giannoni (2014) for discussion). π^* is inflation target in case of IT and increase in targeted price level under PLT (it is assumed to be same in both policy regimes, hence the term π^* is used in both cases without differentiation). Price level target then develops according to[4]

$$\overline{p}_t = \overline{p}_{t-1} + \pi^{*q} \tag{3.5}$$

See Appendix A.1 for an explicit solution of the two versions of the model for given expectations about output gap and inflation (to be defined below). Furthermore, quadratic loss function will be defined to compare the two monetary policy regimes:

$$L = \sum_{t=1}^{T} \left[(\pi_t - \pi^*)^2 + \lambda y_t^2\right] \tag{3.6}$$

The function thus punishes deviations of inflation from the target and of output from its potential, with larger deviations having higher importance (given the quadratic nature of the function). Parameter λ captures relative weight of output gap stability and T is number of periods over which the comparison is performed. This type of loss function is of standard form, see e.g. Clarida *et al.* (1999) or Walsh (2003). The loss function is usually presented in a theoretical form with the evaluation over infinite time period (and with a discount factor), but as the simulations in chapter 4 will be obviously conducted with finite number of periods, the function is presented here in the actual form which will be used for the comparison of the regimes. While the loss function is often introduced *ad hoc*, Rotemberg & Woodford (1999) show how this type of function can be derived from microeconomic foundations in utility-based framework

[4] To avoid any confusion about development of price level, explicit derivation is following:

$$p_t - p_{t-1} = log(P_t) - log(P_{t-1}) = log\left(\frac{P_t}{P_{t-1}}\right) = log\left(1 + \frac{P_t - P_{t-1}}{P_{t-1}}\right) \approx \frac{P_t - P_{t-1}}{P_{t-1}} = \pi_t^q$$

where P_t is price level at time t and the approximation follows from Taylor approximation for sufficiently low inflation rate. Superscript q distinguishes that the relationship holds for quarterly inflation, while model equations contain annualized inflation. Section 3.3.1 discusses this a bit more.

using second-order Taylor approximation. Note that the loss function is the same for both policy regimes—even loss under PLT is defined in terms of inflation volatility, which is the ultimate goal; PLT is approached as just a tool to possibly deliver more stable inflation (and output gap).

So far the model has a standard form, but now let us turn into description of expectations formation. Following De Grauwe (2012), economic agents are assumed to choose between two simple rules for forecasts of inflation and output gap. For both variables, there is a fundamentalist rule (corresponding to variable at its equilibrium level) and an extrapolative rule (forecast for the next period is equal to realization of given variable in the previous period, i.e. the agents use adaptive expectations). Let us focus on output gap forecasts at first. Formally, fundamentalist and extrapolative rules (respectively) for output gap have the form

$$\widetilde{E}_t^f y_{t+1} = 0 \tag{3.7}$$

$$\widetilde{E}_t^e y_{t+1} = y_{t-1} \tag{3.8}$$

While economic agents are not assumed to have rational expectations, they are not choosing forecasting rule completely randomly—they take into account past performance of the rules; we are speaking about bounded rationality here. Subsequent formalization of switching between the rules is based on discrete choice theory as described e.g. by Anderson *et al.* (1992) or Brock & Hommes (1997). The agents evaluate past forecast performances of both rules by calculation of utilities (defined as negative weighted mean squared forecast error of given rule) as follows:

$$U_{f,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^f y_{t-k-1} \right]^2 \tag{3.9}$$

$$U_{e,t}^y = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}_{t-k-2}^e y_{t-k-1} \right]^2 \tag{3.10}$$

where $U_{f,t}^y$ and $U_{e,t}^y$ are the utilities from fundamentalist and extrapolative rules in output gap forecasting, respectively (calculated in period t); and ω_k are geometrically declining weights—errors made in distant past have lower weight than recent errors—which captures tendency to forget. When we define

$\omega_k = (1-\rho)\rho^k$ with $0 < \rho < 1$, we can rewrite equations 3.9 and 3.10 as follows (Appendix A.2 shows this explicitly):

$$U^y_{f,t} = \rho U^y_{f,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 \tag{3.11}$$

$$U^y_{e,t} = \rho U^y_{e,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^e_{t-2} y_{t-1}\right]^2 \tag{3.12}$$

The coefficient ρ then captures memory of people. $\rho = 0$ means no memory—only the most recent forecast error is taken into account. With increasing ρ (up to 1), the importance of more distant errors grows as well.

The utilities represent deterministic components in the choice between the two rules, but there are also stochastic elements $\epsilon_{f,t}$ and $\epsilon_{e,t}$. Resulting probability of choosing the fundamentalist rule is

$$\alpha_{f,t} = P\left[U^y_{f,t} + \epsilon_{f,t} > U^y_{e,t} + \epsilon_{e,t}\right] \tag{3.13}$$

Such specification is based on an idea that the choice between the rules is influenced by both actual performance of the rules as well as by current mood of individual decision makers, which is captured by the stochastic components. Furthermore, $\epsilon_{f,t}$ and $\epsilon_{e,t}$ are assumed to be logistically distributed, which leads to probability of selecting the fundamentalist rule as follows:

$$\alpha_{f,t} = \frac{\exp(\gamma U^y_{f,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})} \tag{3.14}$$

and to probability of choosing the extrapolative rule:

$$\alpha_{e,t} = P\left[U^y_{e,t} + \epsilon_{e,t} > U^y_{f,t} + \epsilon_{f,t}\right] = \frac{\exp(\gamma U^y_{e,t})}{\exp(\gamma U^y_{f,t}) + \exp(\gamma U^y_{e,t})} = 1 - \alpha_{f,t} \tag{3.15}$$

where parameter γ measures intensity of choice or in other words willingness to learn from past errors; and it is given by variance of $\epsilon_{f,t}$ and $\epsilon_{e,t}$. When the variance is zero, $\gamma = \infty$ and the choice between the rules is given purely deterministically by comparison of $U^y_{f,t}$ and $U^y_{e,t}$. When the variance goes to infinity, $\gamma = 0$ and the choice becomes completely random—probability of choice of each rule is 0.5.

The market expectations are then given by

$$\widetilde{E}_t y_{t+1} = \alpha_{f,t}\widetilde{E}^f_t y_{t+1} + \alpha_{e,t}\widetilde{E}^e_t y_{t+1} = \alpha_{f,t} 0 + \alpha_{e,t} y_{t-1} \tag{3.16}$$

The approach applied here thus on one hand relaxes strong assumption of rational expectations—the economic agents are not required to understand large complex model and to behave completely rationally, they just choose between two simple rules; but on the other hand, the agents still have some level of rationality—they do not behave completely randomly, but they are willing to learn from the past and opt for forecasting rule which has led to better results so far.

The same approach is applied for inflation forecasting. The extrapolative rule under inflation targeting is defined by

$$\widetilde{E}_t^{e,IT}\pi_{t+1} = \pi_{t-1} \tag{3.17}$$

and the fundamentalist rule by

$$\widetilde{E}_t^{f,IT}\pi_{t+1} = \pi^* \tag{3.18}$$

The situation is, however, more complicated under price level targeting. The model is constructed in a way that an agent at time t forms expectations about $t+1$ knowing only past and targeted values of all variables, not the current ones. As $t-1$ and $t+1$ are not adjacent periods, it actually matters whether the agent makes expectations about future prices or inflation.[5] In the baseline model, let us assume that expectations are formed about price level; the other case will be described and examined in section 4.2.[6] The model will be solved for p_t using relationship $\widetilde{E}_t\pi^q_{t+1} = \widetilde{E}_t p_{t+1} - p_t$. Agents therefore form expectations $\widetilde{E}_t p_{t+1}$, while inflation expectations $\widetilde{E}_t\pi^q_{t+1}$ are created only *ex post*, after p_t is determined. Fundamentalist rule in this case is then

$$\widetilde{E}_t^{f,PLT}p_{t+1} = \overline{p}_{t+1} \tag{3.19}$$

while for the extrapolative rule there are more possibilities how to relate past and future price level, i.e. how to form $\widetilde{E}_t^{e,PLT}p_{t+1}$. With targeting constant price level this would be simply past price level, but more options arise if there

[5]If the agent knew current values, there would be no difference because of relationship of prices and inflation $p_{t+1} = p_t + \pi^q_{t+1}$. But when he or she needs to rely only on past values, the difference occurs.

[6]While assumption that expectations are formed about inflation might seem a bit more reasonable (see chapter 4 for discussion), the case of expectations about price level is examined at first in order to align the thesis with previous research, namely Ho *et al.* (2019), who examine only this scenario. Section 3.4 discusses their paper in more detail.

is a drift in targeted path of prices. For clarity, the options will be specified in chapter 4 when they will be examined one by one.

Forecast performances of both rules are represented by their respective utilities just as for output gap. For IT these are

$$U^{\pi}_{f,t} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \widetilde{E}^{f,IT}_{t-k-2}\pi_{t-k-1}\right]^2 \tag{3.20}$$

$$U^{\pi}_{e,t} = -\sum_{k=0}^{\infty} \omega_k \left[\pi_{t-k-1} - \widetilde{E}^{e,IT}_{t-k-2}\pi_{t-k-1}\right]^2 \tag{3.21}$$

and similarly for PLT

$$U^{p}_{f,t} = -\sum_{k=0}^{\infty} \omega_k \left[p_{t-k-1} - \widetilde{E}^{f,PLT}_{t-k-2}p_{t-k-1}\right]^2 \tag{3.22}$$

$$U^{p}_{e,t} = -\sum_{k=0}^{\infty} \omega_k \left[p_{t-k-1} - \widetilde{E}^{e,PLT}_{t-k-2}p_{t-k-1}\right]^2 \tag{3.23}$$

After introduction of stochastic logistically distributed elements just as for output gap, respective probabilities of choosing fundamentalist and extrapolative rule in inflation/price level forecasting are (for both regimes)

$$\beta_{f,t} = \frac{\exp(\gamma U^{\{\pi,p\}}_{f,t})}{\exp(\gamma U^{\{\pi,p\}}_{f,t}) + \exp(\gamma U^{\{\pi,p\}}_{e,t})} \tag{3.24}$$

$$\beta_{e,t} = \frac{\exp(\gamma U^{\{\pi,p\}}_{e,t})}{\exp(\gamma U^{\{\pi,p\}}_{f,t}) + \exp(\gamma U^{\{\pi,p\}}_{e,t})} \tag{3.25}$$

Finally, market forecast of inflation under IT is given by

$$\widetilde{E}^{IT}_t \pi_{t+1} = \beta_{f,t}\widetilde{E}^{f,IT}_t \pi_{t+1} + \beta_{e,t}\widetilde{E}^{e,IT}_t \pi_{t+1} = \beta_{f,t}\pi^* + \beta_{e,t}\pi_{t-1} \tag{3.26}$$

and under PLT by

$$\widetilde{E}^{PLT}_t p_{t+1} = \beta_{f,t}\widetilde{E}^{f,PLT}_t p_{t+1} + \beta_{e,t}\widetilde{E}^{e,PLT}_t p_{t+1} = \beta_{f,t}\overline{p}_{t+1} + \beta_{e,t}\widetilde{E}^{e,PLT}_t p_{t+1} \tag{3.27}$$

where $\widetilde{E}^{e,PLT}_t p_{t+1}$ will be specified later. Furthermore, parameter $\beta_{f,t}$ can be also interpreted as credibility of given monetary policy regime.

The rules described above (except for the rules specific to PLT) are common in the strand of literature following De Grauwe (2012), but they have one drawback. It would be preferred to derive heuristic rules at micro-level with individuals having cognitive limitations. Nevertheless, there is no recognized approach how to do this because of our limited knowledge of information processing by human brains. Therefore, the rules are not micro-founded, but only imposed *ex post* into model equations. On the other hand, while particular specification of the rules may seem a bit arbitrary, their simplicity is at the same time an attractive feature, as the economic agents are using only basic information and do not need to understand all complex relationships in the economy. This is in fact the core of this approach as opposed to rational expectations models. Moreover, slightly *ad hoc* character of the rules enables an easy way to test robustness of the results—there can be more than just two rules, agents may have biased estimates about output gap etc. If the results do not change too much when forecasting rules are adjusted, then the results can be considered as robust.

Furthermore, while the exact formulation of forecasting rules is a bit arbitrary, there is some empirical support for this approach of inflation expectations formation. For example, Branch (2004) concludes that agents switch between different discretely defined inflation forecasting rules based on their mean squared error performance (the results are based on survey data with assumption of three possible forecasting rules: naive expectations, adaptive expectations, and expectations based on four-variable VAR model). Hommes (2011) conducted laboratory experiment, and first, he provides further evidence of heterogeneity in expectations and of deviations from the RE hypothesis; second, he finds that heuristic switching model fits data reasonably well for both output gap and inflation. Pfajfar & Zakelj (2014) provide yet another support for switching between different inflation forecasting models in laboratory setting, although the authors focus on adaptive learning approach rather than heuristics.

Note that there are fundamentalist and extrapolative rules for output gap forecast and fundamentalist and extrapolative rules for inflation forecast, but economic agents choose between the rules separately for output gap and inflation (even though intensity of choice parameter γ and memory parameter ρ are assumed to be the same for evaluation of forecasting rules in cases of both variables). Certain correlation can be expected—e.g. when the economy operates for some time below potential, there will be negative output gap and inflation below target, so fundamentalist rules will lead to imprecise forecasts, and probability of using extrapolative rule for both output gap and inflation forecasting will increase. Nevertheless, it is in general perfectly possible that an agent uses the fundamentalist rule for output gap forecast and the extrapolative rule for inflation forecast, or *vice versa*.

Finally, let us define variable AS_t, which stands for *animal spirits* (see section 3.4 for discussion of the term):

$$AS_t = \begin{cases} \alpha_{e,t} - \alpha_{f,t} & \text{if } y_{t-1} > 0, \\ -\alpha_{e,t} + \alpha_{f,t} & \text{if } y_{t-1} < 0. \end{cases} \tag{3.28}$$

The variable AS_t hence ranges between -1 and 1 and is equal to 0 when fraction of agents using each rule is the same, i.e. 0.5. When $y_{t-1} > 0$, extrapolative rule is optimistic, while fundamentalist rule is pessimistic. The more agents use extrapolative rule (the higher $\alpha_{e,t}$), the higher is the fraction of optimists and the higher is AS_t. Conversely, the fundamentalist rule is the optimistic one for $y_{t-1} < 0$, while the extrapolative rule is pessimistic. AS_t is now high for high $\alpha_{f,t}$, i.e. when the fundamentalists prevail. The variable AS_t hence captures degree of optimism and pessimism in the economy. Values close to 1 indicate substantial wave of optimism, values close to -1 then mean strong pessimism. Values around 0 suggest neutral, balanced state of the economy.[7]

3.3.1 Calibration

Now let us discuss calibration of the model. Table 3.1 contains values of parameters in the baseline version. The calibration is such that one time period in the model corresponds to one quarter.[8] Most of the parameters are the same as in De Grauwe (2012) or Ho *et al.* (2019). Parameters a_1 and a_2 in dynamic IS curve and b_1 and b_2 in NK Phillips curve are consistent with values usually used in standard DSGE models. Parameter ρ with value 0.5 (used in the calcula-

[7] De Grauwe (2012) defines *animal spirits* in slightly different way, so that the variable has interpretation of probability and thus ranges between 0 and 1. The interpretation is similar—low values suggest pessimism, high values optimism, and values in the middle neutral state. Nevertheless, the definition used here is a bit more straightforward and it is common in subsequent literature, e.g. in De Grauwe & Ji (2019) or De Grauwe & Ji (2020).

[8] Variables will be presented in annualized form to be consistent with usual convention. Be aware, however, that the relationship between price level and inflation holds for quarterly data; e.g. with 2% (annual) long-term inflation target, log-linearised targeted price level between two subsequent periods increases by 0.005 rather than by 0.02. If an equation throughout the thesis holds for quarterly inflation, it is explicitly denoted by a superscript q. Annualized inflation is left without any superscript.

tion of utilities from the forecasting rules) means that the last forecast error has weight of one half, while all the previous forecast errors have joint weight of the remaining one half. Intensity of choice parameter γ corresponds to De Grauwe (2012) as well, although it has different numerical value for technical reasons.[9] Similar technical adjustment has been made for standard deviations of shocks in model equations.

Table 3.1: Calibration

Parameter	Value
a_1	0.5
a_2	-0.2
b_1	0.5
b_2	0.05
ρ	0.5
γ	20000
σ_ϵ	0.005
σ_η	0.005
σ_u	0.005
$\overline{r}$	0.02 (2%)
π^*	0.02 (2%)
λ	0.25
c_1^{IT}	1.5
c_1^{PLT}	0.3
c_2	0.7
c_3	0.3

σ_ϵ, σ_η, and σ_u are standard deviations of random disturbances in model equations, i.e. in dynamic IS curve, New Keynesian Phillips curve and central bank interest rate rule, respectively. Shock u_t relates to both IT and PLT rules 3.3 and 3.4.

Furthermore, steady state real interest rate $\overline{r}$ of 2% is calibrated according to, among others, Taylor (1993). Inflation target (or rate of increase in targeted price level) is set to 2% as this is standard value used in practice by many inflation targeting central banks (but case of 0% target will be considered too). Weight of output gap in the loss function λ corresponds to e.g. Walsh (2003). Parameters c_2 and c_3 are based on own simulation and are assumed to be the same for both monetary policy regimes (the model has been simulated for a sequence of reasonable values of these parameters; the loss function generally decreases with higher value of these parameters, but the rate of decrease starts to be negligible at values presented in the table under both regimes). Parameter c_1 has been calibrated in the same manner—for values approximately below 1 the IT model diverges (the model is in fact stable even for slightly lower values, about 0.95; but overall logic is in the spirit of standard Taylor principle), for values above 1 the loss function decreases with higher c_1 up to 1.5, and after that it is approximately constant. Parameter c_1 under PLT may not be the same as under IT and its calibration favours values around 0.3, which will be used in the baseline scenario. Nevertheless, changes

[9] De Grauwe (2012) sets the model in a way that inflation value of, say, 2 corresponds to 2% inflation rate. But as price level is an explicit part of the model here, 2% inflation rate is captured by value 0.02 to make the log-linearised relationship between price level and inflation work. Output gap of 2% is then depicted by value 0.02 as well in order to be consistent. And with inflation and output gap values divided by 100, utilities from different forecasting rules are as a result divided by 10000 because of their quadratic form. Parameter γ therefore has to be multiplied by 10000 to compensate for this in the calculation of probabilities of choosing different forecasting rules. As De Grauwe (2012) calibrates value of this parameter to 2, here the parameter is equal to 20000.

in size of the parameter are not decisive for model convergence and they lead to similar qualitative results.

Before turning into own analysis, let us now discuss main characteristics of heuristic models in the next section as described by De Grauwe (2012) and subsequent research. The goal is to understand general implications of this method before applying it to price level targeting.

3.4 Model characteristics

The heuristics in the expectations formation lead to waves of optimism and pessimism. When the economy is hit by a positive demand shock, inflation rises and output gap becomes positive. Extrapolative rules become more precise because of deviations from equilibrium values, so more economic agents start using it. This leads to further increase in inflation and output gap, so that even more people switch to the extrapolative rules with expectations of high inflation and positive output gap. Boom is created. Strong intervention of the central bank, a negative demand shock, or the stochastic element in the choice between the rules can reverse the trend and the economy either returns to the equilibrium with prevailing fundamentalist rules, or it can even end up in a negative self-enhancing cycle, where the extrapolative rules are again favoured, but now with expected and actual inflation below the target and with negative output gap. The economy then enters into a recession.

Business cycles are thus caused by two mechanisms. One is the same as in mainstream DSGE models—an exogenous shock, which influences the economy for several periods due to price rigidity and lagged values in demand and supply equations caused by habit formation in consumption and by indexation of prices. (Autocorrelated error terms are another factor contributing to the cycles in case of standard New Keynesian models, but the behavioural model here assumes white noise shocks.) The other mechanism is specific for the heuristic model—self-fulfilling waves of optimism and pessimism. To illustrate their importance, De Grauwe (2011) compares the simplest version of the behavioural model and of textbook DSGE model (i.e. without lags in supply and demand equations and with white noise error terms). While the rational model leads to autocorrelation in output gap close to zero (so that there is no cycle), the autocorrelation in the simple heuristic model is 0.77. This is still lower than actual value found in US data for 1960-2009, which is 0.94 (that is why the lagged values are still included in equations 3.1 and 3.2), but it shows that substantial output gap autocorrelation can be generated just by a combination of price rigidity and waves of optimism and pessimism.

De Grauwe (2012) calls the waves *animal spirits* (and he also formalizes the term, see equation 3.28), which refers to famous concept of Keynes (1936), more recently emphasized also by Akerlof & Shiller (2009). Note however, that Keynes used the term in quite positive sense of spontaneous optimism, which helps the economy; on the other hand, Akerlof & Shiller (2009) and De Grauwe (2012) consider *animal spirits* to be behavioural tendencies which drive the economy out of the equilibrium, so they use the term with rather negative connotation.

Furthermore, the model generates non-normal distribution of output gap with kurtosis of 4.4 (which means there are fat tails) and Jarque-Bera test rejects normality formally. Periods of large bubbles as well as recessions are hence more probable than in the mainstream models. Moreover, the model itself assumes normal distribution, so it endogenously explains the non-normality, not just assumes it. Correlation of output gap and *animal spirits* variable (in the original definition of De Grauwe (2012)) equals to 0.86 in a simulation with baseline calibration. There is actually Granger causality between output gap and *animal spirits* in both directions. Regarding inflation, about 50% of economic agents use the extrapolative rule and about 50% the fundamentalist rule for most of the time; and inflation fluctuates approximately from -1pp to +1pp around the inflation target. On several occasions, however, most or all of the agents use the extrapolative rule—inflation targeting regime loses its credibility. Inflation then fluctuates substantially more, up to about 3.5pp from the target in the simulation presented in De Grauwe (2012).

When full credibility of inflation targeting is assumed (all agents forecast inflation at its target by construction and only output gap forecasts are made as described in the previous section), output gap is now much closer to normal distribution and there are no longer frequent periods of large booms and busts; waves of optimism and pessimism are mitigated. The reason is that the central bank can under such setting care less about well-anchored inflation and can focus on output gap smoothing. While this is only theoretical approach, it captures the importance of monetary policy credibility. Another interesting result of the model (no longer under the perfect regime credibility) can be found for assumption $c_2 = 0$ in equation 3.3 (i.e. output gap does not enter policy rule of the central bank). Usually, there is a perceived trade-off between inflation and output stabilization, so inflation under such assumption is considered to have the lowest volatility possible, but at the expense of high output gap fluctuations (e.g. Fuhrer (1997) estimates the trade-off and resulting optimal policy frontier for the US). But no regard for output gap in the model presented here leads to overall high instability of the economy; deviations of inflation from its target are actually much higher than in the baseline scenario. Because the output extrapolative rule starts to prevail and there are even more substantial waves of optimism and pessimism than before, neither inflation nor output gap is well-anchored, and as a result, both variables are more volatile. Thus, some amount of output stabilization in the Taylor rule in fact helps even with inflation stabilization. The usual inflation versus output gap variability trade-off then arises only for coefficient c_2 above certain level (about 0.7).

Another characteristic worth mentioning concerns impulse responses—even though mean impulse responses have a standard shape, they have large standard deviations. The thing is that effects of a stochastic shock depend on market sentiment at time when the shock occurs. E.g. an interest rate shock leads to movement of both output and inflation in expected direction, but the effect is substantially larger in periods with high optimism or pessimism than in tranquil periods with many agents using fundamentalist rule. This is especially true for output gap, while for inflation the dependence of the shock effect on market sentiment is weaker (but still present). The uncertainty about the effects of shocks decreases in the long-run, as the importance of current market sentiment diminishes. Nevertheless, monetary policy is state-dependent in the short-run. Already mentioned paper of Falck *et al.* (2019) is an example of empirical evidence of monetary policy state-dependency in reality. Such feature is not present in standard New Keynesian models.

The role of price rigidity is similar to mainstream DSGE models—with fully flexible prices, money neutrality holds and monetary policy influences only nominal variables; monetary policy starts to impact output with increasing price stickiness. In other words, *animal spirits* are not present in case of price flexibility; with increasing level of price rigidity, the *animal spirits* arise and get stronger. The impact of price flexibility on significance of *animal spirits* and on trade-offs faced by central banks is further examined by De Grauwe & Ji (2020).

Sensitivity analysis reveals that *animal spirits* emerge only for positive intensity of choice parameter γ. When $\gamma = 0$, there is no willingness to learn from past mistakes and people decide purely randomly—there is no rationality in their behaviour. Then there is no space for self-enhancing waves of optimism and pessimism. At the same time, certain level of forgetfulness (meaning coefficient ρ is at least slightly lower than 1) is needed for *animal spirits* to arise. Together this means that both certain level of rationality as well as certain cognitive limitations are needed for existence of *animal spirits*; they are not present in case of neither complete irrationality nor full rationality.

Finally, De Grauwe (2012) also examines robustness of results with respect to different forecasting rules. Firstly, he keeps the extrapolative rule for output gap forecasting the same as in equation 3.8, but he uses two fundamentalist rules—one is biased upwards and one downwards from zero output gap by a constant. The interpretation is that output gap is in reality a latent variable. It is always unmeasurable, and estimation of its current value is especially imprecise; agents hence likely do not know which value of output corresponds to zero output gap. The adjustment of rules leads to qualitatively same results as the baseline model, but the waves of optimism and pessimism and corresponding booms and busts are now even more pronounced than before. Secondly, he defines two biased fundamentalist rules as in the first case, but replaces the extrapolative rule with neutral one (forecasting zero output gap). This specification leads to results very similar to the baseline model.

While the results above are based on textbook description by De Grauwe (2012), some versions of the model have been used to examine further issues as well. De Grauwe & Ji (2019) and Hommes & Lustenhouwer (2019a) focus on role of the zero lower bound. The former paper concludes that 2% inflation target leads to high probability of hitting the ZLB and to protracted periods of pessimism and recession. Preferred level of the inflation target would be in range of 3%-4%. The latter paper similarly acknowledges the danger of binding ZLB constraint for a long time and related deflationary spiral, and it recommends higher inflation target or aggressive monetary easing when there is a threat of hitting the lower bound. Furthermore, the authors conclude that range of policy parameters leading to local stability of steady state is wider than under rational expectations, i.e. satisfying Taylor principle is not necessary for local stability.

Zero lower bound is taken into account also in analysis of alternative monetary policies by Proano & Lojak (2020). The authors extend the model by inclusion of fiscal sector and risk premia. While fiscal policy is important to help the economy at the ZLB, its significance is diminished when monetary policy uses non-standard combination of varying inflation target and interest-rate rule containing risk premium.

Slightly different approach is applied by Hommes & Lustenhouwer (2019b), who do not restrict themselves on just two forecasting rules, but agents choose their inflation expectations from normal distribution with mean equal to inflation target and with variance capturing how well are the inflation expectations anchored. With high variance, even small one-off shock can induce strong waves of optimism or pessimism, unless inflation coefficient in the Taylor rule is sufficiently high. Similarly to the papers above, there is a risk of prolonged wave of pessimism and deflationary spiral once the ZLB is hit; this can be avoided by sufficiently high inflation target or by strongly anchored expectations.

To author's best knowledge, the only previous work using the behavioural model with heuristics to the issue of price level targeting is paper of Ho *et al.* (2019). The authors use an open-economy version of the model to examine different monetary policy regimes, and they conclude that PLT leads to better economic stability and social welfare than IT. In particular, the most preferred regime is CPI-level targeting, which is distinct from domestic PLT in open economy; but even the latter outperforms inflation targeting. While the general approach applied in that paper is similar to methodology of this thesis, these works differ in several ways. Ho *et al.* (2019) examine only one very specific case, where agents under PLT forecast future price level (not future inflation), and at the same time targeted price level is constant. The analysis conducted here will start from the same basis, but after that it will examine implications of different assumptions—case with people making forecasts about future inflation will be explored as well and more versions of forecasting rules (not present elsewhere in the literature) will be considered. The model used here also slightly differs in exact formulation of central bank interest rate rules, it allows for positive inflation target and positive natural real interest rate. Moreover, it turns out that general implications of the analysis here will be quite different from the results of Ho and his co-authors.

To conclude, heuristic modelling offers a simple intuitive way of departure from the rational expectations hypothesis. Economic agents form expectations based on a few forecasting rules. Therefore, the approach takes into account cognitive limitations, while people are not considered to be completely myopic either—they still decide according to past performance of individual rules. One drawback of the heuristics lies in *ad hoc* nature of the forecasting process, which is not micro-founded; on the other hand certain empirical and experimental evidence favouring this type of forecasting has been presented. The model leads to several appealing consequences not found in mainstream DSGE models, such as state-dependency of monetary policy or fat tails in output gap. At the same time, its overall superiority to the mainstream models has not been established, as few studies attempted to fit data and compare the heuristic model with its rational expectations counterpart. One such analysis has been conducted by Liu & Minford (2014), who actually reject the behavioural model on US data, while the RE model passes the test. On the other hand, Jang & Sacht (2016) prefer heuristic approach for the euro area using method of moments, while they admit that this model has problem to deal with structural break in US data at the beginning of Great Moderation. Furthermore, Kukacka *et al.* (2018) use simulated maximum likelihood estimation for the euro area and the US and they also incline to favour the behavioural model, even though their conclusion is not unequivocal. There is hence no clear consensus regarding comparison of models with fully versus boundedly rational expectations. The heuristics approach surely has certain relevance and it can serve as a tool to look into PLT issues from different perspective, at the same it should not be considered as clearly superior to the mainstream models and the results should thus be treated with some caution.

Chapter 4

Results

This chapter presents results obtained from simulating the model described in chapter 3. Several cases will be examined differing in targeted inflation, particular specification of forecasting rules, and most importantly, in the way in which expectations under PLT are created. The first section is based on assumption that agents form expectations about price level (this version of the model has been described in section 3.3). Then case of expectations about inflation will be examined. Be aware that the difference holds only for PLT; agents in IT always form expectations about inflation.

In each case, simulation will be conducted for 1000 time periods for both monetary policy regimes, and loss function described by equation 3.6 will be computed.[1] Furthermore, models of both regimes are simulated under the same set of random disturbances, so that resulting variation across the regimes can be attributed to actual difference between them and not to different shocks. Unless stated otherwise, the whole process is repeated 1000 times to gain robustness to a particular simulation. Therefore, 1000 values of the loss function for each regime will be generated. Presented graphs depict one random draw from the 1000 simulations. Whole analysis have been conducted in MATLAB.

To have a formal framework for comparison of loss functions, Wilcoxon signed rank test will be applied. The test can be used to assess whether two paired sets of measurements come from the same underlying distribution. In particular, difference between the two measurements come from a distribution with zero median under the null hypothesis. In context of the analysis herein, each loss value under IT has a corresponding value under PLT based on the same set of realizations of the underlying model disturbances. Rejection of

[1]First two periods are pre-set at equilibrium values, as lagged variables are present in the model; the simulation itself starts in the third period.

the null hypothesis would hence suggest that difference between two regimes is statistically significant.[2]

4.1 Expectations about price level

4.1.1 Zero target

Let us start with a case of zero inflation target under IT and constant targeted price level under PLT. The extrapolative rule under PLT then can be defined as

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} \tag{4.1}$$

Economic agents hence choose between forecasting price level in the next period to be equal either to targeted price level in that period (fundamentalist rule 3.19) or to price level in the previous period. The agents under IT simply choose between the target and past inflation rate as described in section 3.3.

Table 4.1 presents basic descriptive statistics of loss function under both regimes. We can see that price level targeting here clearly outperforms inflation targeting, as its loss function is lower by a substantial margin in all simulations. Virtually zero p-value of Wilcoxon test confirms that better performance of PLT is statistically significant (although this is pretty obvious with loss of IT being higher in all 1000 simulations).

Table 4.1: Loss functions under constant targeted prices

	IT Loss	PLT Loss
Mean	0.4	0.03
Standard deviation	0.136	0.003
Range	0.12-1.07	0.025-0.048
Higher loss	1000	0
Wilcoxon test p-value	<0.001	

The table is based on 1000 simulations. *Higher loss* row counts how many times loss under given regime was higher than corresponding loss under the other regime. *Wilcoxon test p-value* row shows p-value from Wilcoxon signed rank test applied on paired loss functions.

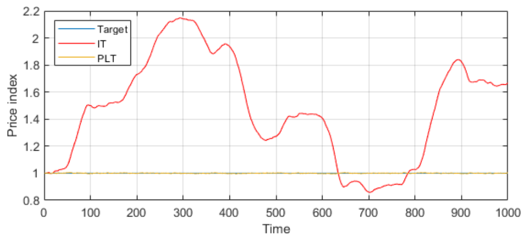
While the simulations are conducted without constraint of ZLB, this issue can be discussed at least indirectly by looking at frequency with which nominal interest rate decreases below zero (note that with zero inflation target, steady state nominal interest rate is equal to real rate, i.e. 2%). The IT model leads to negative rate in about 15.7% of time periods (on average over 1000 simulations), while the PLT one only in approximately 0.6% of time. The difference is thus

[2] Zero mean of differences between two paired samples could be also assessed by paired t-test, but this requires normality of the differences. Jarque-Bera test applied on differences between loss function under IT and PLT, however, rejects normality, so choice has been made in favour of Wilcoxon signed rank test, which does not need such assumption.

rather large and almost zero p-value of Wilcoxon test formally confirms its statistical significance. This result should be approached very cautiously, as the constraint itself is not part of the model—the nominal interest rate is allowed to go below zero without bounds, so the actual dynamics of the economy stuck at the ZLB is not taken into account. While limiting the nominal rate from below would be technically feasible, the actual asymmetry of the economic behaviour at the ZLB would likely not be captured well by the simple log-linearised model. Nevertheless, the substantial difference between the two regimes in terms of average time with negative rate still indicates much lower probability of hitting the ZLB under PLT than under its IT counterpart.

In addition, there is one more benefit of PLT—as figure 4.1 shows, development of price level under PLT is very stable around targeted path, while it is fluctuating significantly in the IT regime. The loss function penalizes high volatility of inflation (as well as output gap), but it does not capture the long-term uncertainty about price level under IT—yet another benefit of PLT, as was discussed in chapter 2.1.

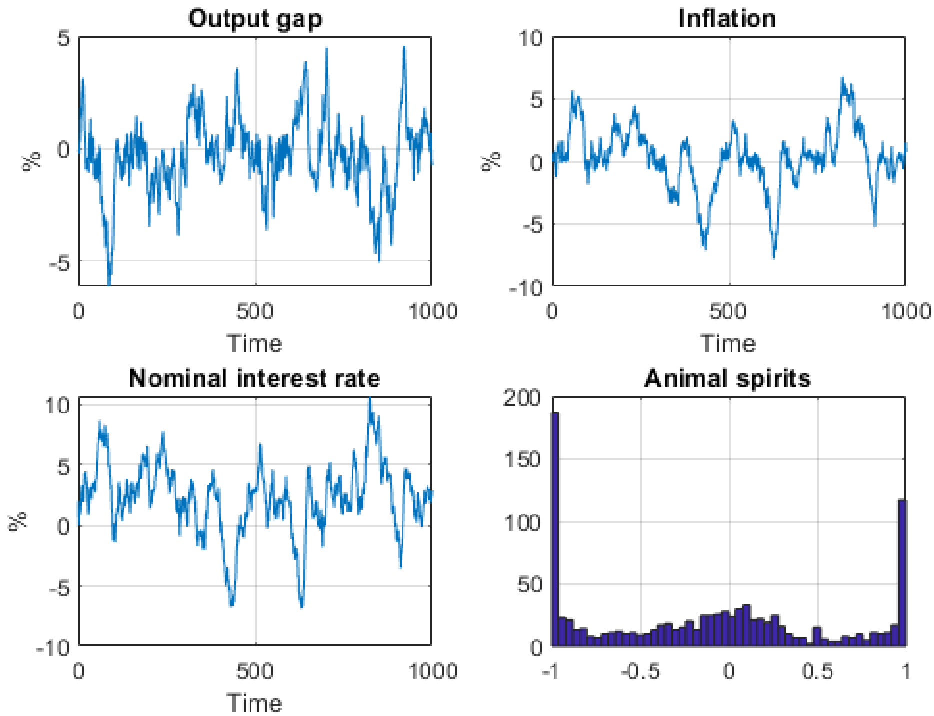
Figure 4.1: Price level



Source: MATLAB output

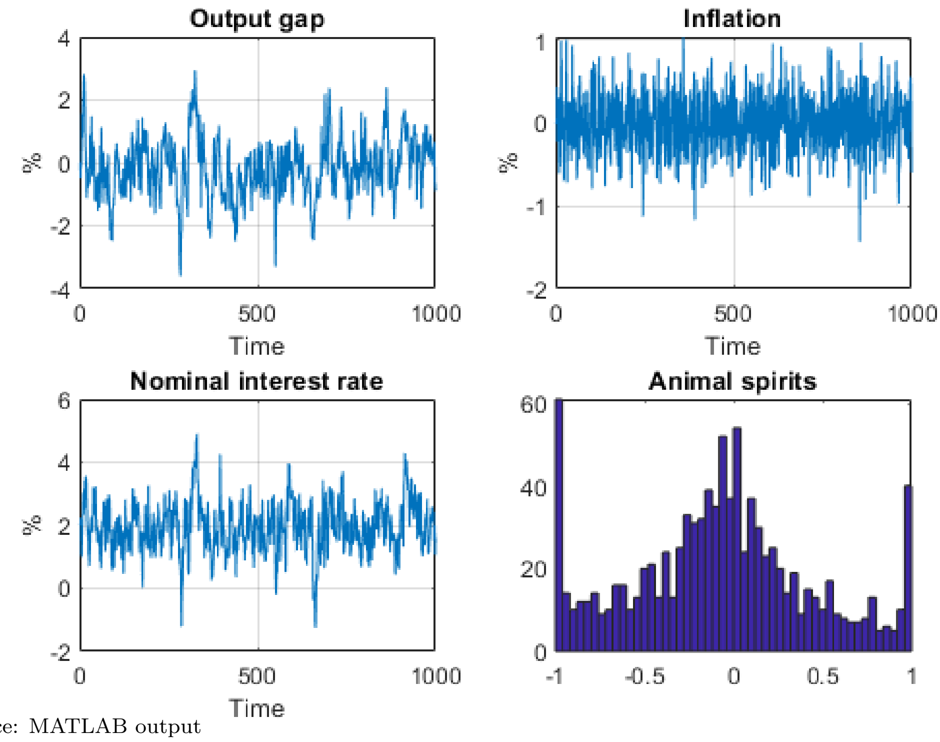
Figures 4.2 and 4.3 depict several key variables for both regimes. *Animal spirits* variable is concentrated around extreme values of -1 and 1 in case of IT, suggesting strong waves of optimism and pessimism. PLT regime leads to several observations of *animal spirits* at the extreme values as well, but the variable is around zero for most of the time, which corresponds to balanced state of the economy. Notice also difference in scales for output gap, inflation, and nominal interest rate—all variables are better anchored around steady state values for PLT. Moreover, there is much higher persistence of the variables under IT than under PLT; the difference between the regimes is particularly significant for inflation persistence—autocorrelation coefficient at the first lag is 0.93 for IT and only 0.24 for PLT. Figure 4.4 depicts autocorrelation function for more lags and confirms that price level targeting substantially decreases persistence in inflation due to its compensation for past deviations of inflation from the long-term target. Finally, figure 4.5 shows fraction of agents using the extrapolative rules for both output gap and inflation and for both policy regimes. Interestingly, probability of using one or the other rule in price level forecasting fluctuates around 0.5 under PLT, while switching between the rules is much more pronounced in the other cases.

Figure 4.2: IT with zero inflation target



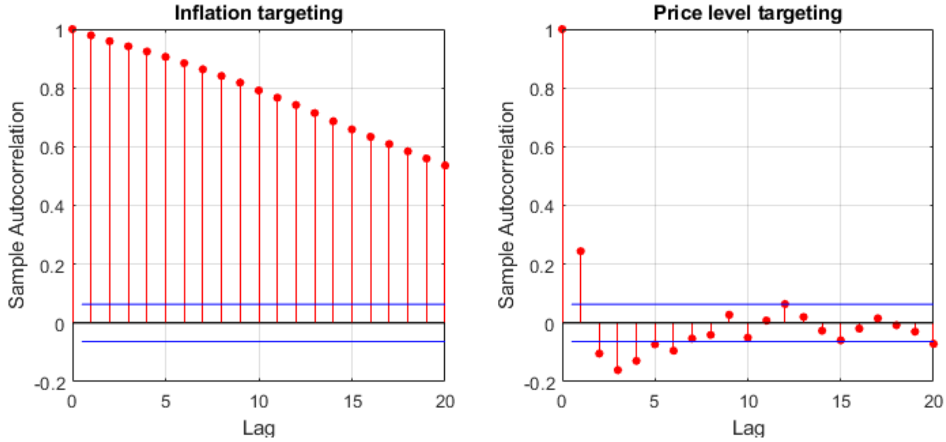
Source: MATLAB output

Figure 4.3: PLT with constant price level target



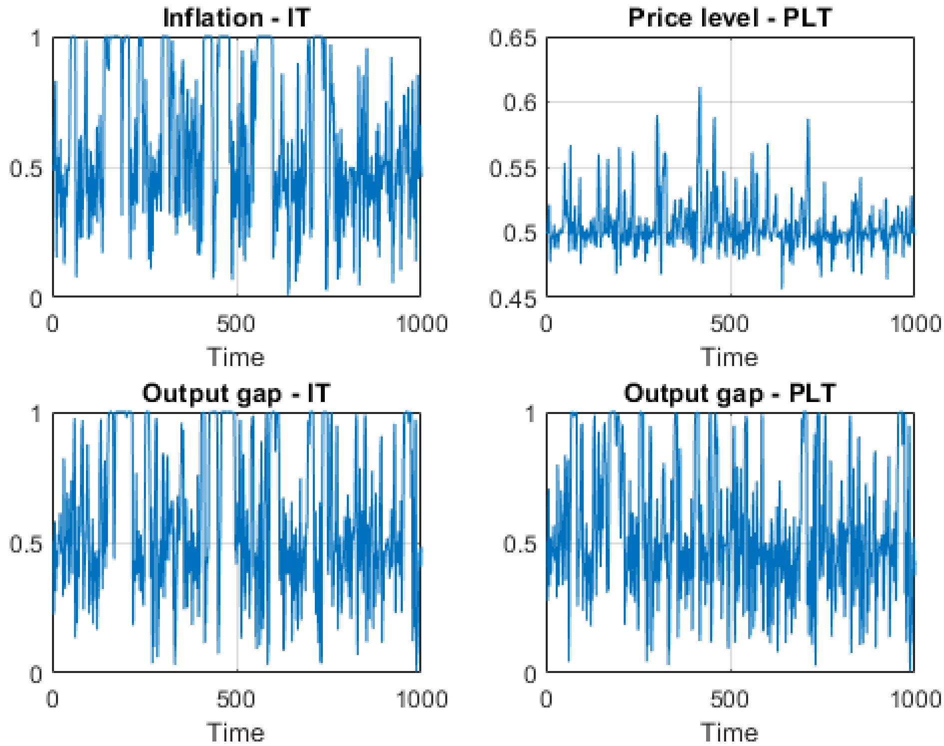
Source: MATLAB output

Figure 4.4: Inflation autocorrelation function for zero target



Source: MATLAB output

Figure 4.5: Fraction of extrapolatives with zero target



Source: MATLAB output

The results are not surprising as assumptions used in this scenario are in fact very similar to those of Ho *et al.* (2019), who also concluded that PLT outperforms IT in such setting, as mentioned in the previous chapter. However, there is a question regarding reliability of such results. While PLT performs better than IT even for $c_1^{PLT} = c_1^{IT} = 1.5$, one startling feature is the fact that loss function under PLT is lowest for values of parameter c_1^{PLT} in range of about $0 - 0.5$, i.e. the regime leads to favourable results even for $c_1^{PLT} = 0$. On the other hand, $c_1^{IT} > 1$ is (approximately) needed for convergence under IT. But for $c_1 = 0$, both interest rate rules 3.3 and 3.4 are identical and the models differ only in expectations formation. The thing is that both forecasting rules under PLT have embedded certain level of regime credibility. Suppose there is a negative shock to inflation, so that price level in one period becomes lower than the target. While fundamentalist rule (forecasting return to the target) is the one consistent with the regime, even extrapolative rule 4.1 implicitly assumes zero inflation, which is itself quite stabilizing for the economy (and it actually corresponds to the fundamentalist rule under IT). Better anchor for inflation expectations is the key theoretical advantage of PLT, but it is hard to believe that if central bank did not react to development of inflation and price level at all, the expectations would remain anchored in this way. Therefore, the next section will introduce increasing targeted inflation; an extrapolative agent under PLT will then have to make an implicit assumption about inflation at time t, which influences the dynamics.

4.1.2 Positive target

Now suppose that $\pi^* = 2\%$. Forecasting future price level to be equal to past price level no longer makes sense; even extrapolative agents have to address increasing trend in prices.[3] Two main options arise. First, the extrapolative rule can be defined as

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} + 2 * \pi^{*q} \tag{4.2}$$

As opposed to the fundamentalist rule, there is no compensating for past deviations in inflation, but at the same time it still contains substantial credibility of the central bank, as inflation at times t and $t+1$ is assumed to be at its long-term target. Rule 4.1 is actually a special case of 4.2 with target set to 0. Results with this rule are very similar to those in previous section with PLT leading to lower loss in all simulations, so details are not presented here to save space; let us discuss just one aspect here. In particular, the number of times with negative nominal interest rate is now for both regimes lower than in section 4.1.1 because of the positive inflation target (steady state nominal

[3]Technically, if extrapolative rule did not account for increasing prices, the rule would soon become extremely imprecise, so all agents would become fundamentalists. Such case would then correspond to perfect credibility of the regime; but it would not be a result of a good performance of the regime—just of not allowing for any reasonable alternative to the fundamentalist rule.

rate is now 4%), but the difference is still both economically and statistically significant—the rate is negative on average in 5% cases for IT and in virtually 0% cases for PLT. Overall, the PLT extrapolative rule is again quite similar to the fundamentalist rule under IT and hence it is itself quite stabilizing; model with the rule 4.2 is thus prone to similar criticism as the one used in the previous section.

Alternative version of the rule could be

$$\widetilde{E}_t^{e,PLT} p_{t+1} = p_{t-1} + 2 * \pi_{t-1}^q \tag{4.3}$$

This rule is of purely extrapolative form; it abandons partial credibility of rules 4.1 and 4.2, and it is hence not subject to the aforementioned criticism. Agents simply assume that the economy will develop in the same way as in the past. The rationale of such rule corresponds better to the logic behind the IT extrapolative rule in equation 3.17.

Inflation targeting model is in all cases considered so far almost the same (it differs only in value of the target), so it leads to very similar results as before—while the regime is subject to certain volatility and long-term uncertainty about price level, it is still fairly stable and functions quite well. On the other hand, price level targeting with extrapolative rule 4.3 is now susceptible to model divergence. At the same time, in cases where the model does not diverge, PLT still outperforms IT. To analyse probability of divergence versus stability with higher precision, the model has been now simulated 100.000 times.

The divergence of PLT model occurs in 18.9% of cases (IT is stable in all simulations). This number captures simulations in which the MATLAB software actually provided *not available* result. Nevertheless, examination of table 4.2 reveals that while most of the remaining simulations leads to value of loss function closely distributed around median, there are several extremely large observations. Mean value is actually higher than 0.97 quantile; and the steep increase in loss value occurs around this quantile. Clearly, the extreme observations also represent model divergence—the divergence is not that substantial that the software would provide *not available* result, but economically it still captures very large instability. Therefore, all simulations with

Table 4.2: PLT loss functions—descriptive statistics

Statistic	Value
0.05 quantile	0.036
Median	0.042
0.95 quantile	0.055
0.96 quantile	0.068
0.97 quantile	0.201
0.98 quantile	1.2
0.99 quantile	7.44
Maximum	186.6
Mean	0.41

Loss value is not available for 18.9% simulations due to model divergence. The numbers are based on remaining 81.1% cases.

loss function higher than 0.97 quantile will be considered as divergent (these account for additional 2.4% of original 100.000 simulations).

Table 4.3: Loss functions with increasing targeted prices

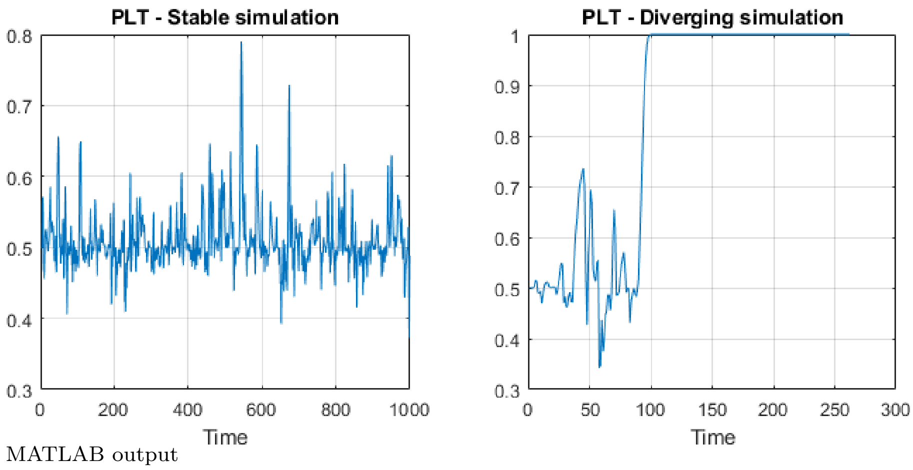
	IT Loss	PLT Loss
Divergence	0	21.3%
Mean (all simulations)	0.41	n/a
Mean (stable simulations)	0.39	0.04
Higher loss	78.7%	0
Wilcoxon test p-value	<0.001	

Higher loss row considers all 100.000 simulations, while Wilcoxon signed rank test has been conducted only for the stable ones.

With this adjustment in mind, let us look at actual results. Table 4.3 summarizes that price level targeting regime leads to better results with 78.7% probability, i.e. in all cases where it behaves stably. But with 21.3% probability, PLT behaves very poorly as it leads to divergence of the economy, while IT still performs relatively well. Notice difference in mean loss function of IT regime calculated from all simulations (0.41) and in mean loss calculated from simulations with stable PLT regime (0.39). While the difference is small, it is not pure coincidence as it is computed from so many simulations, and it still suggests that simulations causing PLT to diverge were subject to slightly larger underlying model shocks—but while the shocks caused the economic volatility under IT to increase only slightly, they led to explosive path under PLT.

The mechanism of divergence is such that if a shock causes the economy to deviate from steady state, the fundamentalist rule becomes imprecise and the extrapolative rule prevails. This drives the economy even farther from the steady state. For sufficiently low deviations, the process can be reversed by a combination of a shock in the opposite direction together with a reaction of the central bank. But once the deviation of price level from its target is a bit higher, no random shock of reasonable size can make the fundamentalist rule favourable again, and adjustment of interest rate by the central bank itself is not enough. The economy is stuck in a situation where the only reasonable forecasting rule is the extrapolative one, and this drives self-fulfilling wave leading to the divergence from the steady state. The difference between the two regimes lies in the compensations for missing desired inflation rate in the past. As IT does not contain such feature, even when inflation is farther from the target, leading to higher share of extrapolatives, reaction of the central bank can quite easily influence inflation in the desired direction and make the fundamentalist rule viable again. But since PLT corrects for past deviations, the adjustment of inflation needs to be large, which further destabilizes the economy if the deviations were high enough. Therefore, everyone continues to use the extrapolative rule.[4] Figure 4.6 shows a typical development of fraction of extrapolatives under PLT in a stable simulation and in a diverging simulation. Furthermore, the main result is not sensitive to value of parameter c_1^{PLT}—the divergence of the PLT model occurs with probability in range 15-25% for all reasonable values of c_1^{PLT}.

Figure 4.6: Fraction of extrapolatives with positive target



Source: MATLAB output

Note that with one period corresponding to one quarter, 1000 periods means 250 years, so probability of divergence 21.3% within this time span is not that high. After re-simulating the model for 200 periods or 50 years, probability of divergence is about 3.2% (which is less than one fifth of 21.3%, as the simulations begin with the economy exactly in the steady state). Overall, in the time range of 50 years, PLT outperforms IT with 96.8% probability, while it leads to very bad outcome in the remaining cases. The risk of divergence is therefore neither very high nor completely negligible.

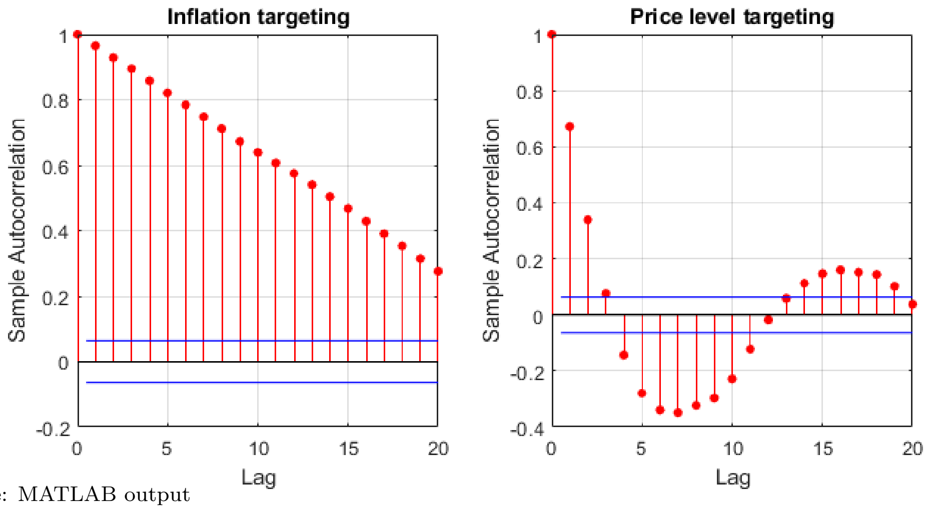
Appendix B contains figures depicting development of economic variables in the currently discussed model (simulation with stable PLT regime has been chosen). Again, *animal spirits* are concentrated mostly around 0 for PLT, while extreme values prevail for IT. PLT also leads to lower persistence in the variables than IT, even though the first lag autocorrelation in inflation 0.66 for PLT is now higher than before (the number was 0.24 in section 4.1.1). Furthermore, price level is again very predictable for the PLT regime, while there is substantial uncertainty about its development under IT.

Finally, figure 4.7 depicts that autocorrelation in inflation under PLT is now statistically significant even for more lags than the first one; and it in fact turns

[4]See chapter 5 for detailed discussion of why the prevailing extrapolative rule actually leads to divergence of the PLT model.

negative in the fourth period. The shape of the autocorrelation function nicely illustrates that the key mechanism of PLT—higher inflation now causing lower inflation in the future, and *vice versa*—functions well.[5]

Figure 4.7: Inflation autocorrelation function (rule 4.3)



Source: MATLAB output

So far, the analysis compared models with just two forecasting rules in order to contrast characteristics of the individual rules. It is, however, easily possible to extend the model for more rules by combination of previous cases. The logic of the heuristic modelling is to replace rational expectations by just a few rules, but as long as the rules are simple and reasonable, there can surely be more of them. In fact, as with one version of the rule PLT clearly outperforms IT, while with the other it may diverge, such approach lets the model decide which version will prevail; and the results can be thus considered as more conclusive than individual analyses before.

IT will be modelled in the same way as before with two rules only; the focus will be on PLT. In particular, let us consider one fundamentalist and two extrapolative rules—one with partial credibility (depicted as e_pc) and one purely extrapolative(e_pure). All of the rules have been presented before, but for clarity:

$$\widetilde{E}_t^{f,PLT} p_{t+1} = \overline{p}_{t+1} \tag{4.4}$$

$$\widetilde{E}_t^{e\_pc,PLT} p_{t+1} = p_{t-1} + 2 * \pi^{*q} \tag{4.5}$$

[5]This pattern is present even in figure 4.4 in section 4.1.1, but here it is more pronounced and much better visible.

$$\widetilde{E}_t^{e\_pure,PLT} p_{t+1} = p_{t-1} + 2 * \pi_{t-1}^q \tag{4.6}$$

Utilities of all three rules are computed based on weighted mean squared forecast errors, just as in equations 3.22 and 3.23. Probabilities of choosing each rule are then

$$\beta_{f,t} = \frac{\exp(\gamma U_{f,t}^p)}{\exp(\gamma U_{f,t}^p) + \exp(\gamma U_{e\_pc,t}^p) + \exp(\gamma U_{e\_pure,t}^p)} \tag{4.7}$$

$$\beta_{e\_pc,t} = \frac{\exp(\gamma U_{e\_pc,t}^p)}{\exp(\gamma U_{f,t}^p) + \exp(\gamma U_{e\_pc,t}^p) + \exp(\gamma U_{e\_pure,t}^p)} \tag{4.8}$$

$$\beta_{e\_pure,t} = 1 - \beta_{f,t} - \beta_{e\_pc,t} \tag{4.9}$$

Resulting market forecast of price level is

$$\widetilde{E}_t^{PLT} p_{t+1} = \beta_{f,t} \widetilde{E}_t^{f,PLT} p_{t+1} + \beta_{e\_pc,t} \widetilde{E}_t^{e\_pc,PLT} p_{t+1} + \beta_{e\_pure,t} \widetilde{E}_t^{e\_pure,PLT} p_{t+1} =$$
$$\beta_{f,t} \overline{p}_{t+1} + (\beta_{e\_pc,t} + \beta_{e\_pure,t}) p_{t-1} + 2(\beta_{e\_pc,t} \pi^{*q} + \beta_{e\_pure,t} \pi_{t-1}^q) \tag{4.10}$$

All remaining aspects of the model are the same as before; the extension for more rules is hence very straightforward. When the model is simulated, it turns out that the PLT regime is now stable and superior to IT, as table 4.4 shows—divergence is no longer a problem. PLT works only slightly worse than in case without the purely extrapolative rule, and it outperforms IT by a large margin.

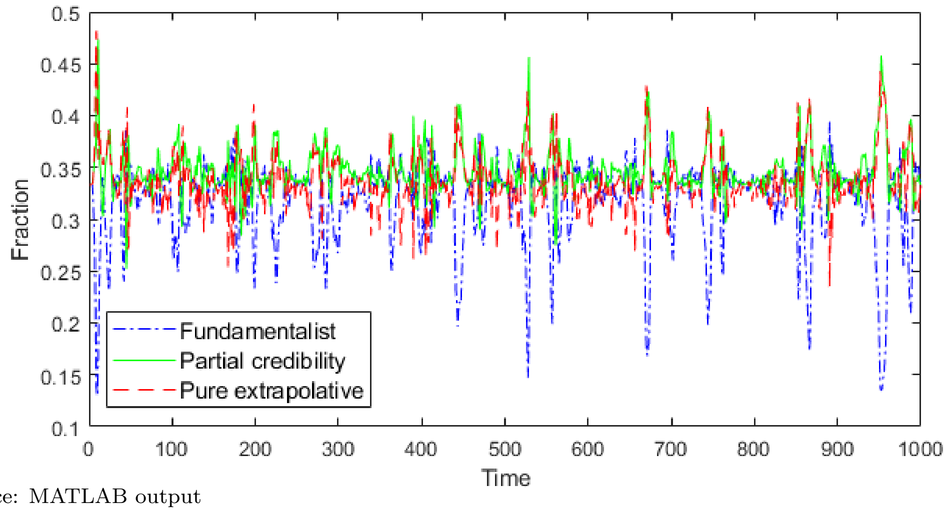
Table 4.4: Loss functions with three forecasting rules

	IT Loss	PLT Loss
Mean	0.4	0.04
Standard deviation	0.128	0.005
Range	0.15-0.88	0.027-0.077
Higher loss	1000	0
Wilcoxon test p-value	<0.001	

IT model is the same as before, so the results correspond to table 4.1. The simulations have been, however, conducted again, so values sensitive to individual observations (such as range) slightly differ.

Figure 4.8 shows share of economic agents using each rule. The fundamentalist rule tends to have slumps with low share of users from time to time, but it always gains its credibility back; and such development is sufficient to stabilize the economy. Further graphs are not shown here, as they are visually similar to those presented in Appendix B for the model with rule 4.3; the only difference is in a bit lower inflation persistence (first lag autocorrelation of 0.46). Overall, as opposed to section 4.1.1, where the results could have seem caused by construction due to stabilizing nature of even the extrapolative PLT rule, results of the analysis with three rules actually provide stronger support for price level targeting, as the regime performs well even in presence of the purely extrapolative rule.

Figure 4.8: Probabilities of using each of three rules



Source: MATLAB output

4.2 Expectations about inflation

The analysis so far assumed that agents under PLT regime at time t form expectations about price level at time $t+1$; the expectations then determine (together with other variables) price level at time t, and only *ex post*, when p_t is known, expectations about inflation at time $t+1$ are formed. Now let us assume the opposite case—even with central bank targeting price level, agents still create expectations about inflation at $t+1$; based on that, current inflation and price level are determined, and expectations about future price level are now formed only *ex post*. The reasoning behind such approach may be such that agents still care about inflation rather than price level (which is captured also by loss function 3.6, which contains deviations of inflation from target, not of price level). Price level targeting regime is considered just as a tool to deliver possibly more stable inflation, which is still the ultimate goal. The agents hence still think about how large inflation rate will be in the next period, not what will be the price level.

Conversely to the previous case, extrapolative rule is now the one which is straightforward:

$$\widetilde{E}_t^{e,PLT}\pi_{t+1} = \pi_{t-1} \tag{4.11}$$

This is the same as the extrapolative rule under IT. Therefore, extrapolative agents now really do not distinguish between monetary policy regimes, they simply use adaptive expectations; and only fundamentalists actually take into account whether the central bank targets inflation or price level. This is quite favourable feature of this approach as opposed to that with expectations formed about price level.

On the other hand, there are now more potential specifications of the fundamentalist rule. The goal is to have inflation rate such that targeted price level in the next period is met, but this is not directly possible as the agents know only past values of variables. Therefore, an assumption about current inflation and current price level needs to be made. Let us assume firstly that current inflation is equal to the long-term target ($\widetilde{E}_t^f \pi_t = \pi^*$) and only in the next period, inflation will adjust to meet the targeted price level path. This leads to following rule

$$\widetilde{E}_t^{f,PLT} \pi_{t+1}^q = \overline{p}_{t+1} - \widetilde{E}_t^f p_t = \overline{p}_{t+1} - p_{t-1} - \widetilde{E}_t^f \pi_t^q = \overline{p}_{t+1} - p_{t-1} - \pi^{*q} = \overline{p}_{t-1} - p_{t-1} + \pi^{*q} \tag{4.12}$$

where the last equality follows from applying equation 3.5 twice.[6] Notice also, that while this rule is much more simple than assuming rational expectations, it is still more complex than the inflation targeting fundamentalist rule 3.18. Therefore, even under assumption that the agents form expectations about inflation in both regimes, the model contains higher requirements on cognitive abilities of people under PLT than under IT.

Another reasonable specification of the fundamentalist rule is under assumption that $\widetilde{E}_t^f \pi_t = \widetilde{E}_t^f \pi_{t+1}$, where value of these two terms is such that $p_{t-1} + \widetilde{E}_t^f \pi_t^q + \widetilde{E}_t^f \pi_{t+1}^q = \overline{p}_{t+1}$. In other words, the agents assume that deviation of price level from its target at time $t-1$ is corrected for during two periods

[6] Note that for some initial price level p_1 (leading to first considered inflation rate being π_2), price levels can be expressed by repeated substitution as $\overline{p}_{t-1} = p_1 + (t-2)\pi^{*q}$ and $p_{t-1} = p_1 + \sum_{j=2}^{t-1} \pi_j^q$. The rule in equation 4.12 can then be rewritten as

$$\widetilde{E}_t^{f,PLT} \pi_{t+1}^q = \overline{p}_{t-1} - p_{t-1} + \pi^{*q} = \pi^{*q} + \sum_{j=2}^{t-1} (\pi^{*q} - \pi_j^q)$$

This formulation nicely illustrates how price level targeting compensates for past deviations of inflation rate from the long-term target.

with the same inflation rate. The rule itself then has the form

$$\widetilde{E}_t^{f,PLT}\pi_{t+1}^q = \frac{\overline{p}_{t+1} - p_{t-1}}{2} \tag{4.13}$$

Forecasting rules in both cases are now even for PLT evaluated according to equations 3.20 and 3.21, as these measure deviations of expected and actual inflation. Equations 3.22 and 3.23 with deviations between expected and actual price level are no longer used.

Thus, we have two different specifications of the fundamentalist rule. Nevertheless, it turns out that regardless of which one is used, PLT leads to divergence of the model in all 1000 simulations. Model with rule 4.12 diverges within 102-339 periods with average of 166.8 periods. Model with rule 4.13 then diverges within 102-367 periods, on average after 167.8 periods; the difference is therefore negligible.[7] Similarly to previous discussion, the extrapolative rule again gains dominance over time, and once this occurs, it is very difficult to reverse the process. All agents extrapolate past inflation rate into the future, monetary policy has no credibility, and the economy is on explosive path. The difference from the case of positive probability of divergence in the previous section is the fact that now the divergence occurs in every simulation. Variables in inflation targeting model on the other hand steadily fluctuate around their steady states; the model behaves in the same way as in section 4.1.2, as no change has been made in it.

The analysis above therefore shows that under assumption that people create expectations about inflation regardless of monetary policy regime, PLT leads to divergence of the economy due to prevalence of extrapolative rule, while IT is quite stable. These results are robust to particular specification of the fundamentalist rule under PLT. It is again very straightforward to extend the analysis by combining both versions of the fundamentalist rule together with the extrapolative rule into one model, just as in section 4.1.2. But since extrapolative rule prevails in both individual scenarios, it can be expected that such model will lead again to divergence; and actual simulations confirm the intuition. The divergence now occurs slightly later, on average after 191.6 periods (with range 108-567), but the extrapolative rule again always sooner or later acquires complete dominance and leads the economy away from the equi-

[7] The numbers capture the first period for which the software provided *not available* results; the economy has always been on an explosive path for several time periods before that.

librium. This result confirms clear dominance of IT over PLT in the model with expectations formed about inflation.

Chapter 5

Robustness Analysis and Discussion

The results presented in chapter 4 under assumption that economic agents in the PLT regime form expectations about future price level therefore favour price level targeting. The model has positive probability of divergence under a particular specification, but this is not present in the setting with three potential price level forecasting rules. On the other hand, when expectations are assumed to be formed about inflation, the PLT regime performs poorly, as it diverges in all simulations. But while the results crucially depend on assumption about the expectations formation, they are fairly robust to particular values of model parameters (with just a few exceptions).

The model has been simulated repeatedly with always one departure from the baseline calibration—all coefficients were one by one increased and decreased by a reasonable amount as compared with their baseline value, and the effect on the results has been examined. This holds not only for parameters in model equations, but also memory parameter ρ. Furthermore, autocorrelation into error terms in the model equations has been introduced (with autoregressive coefficient of various sizes). Most of these changes did not influence the implications of the results under the baseline calibration. PLT still outperforms IT in the version used in section 4.1.1 and with extrapolative rule 4.2 by substantial margin (which is moreover not dependent on weight of output gap in the loss function λ); probability of PLT divergence with rule 4.3 does not fluctuate much from the value for the baseline calibration (21.3%) and PLT is superior in cases where it is stable; and PLT model always diverges in the model used in section 4.2.

The exceptions which actually influence results are the following: size of standard deviation σ_η of shock η in New Keynesian Phillips curve 3.2; adding AR(1) process to the same shock (let us label AR coefficient as ξ_η); and intensity of choice parameter γ. Moreover, these parameters influence only results of two models in section 4.1.2 containing purely extrapolative rule 4.3 (either separately or together with partially extrapolative rule); and γ parameter also influences divergence of model in section 4.2. Therefore, let us focus sensitivity analysis on these cases only and let us start with σ_η and ξ_η.

Table 5.1: Sensitivity to characteristics of shock η

	$\sigma_\eta = 0.001$	$\sigma_\eta = 0.01$	$\xi_\eta = 0.1$	$\xi_\eta = 0.2$	$\xi_\eta = 0.5$	$\xi_\eta = 0.9$
PLT divergence (4.3)	0	1000	410	745	1000	1000
PLT divergence (3 rules)	0	717	0	9	768	1000
Median loss (PLT 4.3)	0.019	n/a	0.047	0.05	n/a	n/a
Median loss (PLT 3 rules)	0.019	0.14	0.041	0.047	0.139	n/a
Median loss (IT)	0.023	1.61	0.5	0.65	1.76	35.7
Loss(IT) < Loss(PLT 4.3)	0	n/a	33	34	n/a	n/a
Loss(IT) < Loss(PLT 3 rules)	0	32	0	0	35	n/a

The table analyses two versions of PLT model, one using extrapolative rule 4.3 and one using this rule together with 4.2, i.e. the model with three rules. A reminder: standard deviation $\sigma_\eta = 0.005$ and autoregressive coefficient $\xi_\eta = 0$ in the baseline calibration. Each case has been simulated 1000 times.

Results from table 5.1 suggests that with increasing standard deviation of NKPC equation shock and with increasing autoregressive component in the same shock, both monetary policy regimes deteriorate. Most importantly, susceptibility of PLT to instability grows substantially; and while it still generally outperforms IT in cases where it is stable, price level targeting is clearly much less robust to the characteristics of η shock. With only small autoregressive coefficient in the shock, it already starts to perform poorly (especially in the specification using rule 4.3 only), and with a moderate size of 0.5 it diverges for most of the time regardless of particular model specification.

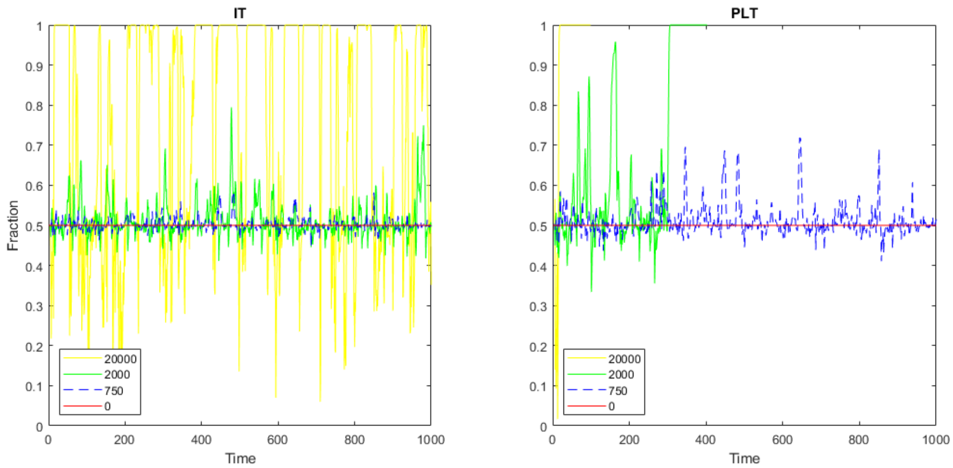
Now let us turn to the intensity of choice parameter γ. As a brief reminder—lower value of the parameter means higher role of randomness in choice between forecasting rules (as discussed in chapter 3.3, $\gamma = 0$ leads to purely stochastic choice between the rules with no regard to their performance). Periods with all agents using just one rule hence cease to exist with decreasing value of the parameter, and probabilities of using one or the other rule are closer to 0.5 (with the extrapolative rule being slightly preferred for most of the time unless $\gamma = 0$). In this case, the sensitivity analysis includes again the model with the extrapolative rule 4.3, and now also the model with expectations about inflation from section 4.2 with the fundamentalist rule 4.13. Model with rule 4.12 would lead to similar results as one with rule 4.13, while remaining model specifications were stable even for initial $\gamma = 20000$. Table 5.2 compares simulations for different values of γ. Stability of PLT increases with decreasing γ, but notice that lower parameter also improves performance of IT, as it eliminates *animal spirits* and enhances viability of the fundamentalist rule. The model under expectations created about inflation, which previously diverged in all cases, starts to be stable with high level of randomness in choice of the forecasting rule; but even when it is stable, it still performs worse than IT in all simulations. Conversely, the other model using rule 4.3 dominates IT as long as it is stable—just as before. To gain more intuition about what different values of γ mean, see figure 5.1, which depicts fraction of extrapolatives in forecasting inflation under several values of the parameter.

Table 5.2: Sensitivity to γ

γ	0	500	1000	5000	10000	20000
PLT divergence (4.3)	0	0	0	0	2	194
PLT divergence (4.13)	0	0	43	1000	1000	1000
Median Loss (PLT 4.3)	0.034	0.034	0.035	0.036	0.039	0.043
Median Loss (PLT 4.13)	0.078	0.078	0.078	n/a	n/a	n/a
Median Loss (IT)	0.069	0.07	0.07	0.11	0.26	0.4
Loss(IT)<Loss (PLT 4.3)	0	0	0	0	0	28
Loss(IT)<Loss (PLT 4.13)	959	942	885	n/a	n/a	n/a

Last column with $\gamma = 20000$ corresponds to the baseline calibration. Particular numbers presented here may slightly differ from those presented in chapter 4, as the simulations for the sensitivity analysis has been conducted separately; however, the difference is negligible.

Figure 5.1: Fraction of extrapolatives for different values of γ



Source: MATLAB output

In total, the results of whole analysis presented in the thesis seem to be mixed. When economic agents form expectations about price level, price level targeting is generally superior to inflation targeting. This holds whether the target is zero (as in section 4.1.1) or positive (as in section 4.1.2). Such conclusion confirms results of Ho *et al.* (2019) that PLT in heuristic model leads to higher social welfare than IT in this specific setting. PLT may diverge with probability of 21.3% in one particular specification, but this is, first, not as high as it might seem since it captures very long time span (the number is based on 1000 time periods, corresponding to 250 years), and second, this feature disappears when the model is expanded to contain one additional forecasting rule. On the other hand, the outcomes of section 4.1 are based on the particular assumption about expectations. Under alternative case with expectations being created about inflation, it turns out that PLT is very prone to losing credibility and it diverges in all simulations. Furthermore, it has been shown that even in the first case, PLT starts to behave poorly with high standard deviation of shock in NK Phillips curve or with autoregressive specification of that shock. Therefore, while potential benefits of PLT have been confirmed, complete results are rather in favour of the IT regime, which is significantly more robust choice.

The overall message of the thesis is hence different to the one by Ho *et al.* (2019), who provide stronger support for PLT. On the other hand, the results (especially those presented in section 4.2) are in line with paper of Yetman (2005) discussed in section 2.2, who introduced rule-of-thumb consumers using adaptive expectations (while the rest uses rational expectations). As opposed to Yetman, this thesis uses endogenous time-varying share of people using individual rules, and even fundamentalist consumers use simple rule rather than the rational expectations; but both works conclude that IT is more robust to share of people using adaptive expectations than PLT.

One crucial question obviously arises—what in fact causes so contradictory results of the model depending on whether the agents form expectations about price level or inflation? For a potential explanation, let us firstly clearly state actual difference between the two approaches. When it is assumed that expectations are created about price level, people simply forecast future price level based on the past with no regard to present. Fundamentalists just assume that price level will be at the target and they do not care whether the compensation for a past inflation deviation will occur in the current time period, in the next one, or gradually in both. There is hence more flexibility and the rule remains viable and stabilizes the economy. On the other hand, when people form expectations about inflation, the fundamentalist rule requires an implicit assumption about current inflation rate in order to make forecast about the future one. Even when PLT is itself working well and it delivers targeted price level, if the core of the compensation for past inflation deviations occurs in the current period, while people assumed it would occur in the next period (or *vice versa*), the fundamentalist rule becomes imprecise. Moreover, for some particular realization of current inflation rate, the fundamentalist rule can in fact become inconsistent with the policy regime and thus not stabilizing for the economy. Therefore, good performance of the fundamentalist rule needs sufficiently precise forecast of both present and future state of the economy, which poses stronger requirements on the economic agents. The extrapolative rule with its simple nature then may become more attractive and it predominates over time. This part of the answer hence explains why PLT in case of expectations created about inflation leads to higher prevalence of the extrapolative rule than under the assumption of expectations formed about price level.

Second part of the answer is related to actual performance of the policy regimes under the extrapolative rules. To examine this issue, the model has been generated without the fundamentalist rules in inflation forecasting. In other words, all economic agents use adaptive expectations and the two regimes therefore differ only in central bank policy rule; monetary policy has no credibility. The fundamentalist rule comprises a stabilizing element in the original model, so the performance of inflation targeting is worse without it; but the regime still leads to relatively stable development of the economy. In fact, some of classic inflation targeting studies by Svensson (1997; 1999a) were conducted under adaptive expectations.[1] On the other hand, price level targeting is unstable under adaptive expectations—when the reaction function of the central bank contains price level deviation rather than inflation, it leads to divergence of the economy. Such conclusion holds regardless of whether the adaptive expectations are about price level or inflation; and it does not depend on value of c_1^{PLT} parameter in the Wicksellian (PLT) policy rule either. Furthermore, the outcome is the same whether it is assumed that output gap forecasts are created according to the heuristic switching mechanism just as before, or whether adaptive expectations are assumed even in case of output gap.

To describe actual dynamics of the model under adaptive expectations, let us assume that the economy is in a boom due to some shocks; inflation is above target and output gap is positive; the central bank then reacts by increasing nominal interest rate. As a result, inflation and output gap fall and return to steady state values; but it takes a few time periods because of inflation

[1] The two papers of Svensson use slightly more elaborate version of adaptive expectations; expected inflation is given by weighted combination of current inflation and current output gap. But still, the expectations are given simply by one backward-looking equation.

persistence, so deviation of price level from its targeted path accumulates. When inflation and output gap are at the steady state, central bank targeting inflation returns also the nominal rate to its steady state value and the economy remains stable. But central bank targeting price level needs to depress inflation further in order to meet its price level target, so the nominal interest rate remains high; and output gap declines as a by-product. The nominal rate now does not smooth the cycle caused by shocks, but on the contrary, it enhances it. When the price level finally meets the target and the nominal rate declines, inflation is below the steady state; and because of the inflation persistence, it remains there for some time, generating negative deviation of price level from its target. The central bank now decreases the nominal rate below the steady state level and the mechanism is repeated. Every time, correcting for deviation in price level requires larger deviation of inflation from its long-term target, which then generates even larger deviation in price level in the opposite direction. Each wave of the economic cycle is therefore more pronounced than previous one and the model is unstable. In addition, even though inflation persistence is key part of the mechanism, the final outcome does not depend on parameter b_1 in New Keynesian Phillip curve (equation 3.2). As $\widetilde{E}_t\pi_{t+1} = \pi_{t-1}$ under adaptive expectations, term $b_1\widetilde{E}_t\pi_{t+1} + (1 - b_1)\pi_{t-1}$ in that equation is independent on value of b_1. The inflation persistence is thus inherent in the adaptive expectations framework, not given by assumption.

To summarize the arguments, PLT performs well as long as it retains certain credibility, but it is, first, more vulnerable to losing the credibility than IT (and the vulnerability is higher in the case of expectations formed about inflation), and second, it behaves much worse when this occurs and adaptive expectations prevail among people. Response to price level in the policy rule needs to be combined with regime-consistent expectations for PLT to function well.

On the other hand, the analysis here might be a bit unjust towards PLT, which constitutes certain limitation of the applied method. Firstly, higher vulnerability of PLT under the assumption of expectations created about inflation may be partly caused artificially by sequential nature of the model. If people knew current state of the economy, expectations about future inflation and price level would be equivalent; and in reality, people presumably have more information about present than the model suggests. (On the other hand, we can expect that the information is imperfect and official macroeconomic data usually come with a lag.) Secondly, if PLT delivered targeted outcome, people might retain the belief in the regime even if the compensation for past deviations occurred differently distributed over time than their simple fundamentalist rule predicted. Thirdly, regarding the behaviour under adaptive expectations, the Wicksellian rule contains only current values of economic variables; forward-looking central bank could take the inflation persistence into account and normalize the interest rate sooner than when price level is at the target.

Therefore (and considering that IT faces substantial difficulties in the form of the zero lower bound), disregarding PLT completely based on its low stability could be a mistake, as it offers a potential solution to at least partially deal with the ZLB problem. At the same time, the model does reveal sensitivity of the PLT regime to particular model specification and to certain model parameters; and it implies that PLT leads to substantially higher requirements on forward-looking behaviour of the central bank.[2] Thus, while completely disregarding PLT might be a mistake, the results at the very least call for high caution before potential implementation of PLT in practice, and they also corroborate the paramount importance of credibility for proper conduct of the regime.

Moreover, the thesis focused on comparison of pure price level and inflation targeting, note however, that these are not the only possibilities for monetary policy regimes. Apart from traditional options such as monetary targeting, exchange rate peg etc., there are also proposals for policies which use certain elements of IT or PLT without being pure forms of these regimes. These can be, among others, nominal GDP targeting, average inflation targeting, or speed limit policies[3]; and under certain assumptions, these regimes can in fact outperform both IT and PLT (for example, see Billi (2017) for nominal GDP targeting, Nessen & Vestin (2005) for average inflation targeting, and Walsh (2003) for speed limit policies). Detailed treatment of these regimes is beyond scope of this text, but especially average inflation targeting could be a potential way to utilize advantages of PLT without endangering credibility of monetary policy. These alternatives therefore surely deserve further attention in economic research as well.

[2]While forward-looking element in policymaking is in practice important for IT as well, the results suggest that it would be a crucial aspect in case of PLT with low credibility—only well-timed reaction of the central bank could alleviate the explosive dynamics of the model with adaptive expectations, and any small mistakes in such scenario could have large consequences.

[3]Speed limit policies are based on targeting inflation and *change* in output gap.

Chapter 6

Conclusion

The thesis has analysed price level targeting, which has been previously found to outperform inflation targeting in various theoretical studies; and the difference between the two regimes is even more pronounced when zero lower bound on nominal interest rates is considered. Such results are, however, to a large extent given by strong assumption of rational expectations, which seems to be unrealistic in practice. Therefore, a simple New Keynesian 3-equation model which abandons rational expectations hypothesis has been applied in the thesis. Economic agents instead form expectations about future inflation (or price level) and output gap using heuristics—they choose between a few simple forecasting rules based on their past performance. Two rules have been used in the baseline setting—fundamentalist rule consistent with given monetary policy regime, and extrapolative rule corresponding to adaptive expectations. Furthermore, two main approaches to PLT have been discussed—one with assumption that people form expectations about future price level, and the other assuming expectations about inflation (which is *ex ante* not equivalent due to sequential nature of the model—people create expectations about future based on past, which then determines present).

It has been shown that PLT outperforms IT under assumption that expectations are formed about price level. Nevertheless, the case with expectations about inflation causes PLT to diverge regardless of particular specification of the fundamentalist forecasting rule. When a substantially high shock hits the economy, the extrapolative rule begins to prevail and monetary policy completely loses its credibility. If that happens, the PLT regime behaves very poorly, self-fulfilling wave arises and gets the economy on an explosive path, which is then very difficult to reverse. On the contrary, inflation targeting is less prone to losing credibility, and even if that happens and the extrapolative rule predominates, IT still leads to relatively stable development of the economy. Moreover, even in models in which PLT is stable and superior under the baseline calibration, it is very sensitive to size of standard deviation and autoregressive nature of shock in NK Phillips curve. Inflation targeting is hence under model uncertainty safer choice and, according to results of the analysis, it is therefore overall preferred to price level targeting (even though the preference is not unequivocal). Results are in general robust to remaining model parameters and hinge especially on the actual assumptions about expectations formation.

The results are thus rather contradictory to much of existing research, which favours PLT—but the research is largely determined by the assumption of rational expectations. On the other hand, the thesis is in accordance with approach common among policymakers, who admit theoretical advantages of PLT, but who are also aware that the rational expectations hypothesis might be too strong assumption and that the regime might not gain and retain sufficient credibility. This is exactly what has been shown here.

At the same time, it needs to be said that the model used here is very simplified. It is a model of closed economy, it takes into account neither length of monetary policy horizon nor many other characteristics or sectors of the economy, and central bank policy rules are backward-looking. Furthermore, forecasting rules have been introduced rather *ad hoc* and not based on microeconomic foundations, because there is no consensus how this should be done. Dealing with all of these issues can be a path for future research. Moreover, heuristic modelling comprises rather novel approach in an attempt to address several drawbacks of mainstream DSGE models, but much work needs to be done to assess overall empirical relevance of heuristic models in detail. Therefore, it would be premature to claim based on results of this thesis that price level targeting is definitely not a good option for monetary policy, which would lead to high risk of explosive path of the economy in practice. The policy regime surely offers substantial potential benefits, which might be especially useful in situation where many economies across the world are stuck at the zero lower bound on nominal interest rates. But at the very least, the results suggest that policymakers should be very cautious before actually adopting PLT in practice, and that introducing only its certain elements, e.g. in a form of average inflation targeting, might be a better option than pure price level targeting itself.

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Appendix A

Mathematical derivations

This appendix shows several explicit derivations, which were skipped in the main body of the thesis.

A.1 Model solution

The applied model consists of three key equations, as presented in chapter 3. These are for inflation targeting regime

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 (i_t - \widetilde{E}_t \pi_{t+1} - \overline{r}) + \epsilon_t \tag{A.1}$$

$$\pi_t = b_1 \widetilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + b_2 y_t + \eta_t \tag{A.2}$$

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1(\pi_t - \pi^*) + c_2 y_t\right] + (1-c_3) i_{t-1} + u_t \tag{A.3}$$

For given expectations of output gap and inflation, the model can be easily solved by plugging equation A.3 into A.1, which leads to

$$\begin{split} y_t = a_1 \widetilde{E}_t y_{t+1} + (1-a_1) y_{t-1} + a_2 c_3 \pi^* + a_2 c_3 \overline{r} + a_2 c_1 c_3 (\pi_t - \pi^*) + a_2 c_2 c_3 y_t + \\ a_2 (1-c_3) i_{t-1} + a_2 u_t - a_2 \widetilde{E}_t \pi_{t+1} - a_2 \overline{r} + \epsilon_t \end{split} \tag{A.4}$$

This equation together with equation A.2 can be further rearranged:

$$\pi_t - b_2 y_t = b_1 \widetilde{E}_t \pi_{t+1} + (1-b_1)\pi_{t-1} + \eta_t \tag{A.5}$$

$$-a_2c_1c_3\pi_t+(1-a_2c_2c_3)y_t = -a_2\widetilde{E}_t\pi_{t+1}+a_1\widetilde{E}_ty_{t+1}+(1-a_1)y_{t-1}+a_2(1-c_1)c_3\pi^*+ a_2(1-c_3)(i_{t-1}-\overline{r})+a_2u_t+\epsilon_t \quad (A.6)$$

Which can be rewritten in matrix notation as

$$\begin{pmatrix} 1 & -b_2 \\ -a_2c_1c_3 & 1-a_2c_2c_3 \end{pmatrix}\begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1 \end{pmatrix}\begin{pmatrix} \widetilde{E}_t\pi_{t+1} \\ \widetilde{E}_ty_{t+1} \end{pmatrix} + \begin{pmatrix} 1-b_1 & 0 \\ 0 & 1-a_1 \end{pmatrix}\begin{pmatrix} \pi_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ a_2(1-c_1)c_3 \end{pmatrix}\pi^* + \begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix}(i_{t-1}-\overline{r}) + \begin{pmatrix} \eta_t \\ a_2u_t+\epsilon_t \end{pmatrix} \quad (A.7)$$

or simply

$$\mathbf{A}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \upsilon_t \quad (A.8)$$

The solution is then given by

$$\mathbf{Z}_t = \mathbf{A}^{-1}\left[\mathbf{B}\widetilde{E}_t\mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \mathbf{b}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \upsilon_t\right] \quad (A.9)$$

The solution exists if $\mathbf{A}$ is not singular, i.e. if $1 - a_2c_2c_3 - a_2b_2c_1c_3 \neq 0$, which can be rewritten as $1 \neq a_2c_3(c_2 + b_2c_1)$. The solution yields y_t and π_t, which can be plugged into interest rate rule A.3 to generate i_t.

Under assumption that agents forecast inflation even under PLT (as examined in section 4.2), the same approach can be applied to price level targeting, which differs only in the interest rate rule:

$$i_t = c_3\left[\pi^* + \overline{r} + c_1(p_t - \overline{p}_t) + c_2y_t\right] + (1-c_3)i_{t-1} + u_t \quad (A.10)$$

As equation A.2 determines current inflation rate, we need to put inflation also into the interest rate rule, which can however be done easily as $p_t = p_{t-1} + \pi_t^q$. But as the model equations contain annualized inflation, we need to use the form $p_t = p_{t-1} + \frac{1}{4}\pi_t$. The interest rate rule can be therefore written in form

$$i_t = c_3\left[\pi^* + \overline{r} + c_1(\frac{\pi_t}{4} + p_{t-1} - \overline{p}_t) + c_2y_t\right] + (1-c_3)i_{t-1} + u_t \quad (A.11)$$

where p_{t-1} and $\overline{p}_t$ are known values. This rule can be then plugged into equation A.1 and rearranged just as in case of inflation targeting above. This yields

$$\begin{pmatrix} 1 & -b_2 \\ -\frac{a_2 c_1 c_3}{4} & 1 - a_2 c_2 c_3 \end{pmatrix} \begin{pmatrix} \pi_t \\ y_t \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ -a_2 & a_1 \end{pmatrix} \begin{pmatrix} \widetilde{E}_t \pi_{t+1} \\ \widetilde{E}_t y_{t+1} \end{pmatrix} + \begin{pmatrix} 1 - b_1 & 0 \\ 0 & 1 - a_1 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ a_2 c_3 \end{pmatrix} \pi^* + \begin{pmatrix} 0 \\ a_2(1 - c_3) \end{pmatrix} (i_{t-1} - \overline{r}) + \begin{pmatrix} 0 \\ a_2 c_1 c_3 \end{pmatrix} (p_{t-1} - \overline{p}_t) + \begin{pmatrix} \eta_t \\ a_2 u_t + \epsilon_t \end{pmatrix} \tag{A.12}$$

or

$$\widetilde{\mathbf{A}}\mathbf{Z}_t = \mathbf{B}\widetilde{E}_t \mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1} - \overline{r}) + \mathbf{d}(p_{t-1} - \overline{p}_t) + \upsilon_t \tag{A.13}$$

which differs from the case of inflation targeting by adjustments in matrix $\widetilde{\mathbf{A}}$, vector $\widetilde{\mathbf{b}}$ (the tilde captures that it is slightly different than matrix $\mathbf{A}$ and vector $\mathbf{b}$ in equation A.8), and by the term $\mathbf{d}(p_{t-1} - \overline{p}_t)$. The solution is given by

$$\mathbf{Z}_t = \widetilde{\mathbf{A}}^{-1} \left[\mathbf{B}\widetilde{E}_t \mathbf{Z}_{t+1} + \mathbf{C}\mathbf{Z}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1} - \overline{r}) + \mathbf{d}(p_{t-1} - \overline{p}_t) + \upsilon_t \right] \tag{A.14}$$

with condition $1 \neq a_2 c_3 (c_2 + \frac{b_2 c_1}{4})$.

Derivation for PLT model is slightly different when it is assumed that people form expectations about price level (as in section 4.1). Again, we need to explicitly account for difference between annualized and quarterly inflation and to rewrite model equations using $\widetilde{E}_t \pi_{t+1} = 4(\widetilde{E}_t p_{t+1} - p_t)$ and $\pi_t = 4(p_t - p_{t-1})$. This leads to

$$y_t = a_1 \widetilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 i_t - 4a_2 \widetilde{E}_t p_{t+1} + 4a_2 p_t - a_2 \overline{r} + \epsilon_t \tag{A.15}$$

$$4(p_t - p_{t-1}) = 4b_1 \widetilde{E}_t p_{t+1} - 4b_1 p_t + 4(1 - b_1)(p_{t-1} - p_{t-2}) + b_2 y_t + \eta_t \tag{A.16}$$

$$i_t = c_3 \left[\pi^* + \overline{r} + c_1 (p_t - \overline{p}_t) + c_2 y_t \right] + (1 - c_3) i_{t-1} + u_t \tag{A.17}$$

Plugging nominal interest rate from A.17 to A.15 just as before yields

$$y_t = a_1\widetilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2c_3\pi^* + a_2(c_3-1)\overline{r} + a_2c_1c_3p_t - a_2c_1c_3\overline{p}_t + a_2c_2c_3y_t + a_2(1-c_3)i_{t-1} + a_2u_t - 4a_2\widetilde{E}_tp_{t+1} + 4a_2p_t + \epsilon_t \quad \text{(A.18)}$$

Then let us put all terms with y_t and p_t in equations A.16 and A.18 on the left hand side and all remaining terms on the right hand side. This results in

$$\begin{pmatrix} 4(1+b_1) & -b_2 \\ -a_2(4+c_1c_3) & 1-a_2c_2c_3 \end{pmatrix}\begin{pmatrix} p_t \\ y_t \end{pmatrix} = \begin{pmatrix} 4b_1 & 0 \\ -4a_2 & a_1 \end{pmatrix}\begin{pmatrix} \widetilde{E}_tp_{t+1} \\ \widetilde{E}_ty_{t+1} \end{pmatrix} + \begin{pmatrix} 4(2-b_1) & 0 \\ 0 & 1-a_1 \end{pmatrix}\begin{pmatrix} p_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} -4(1-b_1) & 0 \\ 0 & -a_2c_1c_3 \end{pmatrix}\begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix} + \begin{pmatrix} 0 \\ a_2(1-c_3) \end{pmatrix}(i_{t-1}-\overline{r}) + \begin{pmatrix} 0 \\ a_2c_3 \end{pmatrix}\pi^* + \begin{pmatrix} \eta_t \\ a_2u_t+\epsilon_t \end{pmatrix} \quad \text{(A.19)}$$

or

$$\overline{\mathbf{A}}\widetilde{\mathbf{Z}}_t = \overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t \quad \text{(A.20)}$$

with $\mathbf{P}_t = \begin{pmatrix} p_{t-2} \\ \overline{p}_t \end{pmatrix}$. This can be solved again as

$$\widetilde{\mathbf{Z}}_t = \overline{\mathbf{A}}^{-1}\left[\overline{\mathbf{B}}\widetilde{E}_t\widetilde{\mathbf{Z}}_{t+1} + \overline{\mathbf{C}}\widetilde{\mathbf{Z}}_{t-1} + \widetilde{\mathbf{b}}\pi^* + \mathbf{c}(i_{t-1}-\overline{r}) + \widetilde{\mathbf{d}}\mathbf{P}_t + \upsilon_t\right] \quad \text{(A.21)}$$

with condition $4(1+b_1)(1-a_2c_2c_3) \neq a_2b_2(4+c_1c_3)$.

A.2 Forgetfulness

This part shows equality between equations 3.9 and 3.11, which was claimed to hold in chapter 3. Let us start with the latter equation and substitute for $U^y_{f,t-1}$, $U^y_{f,t-2}$ etc.

$$\begin{aligned} U^y_{f,t} &= \rho U^y_{f,t-1} - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \\ &\rho\left[\rho U^y_{f,t-2} - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3} y_{t-2}\right]^2\right] - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \\ &\rho\left\{\rho\left[\rho U^y_{f,t-3} - (1-\rho)\left[y_{t-3} - \widetilde{E}^f_{t-4} y_{t-3}\right]^2\right] - (1-\rho)\left[y_{t-2} - \widetilde{E}^f_{t-3} y_{t-2}\right]^2\right\} \\ &\qquad - (1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 \end{aligned} \tag{A.22}$$

Further repeating the substitution $j-1$ times then leads to

$$\begin{aligned} &\rho^j U^y_{f,t-j} - \rho^{j-1}(1-\rho)\left[y_{t-j} - \widetilde{E}^f_{t-j-1} y_{t-j}\right]^2 - \rho^{j-2}(1-\rho)\left[y_{t-j+1} - \widetilde{E}^f_{t-j} y_{t-j+1}\right]^2 - \\ &\ldots - \rho^0(1-\rho)\left[y_{t-1} - \widetilde{E}^f_{t-2} y_{t-1}\right]^2 = \rho^j U^y_{f,t-j} - \sum_{k=0}^{j-1} \rho^k (1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 \end{aligned} \tag{A.23}$$

For j going to infinity, with $0 < \rho < 1$, and after defining $\omega_k = \rho^k(1-\rho)$, this expression yields

$$U^y_{f,t} = -\sum_{k=0}^{\infty} \rho^k (1-\rho)\left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 = -\sum_{k=0}^{\infty} \omega_k \left[y_{t-k-1} - \widetilde{E}^f_{t-k-2} y_{t-k-1}\right]^2 \tag{A.24}$$

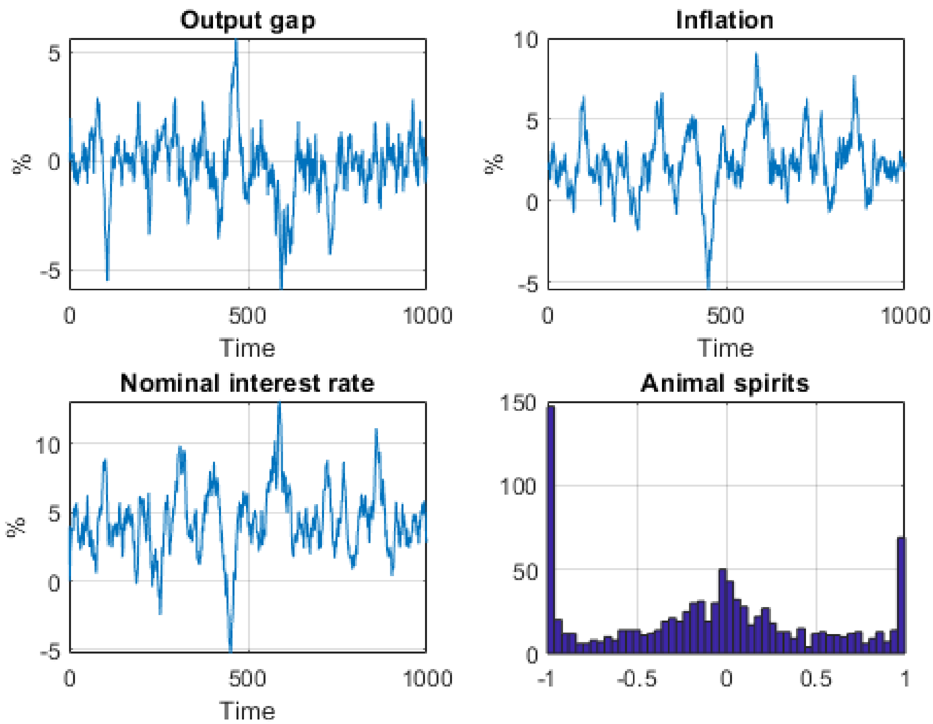
which is identical to equation 3.9. Equivalence of equations 3.10 and 3.12 can be shown in completely same way.

Appendix B

Additional Graphs

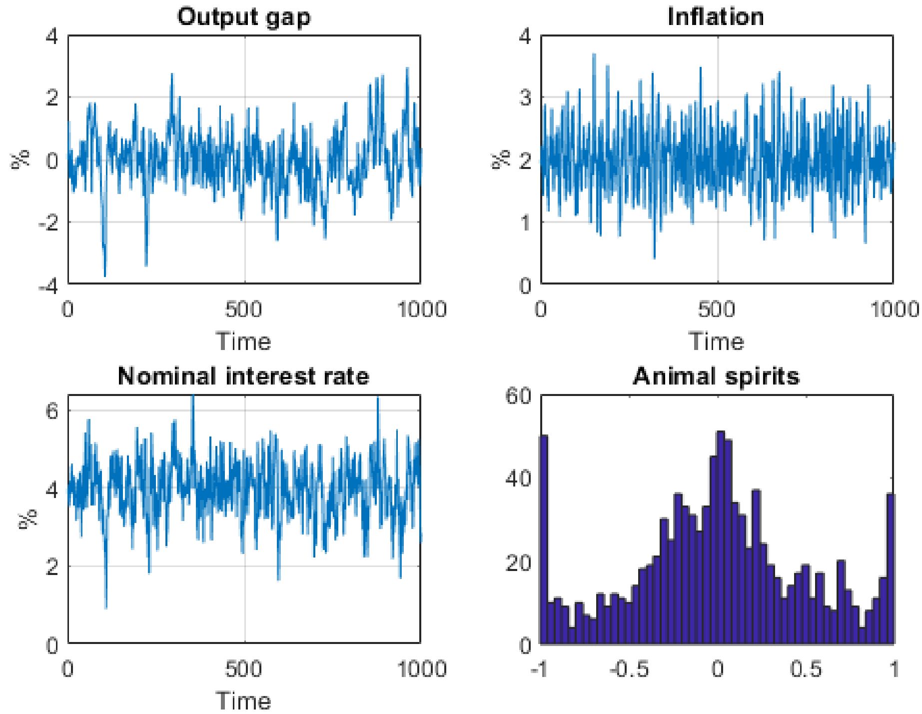
This appendix presents a few additional graphs related to section 4.1.2. In particular, figure B.1 depicts development of macroeconomic variables (and *animal spirits*) in the IT regime with positive inflation target. Figure B.2 contains corresponding graphs under PLT (in the version with expectations formed about price level with purely extrapolative rule 4.3). Figure B.3 then compares development of price level under the two regimes.

Figure B.1: IT with positive target



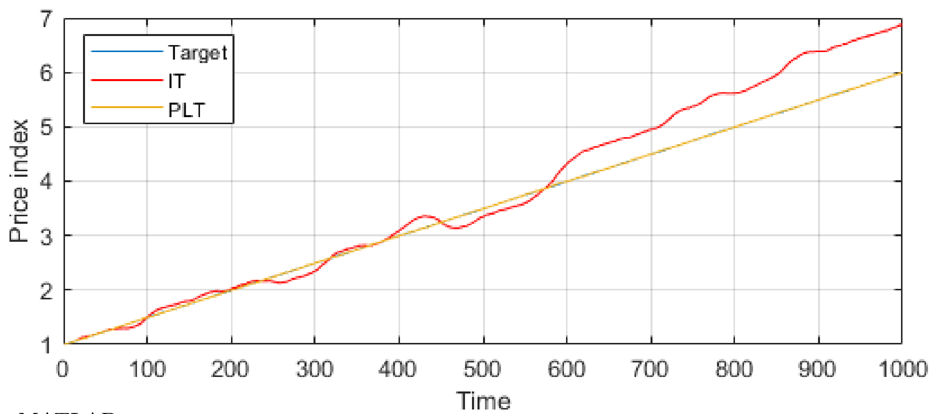
Source: MATLAB output

Figure B.2: PLT with positive target (rule 4.3)



Source: MATLAB output

Figure B.3: Price level (rule 4.3)



Source: MATLAB output