

This doctoral thesis provides several new results in which we leverage the inner structure of non-singular rings, in particular of self-injective von Neumann regular rings. First, we describe categorical and set-theoretical conditions under which all products of compact objects remain compact, where the notion of compactness is relativized with respect to a fixed subclass of objects. A special instance when such closure property holds are the classic module categories over rings of our interest. Moreover, we show that a potential counterexample for Köthe's Conjecture might be in the form of a countable local subring of a suitable simple self-injective von Neumann regular ring.