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2020 **Lukáš Tomek**

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Financial Markets Comovements in Northern Europe

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Abstract

In this bachelor thesis, we study conditional correlation of various sector indices on the stock markets in Northern Europe, namely in Stockholm, Helsinki, Copenhagen and composite indices for Baltic countries. To model conditional correlations, we employ DCC-GARCH framework estimated by maximum likelihood estimator. Validation of estimated models is based on residuals. We discovered that there is low level of correlation between Nordic and Baltic countries and that some sectors exhibits very high level of correlation, while other tends to have correlation close to zero or even negative for some time periods. Moreover, we observe that some industries have very persistent correlation structure, while others tends to react to the price shocks drastically.

Abstrakt

Tato bakalářská práce se zabývá podmíněnou korelací různých sektorových indexů na burzách cenných papírů v serverní Evropě, konkrétně ve Stockholmu, v Helsinkách, v Kodani a společných indexů pro Baltskě státy. Pro modelování podmíněných korelací používáme DCC-GARCH model odhadnutý pomocí metody maximální věrohodnosti. K validaci modelů používáme analýzu residuí. Objevili jsme, že korelace mezi severskými a baltskými indexy je velmi nízká a vysoké korelace mezi jistými sektory, zatímco jiné korelace mezi řadami jsou velmi blízko nule a dokonce negativní pro některá období. Navíc můžeme pozorovat přetrvávající trend v korelaci, zatímco další reaguje drasticky na cenové šoky.

Keywords

GARCH, DCC-GARCH, Portfolio Diversification, Conditional Correlation, Northern Europe, Financial Markets, Sector Diversification

Klíčová slova

GARCH, DCC-GARCH, Diverzifikace Portfolia, Podmíněná Korelace, severní Evropa, Finanční Trhy, Sektorová Diverzifikace

Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, 6 May 2020

Signature

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Thesis Proposal

Research question and motivation

To understand the connections and links on the financial market is crucial for all market players. This thesis will focus mainly on stock markets in Northern Europe (i.e. in Sweden, Denmark, Finland, Baltic countries, etc.). The primary goal of this thesis is to discover and describe relationships in these markets, with an emphasis on mutual interactions between countries and economic sectors in Northern Europe. To estimate patterns, we will model volatility using GARCH approach and estimate pairwise conditional correlation.

Contribution

The results of this analysis help us to understand patterns among financial markets. This knowledge is crucial for modern portfolio theory, which is one of the foundations for trading in financial markets. The finding might be used for portfolio diversification to minimize concentration risk and maximize long-run profit. This thesis may also serve as a grounding for further studies and analysis the way that financial market works, especially in terms of mutual inner interactions. In past, there were various attempts to find patterns in stock markets correlations. Gomes & Taamouti (2016) discovered a statistically significant high correlation between western European countries. They also bring evidence that comovements are driven mainly by global factors. Bekaert et al (2009) shown that sector comovements in a country are stronger than cross country comovements. Dutt & Mihov (2013) claims that countries with similar industry structure are more correlated than other countries. However, Griffin amp; Karolyi (1998) found that cross county correlations cannot be explained by industrial sector decomposition. Cao et al (2013) studied sector indices in China and found that sectors are more correlated during dynamic growth of China's economy in 2007-2008, but the comovements almost disappear after that period. Even though many studies have been made on sector correlation, we still do not have a unified robust theory which would fit perfectly to the real-world data.

Methodology

We will use GARCH approach to model variances and covariances of financial markets sectors, countries etc. and then standardized into correlations. GARCH models are widely used to model volatility based on historical data. The standard historical data from stock markets will be used. Historical prices of stocks and indices are available on NASDAQ Nordic website and partially on the Yahoo inance website. These data will be employed to model correlations. We will introduce proper hypotheses based on existing literature on correlations between financial markets. We expect to find strong cross-country correlations and even stronger between industrial sectors in each country. We also expect a statistically significant correlation between the same industry across different countries.

Expected Outline

1. Introduction
2. Theoretical background and literature review
3. Methodology
4. Data description
5. Result discussion
6. Overview of applications
7. Summary

Contents

1 Introduction

From the dawn of our civilisation, markets were one of the key element of our culture. Exchange of goods and services quickly became the cornerstone for all the cities and even states. In our modern and complex times, the word market is widely used to describe colourful areas of human mutual interactions. Today, researchers use market and economic principles to explore and to interpret worlds of politics, education, crime or even dating. Therefore it is no surprise that there exist markets to trade financial assets like stocks, bonds, currencies, commodities and various financial derivatives.

As on any other markets, financial markets serve as places to connect buyer and seller for the benefits of both sides. The financial market participants tend to maximise their profits. However, financial markets are tightly linked with high risk and uncertainty, therefore it is often the case to rather minimise risk while keeping the profits positive. Investors worked from very beginning on various techniques to tackle the risk. Some of them tend to diminish uncertainty to trade as save as possible, other seek risk for higher profits. For both of these approaches, it is crucial to understand the hazard.

In this thesis, we will explore and investigate risk in Northern Europe by studying the behaviour of the financial assets. Our main goal is to find mutual comovements among economy sectors on financial markets. By studying mutual interactions, one is able to adjust their strategy to maximise profit or minimise risk. In these times, investors and researchers chase for the very best methods and schemes of trading on various financial markets. In this thesis, we will aim specifically on stock markets in Northern Europe.

This thesis is organised as follows. We start with theoretical background, in which we explore professional literature in the topics of portfolio diversification, modelling second moments of financial time series and also unique properties of the financial markets in Northern Europe. In the methodology section, we introduce a univariate model for estimating time-varying volatility, which we extend to the multivariate frame to model joint second moment as well. We also describe the estimation and validation procedure. Finally, we describe the data and interpret the results of our estimated models.

2 Literature Review

2.1 Portfolio Diversification

Portfolio diversification is one of the key decision making tool for investors. This tool have been widely used to handle risk and uncertainty in portfolios. According to Koumou & Dionne (2019), there are four fundamental principles of modern portfolio diversification, namely *law of large numbers*, *capital assets pricing models*, *risk parity* and *correlation*. In this thesis, we are interested in the last of the principles.

In the most general scope, correlation can be defined as any linear statistical relation. Correlation is a good tool to describe linear relationship between variables. It has many interesting properties, which might be very useful for the in-depth analysis. First of all, correlation is (measurable) quantity. Even though there exist various approaches to defining correlation, it can be defined properly based on mathematical axioms. Robust definition allows us to grasp the concept seriously and incorporate it into any quantitative analysis. Even thought correlation can be defined in many ways (i.e. Pearson's, Spearman's), we will stick to the classical definition with standard properties stated further in this section. Secondly, correlation is standardised measure of linear dependency. This means that correlation is limited to certain values. These boundaries allow us to actually interpret and/or compare its quantity. In comparison to non-standardised measure (for instance *covariance*), we can directly construe the strength and nature of the relationship.

As mentioned above, correlation has many mathematical properties. Correlation of two random variables X and Y is defined as

$$\rho = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} \tag{1}$$

where σ_X is standard deviation of random variable X and σ_{XY} is covariance of random variables X and Y. Rescaling covariance this way bounds correlation coefficient to the interval between -1 and 1. In this scale, correlation 0 means that there is absolutely no linear relationship between two random variables, correlation equal to 1 (resp. -1) means that there is absolute positive (negative) linear relationship. Since we have a very specific scale, it allows us interpret values in our result. Even though this might seems trivial, it is important to clarify properties we will use in this thesis.

Investors all over the world use and develop various techniques of portfolio diversification, which are used to fulfil two main tasks: to maximise profit and to minimise risk. Generally, investors can diversify their portfolios by asset type (stocks, bonds, commodities, real estate etc.), by country (different countries has different specifications) or by many other characteristics which investors find useful or important. Each of these approaches has a specific purpose and helps to eliminate a different type of risk or uncertainty. In this thesis, we will examine mutual behaviour between both economic sectors and countries.

One of the first academic take on *modern portfolio theory* was series of articles by Markowitz (1952, 1959), in which he described optimal mean-variance portfolio and also developed mathematical tools, for example *critical line method* used for identification of what is known as *Markowitz frontier*. This efficient frontier helps to identify portfolio with highest returns for given risk (level of variance). Markowitz continued to explore and enhance his method for non-quadratic utility functions in Levy & Markowitz (1979). This article was later revisited and expended with the idea for infinite number of possible options by Kroll et al. (1984). Fortunately, with improving technology, many papers managed to present more useful algorithms and approaches, for example Sharpe (1978).

The key idea of modern portfolio theory claims that whole portfolio is less risky than just a sum of all the assets it contains. This information lead all players on financial markets to create a huge variety of portfolios, exploring its perks and characteristics and building less risky or more profitable portfolios. Many techniques were opened up and evolved all along the way with growing technology and expansion of financial products, their derivatives or even completely new investment option like cryptocurrencies. Such a complexity of investments brings up new challenges for portfolio theory. Despite our advancement in computational power, it is getting harder and nearly impossible to work with this enormous data. For that purpose, we have to use new and advanced mathematical and econometric tools (which is very demanding and not a goal of this thesis) or compromise the data we process.

Dividing economy into sectors might be useful to solve the problem of overwhelming data by grouping similar securities into one index. There exist various methodologies for this classification, but we can generally state that they are built on similar processes, products and behaviour. Sector indices are common indicators presented by many stock exchanges all around the world. They represent various sectors on financial market, which helps us to have aggregated information about many stocks. Having just a few indices mirroring major information on market is really powerful and allows us to use well established and (relatively) clear approach of modelling correlation.

Sector effects were investigated many times in the past. Drummen & Zimmermann (1992) explore daily returns on European stock markets and claim that country effect is most important for the variance of returns, while sector effect being relatively minor. In the same year, Roll (1992) published their findings on sector effect. Roll claims that behaviour of given sector index is driven by composition, country and exchange rate. Other studies indicates a very similar trend. For example, Lessard (1974) analysed data between years 1959 - 1973 and presents that industry effect has minimal impact on volatility of returns, while national effect being the driving force for the second moment.

However, in vast majority of studies publish lately authors hint that sector effect is becoming more and more significant, while geographical diversific-ation lowers its importance. This idea was originally revealed by Hauser & Vermeersch (2002). They used data from G7 countries and Switzerland between years 1980-2001 split into 4 non-overlapping periods. In the first three periods, country diversification outperforms sector effect. However, in the fourth period (1995-2001), sector and country diversification are both efficient. In this particular study, authors explain this shift by globalisation and similarity in economies of selected countries. Since we have a comparable setting in this thesis, these findings will be very useful for hypothesis specification. Many further studies also prove this idea. Shawky et al. (2012) investigated effect of diversification on the performance of a hedge funds. This study claims that sector and asset type diversification outperforms any other diversification. Moreover, they found strictly positive and significant relationship between sector diversification and returns of given hedge fund.

Campa & Fernandes (2006) argues that globalisation and financial integration is the leading force diminishing the country effect and giving more weight to sectors. Ferreira & Ferreira (2006) studied country and sector effect in European countries. They managed to replicate the results of previous findings in periods between 1975-1990. However, they found out significant increase of industry effect in the 1990s, which they attributed to the common EURO currency. This particular effect of EURO currency was explored by Flavin (2004). They managed to confirm the shift of importance from country effect to the sector effect. Be that as it may, the latest studies brings up the idea that the global financial crisis in 2008 strengthen the country effect again, while sector effect steps back.

2.2 Modelling Second Moment of Returns

Since the second moment of returns cannot be directly observable, models to estimate it reliably and precisely were always a tough-nut-to-crack. Variance in time series is the key measurement of risk. Moreover, variance is key component of various *Capital Asset Pricing Models* (CAPM) and even the cornerstone of *Value-at-Risk* (VaR) approach. One of the first model that prove itself over time to be able to model conditional second moment of returns was introduced by Engle (1982). Their *Autoregressive Conditional Heteroskedastic* (ARCH) model used data from previous time stamp to model the next day volatility. They successfully used ARCH to explore variance of UK inflation between year 1958-1977. A few years later, ARCH process was enhanced by Bollerslev (1986) into *Generalized Autoregressive Conditional Heteroskedastic* (GARCH) model by adding past *conditional variances* into autoregressive equation. By modelling variance with ARMA process, GARCH model is able to capture instant shock better than ARCH, while keeping longer memory of variance. Since the introduction of GARCH, many scientists and investors have been using it really actively to model the second moments of a broad scope of time series data. Even though GARCH is primarily used in finance due to its properties, other fields also tried to employ it, for example, look at Campbell & Diebold (2005) for meteorology or Galka et al. (2004) for physics and chemistry applications.

Various researchers continuously work towards improving GARCH model by introducing a large variety of add-ons or modifications. Useful transformation war published by Engle & Bollerslev (1986) who connected GARCH model with *Exponentially Weighted Moving Average* (EWMA) model to form *Integrated* GARCH (IGARCH). IGARCH exhibits numerous characteristic which outperforms many other models. Namely, IGARCH needs to estimate one parameter less than a classical GARCH, hence it should be easier to estimate. Additionally, the specification does not allow existence of *unconditional variance*, therefore the long-run forecast does not converges to the unconditional variance. Next interesting improvement was introduction of *Exponential* GARCH (EGARCH) by Nelson (1991), in which they used logarithmic transformation. Another one of the huge improvement were published by Glosten et al. (1993). They incorporated ideas of prospect theory earlier presented by Tversky & Kahneman (1979) to develop *Glosten-Jagannathan-Runkle* GARCH (GJR-GARCH). Their idea of asymmetry of returns enhanced the results of model drastically.

Recent advancements in modelling volatility are slightly turning back from the GARCH approach. Even though GARCH is a truly powerful weapon, some researchers argues that there exist better models or procedures. For example, Starica (2003) shows that GARCH(1,1) is not generating process for returns of major global indices. Mainly, they deny that GARCH(1,1) could be any useful in long-term forecasting. Hansen & Lunde (2005) compared over 300 different ARCH-type models to study their out-of-sample forecasting performance. Nevertheless, they did not manage to show that GARCH(1,1) can be beaten by more complicated models, they still claim that it performs worse than models which are able to integrate *leverage effect*[1]. Despite the fact that Andersen & Bollerslev (1998) and many other researchers defended GARCH processes, new alternative approaches were (and still are) being published to these days.

We are continuously improving our mathematical knowledge and technology in general, henceforth researchers progress further better variance estimation. High frequency data could be considered as the huge jump in terms of understanding risk and volatility. One of the first successful takes on volatility in high frequency data were published by Roll (1992). They inspected what portion of volatility is represented in *bid-ask spread* and introduced formula later known as Roll's model. Andersen et al. (2003) and Barndorff-Nielsen & Shephard (2002) worked on their groundbreaking concept of *realised variance* (RVar) resp. *realised volatility* (RVol). Realised variance for a given day can be described as a sum of squared intra-day changes in that day. Simultaneously Andersen et al. (2000) merged non-parametric approach to volatility with family of GARCH models to analyse dollar exchange rates. Beside that, Bollerslev (2001) publicised impressive essay on past and future of financial econometrics in which they stated that models allowing time-varying second moments will dominate the field.

Even with these advanced techniques, modelling variance is plausible and doable, but for multivariate analysis it opens up few shortcomings. Firstly, we need to mathematically derive the formulas and related properties, which tends to be demanding and exhausting. Secondly, the real problem is the

[1]Leverage effect is well documented phenomenon which says that positive and negative shocks tend to have different impacts on volatility. Existence of leverage effect has been empirically studied many times and it is part of various types of models. In case of GARCH family models, there are interesting adjustment to incorporate leverage effect, for example GJR-GARCH or TAR-GARCH models.

estimation of parameters. Estimation techniques for a multivariate model are usually heavily complicated, both for computing power as well as for researchers to even derive it. For example, *Maximum Likelihood Estimator* does not always exist, therefore we need to substitute with a different (often very specific) approach.

Having the variances not always tells the full story about stock market behaviour, therefore researchers try to expand GARCH framework into more dimensions to model also correlations between assets. There were many different techniques and ideas to create useful Multivariate GARCH model. According to Maďar (2012), three major extensions of GARCH models for multivariate cases exist: generalisation, linear combinations and non-linear combinations. In this thesis, we will focus primarily on the last option. To see details on other categories, look at Maďar (2012). The most common approaches are *Constant Conditional Correlation* GARCH (CCC GARCH) published by Bollerslev (1990) and *Dynamic Conditional Correlation* GARCH (DCC GARCH) presented by Engle (2002). The initial step was made by Bollerslev (1990) who published their expansion of classical GARCH to a multivariate case which allows time-varying conditional covariances while keeping conditional correlations fixed. At the same time, Tse & Tsui (2002) and Engle (2002) published their propositions of DCC GARCH. They show that DCC GARCH can be relatively easily estimated, it keeps the simplicity of interpretation as classical GARCH and captures the second moment better than CCC GARCH.

Having said that, the big problem of multivariate cases lasts. The number of parameters to be estimated growth rapidly with a increasing number of dimension. This problem is known as the *curse of (multi)dimensionality*. If we consider portfolio with 150 unique positions (which is very humble these days), the number of parameters to estimate will be 11 175[2], depending on model specification. This can be overcome by various ways. One of them might be *Heterogeneous Auto-Regressive* (HAR) model with Realised

[2]The exact number of parameters is closely tied with specification of model, but the variance - covariance matrix has n different variances and $\frac{n(n-1)}{2}$ covariances to be estimated.

Variance proposed by Corsi (2009). Unfortunately, it needs high-frequency data which are not accessible very often. Another solution could be to (again) compromise the data, which we will use and explain further in the thesis.

2.3 Northern Financial Markets Specification

Since we will analyse data from a very specific geographical region, it is important to mention some features. All countries (maybe with exception of Baltic Countries, which are represented together in this thesis) are very culturally and historically connected. They are economically developed with high scores on Human Development Index (HDI) which accounts for colourful range of indicators like life expectancy, education level or standards of living[3]. Moreover, as the driving force in their economies could be considered Energy sector. All the countries have developed economies with a low level of unemployment and high GDP per capita. All of the countries are part of European union, OECD and EEA. We could go further in explaining similarities between them, but in conclusion, the countries are really comparable and related.

Nielsson (2007) argues that Baltic and Nordic countries[4] are not connected and there is a very little evidence that their financial markets are converging. This may imply that co-operation between Nordic and Baltic countries is very limited. Brunzell et al. (2014) inspected dividend policy in Northern Europe and found very specific approach to payout ratio. Lekvall et al. (2014) deeply studied and explained truly specific governance model closely aligned to the Nordic culture. Booth et al. (1997) employed EGARCH model to examine volatility shocks spillovers between Scandinavian countries. Surprisingly, they found very few spillovers for bad news and almost none for good news. Malkamäki (1993) studied the influence of global

[3]Details about the exact methodology and full scores can be found in Human Development Reports published annually United Nations Development Programme

[4]The term Nordic countries refers to Scandinavian countries such as Norway, Finland and Sweden together with Denmark and Iceland, while Baltic countries are post-soviet state, namely Estonia, Latvia and Lithuania.

market on Nordic countries. They find that Sweden financial market is the driving force in the North, but the global market has a much more significant impact on the whole region. They attributed this effect to the growth of globalisation and free movement of capital.

3 Methodology

3.1 Idea of GARCH process

Engle (1982) introduced their idea of volatility process allowing for time varying conditional variance known as ARCH model. ARCH model variance as an autoregressive (AR) process of past errors and assume that mean corrected returns are serially uncorrelated, but dependent[5]. Since its first publishing, ARCH-type models rise rapidly on popularity among researchers (as presented in literature review section), mainly for purpose of modelling volatility of financial time series.

Having said that, the original ARCH model exhibits some noticeable shortcomings. First of all, ARCH does not take into account an *leverage effect* - a well documented phenomenon of financial series, which describe different impact of positive and negative shocks on volatility. This effect has been empirically tested many times on various time-series financial data, for further details, see for example Bouchaud et al. (2001). Secondly, ARCH model responds really slowly to volatility shock. The structure of ARCH model does not allow volatility to capture its sudden shocks. Since the volatility often appears in clusters, this might lead to fail of the model. For more details, see Lux & Marchesi (2000). Finally, we usually need to employ high order of ARCH model to somewhat capture the real behaviour of volatility of a given time series. Estimating few dozens of parameters produces various problems, for example it requires huge computational power and models with that many parameters often performs worse in case of forecasting.

The last two of mentioned drawbacks were challenged by Bollerslev (1986) with introduction of GARCH model. They adopt *Autoregressive Moving Average* (ARMA) process instead of AR to model conditional variance, which in fact substitutes ARCH of infinite order by GARCH model of order one. This helps to capture the same effect with just few parameters contrary to infinity. Moreover, adding AR component into model catches *volatility clustering* far better than original ARCH while estimating just few parameters.

[5] There exists any relationship, which is not linear, therefore it is not capture in correlation.

In the methodology segment, we closely follow approach of Orskaug (2009) and Maďar (2012).

3.2 Univariate GARCH

The univariate GARCH(p, q) process is defined as

$$r_t = \mu_t + a_t \tag{2}$$

$$a_t = h_t^{1/2} \epsilon_t \tag{3}$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i} \tag{4}$$

where r_t is logarithmic daily return of a given time series at time t, $\mu_t = \mathbb{E}(r_t|\mathcal{F}_{t-1})$ ($\mathcal{F}_{t-1}$ is a set of all information about the time series $\{r_t\}$ at the time $t-1$). In the second equation, a_t is a mean corrected return of a time series at the time t, h_t is the conditional variance of a given time series (conditioned on the past) and $\{\epsilon_t\}$ is a series of *independent and identically distributed* (iid) random variable from a *stable distribution*[6], in this case we can assume that $\epsilon_t \sim N(0,1)$. Additionally, $\alpha_0, \ldots \alpha_p, \beta_1, \ldots \beta_q$ are non-negative parameters of the model and p, q are orders of the equation (4), which is basically an ARMA-like structure of variance. .

By the deconstruction of the equation (4), we can extract various concept captured by the model. Since conditional variance is dependent on its past values, it is able to capture clusters in volatility really well. If there is a big positive shock to variance at the time $t-1$, this impact will transmit into the value of h_t at the time t. The same logic also applies for all other autoregressive terms in equation (4) up to the order q. Having said that, equation (3) does not necessarily implies that higher h_t leads to higher a_t due to the random effect of ϵ_t. Taking into account that $\{a_t\}$ is dependent, yet serially uncorrelated, implies that relationship is not linear. Moreover note that model uses squared returns to model conditional variance which

[6]Stable distribution is any probability distribution that a linear combination of two random variables with this distribution has same distribution (with re-scaled parameters)

limits us to distinguish between positive and negative returns. Generally, to use GARCH to catch the leverage effect, important adjustments have to be made[7].

It is important to mention some important technical properties of the default GARCH model. Unconditional mean of GARCH(p, q) process is defined as

$$\mathbb{E}(a_t) = 0 \tag{5}$$

which can be obtained directly from definition of expected value

$$\mathbb{E}(a_t) = \mathbb{E}[\mathbb{E}(a_t|\mathcal{F}_{t-1})] = \mathbb{E}[\mathbb{E}(h_t^{1/2}\epsilon_t|\mathcal{F}_{t-1})] \tag{6}$$

knowing that h_t and ϵ_t are independent, we can write

$$\mathbb{E}(a_t) = \mathbb{E}(h_t^{1/2}\mathbb{E}(\epsilon_t|\mathcal{F}_{t-1})) = \mathbb{E}(0h_t^{1/2}) = 0. \tag{7}$$

Equivalently, we can define unconditional variance of GARCH(p, q) process as

$$\sigma^2 = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \alpha_i - \sum_{i=1}^{q} \beta_i} \tag{8}$$

given that

$$\sum_{i=1}^{p} \alpha_i + \sum_{i=1}^{q} \beta_i < 1 \tag{9}$$

and it is derived in the same way as unconditional mean, directly from definition of variance, hence we can write

$$\sigma^2 = Var(a_t) = \mathbb{E}(a_t^2) - \mathbb{E}^2(a_t) \tag{10}$$

which we can simplify to

$$\sigma^2 = \mathbb{E}(a_t^2) = \mathbb{E}(h_t\epsilon_t^2|\mathcal{F}_{t-1}) = \mathbb{E}(h_t) \tag{11}$$

[7]We describe these models earlier in thesis with GJR-GARCH and TAR-GARCH.

where we can plug equation (4) to get

$$\sigma^2 = \mathbb{E}(\alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i}) = \alpha_0 + \sum_{i=1}^{p} \alpha_i \mathbb{E}(a_{t-i}^2) + \sum_{i=1}^{q} \beta_i \mathbb{E}(h_{t-i}). \tag{12}$$

Since we know that expectation of h_t is unconditional variance and that $\{a_t\}$ has constant variance, we can substitute and solve for σ^2 to obtain

$$\sigma^2 = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \alpha_i - \sum_{i=1}^{q} \beta_i}. \tag{13}$$

Estimation of GARCH process faces many challenges and is usually perform by *Maximum Likelihood Estimator* (MLE). The idea behind MLE is to maximise likelihood function to find most probable model from which observed data were generated. To use MLE, we have to assume that a_t conditionally follows some stable distribution. For simplicity, let's assume that mean-corrected returns are normally distributed, explicitly

$$r_t \sim N(\mu, \sigma_t^2) \implies a_t \sim N(0, \sigma_t^2). \tag{14}$$

To use MLE for GARCH, we have to consider the joint distribution $f(a_1, \ldots, a_t, \gamma)$, where γ is a vector of parameters in a given model. In our specific case of GARCH(p, q), we can write $\gamma = (\alpha_0, \ldots, \alpha_p, \beta_1, \ldots, \beta_q)$. Then the log-likelihood function of a GARCH(p,q) process is defined as

$$\mathcal{L}_T(\gamma) = \sum_{t=1}^{T} \big(-\frac{1}{2}\big)\big[\log(2\pi) + \log(\sigma_t^2) + \frac{a_t^2}{\sigma_t^2}\big] \tag{15}$$

where $a_t \sim N(0, \sigma_t^2)$. The core idea behind derivation of maximum log-likelihood function of GARCH(p, q) process is that joint distribution is the product of the marginal and conditional densities, henceforth we can write

$$f(a_1, \ldots, a_t, \gamma) = f(a_0, \gamma) f(a_1, \ldots, a_t | a_0, \gamma) = f(a_0, \gamma) \prod_{t=1}^{T} f(a_t | a_{t-1}, \gamma) \tag{16}$$

where we can substitute density function of a_t to obtain

$$f(a_1, \ldots, a_t, \gamma) = f(a_0, \gamma) \prod_{t=1}^{T} \frac{\exp(-\frac{a_t^2}{2\sigma_t^2})}{\sigma_t \sqrt{2\pi}} \tag{17}$$

finally, we can drop out $f(a_0, \gamma)$ and take logarithms to get

$$\mathcal{L}_T(\gamma) = \sum_{t=1}^{T} \big(-\frac{1}{2}\big)\big[\log(2\pi) + \log(\sigma_t^2) + \frac{a_t^2}{\sigma_t^2}\big] \tag{18}$$

The assumption of normal distribution might seems a bit strict. However, we could use any other stable distribution, which would yield similar result. To choose normal distribution seems safe, because of *law of large numbers* (LLN). Researchers explored asymptotic features of GARCH-type models, for further details see for example Lee & Hansen (1994).

3.3 Multivariate GARCH

In the last decades, investors and researchers are getting more and more interested in connections and integrity among markets. The rise of globalisation lead to new interconnection between various markets, indices, securities and commodities. To understand real behaviour of financial markets, it became important to include mutual relationship between wide variety of financial time series. Correlation has grown into one of the fundamental values and indicators for finance professionals. Nowdays, portfolio asset pricing is heavily dependent on covariance. Extensions to multivariate models can bring new information of connections between different time series, which helps us to understand the real behaviour of financial markets. These techniques are crucial for the right decision making in portfolio management, risk management and other related areas.

According to Maďar (2012), there exists three main approaches to extend GARCH into multivariate framework. First way is to generalise classical univariate GARCH, which leads to VECH and BEKK models. Second approach is to use linear combination of univariate GARCH processes to create o-GARCH-type models. Finally, the model we use in this thesis is the approach of non-linear combination of univariate GARCH models.

One of the first successful takes on non-linear combinations of univariate GARCH models were Bollerslev (1990) and their *Constant Conditional Correlation* GARCH (CCC-GARCH). The idea behind CCC-GARCH is to choose univariate GARCH model for each time series to model conditional variances, then estimate time-invariant positive definite conditional correlation matrix. Finally, we can get conditional covariances as rescaled product of corresponding conditional variances. Formally, CCC-GARCH(p, q) can be define as

$$r_t = \mu_t + a_t \tag{19}$$

$$a_t = H_t^{1/2} z_t \tag{20}$$

$$H_t = D_t R D_t \tag{21}$$

where r_t is a $n \times 1$ vector of returns of n times series at the time t, μ_t $n \times 1$ is a vector of expected returns of n times series at the time t and a_t is a $n \times 1$ vector of mean-corrected returns of n times series at the time t. Therefore, the first equation (19) is just a simple extension of equation (2) into multidimensional form. In the equation (20), H_t is a $n \times n$ conditional covariance matrix of n assets at the time t. The problem with obtaining $H_t^{1/2}$ can be solved by Cholesky decomposition. z_t is a vector of iid random variables from a stable distribution, such as $\mathbb{E}(Z_t) = 0$ and $\mathbb{E}(z_t z_t^T) = I$. Finally, D_t is a diagonal matrix of conditional standard deviations and R is a $n \times n$ constant conditional correlation matrix.

In the original Bollerslev's CCC-GARCH model, the elements of matrix H_t are defined as

$$h_{t,ij} = \rho_{ij}(h_{t,ii} h_{t,jj})^{1/2} \tag{22}$$

where $h_{t,ii}$ is defined as univarite GARCH model, such as

$$h_{t,ii} = c + \sum_{k=1}^{p} A_i a_{t-k}^2 + \sum_{l=1}^{q} B_i h_{t-l} \tag{23}$$

in which c is a $n \times 1$ vector of constant parameters and A_i, B_i are n-dimensional diagonal matrices. Additionally, a_t^2 is defined as Hadamard's element-wise product of a_t vectors, noted as $a_t^2 = a_t \odot a_t$.

CCC-GARCH offered a charming and beautiful way to extend GARCH-type models into multivariate cases due its simplicity both computational as well as interpretation. However, these advantages are outweighed by constrains put on the model. CCC-GARCH assumes that the correlation matrix R is constant over time, which is empirically not achievable. These unrealistic premises lead to the new type of non-linear GARCH models that allows time-varying correlations generally called *dynamic conditional correlation* GARCH (DCC-GARCH). Definition of DCC-GARCH is very similar to CCC-GARCH specification, but correlation matrix R_t is allowed to vary over time. Therefore, this can be stated as

$$r_t = \mu_t + a_t \tag{24}$$

$$a_t = H_t^{1/2} z_t \tag{25}$$

$$H_t = D_t R_t D_t \tag{26}$$

where the only difference from CCC-GARCH is R_t, which is a conditional correlation matrix at the time t. In this thesis, we assume normality of error terms, therefore we can write

$$z_t = D_t^{-1} a_t \sim N(0, R_t) \tag{27}$$

In a DCC-GARCH model, elements of diagonal matrix of standard deviations D_t are defined as $\sqrt{h_{t,ii}}$, where $h_{t,ii}$ is some univariate GARCH model, for example

$$h_{t,ii} = c_i + \sum_{k=1}^{p} A_i a_{t-k}^2 + \sum_{l=1}^{q} B_i h_{t-l} \tag{28}$$

Since this is a classical univariate GARCH model, it has to satisfy all conditions mentioned earlier in this thesis, such as stability condition or non-negativity of parameters.

To ensure that the matrix R_t is properly define, it has to meet two requirements. As we stated earlier, correlation can take only values from -1

to 1, therefore each element of R_t has to meet this condition for every t. Since H_t is a covariance matrix, it has to be positive definite, therefore R_t also has to be positive definite (D_t is positive definite by definition). In the current settings, securing these restrictions to hold is not an easy task, we can break down R_t into

$$R_t = Q_t^{*-1} Q_t Q_t^{*-1} \tag{29}$$

where Q_t is an unconditional covariance matrix of error terms, which can be estimated as

$$Q_t = (1 - k - l)\bar{Q} + k z_{t-1} z'_{t-1} + l Q_{t-1} \tag{30}$$

$$\bar{Q} = \frac{1}{T} \sum_{t=1}^{T} z_t z'_t \tag{31}$$

in which k, l are non-negative scalars satisfying $k + l < 1$ and Q_t^* is a diagonal matrix with elements such as $q^*_{t,ii} = \sqrt{q_{t,ii}}$.

Defining DCC-GARCH model this way seems quite straightforward, the real challenge lies in its estimation. To directly use log-likelihood function for estimation is rather challenging, but Engle (2002) has proposed a DCC-GARCH model in such a way it can be estimated by an elegant two-step procedure. In the first case we will demonstrate in this thesis we assume that z_t is a vector of errors from a standardised joint normal distribution given by following density function

$$f(z_t) = \prod_{t=1}^{T} \frac{\exp\left(-\frac{1}{2} z_t^T z_t\right)}{\sqrt{(2\pi)^n}} \tag{32}$$

then the likelihood function can be stated as

$$L_T(\gamma) = \prod_{t=1}^{T} \frac{\exp\left(-\frac{1}{2} a_t^T H_t^{-1} a_t\right)}{\sqrt{(2\pi)^n} |H_t|^{1/2}} \tag{33}$$

from which we can take logarithm to obtain log likelihood function in the following form

$$\mathcal{L}_T(\gamma) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log(|H_t|) + a_t^T H_t^{-1} a_t\right) \tag{34}$$

that can be decomposed into

$$\mathcal{L}_T(\gamma) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t\right) \tag{35}$$

To estimate this very log likelihood function is not easy, so we will use the two-step solution introduced by Engle (2002). The main idea is to divide parameter vector γ into two parts - parameters for each univariate models for each time series and parameters of the decomposition of R_t. We can denote it as $\gamma = (\phi, \omega)$ where $\phi = (\phi_1, \ldots, \phi_n) = (\alpha_{01}, \ldots, \alpha_{0n}, \alpha_{11}, \ldots, \alpha_{pn}, \beta_{11}, \ldots, \beta_{qn})$ and $\omega = (k, l)$. In the first step, the correlation matrix R_t is substituted with identity matrix I, which drops the parameter vector ω and estimate only parameter vector ϕ. This idea results in following likelihood function

$$\mathcal{L}_T(\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|D_t|) + \log(|I|) + a_t^T D_t^{-1} I^{-1} D_t^{-1} a_t\right) \tag{36}$$

which can be expressed as

$$\mathcal{L}_T(\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left[n\log(2\pi) + \sum_{i=1}^{n}\left(\log(h_{t,ii}) + \frac{a_{t,i}^2}{h_{t,ii}}\right)\right] \tag{37}$$

where we can take out the sum to obtain another interesting view on DCC-GARCH model

$$\mathcal{L}_T(\phi) = -\frac{1}{2}\sum_{i=1}^{n}\left[T\log(2\pi) + \sum_{t=1}^{T}\left(\log(h_{t,ii}) + \frac{a_{t,i}^2}{h_{t,ii}}\right)\right] \tag{38}$$

The last equation provides new point of view on estimation of DCC-GARCH model. The log likelihood function for DCC-GARCH is just a sum of log-likelihood function for each univariate GARCH model in DCC-GARCH structure for n different time series. Therefore, the first step of this procedure estimates parameter vector ϕ and sequentially unconditional variances $h_{t,ii}$, z_t and $\bar{Q}$ from equations (27) and (31) respectively.

The second step is to estimate parameter vector ω given already estimated parameter vector ϕ in the first step, henceforth the log likelihood function for the second phase can be stated as

$$\mathcal{L}_T(\omega|\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log(|D_t|) + \log(|R_t|) + a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t\right) \tag{39}$$

To maximise log likelihood function, we can drop any constant terms, since they do not affect the final results. Moreover, we can substitute equation (27). Knowing that D_t is given by the first step, we can consider it as constant, therefore the simplified form of log likelihood function for second step is following

$$\mathcal{L}_T(\omega|\phi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log(|R_t|) + z_t^T R_t^{-1} z_t\right) \tag{40}$$

This estimation procedure yields consistent and unbiased results under very reasonable and realistic conditions. The specific details are discussed in Engle (2002). The basic idea is that the consistency of the first step implies consistency of the second step. At the beginning of estimation process, we strictly assumed joint standardised normal distribution. However, this presumption does not necessary reflect real-world financial data. To better capture reality, it seems reasonable to consider different stable distribution. The best candidate could be joint Student's t-distribution, most commonly used in real-world applications, given by the density function

$$f(z_t|\nu) = \prod_{t=1}^{T}\frac{\Gamma(\frac{\nu+n}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi^n(\nu-2)^n}}\left(1 + \frac{z_t^T z_t}{\nu-2}\right)^{-\frac{1}{2}(n+\nu)} \tag{41}$$

in which $\Gamma(\cdot)$ denotes Gamma function. For t-distributed error terms, we will follow identical techniques and step as for jointly normally distributed errors, hence we provide very brief overview of the process. The full details so the log likelihood function takes form

$$\mathcal{L}(\gamma) = \sum_{t=1}^{T} \Big[\log\Big(\Gamma(\frac{\nu+n}{2})\Big) - \log\Big(\Gamma(\frac{\nu}{2})\Big) - \frac{n\log\big(\pi(\nu-2)\big)}{2} - \frac{\log\big(|D_t R_t D_t|\big)}{2} - \frac{(\nu+n)\log(1+\frac{a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t}{\nu-2})}{2} \Big] \tag{42}$$

As for previous distribution, this log likelihood function can be maximised by two step procedure proposed by Engle (2002). The first step is very same as in the first case, hence let's decompose only the second step. The log likelihood function to estimate vector $\omega = (k, l, \nu)$ can be stated as

$$\mathcal{L}(\omega|\phi) = \sum_{t=1}^{T} \Big[\log\Big(\Gamma(\frac{\nu+n}{2})\Big) - \log\Big(\Gamma(\frac{\nu}{2})\Big) - \frac{n\log\big(\pi(\nu-2)\big)}{2} - \frac{\log\big(|D_t R_t D_t|\big)}{2} - \frac{(\nu+n)\log(1+\frac{a_t^T D_t^{-1} R_t^{-1} D_t^{-1} a_t}{\nu-2})}{2} \Big] \tag{43}$$

from which we can also drop all constant terms including D_t to obtain simple form that can be maximise much easier

$$\mathcal{L}(\omega|\phi) = \sum_{t=1}^{T} \Big[\log\Big(\Gamma(\frac{\nu+n}{2})\Big) - \log\Big(\Gamma(\frac{\nu}{2})\Big) - \frac{n\log\big(\pi(\nu-2)\big)}{2} - \frac{\log\big(|R_t|\big)}{2} - \frac{(\nu+n)\log(1+\frac{z_t^T R_t^{-1} z_t}{\nu-2})}{2} \Big] \tag{44}$$

Even though that two-step procedure makes estimation of DCC-GARCH models much smoothly, it still carries some important drawbacks. Caporin & McAleer (2013) presented useful overview of shortcomings connected to the modelling of dynamic conditional correlation. Firstly, the estimation process itself might be problematic. While estimating too many parameters or choosing the wrong initial values, optimisation might not manage to reach the global maximum. The next problem with DCC-GARCH framework is that the model is told and established on empirical observations rather than derived. This brings out many statistical properties which are not defined for DCC-GARCH models. These missing properties does not allow for testing, exploring higher moments of conditional correlations and requirements for consistency or efficiency remain mostly unknown or too vague and general. It is important to bear in mind all shortcomings of DCC-GARCH framework while working with them to avoid any crucial mistakes.

3.4 Model Validation

As we stated earlier, DCC-GARCH models are not well suited for validation and testing, therefore we will validate our model mainly based on analysis of standardised residuals. We would like to capture all significant effects in time variation of correlation, henceforth standardised residuals should exhibit certain properties that we can observe or even be able to test them.

The first technique which could help us validate our models is a *quantile-quantile plot* (QQ plot). QQ plot is a graphical way to compare two shapes of distributions by plotting each quantile of one distribution against correspondings quantile of another distribution. In our case, we plot distribution of residuals on x-axis and assumed distribution on y-axis. Since the normality assumption is often too strict and unrealistic, we will use student's t-distribution. In each graph, we are supposed to see matching distribution which is representing by approximately straight line up to some statistical discrepancies.

The second technique is also a graphical way to evaluate standardised residuals of DCC-GARCH model and it is known as auto-correlation function (ACF). ACF is defined as correlation between a random variable and its lagged values up to the lag t. In this thesis, we will examine residuals and its lagged values. Mathematically, auto-correlation function for the lag k can be stated as

$$\rho_{\epsilon_t,\epsilon_{t-k}} = \frac{\sigma_{\epsilon_t,\epsilon_{t-k}}}{\sigma_{\epsilon_t}\sigma_{\epsilon_{t-k}}} \tag{45}$$

which is just a form of equation (1). Auto-correlation function is a very useful tool in so called *Box-Jenkins method* presented by Box & Jenkins

(1970)[8]. Generally, it helps to determine orders of ARMA (or any other forms of ARMA) models. In case of the residuals, we expect to observe only a white noise, therefore no statistically significant correlation between residuals and its lagged values. This would imply that residuals are just a random disturbances and there is no other significant relationship that we could model and we can consider the estimated one as a good model for that time-series.

Finally, we need some more robust way to investigate residuals. We will use widely used Ljung-Box test for auto-correlation to find if there is any serial correlation in residuals. If there is no auto-correlation left in residuals, the model explained the relationships in data. This test was proposed by Ljung & Box (1978) and the test statistics is defined as

$$Q = n(n+2)\sum_{i=1}^{m}\frac{\rho_i^2}{n-k} \tag{46}$$

where n is a sample size, m is a number of lags tested and ρ is a sample auto-correlation at lag k. We can reject the null hypothesis on α level of significance when

$$Q > \chi_{1-\alpha,m} \tag{47}$$

and the null hypothesis is that there is no significant correlation. Alternative hypothesis is that there exists some significant serial correlation in disturbances.

The framework of dynamic correlation extension to GARCH models is quite new approach and even thought that it is widely used, this features do not allow for any robust statistical testing like univariate GARCH models. Nevertheless there exist few concepts that try to test for goodness of fit of given DCC-GARCH model, they are still in very experimental stage and none of them proved to be useful in a long-run. They are usually based on VaR applications and forecasting ability and are far beyond the scope

[8]Box-Jenkins method is a profound approach to build the best model for time series analysis. The method consists three main stages - identification, estimation and diagnostic.

of this thesis. For some details on testing of multivariate GARCH model, see for example Baringhaus & Franz (2004). This lack of statistical-based tools left much work on deep analysis of residuals and overall properties and characteristics of each model to choose the very best model.

4 Data and Hypothesis Overview

4.1 Data Description

We have daily closing price[9] of various indices on Copenhagen Stock Exchange (CSE), Stockholm Stock Exchange (SSE), Helsinki Stock Exchange (HSE) and on NASDAQ Baltic, which operates three stock exchanges in Tallinn, Riga and Vilnius. All of the stock exchanges operate under NASDAQ Nordic, therefore the methodology for both sector (or any other) classification as well as for data published is united and standardised. We will focus primarily on banking, construction & materials, food & beverages, healthcare, industrials and personal & household goods sector indices. Even though NASDAQ Nordic provides more sector indices, we will not focus on them in this thesis. To choose these sectors is a difficult task to do it right, but it has to be executed to tackle curse of dimensionality.

Our choice is based on several criteria. We need only sector indices that exist for the extended period of time to have a sufficient amount of data. Additionally, during its existence, it have to contain at least some securities to have any useful data on changes of prices. The next assumption is to have same sector indices for all considered countries to keep the logic of interpretation. In conclusion, we considered only sector indices which contains higher amount of securities to maintain a variety of logarithmic returns for whole period (which is proved by graphs in the Appendix B, that there are no windows of zero returns that some originally considered indices demonstrated) and fulfil this condition for each country examined.

We provide table with summary statistics including Jarque - Bera test for normality and Ljung - Box test for autocorrelation of logarithmic returns. We use the standard transformation of a daily close price to compute daily logarithmic returns. The transformation can be written as

$$r_t = \log\left(\frac{P_t - P_{t-1}}{P_{t-1}}\right) = \log P_t - \log P_{t-1} \tag{48}$$

[9]According to NASDAQ, the closing price is defined as the last transaction of a given asset completed during trading day on given exchange.

where P_t denotes the closing price of given asset at the time t and r_t is the daily return of given asset at the time t. We have prices from the January 02, 2008 to the May 10, 2019. Therefore, our daily returns can be computed for dates between the January 03, 2008 and the May 10, 2019. We provide plots of all time-series in the Appendix B. We can read important information about the returns just from the presented graphs. At the first sight, we can tell that volatility really varies over time, therefore we can also expect that covariance and correlation also changes in time. Moreover, we can also see that volatility occurs in clusters. Especially, there are huge clusters of high volatility in also every time series during 2008 and 2009 global financial crisis. Even though these observations are interesting, they are not very helpful in terms of stating any assumptions about mutual covariances or correlations. Therefore, we have to state our expectations about multivariate second moments clearly from previous studies in this topic, which we discussed earlier in the previous sections.

4.2 Hypothesis

The latest studies tend to claim that globalisation and connection of countries in various ways like free movement of capital, goods and services across borders, monetary unions or any other economical or even political deal or agreement leads to the rise of correlation between countries. This leads to that the comovements between sectors or industries lean to be the crucial factor for modern portfolio diversification. Moreover, we are interested in similar countries with similar economies (and other common aspects), therefore we can expect high correlations within same sector among countries with overall increasing trend. As opposed to the same industry, we anticipate to find lower correlations among sectors within one country.

Another important factor in the date we are going to examine is the financial crisis in the beginning of the time window. Graphs of logarithmic returns in Appendix B hints that volatility clusters during this crisis differ for various sectors. This effect varies across countries, but overall we can state that food & beverages, construction & materials and personal & household goods indices exhibit much more persistence in volatility. Based on this simple observation, we can assume to find higher correlation between these three sector than among the three others.

However, one could argue that during the crisis, some sectors are more vulnerable than others and we can expect that banks, industrials and constructions & materials behave alike during some economic (financial or any other) crisis or depressions, since their operations can be described as some kind of "luxury" due to its dependence on overall economic growth. Contradictory, the food & beverages, healthcare and personal & household goods might be more stable in terms of variance and are more immune to crisis. If this idea is right, then we could expect higher correlation within those two groups of sectors.

Finally, we can extract some relevant information to enhance our expectations about the empirical results from decomposition of indices. When we take a look at the securities that each index contains, we can find that some of them share a few securities. For example, construction & material and industrials consists some same stocks, as well as food & beverages and personal & household goods in some countries. This inspection would also imply that we are suppose to see higher correlation between these indices.

5 Empirical results discussion

Estimation of multivariate models is often demanding and has to be done right to yield useful results. As we stated earlier in previous sections, there exist a huge problem called curse of dimensionality. Even though we tried to minimise our data by using sector indices to substitute many time series with very few and by wise selection of those indices, we still have 6 sectors for 4 different regions, which is 24 different time series. Unfortunately, this number is still too high for DCC-GARCH models and we are not able to estimate such a model. Therefore, we have to find an another way of modelling correlations using DCC-GARCH family of models.

Knowing that, we propose a solution based on dividing all the time series into several groups. However, we have to execute the selection of these smaller groups correctly to maintain the interpretation effective and practical. Therefore, we decided to introduce 10 different groups, 6 for each sector across countries and 4 for each region across sectors. This method seems really promising, because the estimation of these models should be feasible and the interpretation of these models keeps its simplicity and respect the structure of our data to remain useful for further purposes. Our proposed scheme allows us to answer questions about correlations between a given sector across countries and correlations between a given country across sectors. With this simple approach, we are able to examine the correlation structure as for the whole financial market.

Since we have to estimate 10 different DCC-GARCH models, investigate each of them in detail and evaluate it in various ways, we describe the full process only for the banking sector and for Baltic countries. For the remaining groups of indices, we discuss mainly the results without detail on the whole estimation process, testing and calibrating the models. Overall, we prefer the simplest model, henceforth we favour ARMA$(1, 1)$[10] for estimating vector μ_t, GARCH$(1, 1)$ for modelling univariate volatilities for D_t and DCC-GARCH$(1, 1)$, both with Student's t-distribution[11]. Any model of higher

[10]The ARMA model is specified as $\mu_t = m + \gamma\mu_{t-1} + \delta\eta_{t-1} + \eta_t$

[11]We will refer to this specification as a *default model* later in the thesis.

order will be implemented if and only if various validation techniques fails, because the number of parameters growths rapidly and such a complicated models are useless for forecasting purposes and hard to interpret. Even though these specifications might not be the best model compared by a classical statistical tools like information criteria, it might still be useful for interpretation and forecasting.

5.1 Country Effect for Sectors

We start with banking sectors across regions. First of all, we perform a test for constant correlation by exploring the standardised residuals of CCC-GARCH model. The null hypothesis for this test states that the standardised residuals are jointly iid with variance of identity matrix I. If we reject the null hypothesis, then the correlation is not considered as constant and varies over time, so we have to capture that dynamic, in our case with DCC-GARCH-type model. For the matrix of logarithmic returns, the p - value of this test is significantly lower than 0.01[12], therefore we can safely reject the null hypothesis of a constant correlations and we can estimate DCC-GARCH models.

[12] 7.661032e-05

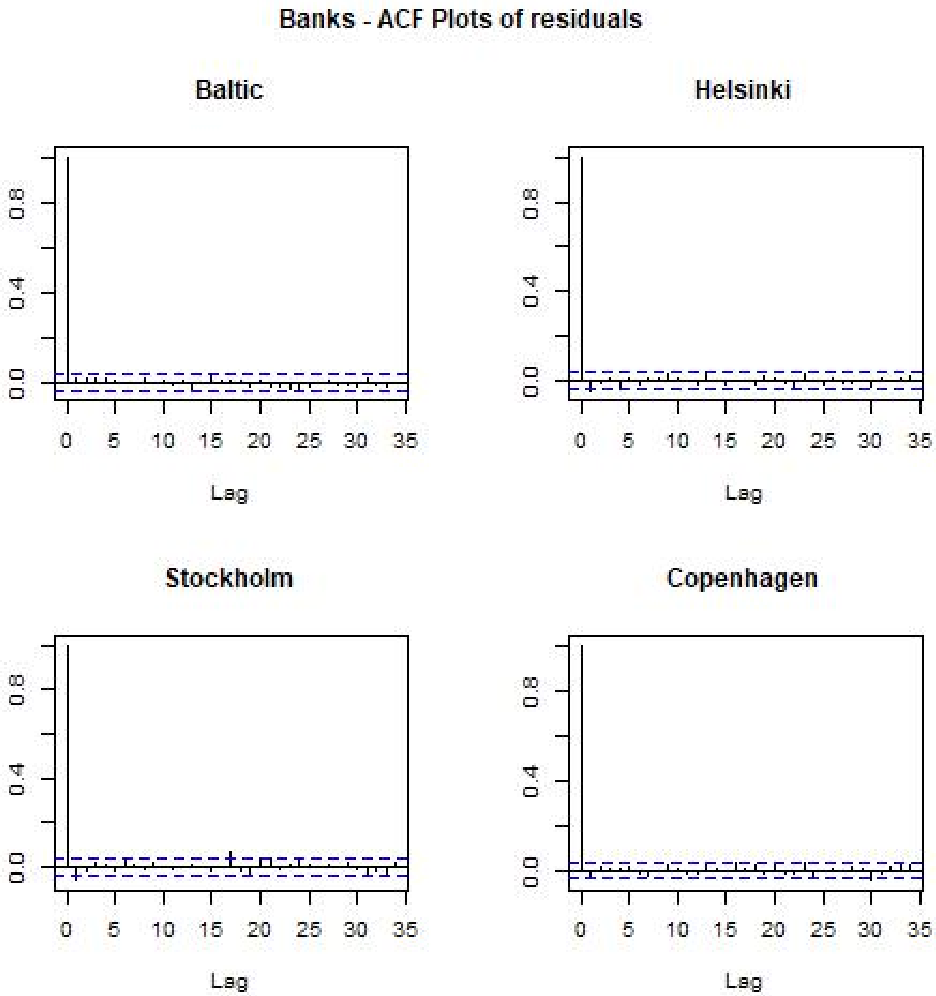

Figure 1: ACF plots of residuals from the estimated DCC-GARCH model of returns from banking indices

As we stated above, we start with our default specification. Estimated parameters with corresponding standard deviations are in tables 1 and 2. In the first table, one can find the estimates for univariate models specified in the equation (28). The second table consists the estimates of a correlation structure given by the equation (29). Moreover, we also include QQ-plots of the standardised residuals and auto-correlation function of the standardised residuals. Ljung-Box test with 12 lags has p - value higher than 0.1, henceforth we cannot reject the null hypothesis of no serial correlation on any significant statistical level. If we take a look at the QQ-plots and ACF plots, we can consider our default model as a useful one that we can interpret the real correlation structure. At the figure 3 are graphs with estimated conditional correlations over time.

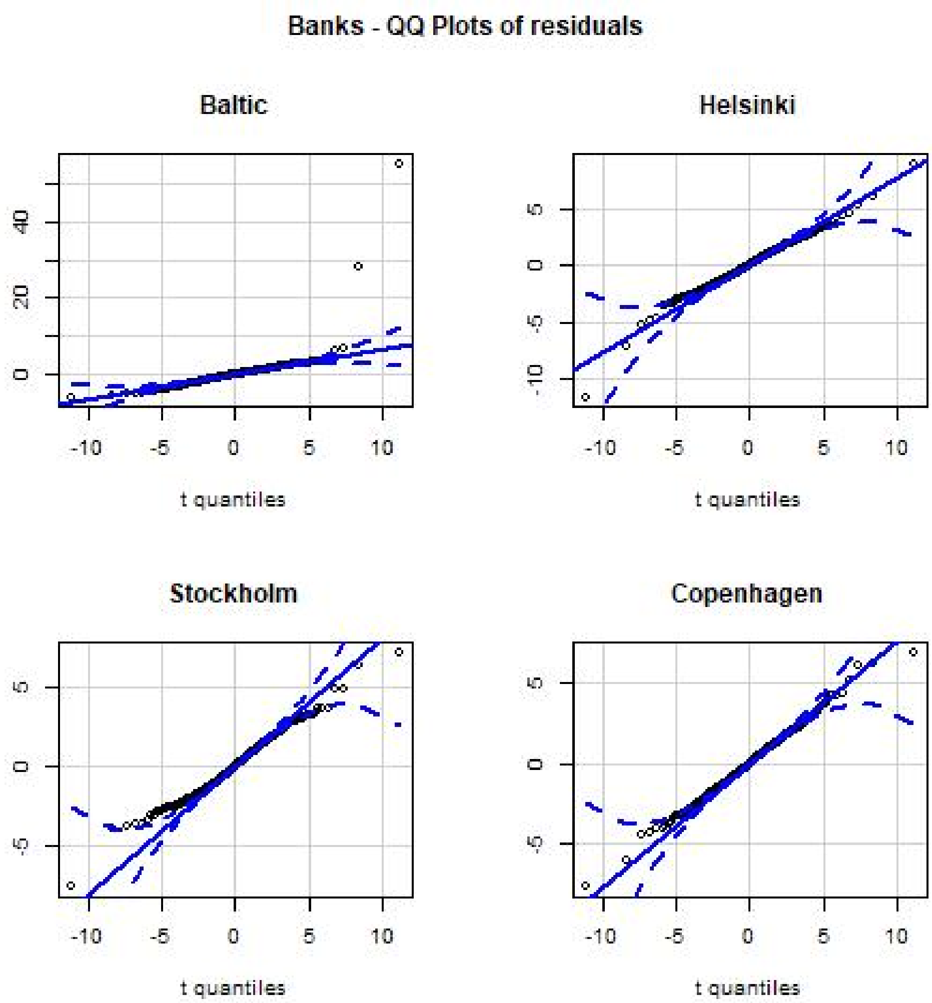

Figure 2: QQ-plots of residuals from the estimated DCC-GARCH model of returns from banking indices

To interpret our models, we have to take a look at both tables with the estimated parameters and also at graphs with modelled dynamic correlations. First of all, we can see that all constant terms m_i and c_i are very close to zero. This correspond with our expectations which we concluded earlier in this thesis from a descriptive statistics that the means of returns are very close to zero.

Table 1: Estimated parameters of univariate structure of DCC-GARCH model for banking indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	0.0000	0.0002	-0.0002	0.0002	-0.0005	0.0002	-0.0006	0.0002
γ_i	0.4626	0.1621	0.1173	0.7358	0.8988	0.0148	0.8833	0.0823
δ_i	-0.4845	0.1587	-0.1079	0.7353	-0.9160	0.0109	-0.9009	0.0761
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.3157	0.0358	0.0976	0.0199	0.0658	0.0123	0.0738	0.0206
B_{ii}	0.6833	0.0438	0.8875	0.0259	0.9308	0.0132	0.9199	0.0223
ν_i	3.3371	0.1938	6.6958	0.7678	6.6379	0.7881	7.2902	0.9795

Note: SD stands for standard deviation of corresponding estimated parameter

Crucial information about variances is hidden in the diagonal matrices A and B. ARCH term for Baltic region is significantly higher than for other countries, therefore Baltic variance is not that persistent and is really influenced by the shocks in returns. This is probably caused by the huge outliers in a data which are visible in logarithmic returns plots in Appendix B. DCC-GARCH model try to compensate for these outliers by allowing a quick response to the price shocks in the ARCH term. The similar effect can be seen in the returns from Copenhagen exchange, but not that strong and the variance is quite stable. On the other hand, returns from Helsinki and Stockholm stock exchanges exhibit more classical characteristics of high persistence in volatility and are more immune to the price shocks. Degrees of freedom estimated for student's t-distribution. Estimated values for ν are quite low, therefore it differs a lot from normal distribution which justify our denial of normality assumption.

Table 2: Estimated parameters of correlation structure

of DCC-GARCH model for banking indices

	Estimate	SD
k	0.0207	0.0047
l	0.9656	0.0095
ν	5.5229	0.2354

Note: SD stands for standard deviation

of corresponding estimated parameter

In terms of a correlation structure, the estimated parameters k and l demonstrate a common patterns that correlation varies over time and it is quite persistent. Estimated degrees of freedom are also significant and therefore decline normality assumption. Graphs of estimated correlations reveals a crucial information about relationship of logarithmic returns. Just as we predicted in the previous section, Baltic countries are far less correlated with other consider countries than the rest countries among themselves. These findings confirms the claims presented by the previous studies mentioned in literature review section. Another interesting phenomenon can be found in the correlation between the returns from banking indices from Helsinki and Stockholm stock exchanges. The dynamics is quite stable with one huge jump caused by outlier. However, the dynamics is quickly back to its original level. Therefore we can tell that the correlation dynamics between the returns from Helsinki and Stockholm stock exchanges shows a high persistence. In conclusion, correlation varies over time and appears in clusters.

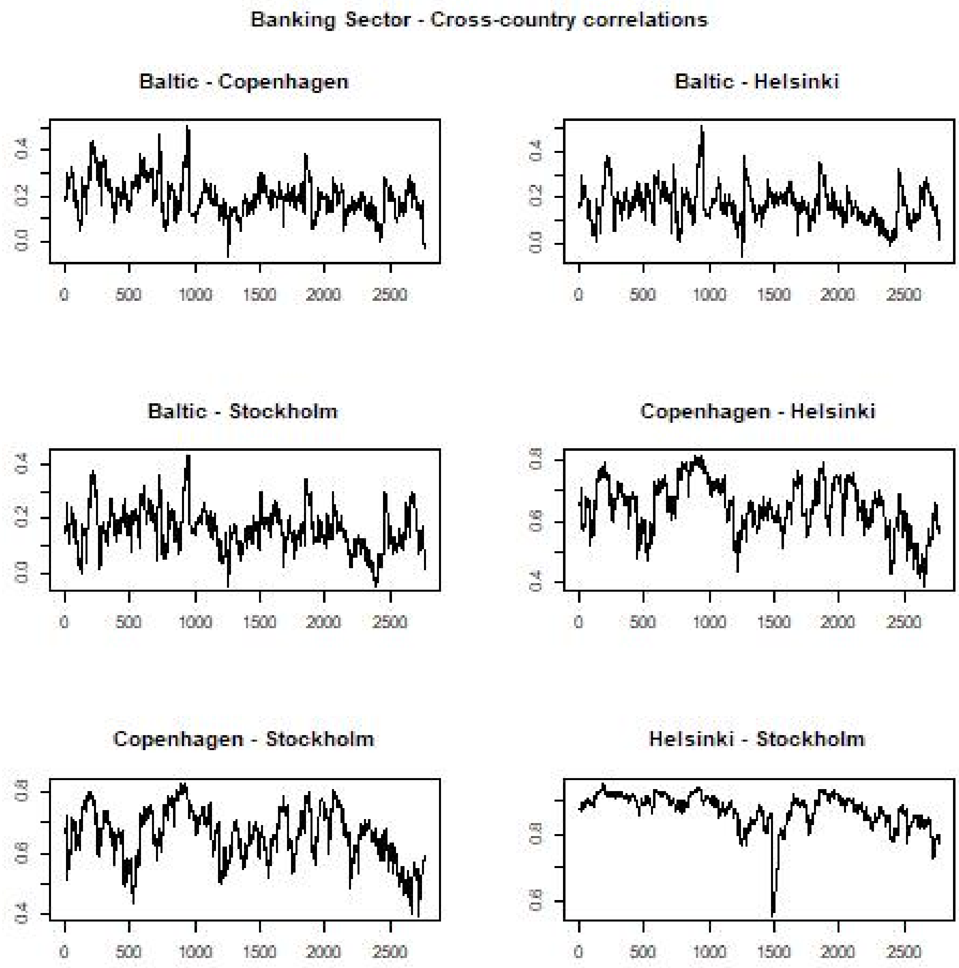

Figure 3: Cross-country conditional correlation of logarithmic returns for banking indices

We provided detailed analyses of the banking indices in the previous paragraphs. To keep conciseness and pointedness, we will not examine each sector group in such a detail. We expect to observe similar patterns in other sectors, henceforth we comment only curious differences compared to the banking sector. Even though we expect to see comparable characteristics like low correlations between Nordic and Baltic countries and a tight connection between Helsinki and Stockholm stock exchanges, we foresee some differences for specific sectors, as we stated earlier in the hypothesis section.

For the rest of studied sector indices, the result of estimated DCC-GARCH models are attached in Appendices C - G, named accordingly to the corresponding sectors. The structure of each appendix comply with the structure for banking sector described earlier. Construction & materials indices exhibit a notable distinction from other sector by having the highest persistence in the Baltic region. Also Food & beverages and Healthcare indices in Stockholm and Helsinki stock exchanges show significantly lower perseverance in volatility, making them less immune to the various price jumps. Moreover, for Healthcare indices one can observe that they are much less resistant to the price shocks by having a quite high estimates of A_{ii} parameters, which describe a sensitivity to the price changes, while having comparably lower B_{ii} parameters which represent the persistence in volatility. This observation would imply that Healthcare indices tends to be more vulnerable to some shocks and our DCC-GARCH model try to compensate for that effect. Moreover, these correlations often fall below 0. Also the estimates of degrees of freedom for student's t-distribution are relatively small, which also imply refuse of normality.

In terms of a correlation structure, remaining sectors shows alike patterns with a high persistence and a high immunity to the jumps in prices. The only exception is the healthcare industry with comparably low immunity to the shocks and we can see in the graphs that there are many huge jumps in the mutual correlations across the regions we investigate. The only economic sector that we were not able to estimate properly with our default model have been Personal & household goods. Several Ljung-Box tests p-values were much lower than 0.01, therefore we can safely reject the null hypothesis of no auto-correlation. To get model which can represent the real-world data, we tried different iterations of DCC-GARCH model to find a better one that would capture the remaining relationships in the residuals but which would stays as easy as possible. We decided to use ARMA(2,2) process to model mean of the returns. This adjustment improve the performance of the model considerably, compared to the changes in univariate GARCH orders, orders of DCC structure or changes made in the type of univariate GARCH model (for example IGARCH). The result of preferred model is in Appendix G

- Personal & Household Goods and the only noticeable difference is a low persistence in Copehagen index volatility.

To conclude, in terms of cross-country dynamic correlations, we managed to confirm our hypothesis about the correlation between Nordic and Baltic regions. Furthermore, we explored persistence of volatilities as well as resistance for the price shocks. We have found similar patterns for the correlation structure of each sectors. Subsequently, we explored additional characteristics of each sector across regions. However, we were not able to observe any significant overall (increasing) trend as we predicted.

5.2 Sector Effect for Countries

In this section, we will examine sector indices in each of the investigated region. We use the same default model to estimate correlations among sectors in each country as well as volatilities of each series. Generally, we expect to witness lower correlations than those in previous section. Again, we start with deep analysis of Baltic countries where we demonstrate our approach to the estimation procedure and model validation. For the rest of the countries, we will provide just estimated models as we did for sectors in the previous section.

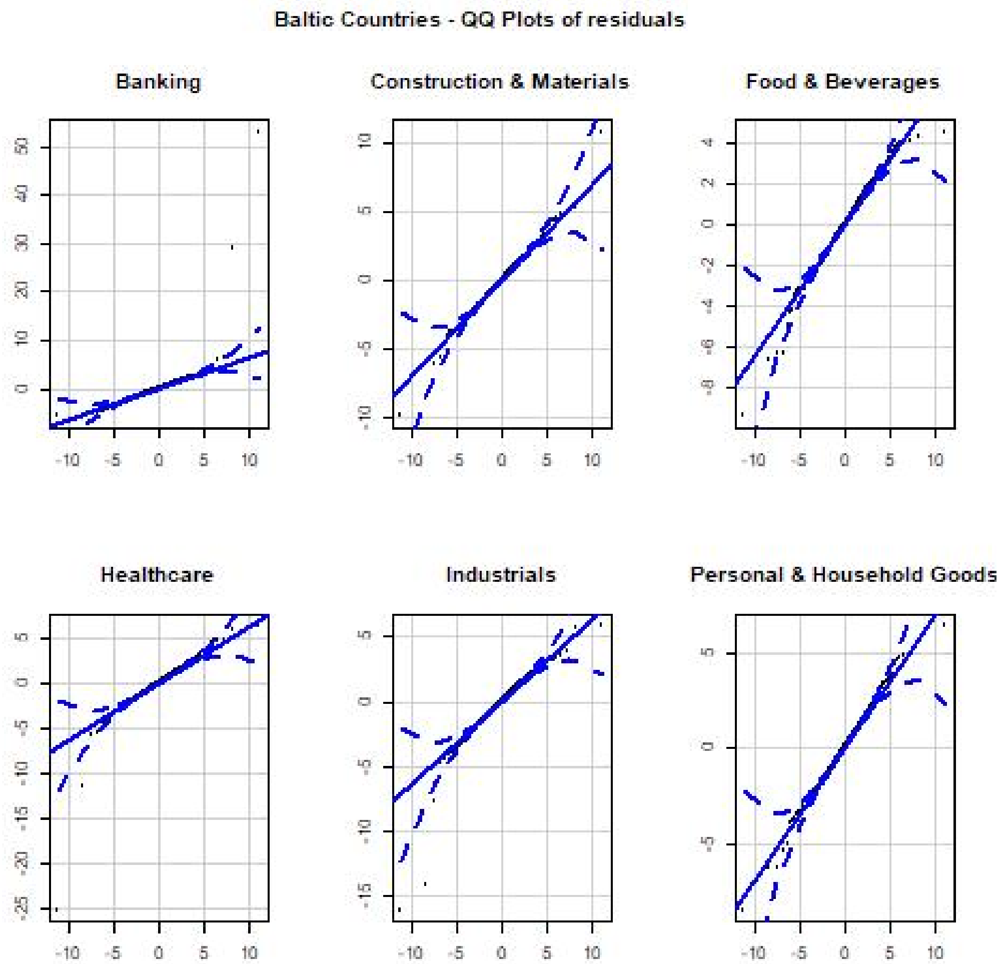

Figure 4: QQ-plots of residuals from the estimated DCC-GARCH model of returns from Baltic sector indices

We start by analysing correlation between sectors in the Baltic countries. We open with DCC-GARCH model under our default specification. The result of estimation can be found tables 3 and 4, as well as QQ-plots of the standardised residuals, ACF plots of the standardised residuals and plots of estimated dynamic conditional correlations in following graphs. We can see from QQ and ACF plots that the standardised residuals that there is probably no serial correlation left and they are very likely randomly distributed. That is also confirmed by Ljung-Box tests with p-values higher than 0.1. Therefore we consider the default model as a good one for interpretation and potential forecasting.

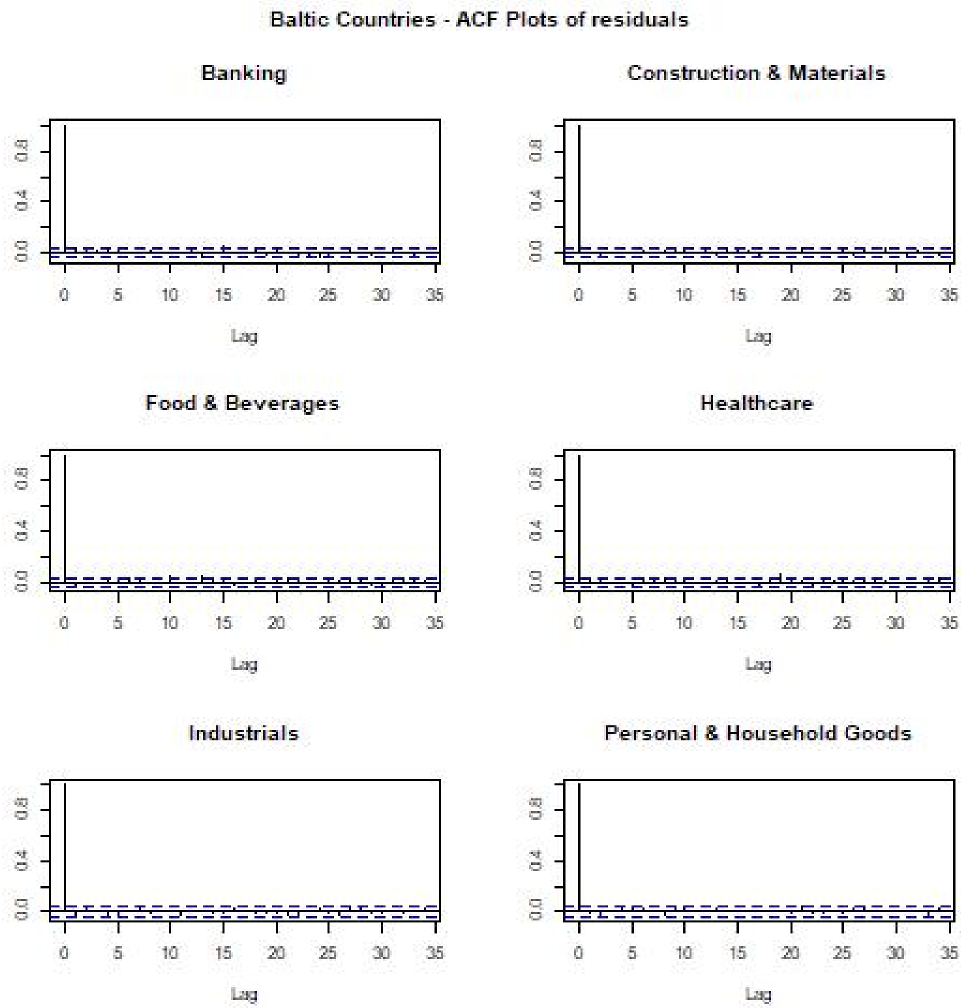

Figure 5: ACF plots of residuals from the estimated DCC-GARCH model of returns from Baltic sector indices

When we take a look at estimated univariate GARCH models, we can see the identical estimates as for country effect in previous section. This is caused by the two-step estimation procedure described in the methodology section in which the first step is to estimate univariate models for each time series of the logarithmic returns. Therefore, the estimation process just keeps the very same first step and the results stay the same. However, comparison of the estimates across sector might help us to observe an additional notion about volatility of each time series. For example, one can detect that the banking and healthcare sectors are highly responsive to the price shocks compared to the other sectors. On the other hand, construction & materials index shows a high resistance to the shocks and displays a huge persistence in volatility, which is against our initial expectations. However, we predicted the same effect for personal & household goods indices, which is confirmed by our estimates. Estimated degrees of freedom are also quite low, which justify use of student's t-distribution.

Table 3: Estimated parameters of univariate structure of DCC-GARCH model for sector indices in Baltic countries

	Banking		Con & Mat		Food & Beverages	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	0.0000	0.0002	-0.0001	0.0002	-0.0002	0.0001
γ_i	0.4620	0.1623	-0.1572	0.1128	0.0805	0.1174
δ_i	-0.4838	0.1591	0.0299	0.1148	-0.1717	0.1168
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.3157	0.0359	0.0528	0.0057	0.1113	0.0286
B_{ii}	0.6833	0.0442	0.9437	0.0064	0.8870	0.0238
ν	3.3369	0.2028	4.0465	0.3482	3.3969	0.3100

	Healthcare		Industrials		Personal & HH Goods	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0001	0.0002	-0.0001	0.0001	-0.0001	0.0002
γ_i	0.0122	0.1841	-0.4955	0.1045	-0.9435	0.0061
δ_i	-0.0953	0.1838	0.4320	0.1092	0.9603	0.0007
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.2002	0.0555	0.1175	0.0483	0.0516	0.0032
B_{ii}	0.7425	0.0705	0.8815	0.0403	0.9474	0.0041
ν	3.1790	0.2235	3.7045	0.3885	3.9972	0.3465

Note: SD stands for standard deviation of corresponding estimated parameter

Table 4: Estimated parameters of correlation structure
of DCC-GARCH model for sector indices in Baltic countries

	Estimate	SD
k	0.0161	0.0025
l	0.9502	0.0102
ν	4.7417	0.1458

Note: SD stands for standard deviation
of corresponding estimated parameter

In the case of a correlation structure, one can notice also relatively high persistence in correlation and the high immunity to the price shocks. Graphs of the dynamic conditional correlations presented in the tables 3 and 4 display that overall level of correlation seems to be significantly lower than the correlations across countries for each sector. The only exceptions are correlations between construction & material and industrials and maybe between food & beverages and personal & household goods. This phenomenon were predicted in the hypothesis section due to the shared securities in both indices. For the other pairs of the logarithmic returns, correlations occurs rather closer to zero, as were anticipated. Furthermore, one can observe that the correlation appears in cluster. This effect is most apparent in 2014 (around 1500 days) with the huge sudden sharp increase in correlation for almost all pairs. In these graphs, one can perceive more similar shocks in correlations.

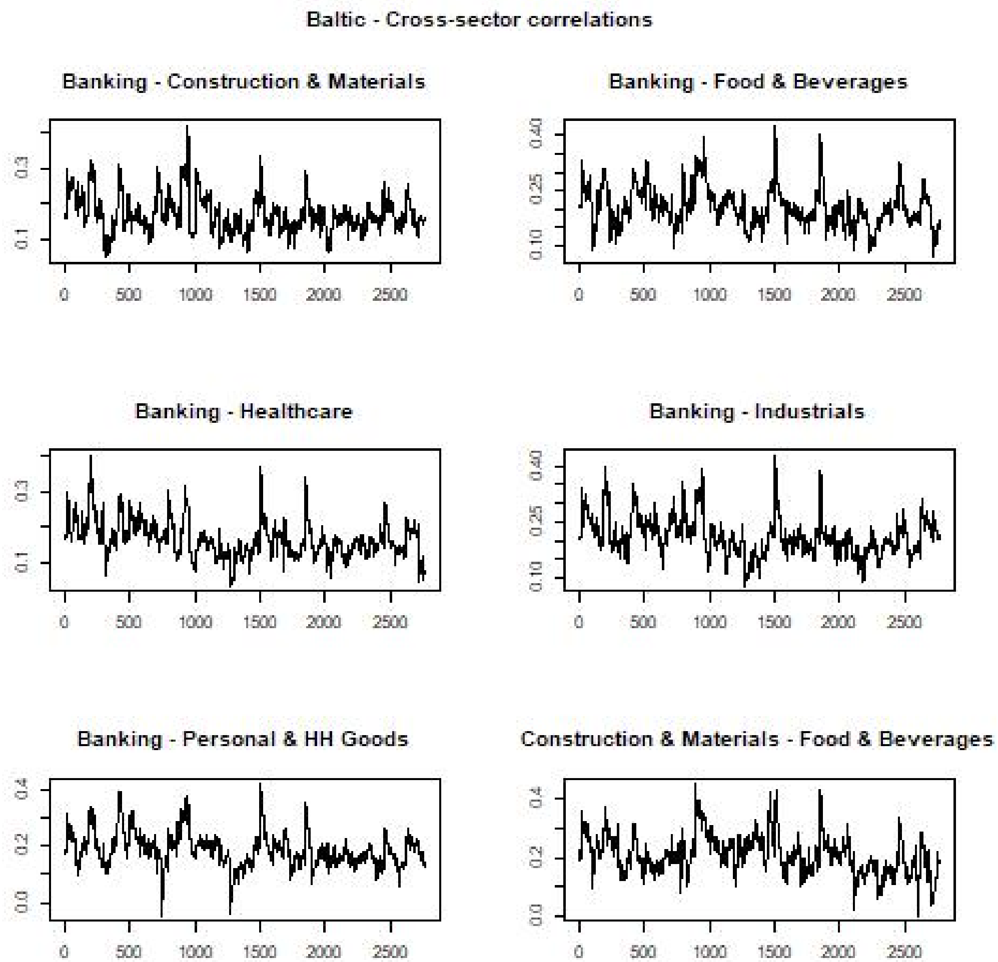

Figure 6: Cross-country conditional correlation of logarithmic returns from Baltic sector indices - part 1

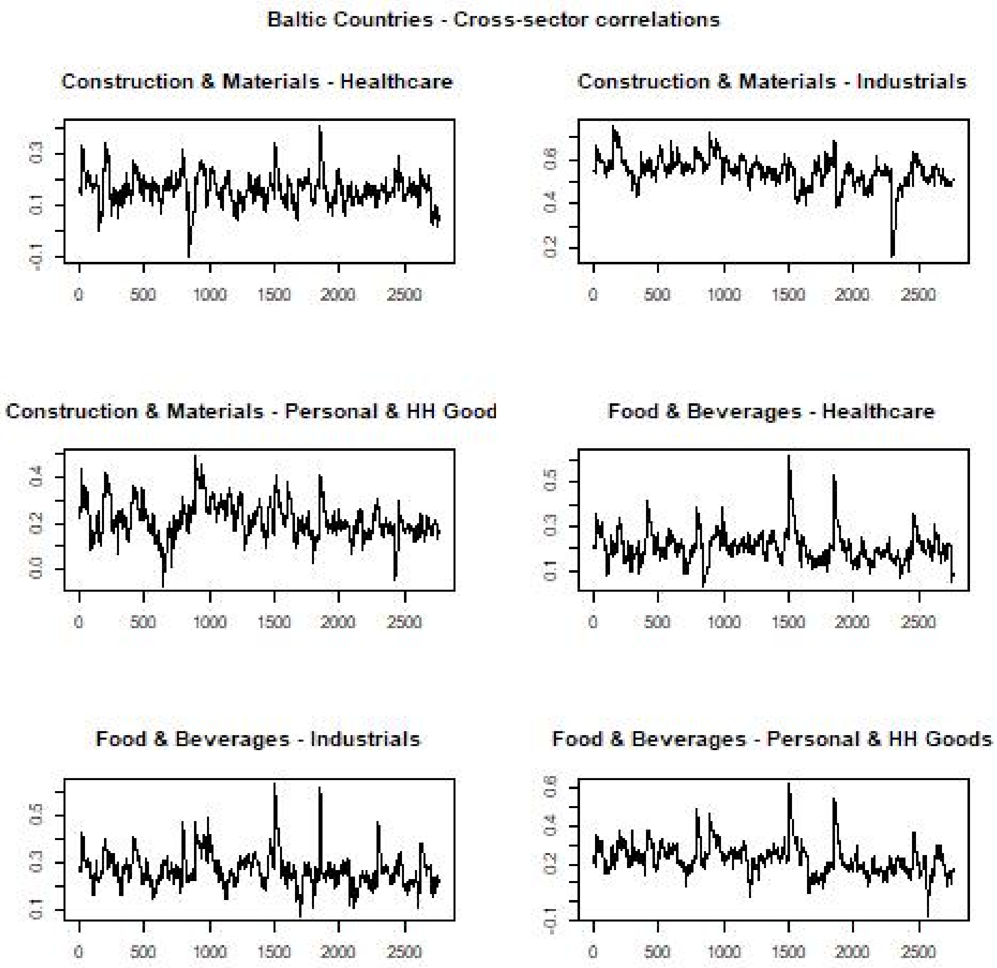

Figure 7: Cross-country conditional correlation of logarithmic returns from Baltic sector indices - part 2

For the Nordic countries, we found that the default DCC-GARCH model is sufficient for interpretation. All the related estimates and graphs are attached at corresponding Appendices H - J. In terms of the univariate GARCH estimates, it is interesting to observe difference in the persistences of volatilities in each country, since each region seems to have distinctive sectors with low and high perseverance. Also the correlation structure display very high persistence. However, correlations are often much higher than in the Baltic region. Having said that, one can observer a decreasing trend in many correlations and the correlation of alike sectors again higher. Unfortunately, we cannot certainly conclude that sector effect is overall better for portfolio diversification. For the Baltic countries, sector diversification might be a really useful tool to minimise risk. However, for other countries, the sector and the country effects are comparable. We can also observe generally high correlations during the financial crisis at the beginning of considered period.

6 Conclusion

In this thesis, we explored logarithmic returns of various sector indices on stock markets in the Northern Europe. Firstly, we explored wide literature on the topics of portfolio diversification, modelling second moments of returns (with emphasis on GARCH family of models) and also on characteristics of economies and financial markets in the Northern Europe. In the methodology section, we introduced univariate GARCH model with different possible specifications and presented the maximum likelihood estimation procedure. Then we extend classical univariate GARCH into the multivariate CCC-GARCH and DCC-GARCH structures, described two-stage estimation process for DCC-GARCH models and specified techniques for model validation. In the second to last section, we examined data and stated our hypothesis both from data as well as from existing literature on this topic. Finally, we presented estimated models, analysed the results and compared them to our hypothesis.

We have find that the correlation of logarithmic returns of sector indices in the Northern Europe varies over time. The correlation between Nordic and Baltic countries is rather low, therefore we cannot claim that these two region are connected in terms of financial markets. Additionally, we identified tightly connected sectors and countries. Generally, one can state that Helsinki and Stockholm exhibits high level of correlation, as well as construction & materials and industrials sectors. Overall, we did not observe any significant trend in the evolution of correlations or any comprehensive differences between the sector and the country effect. The result of this thesis might be helpful to the various investors and portfolio or risk managers.

For the research purposes, one could try enhance the model by introducing jumps to compensate models for outliers. Additionally, it would be beneficial to model the correlations with different techniques to compare and confirm the results of this thesis. The big improvement could be made with high-frequency data to explore intraday changes, for example with a concept of realised variance and HAR model. It is also possible to directly apply the results of this thesis to construct or enhance a portfolio or to perform VaR analysis.

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List of Acronyms

- AFC - Autocorrelation Function
- AR - Autoregressive
- ARCH - Autoregressive Conditional Heteroskedastic (model)
- ARMA - Autoregressive Moving Average
- BEKK - Baba - Engle - Kraft - Kroner (model)
- CAPM - Capital Asset Pricing Model
- CCC-GARCH - Constant Conditional Correlation Generalized Autoregressive Conditional Heteroskedastic (model)
- CSE - Copenhagen Stock Ex-change
- DCC-GARCH - Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroskedastic (model)
- EEA - European Economic Area
- EGARCH - Exponential Generalized Autoregressive Conditional Heteroskedastic (model)
- EWMA - Exponentially Weighted Moving Average
- GARCH - Generalized Autoregressive Conditional Heteroskedastic (model)
- GDP - Gross Domestic Product
- GJR-GARCH - Glosten-Jagannathan-Runkle Generalized Autoregressive Conditional Heteroskedastic (model)
- HAR - Heterogeneous Autoregressive
- HDI - Human Development Index
- HSE - Helsinki Stock Exchange
- IGARCH - Integrated Generalized Autoregressive Conditional Heteroskedastic (model)

- LLN - Law of Large Numbers
- MLE - Maximum Likelihood Estimator
- NASDAQ - National Association of Securities Dealers Automated Quotations
- OECD - Organisation for Economic Co-operation and Development
- QQ-plot - quantile-quantile plot
- RVar - Realised Variance
- RVol - Realised Volatility
- SSE - Stockholm Stock Exchange
- VaR - Value-at-Risk
- VECH - Vectorised Conditional Heteroskedastic (model)

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Appendix A - Descriptive Statistics of Returns

Table 5: Descriptive statistics for daily returns from Baltic stock exchanges

	Min	Median	Mean	Max	SD	Skewness	Kurtosis	JB	LB
B	-0.138	0.000	0.001	0.966	0.027	17.40	592.76	0.00	0.00
CM	-0.124	0.000	0.000	0.129	0.016	0.24	13.54	0.00	0.00
FB	-0.124	-0.000	-0.000	0.070	0.011	-0.91	16.86	0.00	0.00
HC	-0.318	0.000	-0.000	0.143	0.017	-2.13	61.60	0.00	0.00
In	-0.128	-0.000	-0.000	0.093	0.012	-0.42	17.19	0.00	0.00
PHH	-0.110	0.000	0.000	0.121	0.015	0.23	10.45	0.00	0.00

Note: Acronyms in the first column are B - Banks, CM - Construction & Materials, FB - Food & Beverages, HC - Healthcare, In - Industrials, PHH - Personal & Household Goods. JB stands for the p - value of Jarque-Bera test, LB stands for the p-value of Ljung-Box performed on 12 lags.

Table 6: Descriptive statistics for daily returns from Copenhagen stock exchange

	Min	Median	Mean	Max	SD	Skewness	Kurtosis	JB	LB
B	-0.102	0.000	0.000	0.131	0.017	0.14	7.82	0.00	0.00
CM	-0.106	-0.000	-0.000	0.115	0.018	0.28	8.77	0.00	0.00
FB	-0.104	-0.000	-0.000	0.133	0.016	0.45	11.79	0.00	0.00
HC	-0.080	-0.001	-0.001	0.010	0.013	0.41	8.68	0.00	0.60
In	-0.111	-0.001	-0.000	0.139	0.017	0.24	9.34	0.00	0.01
PHH	-0.143	-0.000	0.000	0.678	0.026	7.33	189.67	0.00	0.00

Note: Acronyms in the first column are B - Banks, CM - Construction & Materials, FB - Food & Beverages, HC - Healthcare, In - Industrials, PHH - Personal & Household Goods. JB stands for the p - value of Jarque-Bera test, LB stands for the p-value of Ljung-Box performed on 12 lags.

Table 7: Descriptive statistics for daily returns from Helsinki stock exchange

	Min	Median	Mean	Max	SD	Skewness	Kurtosis	JB	LB
B	-0.160	-0.000	-0.000	0.125	0.020	-0.52	10.37	0.00	0.10
CM	-0.102	-0.000	0.000	0.111	0.016	-0.03	6.61	0.00	0.00
FB	-0.084	-0.000	-0.000	0.068	0.011	-0.16	8.13	0.00	0.01
HC	-0.099	-0.001	-0.000	0.111	0.014	0.25	8.85	0.00	0.36
In	-0.091	-0.001	-0.000	0.082	0.015	-0.01	6.28	0.00	0.00
PHH	-0.095	-0.000	-0.000	0.094	0.013	0.08	8.27	0.00	0.00

Note: Acronyms in the first column are B - Banks, CM - Construction & Materials, FB - Food & Beverages, HC - Healthcare, In - Industrials, PHH - Personal & Household Goods. JB stands for the p - value of Jarque-Bera test, LB stands for the p-value of Ljung-Box performed on 12 lags.

Table 8: Descriptive statistics for daily returns from Stockholm stock exchange

	Min	Median	Mean	Max	SD	Skewness	Kurtosis	JB	LB
B	-0.141	-0.000	-0.000	0.109	0.019	-0.29	10.25	0.00	0.13
CM	-0.108	-0.001	-0.001	0.077	0.015	-0.18	8.59	0.00	0.04
FB	-0.097	-0.001	-0.001	0.063	0.013	-0.06	6.63	0.00	0.21
HC	-0.076	-0.001	-0.000	0.065	0.011	0.26	7.13	0.00	0.04
In	-0.098	-0.001	-0.000	0.095	0.016	0.05	7.28	0.00	0.08
PHH	-0.082	-0.001	-0.001	0.066	0.013	-0.17	7.40	0.00	0.06

Note: Acronyms in the first column are B - Banks, CM - Construction & Materials, FB - Food & Beverages, HC - Healthcare, In - Industrials, PHH - Personal & Household Goods. JB stands for the p - value of Jarque-Bera test, LB stands for the p-value of Ljung-Box performed on 12 lags.

Appendix B - Plots of Logarithmic Returns

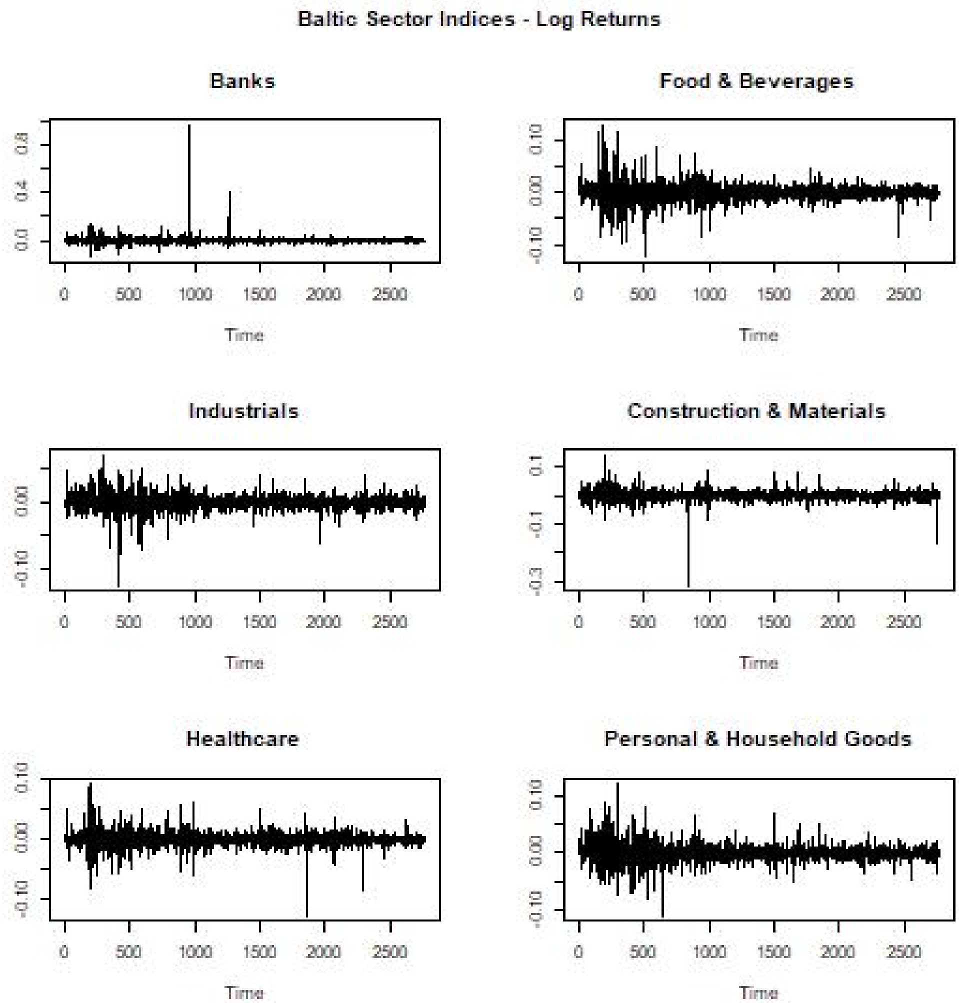

Figure 8: Logarithmic returns from Baltic stock exchanges

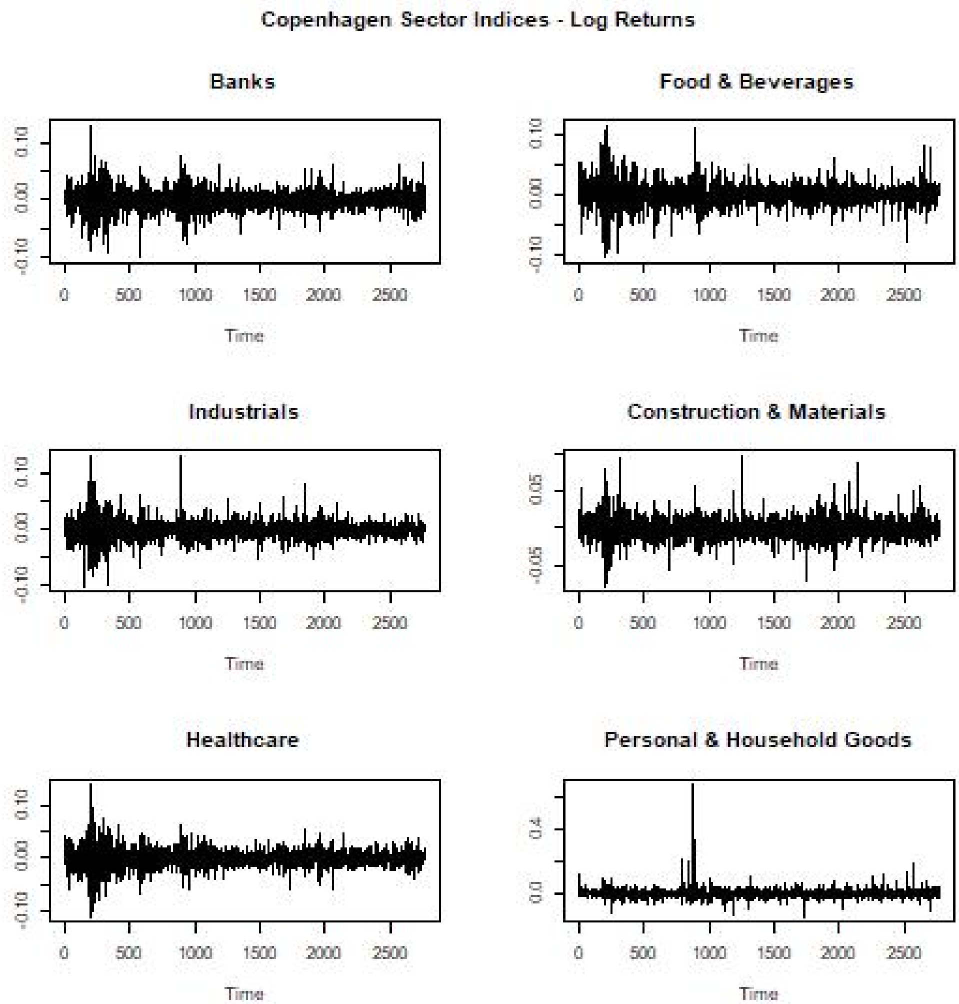

Figure 9: Logarithmic returns from Copenhagen stock exchange

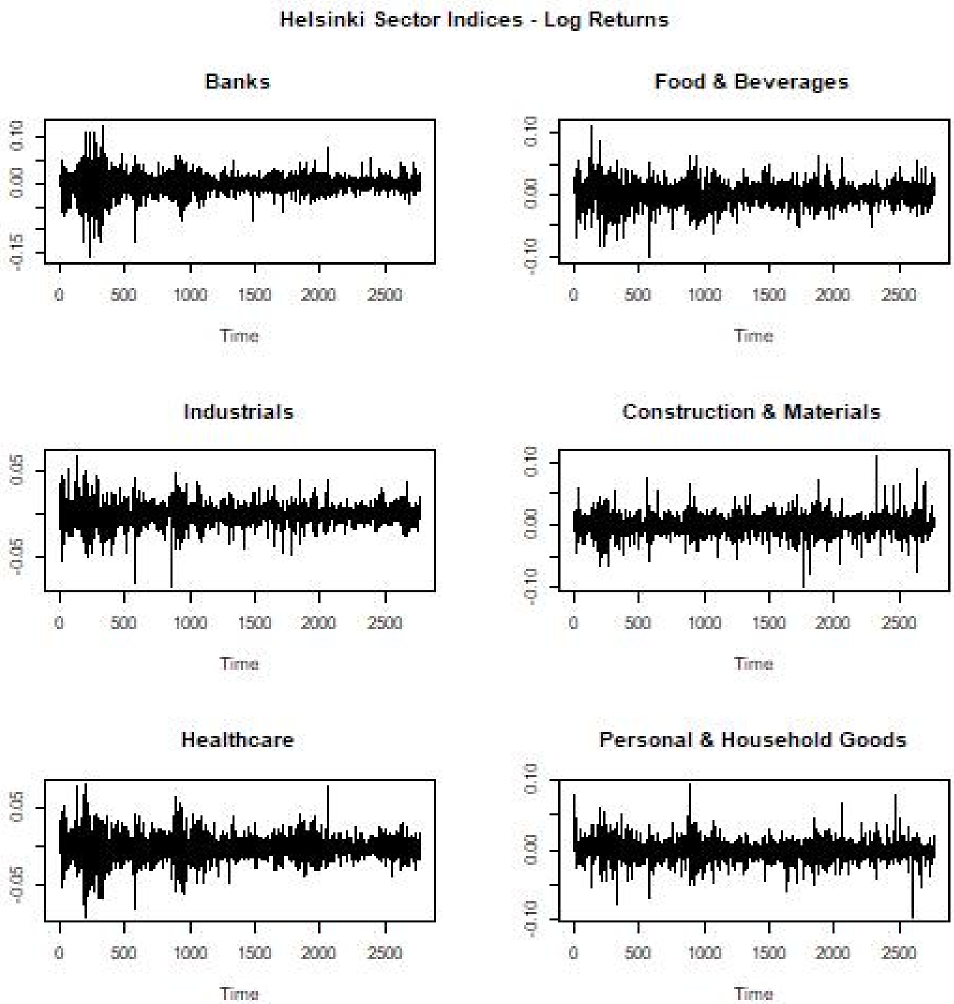

Figure 10: Logarithmic returns from Helsinki stock exchange

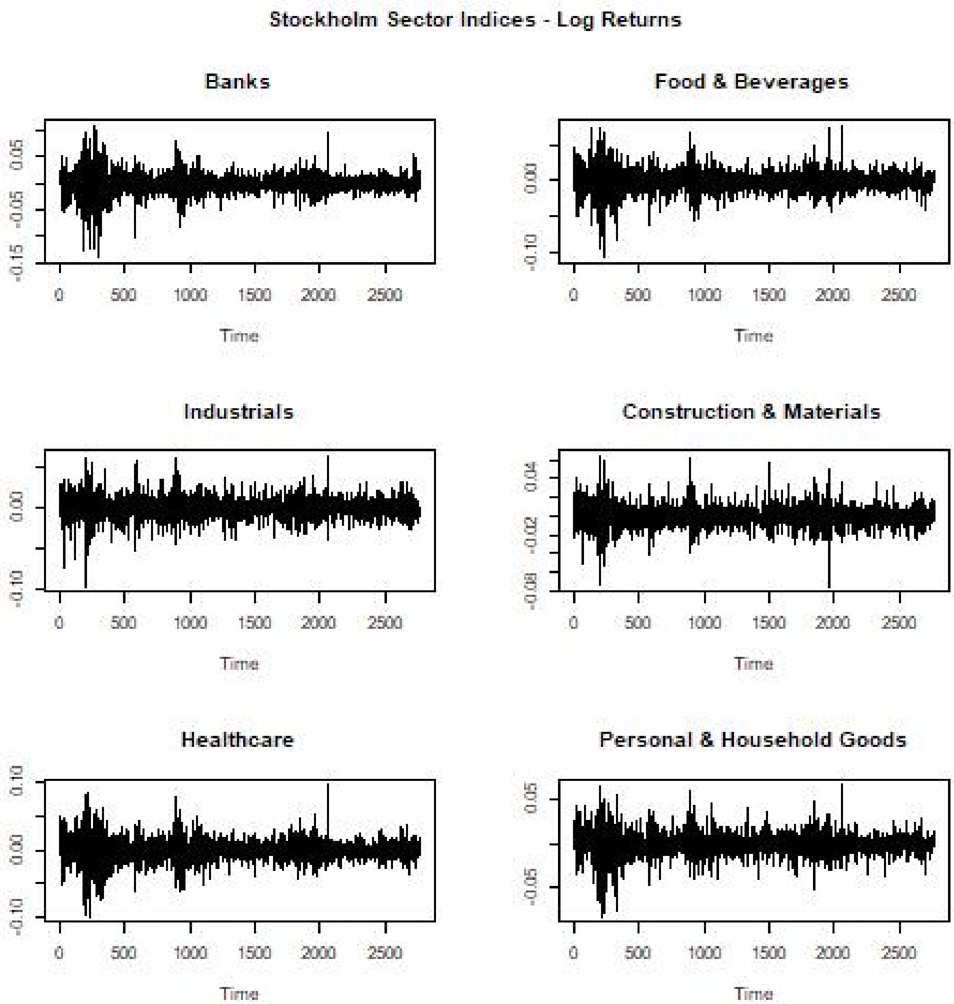

Figure 11: Logarithmic returns from Stockholm stock exchange

Appendix C - Construction & Materials

Table 9: Estimated parameters of univariate structure
of DCC-GARCH model for construction & materials indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0001	0.0002	-0.0005	0.0003	-0.0002	0.0003	-0.0008	0.0002
γ_i	-0.1572	0.1128	0.3847	0.1316	0.3550	0.0999	0.8965	0.0711
δ_i	0.0299	0.1149	-0.3303	0.1331	-0.2721	0.1015	-0.9020	0.0695
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.0528	0.0056	0.0971	0.0580	0.0670	0.0169	0.0768	0.0715
B_{ii}	0.9437	0.0061	0.8822	0.0635	0.9243	0.0194	0.9106	0.0819
ν_i	4.0467	0.3432	5.1075	2.0169	5.7044	0.6025	7.4073	1.7363

Note: SD stands for standard deviation of corresponding estimated parameter

Table 10: Estimated parameters of correlation structure
of DCC-GARCH model for construction & materials indices

	Estimate	SD
k	0.0110	0.0030
l	0.9773	0.0081
ν	6.0285	0.2321

Note: SD stands for standard deviation
of corresponding estimated parameter

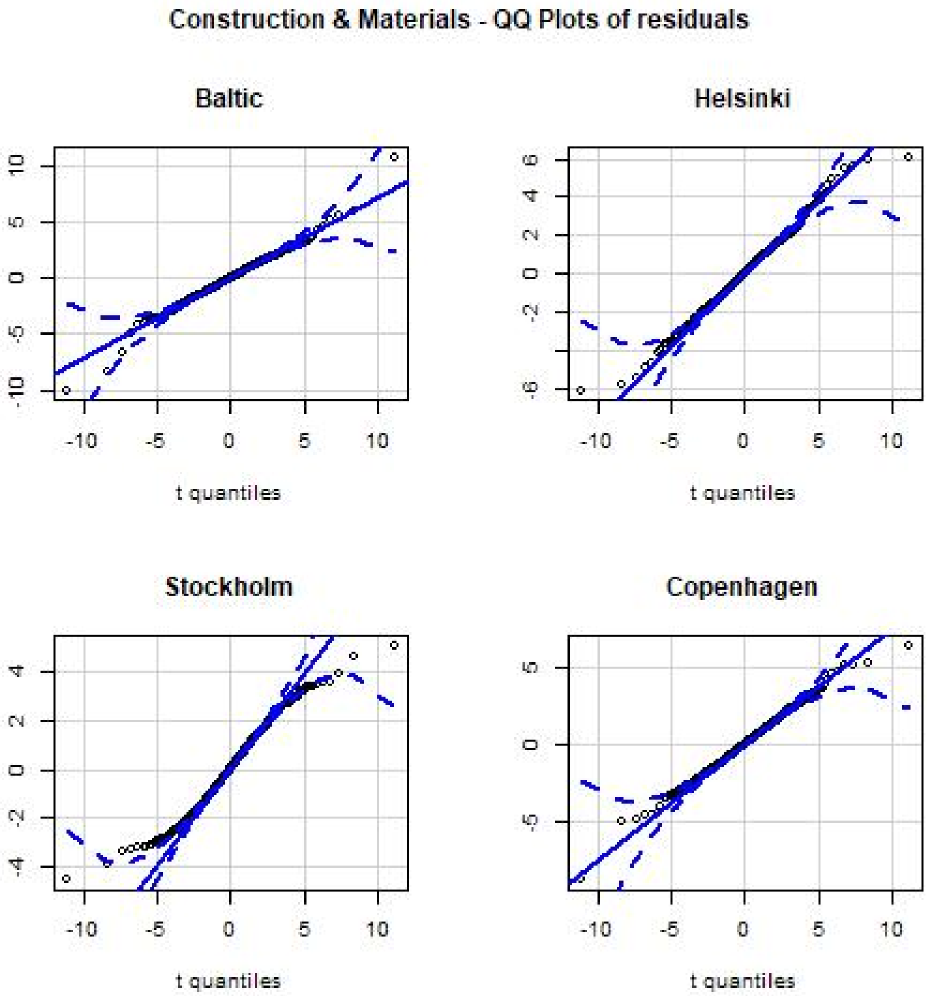

Figure 12: QQ-plots of residuals from the estimated DCC-GARCH model of returns from construction & materials indices

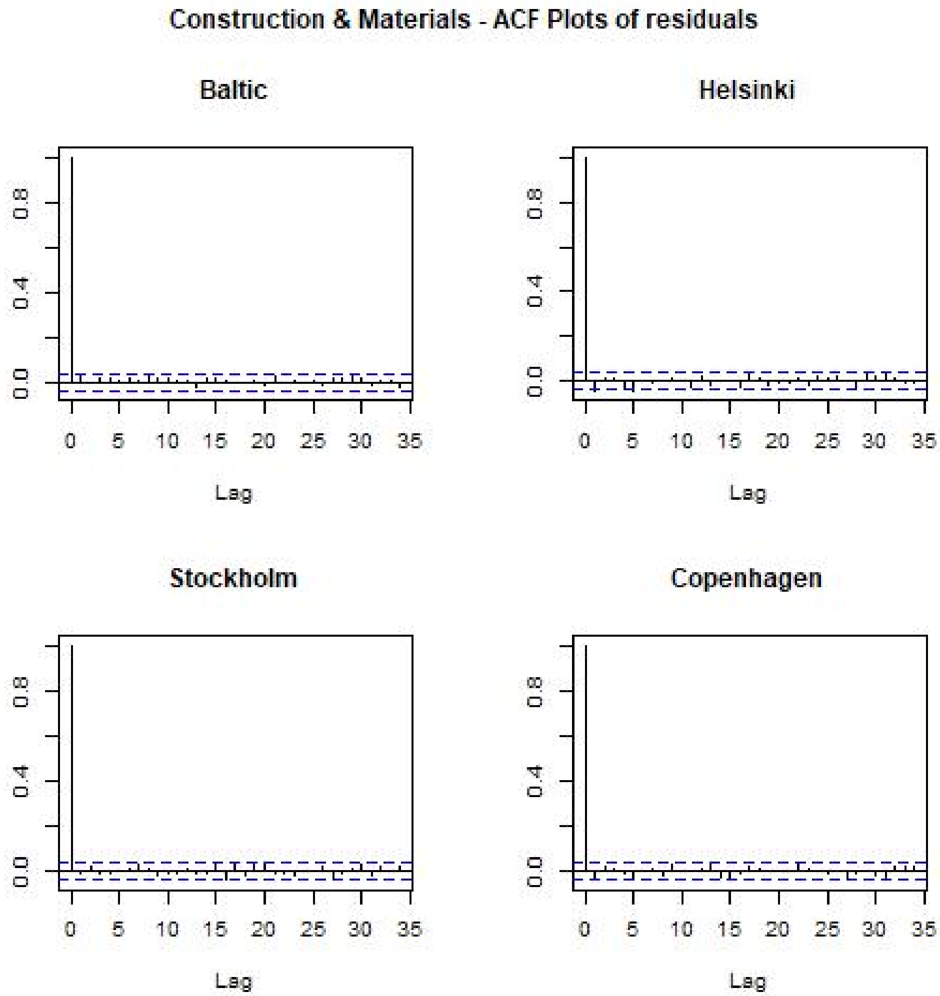

Figure 13: ACF plots of residuals from the estimated DCC-GARCH model of returns from construction & materials indices

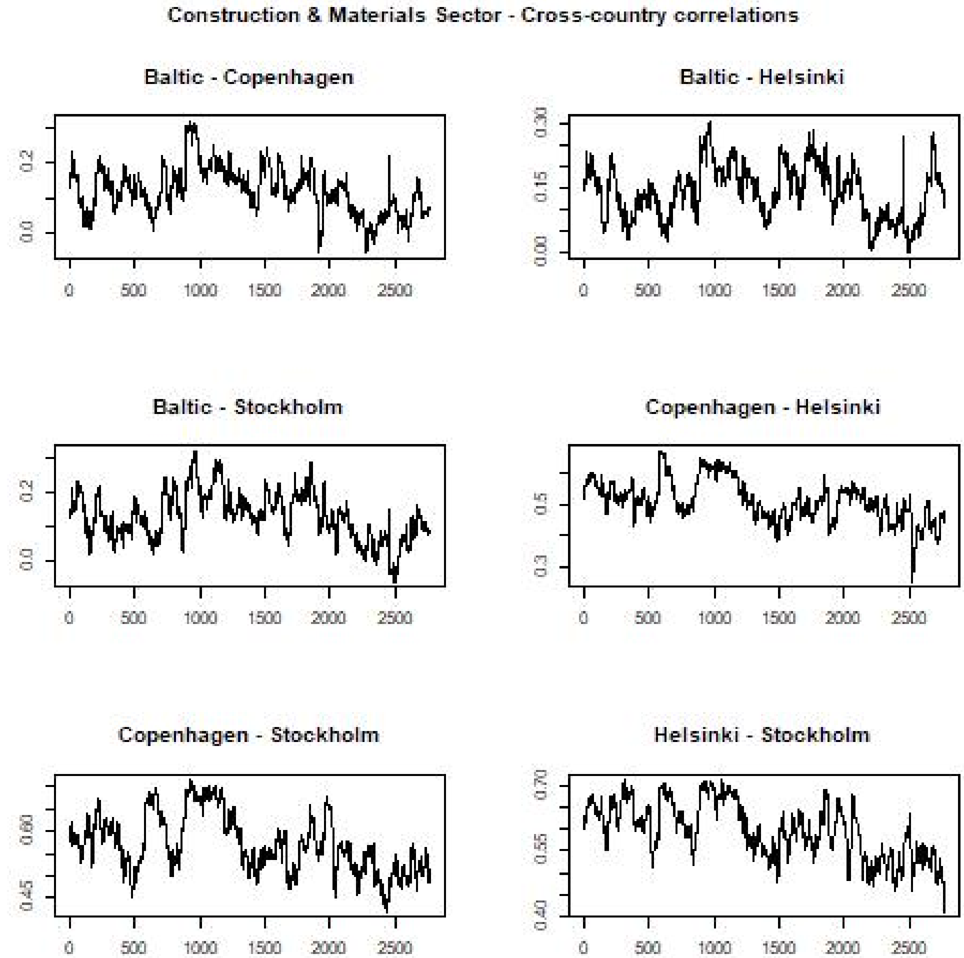

Figure 14: Cross-country conditional correlation of logarithmic returns for construction & materials indices

Appendix D - Food & Beverages

Table 11: Estimated parameters of univariate structure
of DCC-GARCH model for food & beverages indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0002	0.0001	-0.0006	0.0002	-0.0002	0.0002	-0.0007	0.0002
γ_i	0.0793	0.1175	0.8056	0.0604	0.5994	0.1814	-0.1242	0.6362
δ_i	-0.1707	0.1170	-0.8256	0.0566	-0.5691	0.1855	0.1451	0.6335
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.1111	0.0290	0.0398	0.0109	0.1416	0.0401	0.1349	0.0097
B_{ii}	0.8870	0.0240	0.9556	0.0079	0.8220	0.0674	0.7871	0.0141
ν_i	3.3967	0.3106	5.0177	1.0976	4.6375	0.9681	6.5600	0.7178

Note: SD stands for standard deviation of corresponding estimated parameter

Table 12: Estimated parameters of correlation structure
of DCC-GARCH model for food & beverages indices

	Estimate	SD
k	0.0106	0.0026
l	0.9536	0.0136
ν	5.9810	0.2655

Note: SD stands for standard deviation
of corresponding estimated parameter

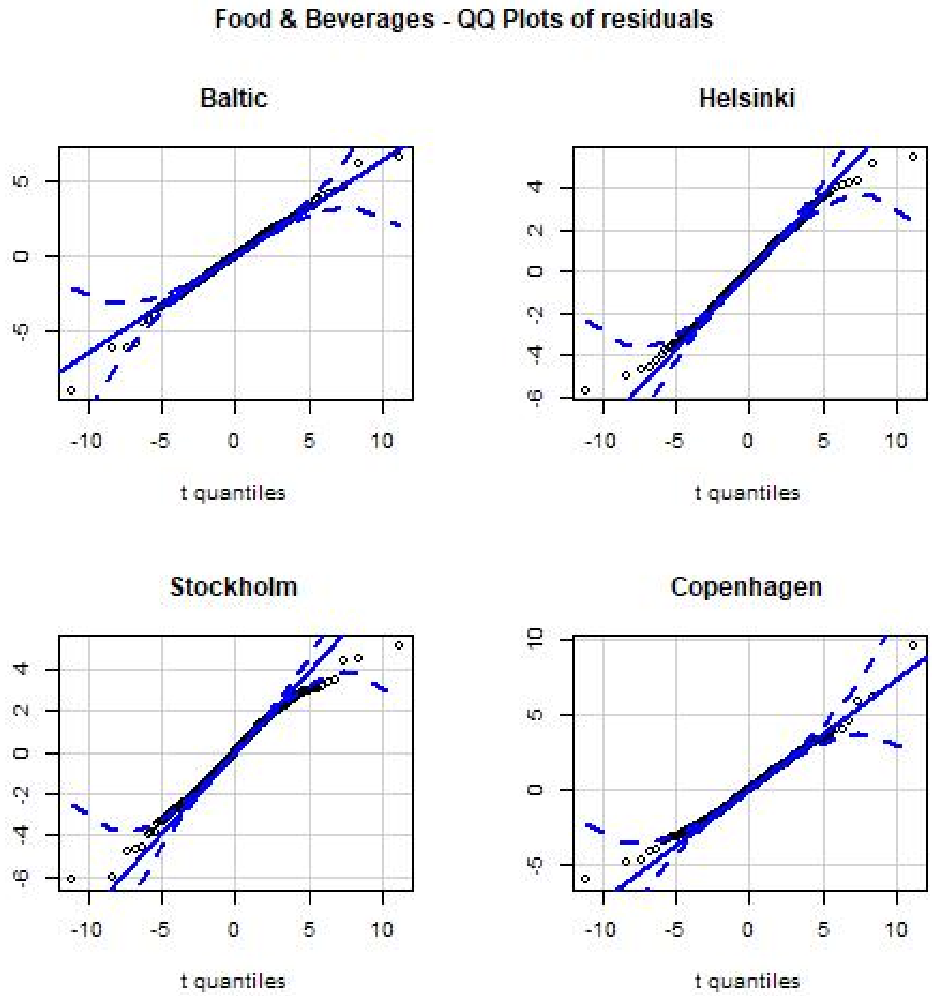

Figure 15: QQ-plots of residuals from the estimated DCC-GARCH model of returns from food & beverages indices

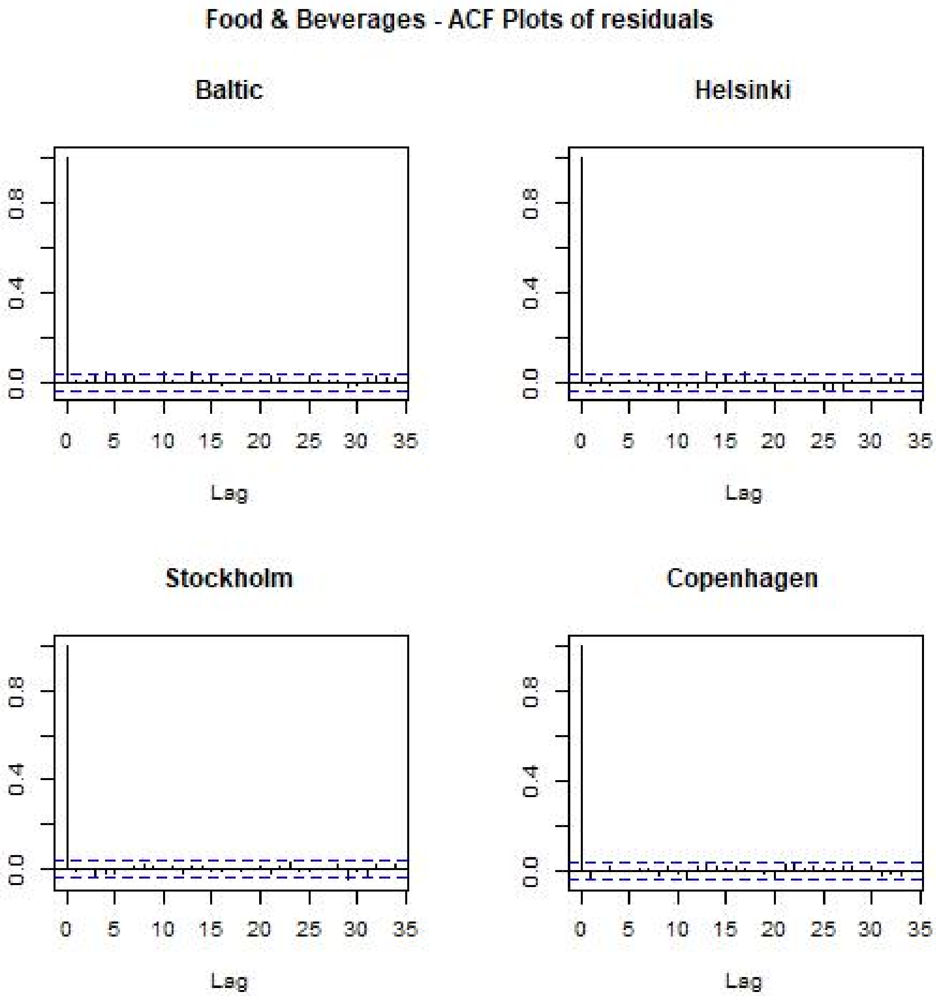

Figure 16: ACF plots of residuals from the estimated DCC-GARCH model of returns from food & beverages indices

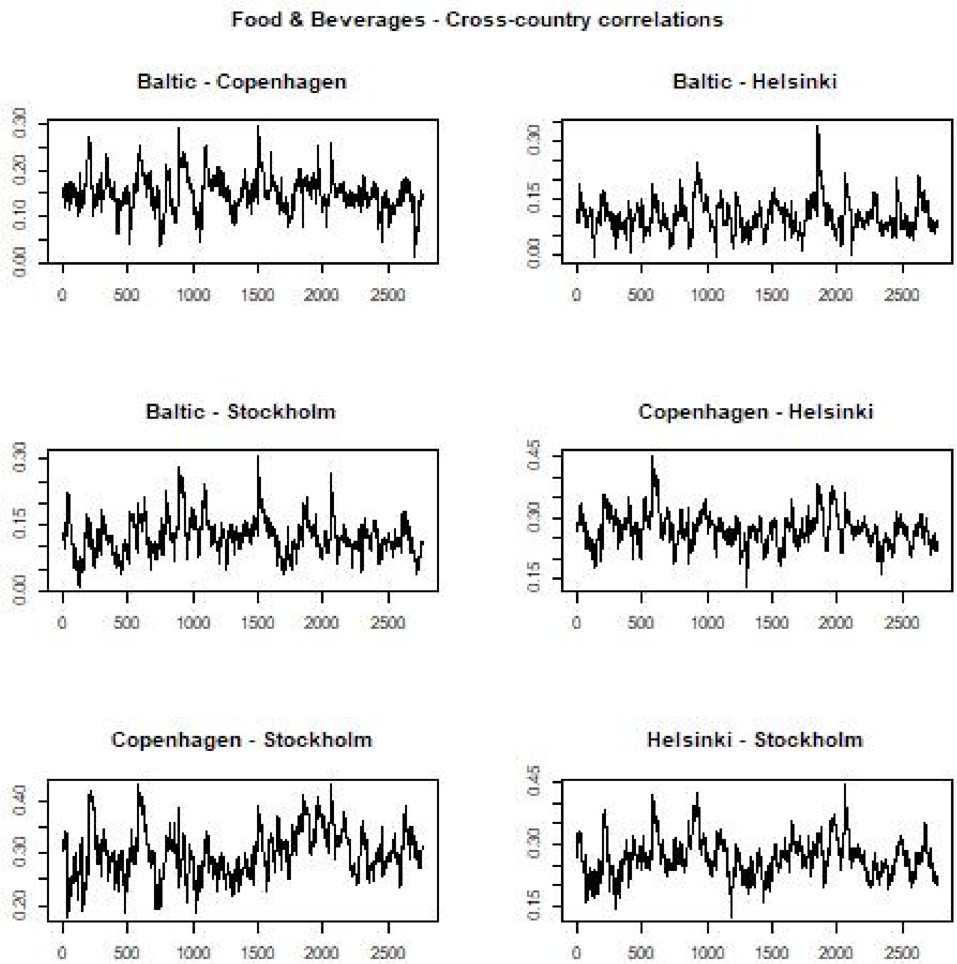

Figure 17: Cross-country conditional correlation of logarithmic returns for food & beverages indices

Appendix E - Healthcare

Table 13: Estimated parameters of univariate structure
of DCC-GARCH model for healthcare indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0001	0.0002	-0.0009	0.0002	-0.0006	0.0002	-0.0008	0.0002
γ_i	0.0120	0.1843	0.8776	0.0509	0.2049	0.6232	0.3429	0.7239
δ_i	-0.0952	0.1839	-0.8991	0.0455	-0.2158	0.6227	-0.3179	0.7305
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.2004	0.0554	0.0913	0.0062	0.1293	0.0435	0.1058	0.0153
B_{ii}	0.7425	0.0705	0.8472	0.0110	0.7617	0.0898	0.8610	0.0285
ν_i	3.1779	0.2136	4.9877	0.4544	4.2220	0.3383	6.6572	0.9223

Note: SD stands for standard deviation of corresponding estimated parameter

Table 14: Estimated parameters of correlation structure
of DCC-GARCH model for healthcare indices

	Estimate	SD
k	0.0319	0.0071
l	0.8589	0.0396
ν	5.1047	0.1813

Note: SD stands for standard deviation
of corresponding estimated parameter

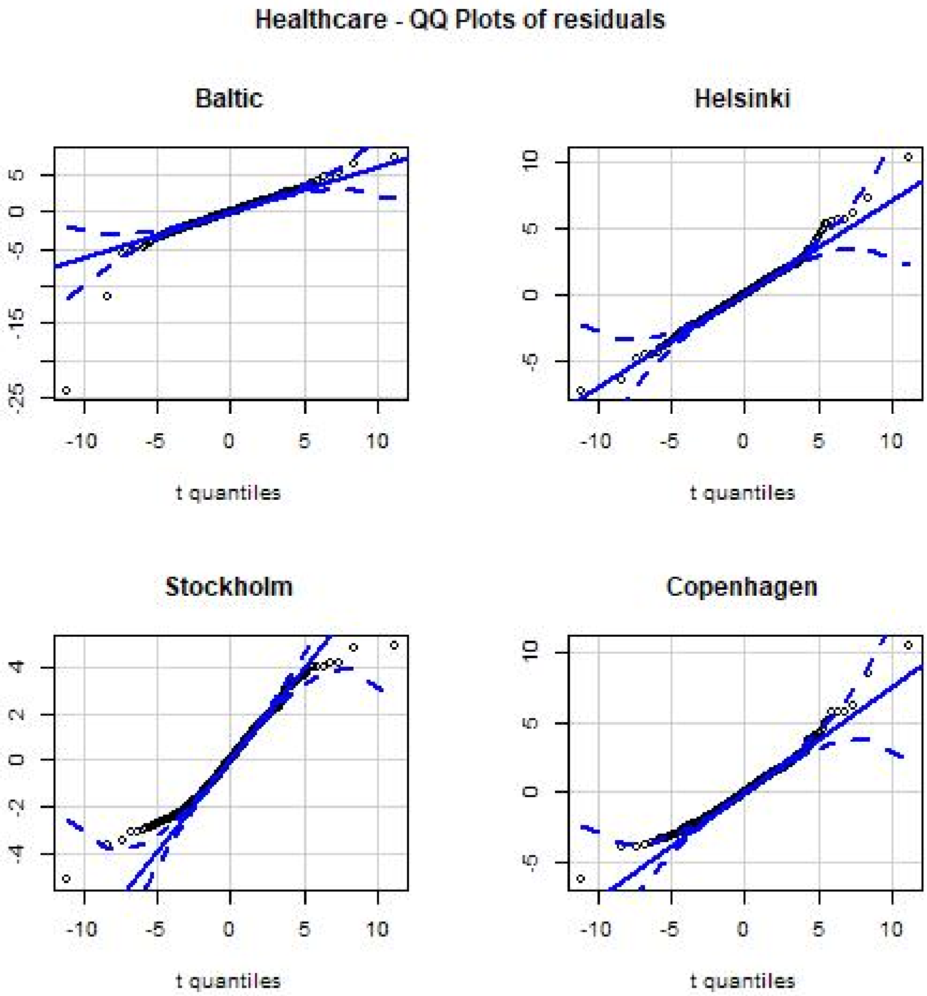

Figure 18: QQ-plots of residuals from the estimated DCC-GARCH model of returns from healthcare indices

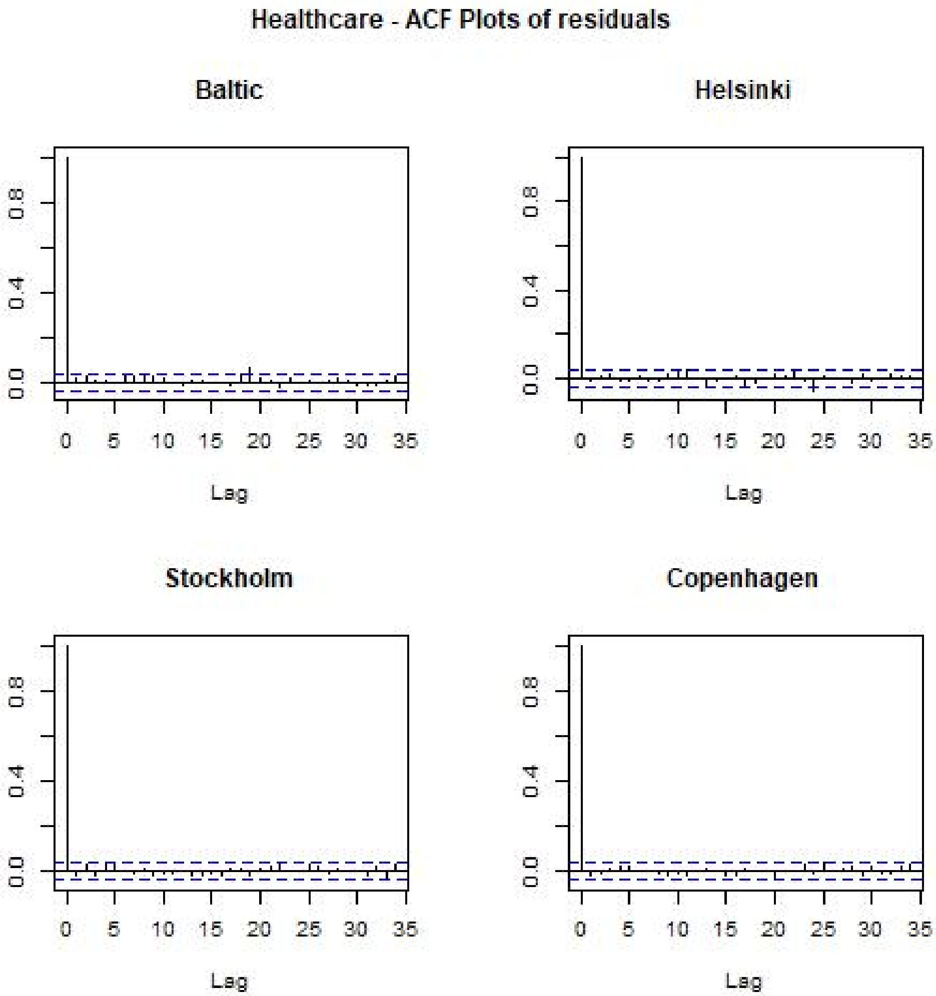

Figure 19: ACF plots of residuals from the estimated DCC-GARCH model of returns from healthcare indices

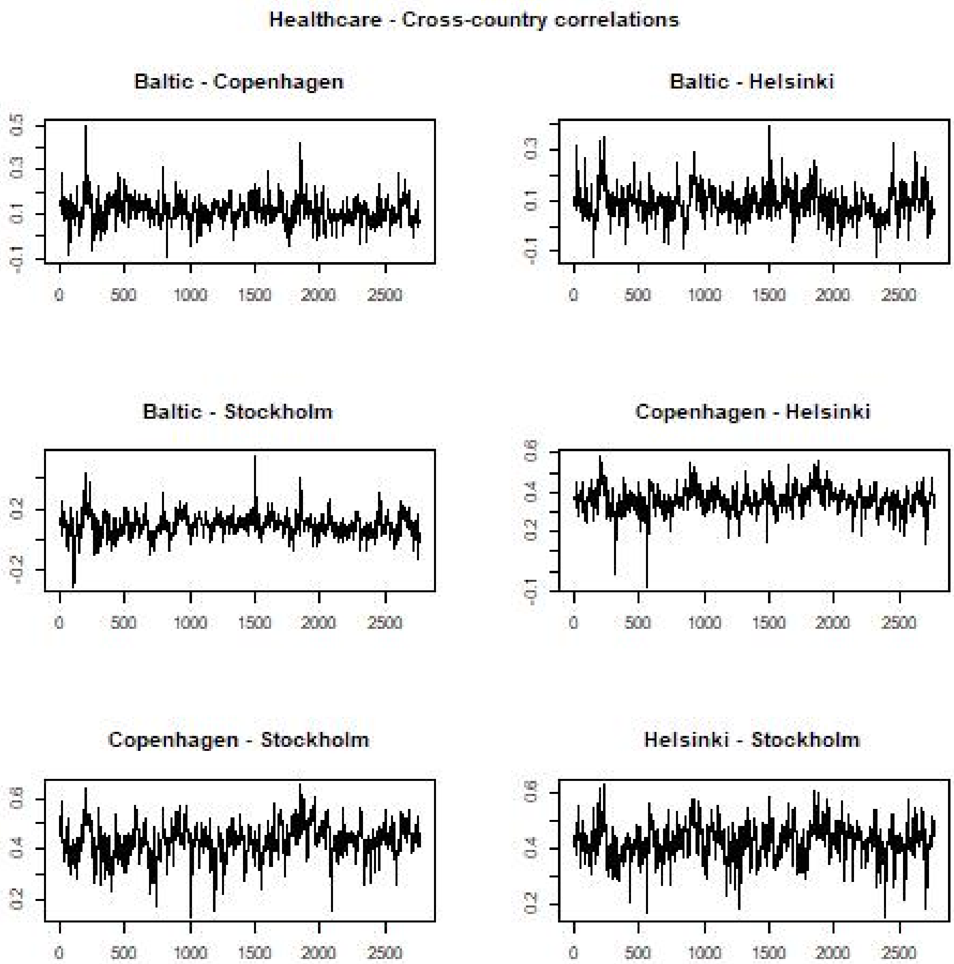

Figure 20: Cross-country conditional correlation of logarithmic returns for healthcare indices

Appendix F - Industrials

Table 15: Estimated parameters of univariate structure
of DCC-GARCH model for industrials indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0001	0.0001	-0.0006	0.0002	-0.0006	0.0003	-0.0005	0.0000
γ_i	-0.4957	0.1043	0.2175	0.1528	0.1173	0.2744	0.9932	0.0024
δ_i	0.4323	0.1090	-0.1835	0.1523	-0.0736	0.2824	-0.9991	0.0001
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.1175	0.0475	0.0676	0.0276	0.0895	0.2476	0.0883	0.0095
B_{ii}	0.8815	0.0393	0.9211	0.0327	0.9029	0.2611	0.9036	0.0102
ν_i	3.7042	0.3884	8.8384	1.5060	11.4500	10.7645	10.9277	2.3291

Note: SD stands for standard deviation of corresponding estimated parameter

Table 16: Estimated parameters of correlation structure
of DCC-GARCH model for industrials indices

	Estimate	SD
k	0.0183	0.0037
l	0.9615	0.0098
ν	7.4491	0.1168

Note: SD stands for standard deviation
of corresponding estimated parameter

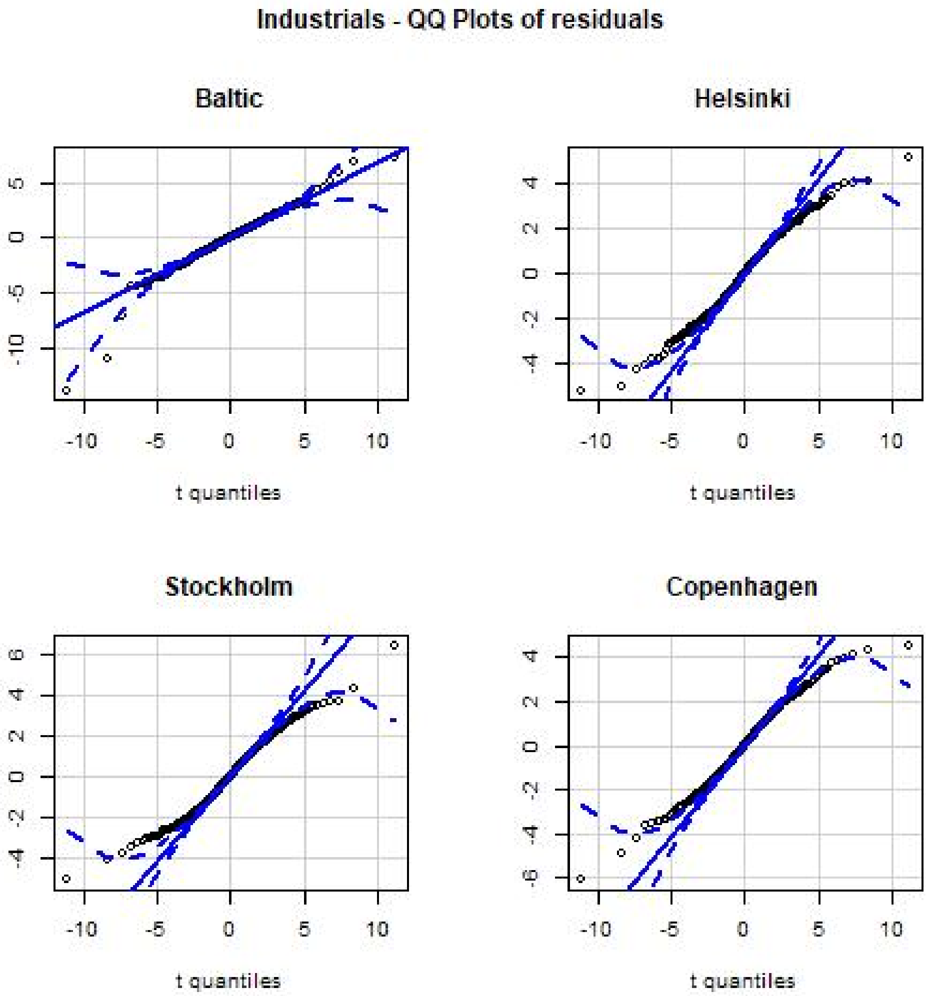

Figure 21: QQ-plots of residuals from the estimated DCC-GARCH model of returns from industrials indices

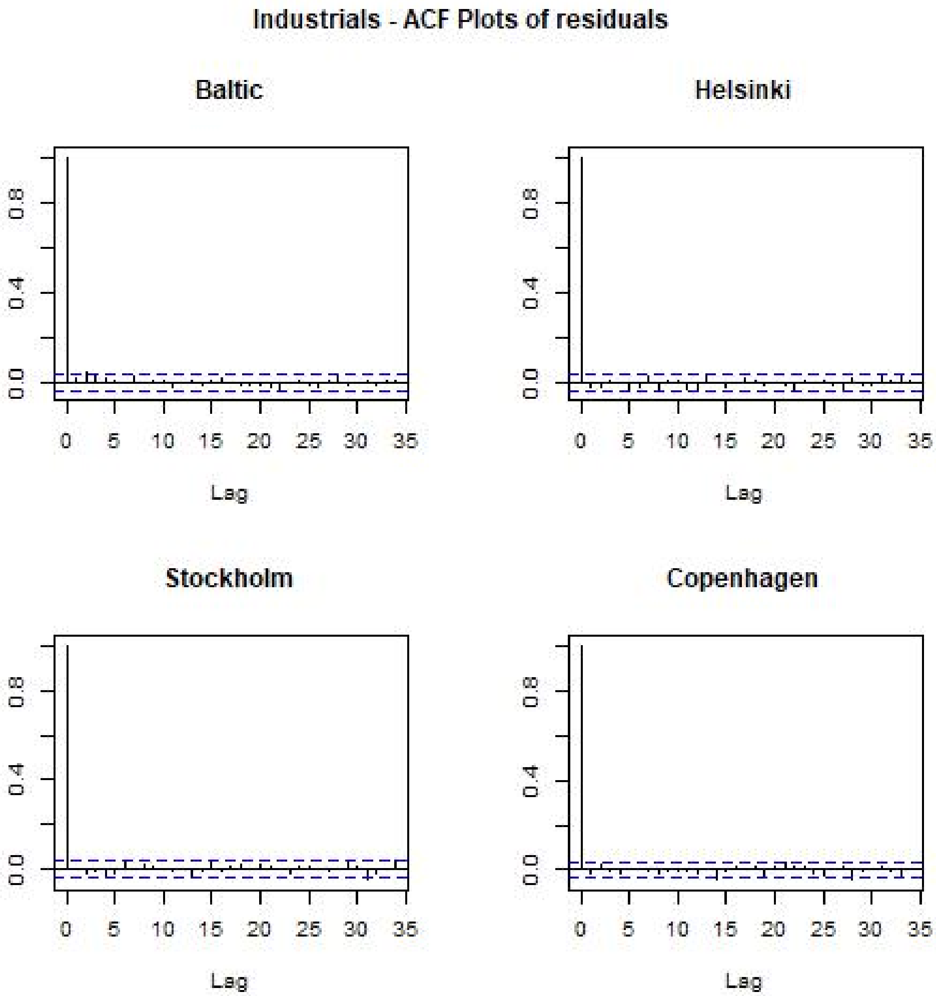

Figure 22: ACF plots of residuals from the estimated DCC-GARCH model of returns from industrials indices

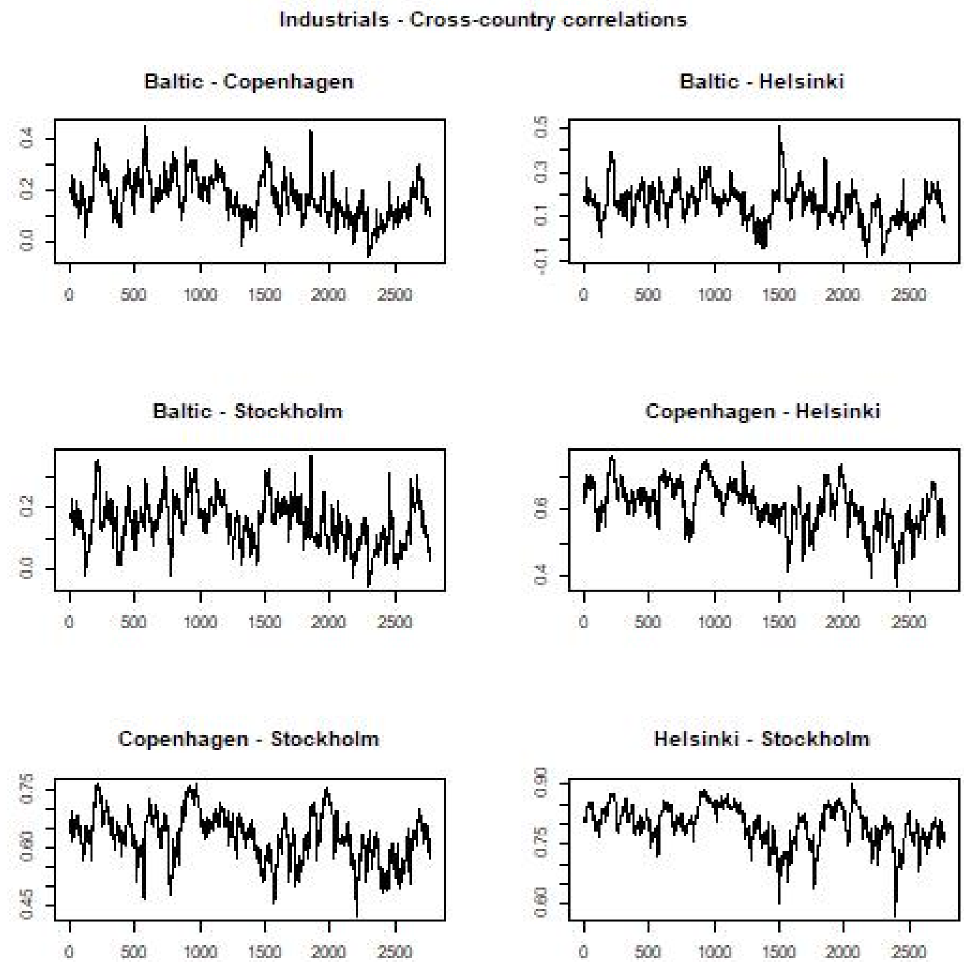

Figure 23: Cross-country conditional correlation of logarithmic returns for industrials indices

Appendix G - Personal & Household Goods

Table 17: Estimated parameters of univariate structure
of DCC-GARCH model for personal & household goods indices

	Baltic		Copenhagen		Helsinki		Stockholm	
	Estimate	SD	Estimate	SD	Estimate	SD	Estimate	SD
m_i	0.0001	0.0004	-0.0005	0.0003	-0.0005	0.0002	-0.0006	0.0002
γ_i	0.0453	0.0064	-0.2225	0.4155	-0.1542	0.1547	0.1677	0.2293
$\gamma_{2,i}$	0.9404	0.0061	-0.4453	0.3964	-0.7569	0.3026	0.4833	0.1891
δ_i	-0.0226	0.0006	0.2485	0.3975	0.1619	0.1478	-0.1838	0.2247
$\delta_{2,i}$	-0.9494	0.0001	0.4934	0.3897	0.7804	0.2875	-0.4898	0.1827
c_i	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.0492	0.0046	0.1922	0.0602	0.1590	0.0318	0.0723	0.0447
B_{ii}	0.9498	0.0051	0.6158	0.1286	0.7719	0.0415	0.9123	0.0533
ν_i	3.9992	0.3053	3.4879	0.2553	4.1231	0.5163	6.6983	1.1165

Note: SD stands for standard deviation of corresponding estimated parameter

Table 18: Estimated parameters of correlation structure
of DCC-GARCH model for personal & household goods indices

	Estimate	SD
k	0.012794	0.00854
l	0.968074	0.034315
ν	5.152326	0.181097

Note: SD stands for standard deviation
of corresponding estimated parameter

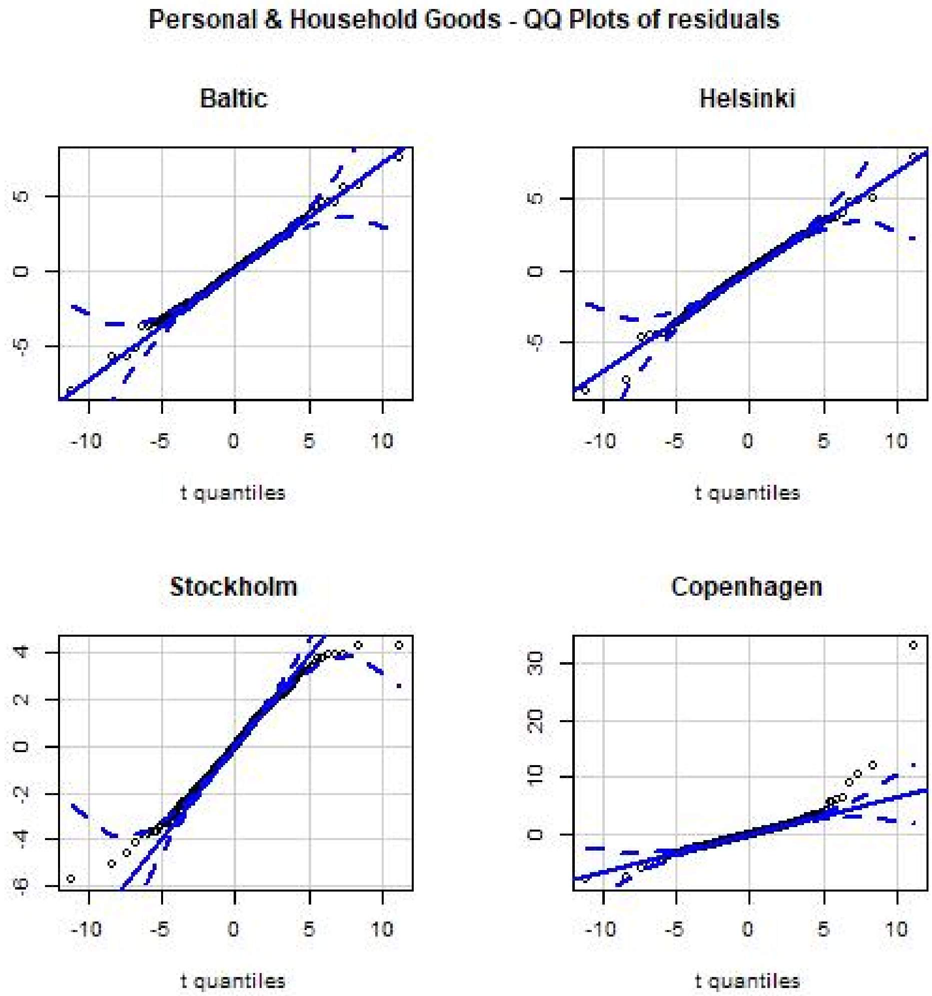

Figure 24: QQ-plots of residuals from the estimated DCC-GARCH model of returns from personal & household goods indices

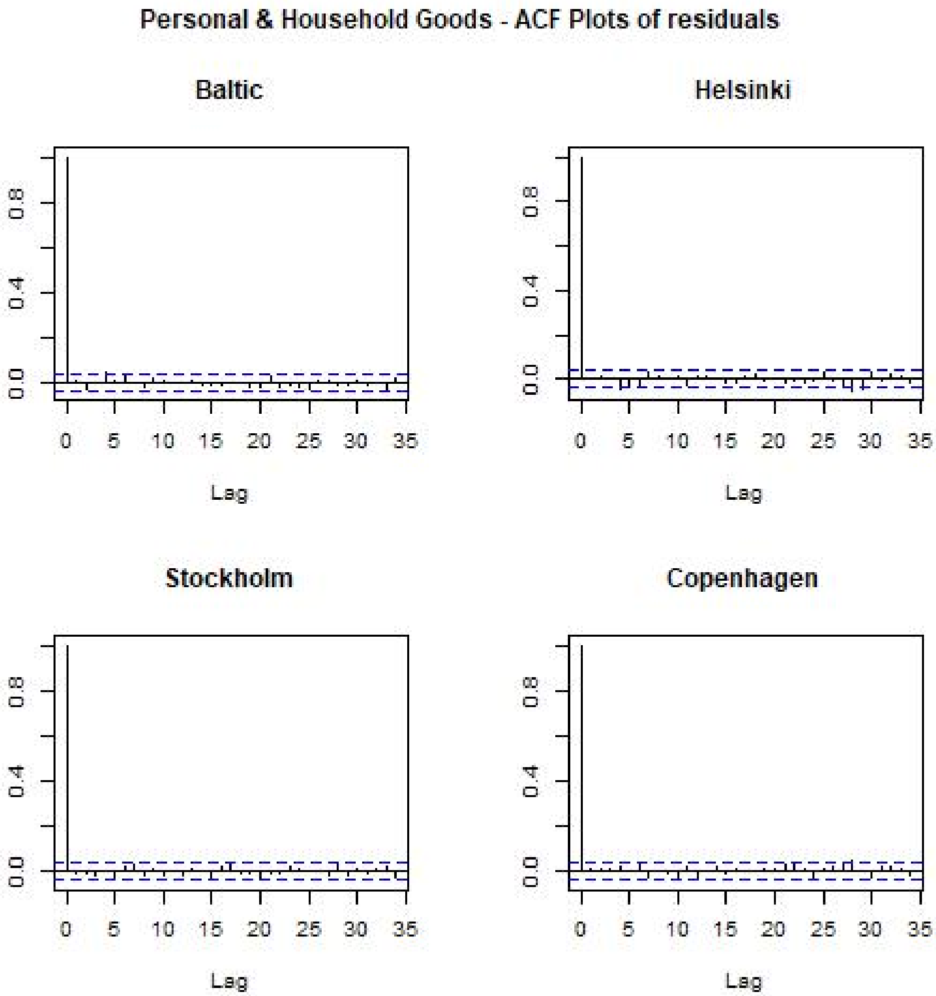

Figure 25: ACF plots of residuals from the estimated DCC-GARCH model of returns from personal & household goods indices

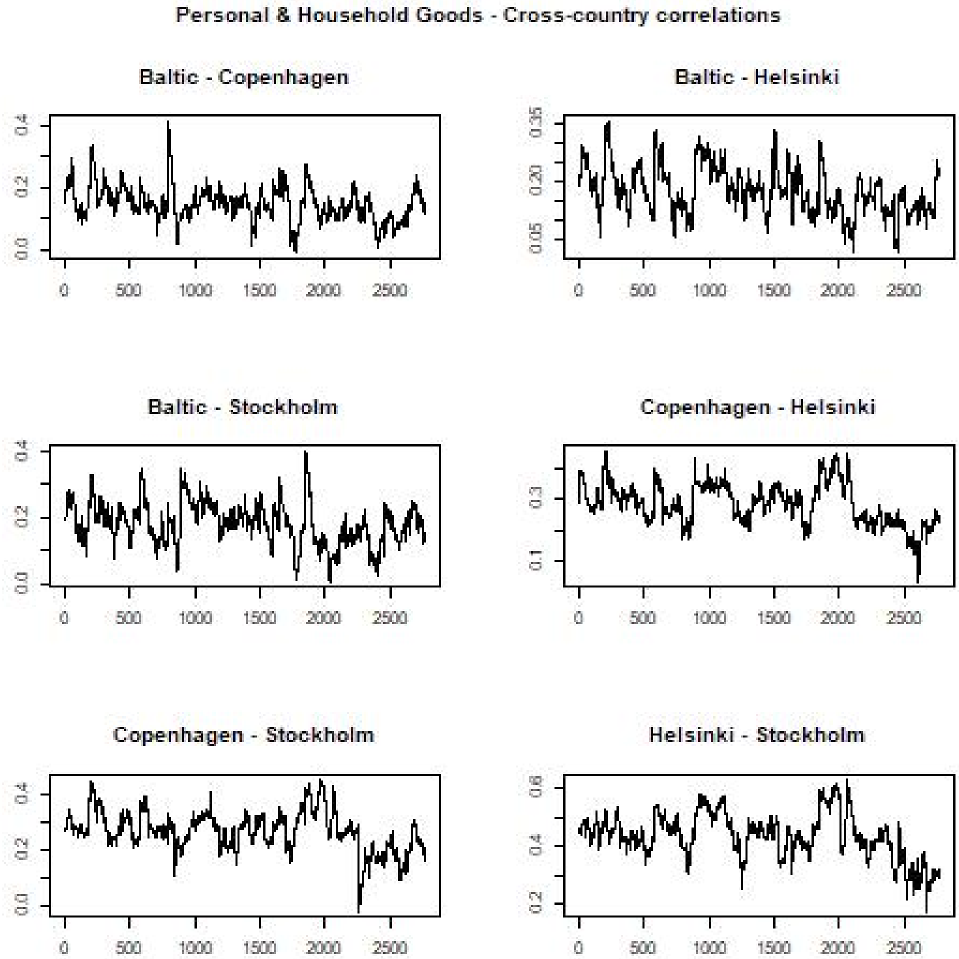

Figure 26: Cross-country conditional correlation of logarithmic returns for personal & household goods indices

Appendix H - Copenhagen

Table 19: Estimated parameters of univariate structure of DCC-GARCH model for sector indices in Copenhagen

	Banking		Con & Mat		Food & Beverages	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0002	0.0002	-0.0005	0.0003	-0.0006	0.0002
γ_i	-0.8935	0.1359	0.3848	0.1316	0.8080	0.0592
δ_i	0.9063	0.1267	-0.3303	0.1330	-0.8278	0.0551
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.0991	0.0188	0.0971	0.0580	0.0398	0.0112
B_{ii}	0.8856	0.0252	0.8822	0.0634	0.9556	0.0083
ν	6.6422	0.7672	5.1072	2.0201	5.0249	1.1329

	Healthcare		Industrials		Personal & HH Goods	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0009	0.0002	-0.0006	0.0002	-0.0005	0.0003
γ_i	0.8774	0.0511	0.2176	0.1528	0.6481	0.3387
δ_i	-0.8988	0.0457	-0.1836	0.1525	-0.6158	0.3502
c_i	0.0000	0.0000	0.0000	0.0000	0.0001	0.0001
A_{ii}	0.0913	0.0063	0.0676	0.0279	0.1857	0.0622
B_{ii}	0.8471	0.0110	0.9211	0.0330	0.6251	0.1406
ν	4.9861	0.4596	8.8408	1.5013	3.4788	0.2666

Note: SD stands for standard deviation of corresponding estimated parameter

Table 20: Estimated parameters of correlation structure of DCC-GARCH model for sector indices in Copenhagen

	Estimate	SD
k	0.0112	0.0027
l	0.9709	0.0099
ν	5.9056	0.2152

Note: SD stands for standard deviation of corresponding estimated parameter

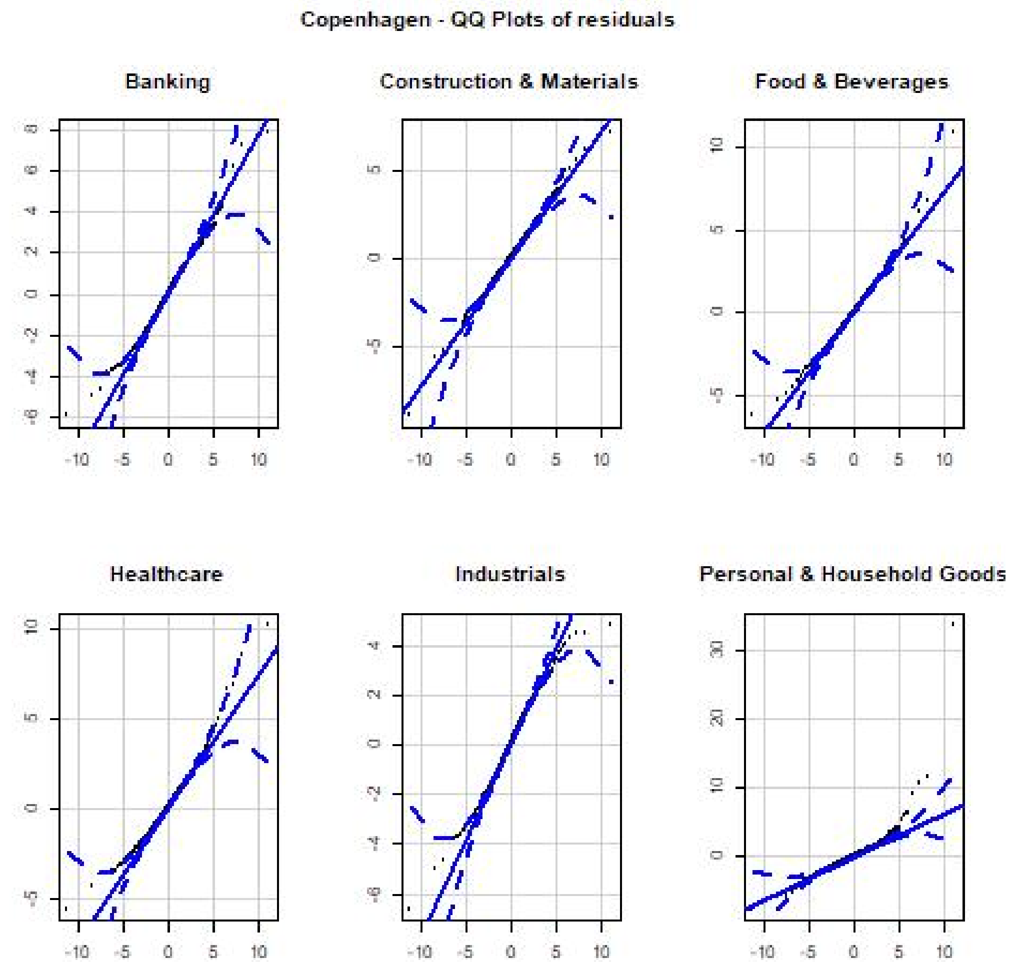

Figure 27: QQ-plots of residuals from the estimated DCC-GARCH model of returns from Copenhagen sector indices

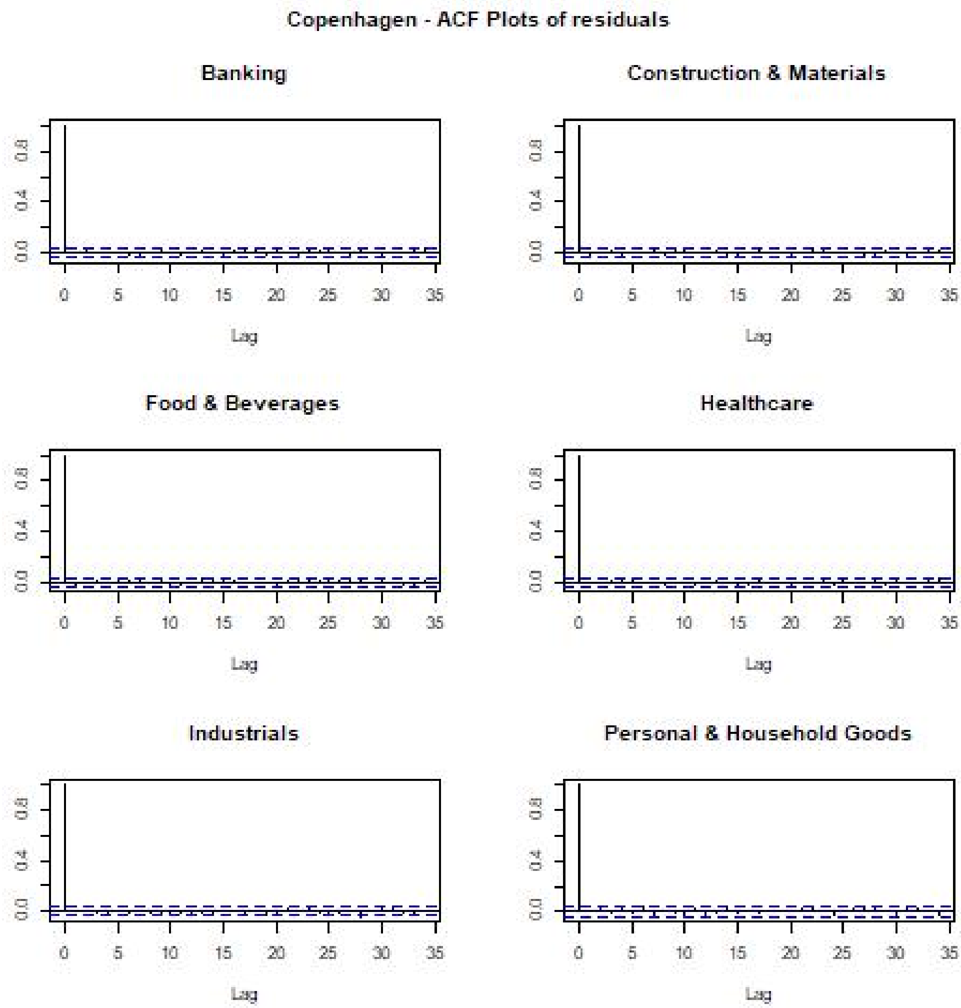

Figure 28: ACF plots of residuals from the estimated DCC-GARCH model of returns from Copenhagen sector indices

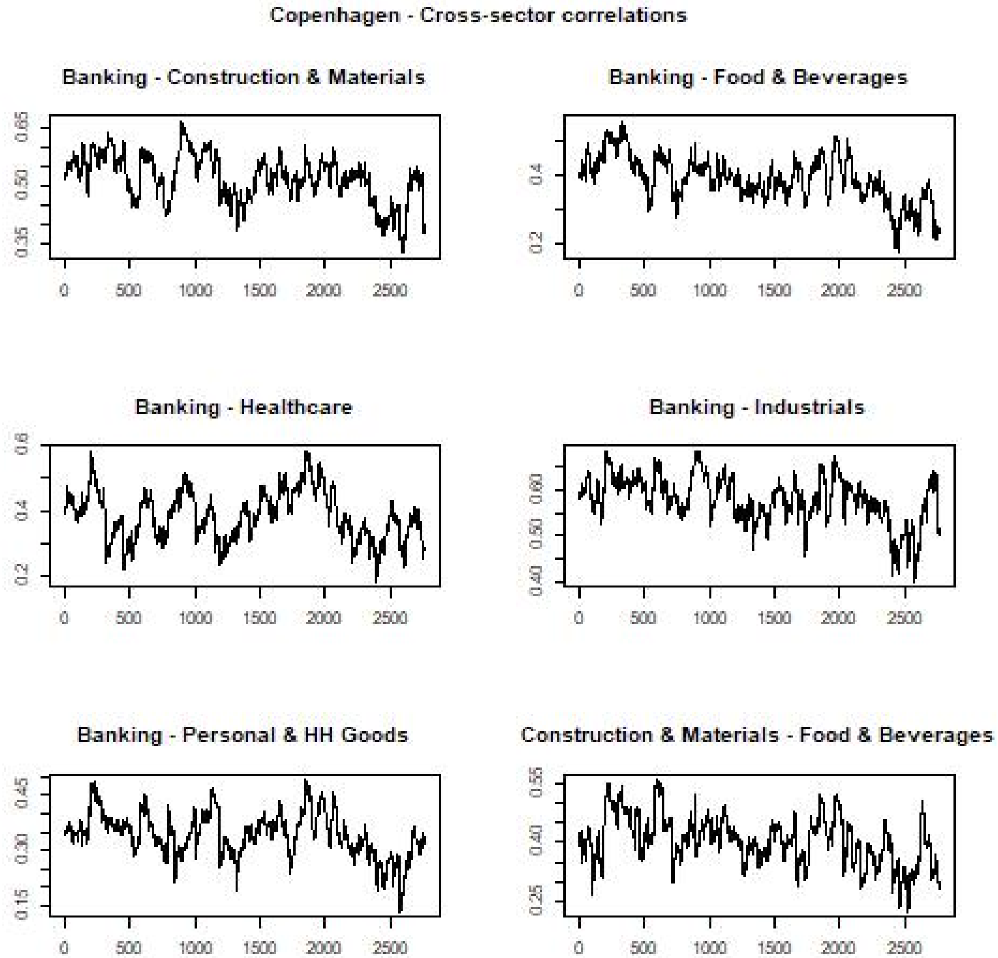

Figure 29: Cross-country conditional correlation of logarithmic returns from Copenhagen sector indices - part 1

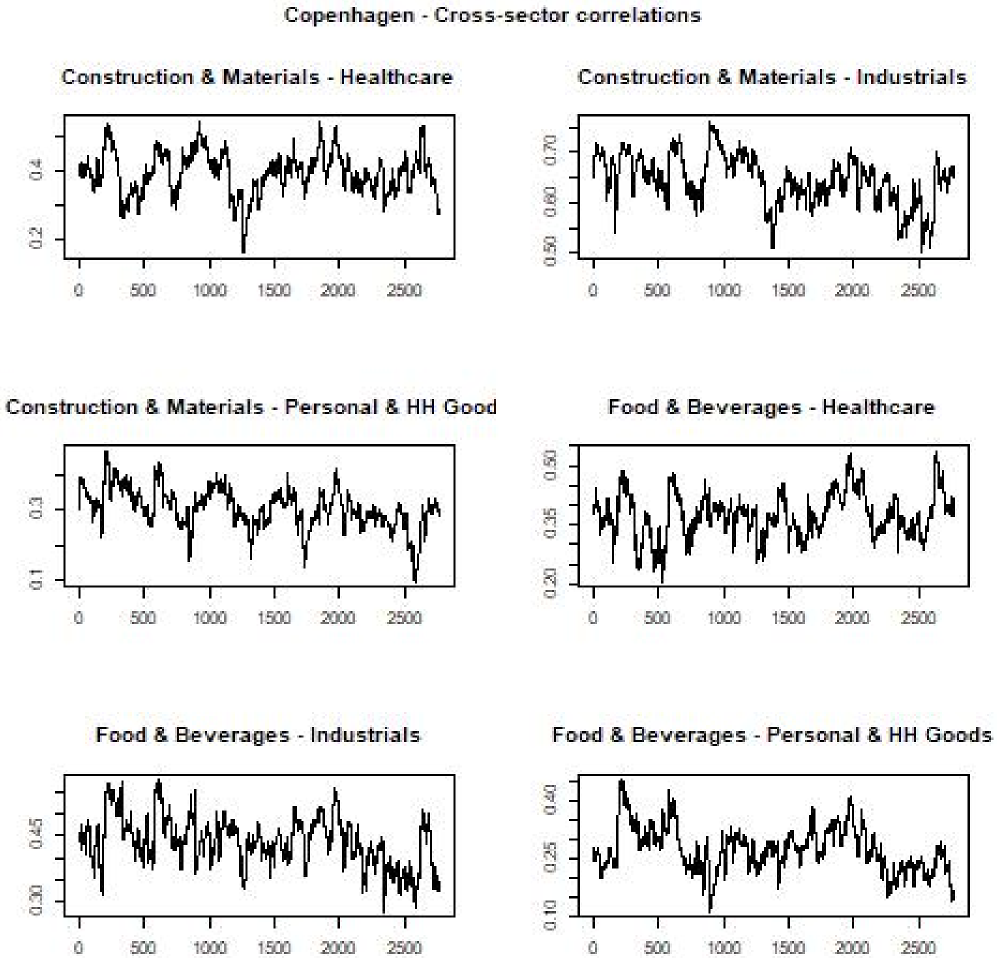

Figure 30: Cross-country conditional correlation of logarithmic returns from Copenhagen sector indices - part 2

Appendix I - Helsinki

Table 21: Estimated parameters of univariate structure of DCC-GARCH model for sector indices in Helsinki

	Banking		Con & Mat		Food & Beverages	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0005	0.0002	-0.0002	0.0003	-0.0002	0.0002
γ_i	0.8989	0.0146	0.3553	0.0997	0.5992	0.1815
δ_i	-0.9161	0.0108	-0.2720	0.1013	-0.5689	0.1855
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.0658	0.0125	0.0672	0.0171	0.1416	0.0403
B_{ii}	0.9308	0.0134	0.9241	0.0196	0.8220	0.0677
ν	6.6385	0.8004	5.7043	0.6053	4.6382	0.9720

	Healthcare		Industrials		Personal & HH Goods	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0006	0.0002	-0.0006	0.0004	-0.0005	0.0002
γ_i	0.2056	0.6223	0.1252	0.2870	0.2749	0.1681
δ_i	-0.2166	0.6217	-0.0813	0.2991	-0.2517	0.1680
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.1293	0.0435	0.0898	0.3162	0.1593	0.0308
B_{ii}	0.7617	0.0900	0.9025	0.3335	0.7725	0.0401
ν	4.2220	0.3465	11.3812	13.3123	4.1102	0.4974

Note: SD stands for standard deviation of corresponding estimated parameter

Table 22: Estimated parameters of correlation structure of DCC-GARCH model for sector indices in Helsinki

	Estimate	SD
k	0.0065	0.0014
l	0.9881	0.0036
ν	6.0735	0.1777

Note: SD stands for standard deviation of corresponding estimated parameter

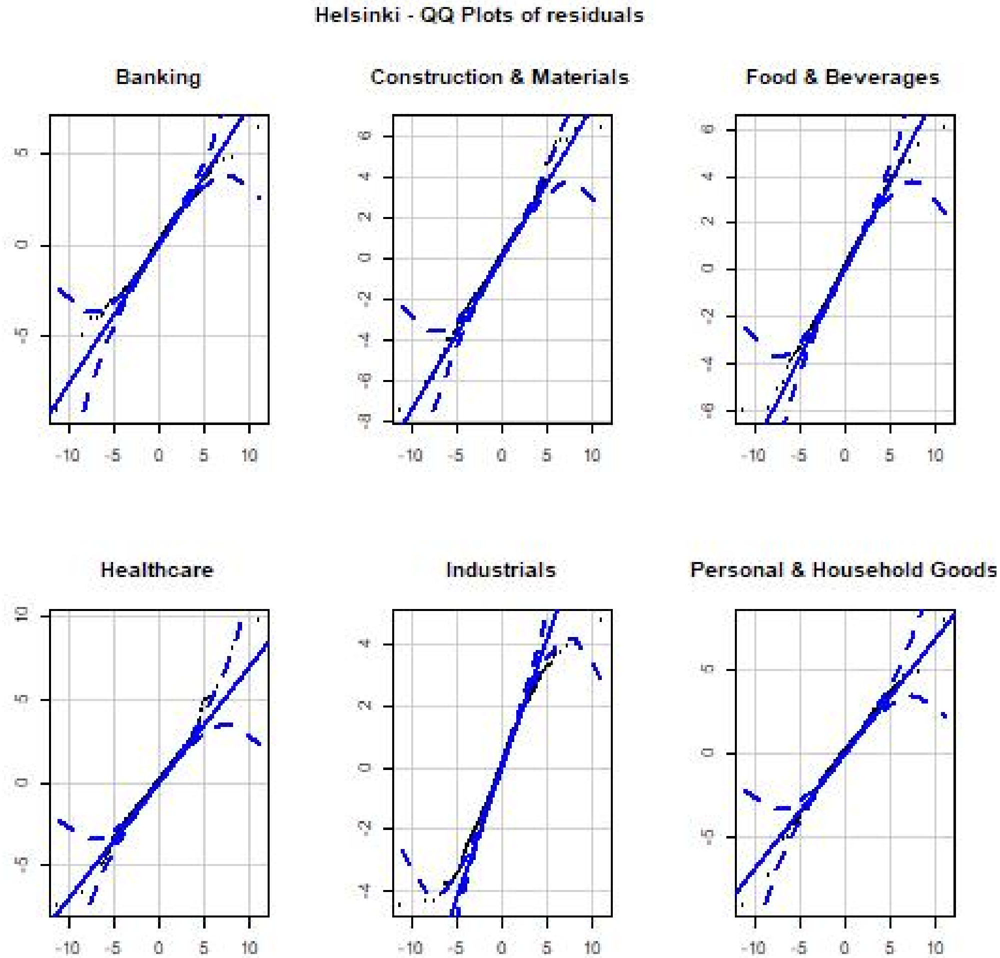

Figure 31: QQ-plots of residuals from the estimated DCC-GARCH model of returns from Helsinki sector indices

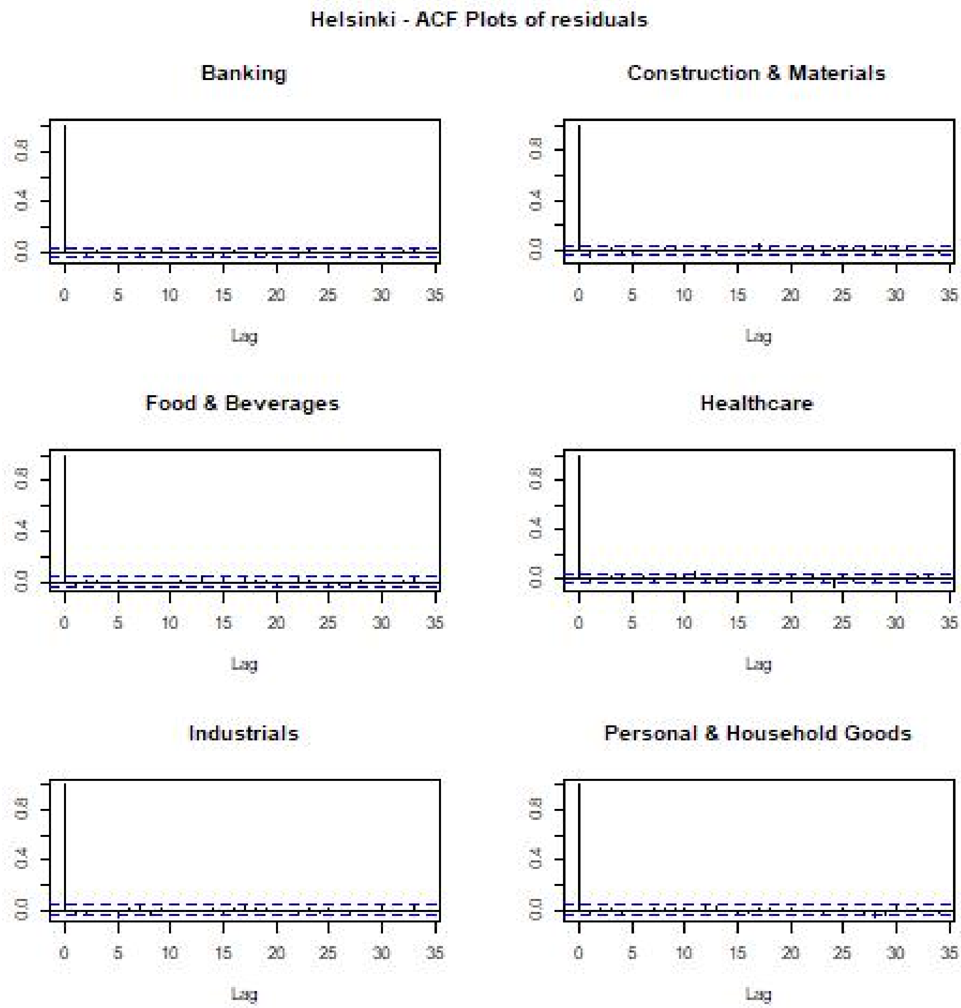

Figure 32: ACF plots of residuals from the estimated DCC-GARCH model of returns from Helsinki sector indices

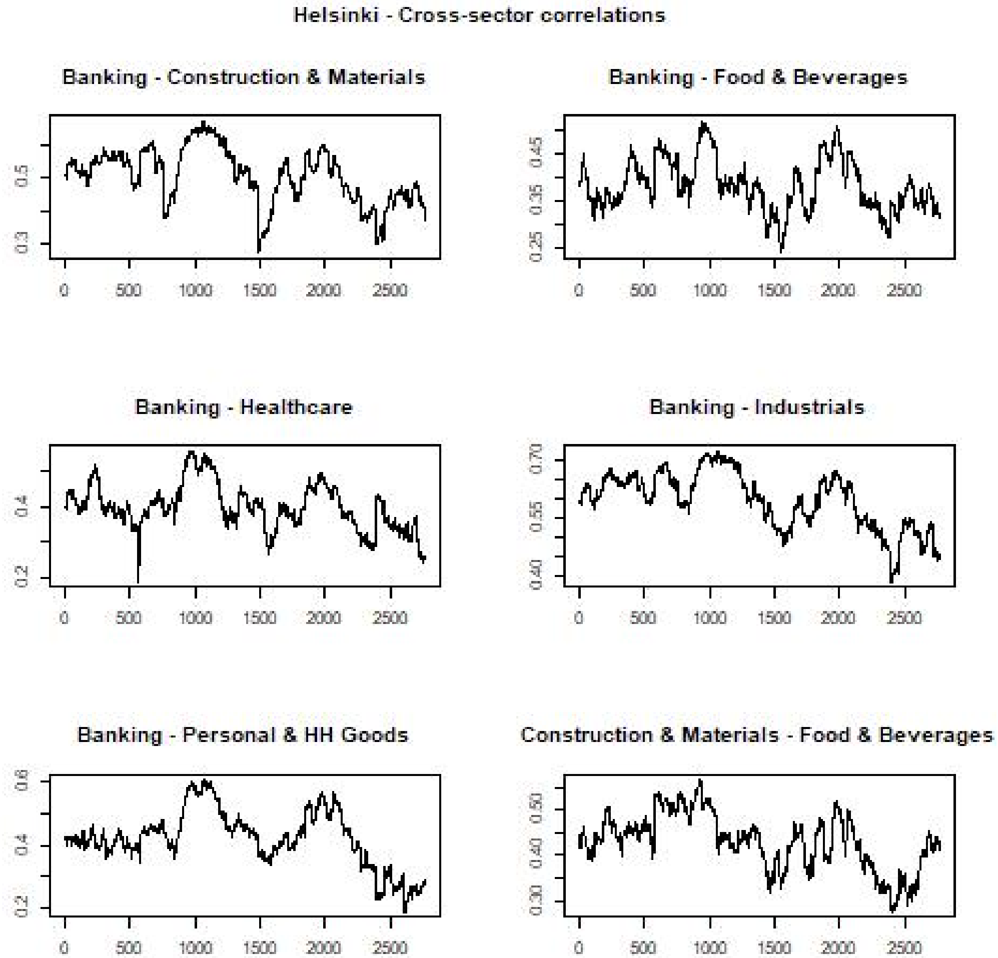

Figure 33: Cross-country conditional correlation of logarithmic returns from Helsinki sector indices - part 1

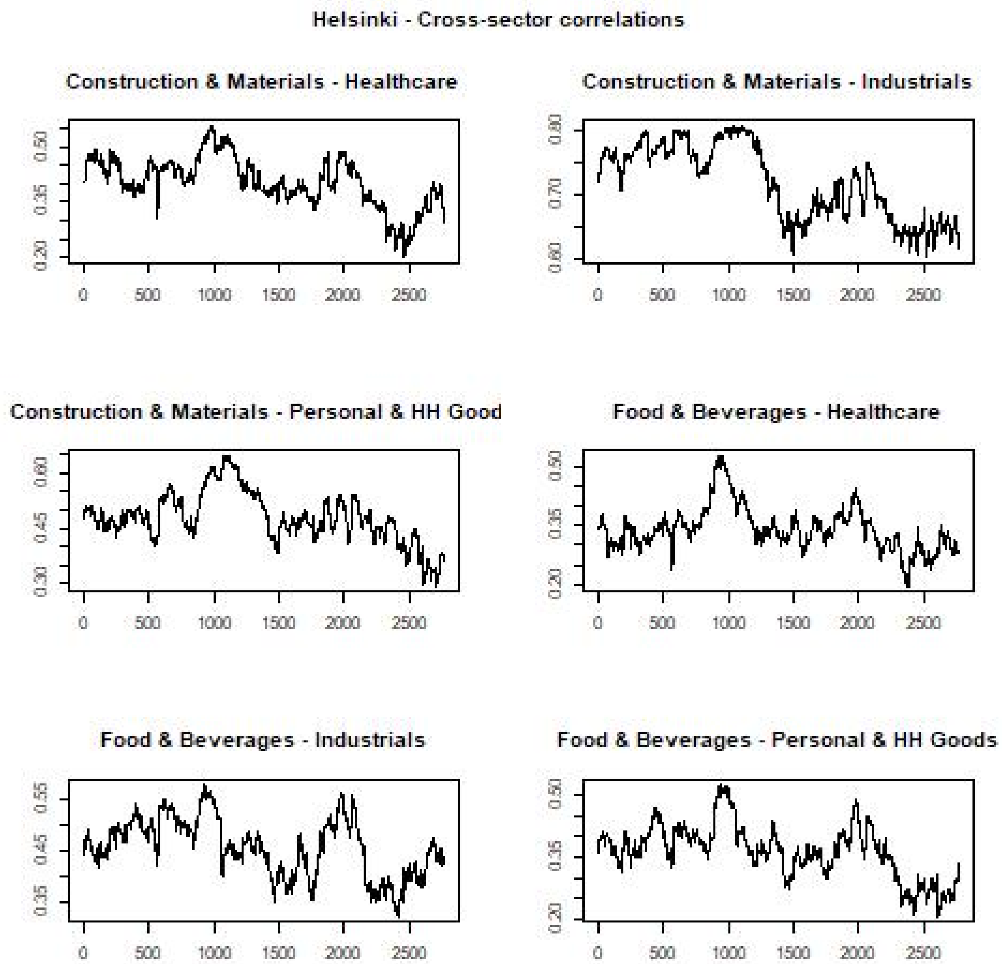

Figure 34: Cross-country conditional correlation of logarithmic returns from Helsinki sector indices - part 2

Appendix J - Stockholm

Table 23: Estimated parameters of univariate structure of DCC-GARCH model for sector indices in Stockholm

	Banking		Con & Mat		Food & Beverages	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0006	0.0002	-0.0008	0.0002	-0.0007	0.0002
γ_i	0.8835	0.0817	0.8961	0.0708	-0.1246	0.6383
δ_i	-0.9012	0.0755	-0.9018	0.0689	0.1455	0.6355
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.0739	0.0207	0.0768	0.0704	0.1349	0.0098
B_{ii}	0.9198	0.0223	0.9106	0.0805	0.7871	0.0141
ν	7.2888	0.9799	7.4117	1.6967	6.5611	0.7231

	Healthcare		Industrials		Personal & HH Goods	
	Estimate	SD	Estimate	SD	Estimate	SD
m_i	-0.0008	0.0002	-0.0005	0.0000	-0.0006	0.0002
γ_i	0.3439	0.7262	0.9932	0.0024	0.7756	0.1808
δ_i	-0.3190	0.7328	-0.9991	0.0001	-0.7900	0.1757
c_i	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
A_{ii}	0.1058	0.0153	0.0883	0.0095	0.0723	0.0438
B_{ii}	0.8610	0.0286	0.9036	0.0101	0.9123	0.0520
ν	6.6590	0.9309	10.9238	2.3415	6.6869	1.0835

Note: SD stands for standard deviation of corresponding estimated parameter

Table 24: Estimated parameters of correlation structure of DCC-GARCH model for sector indices in Stockholm

	Estimate	SD
k	0.0142	0.0021
l	0.9713	0.0059
ν	7.0841	0.2471

Note: SD stands for standard deviation of corresponding estimated parameter

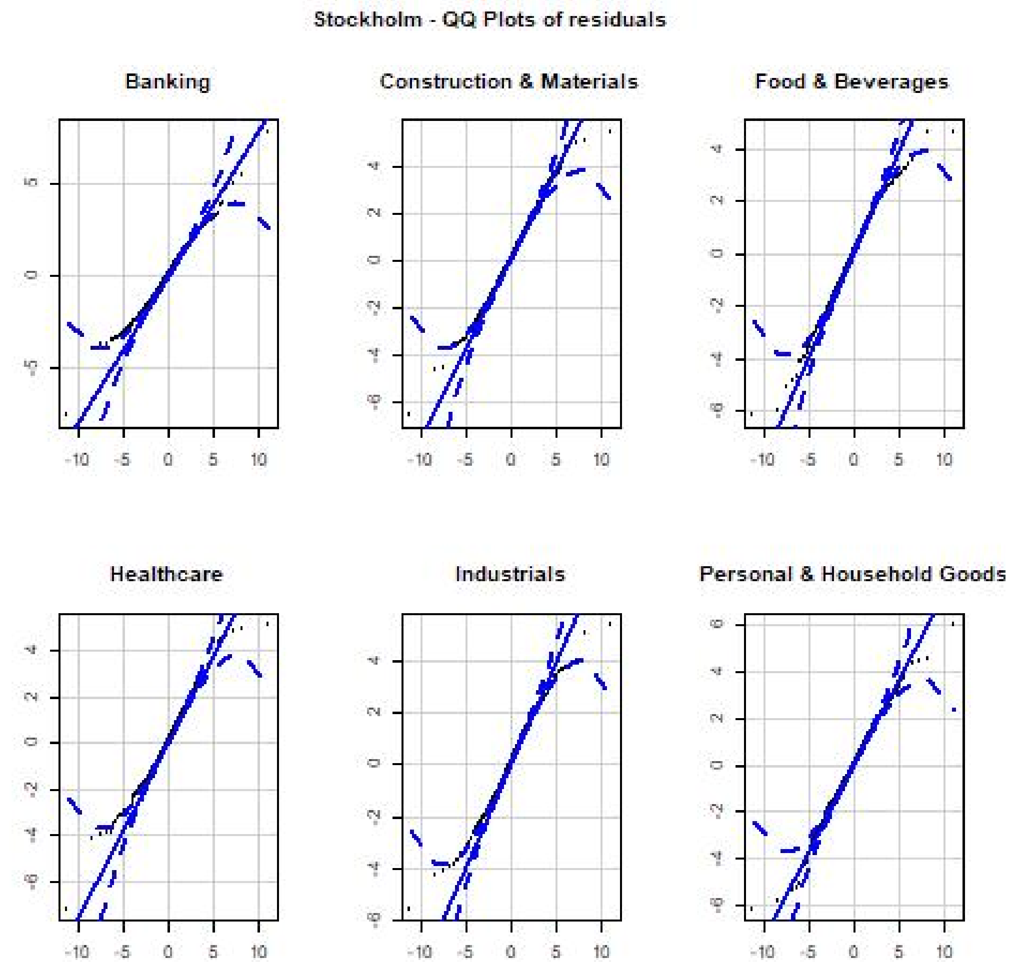

Figure 35: QQ-plots of residuals from the estimated DCC-GARCH model of returns from Stockholm sector indices

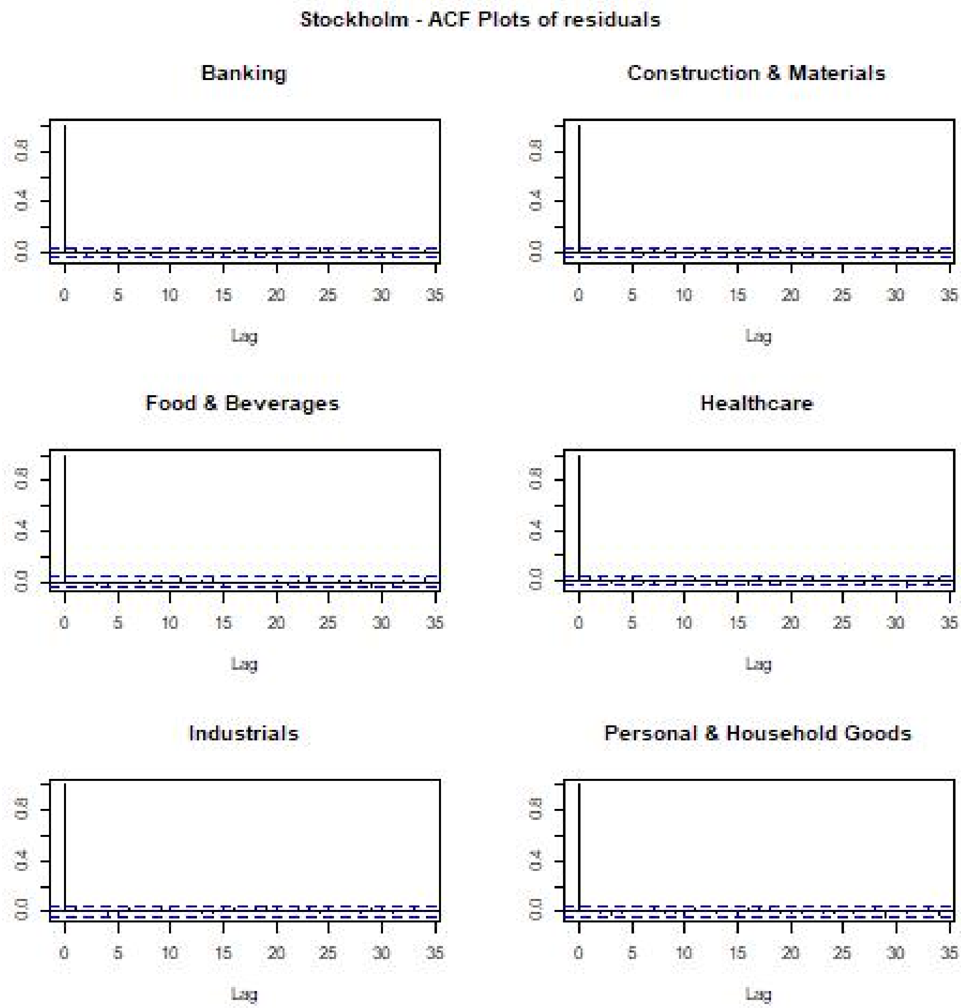

Figure 36: ACF plots of residuals from the estimated DCC-GARCH model of returns from Stockholm sector indices

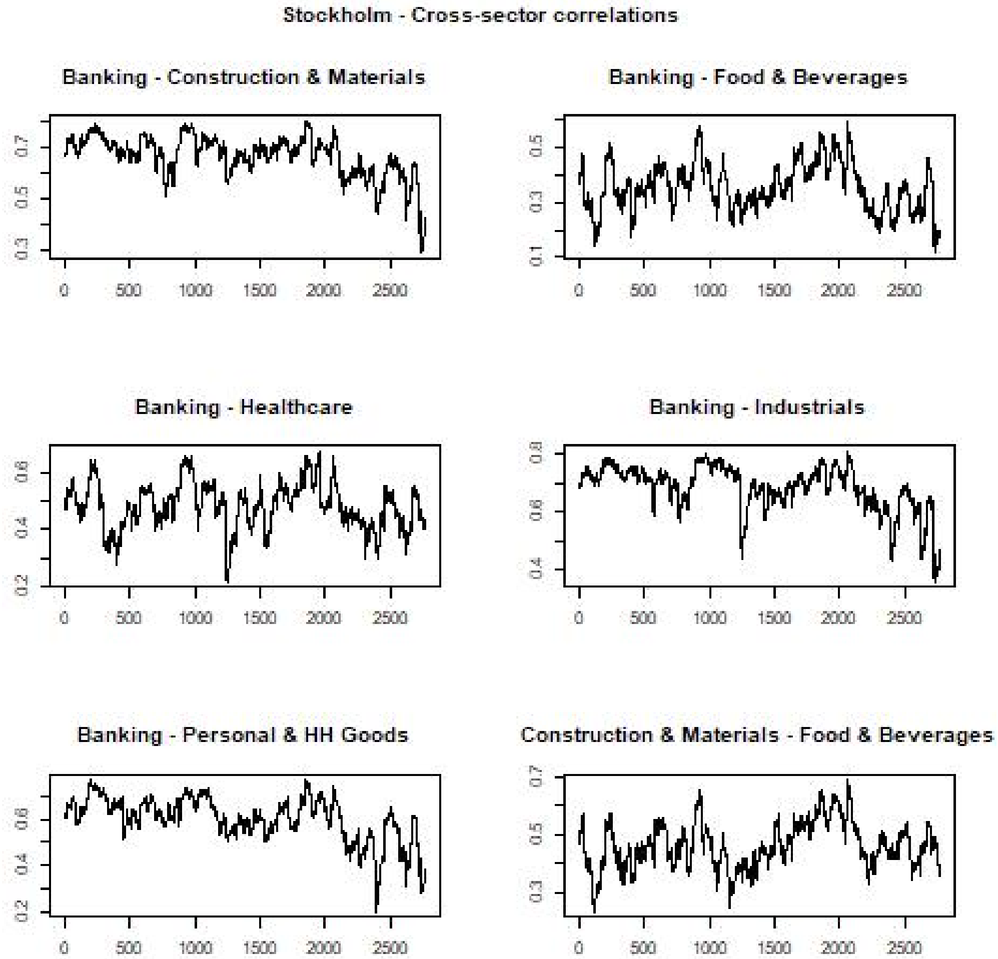

Figure 37: Cross-country conditional correlation of logarithmic returns from Stockholm sector indices - part 1

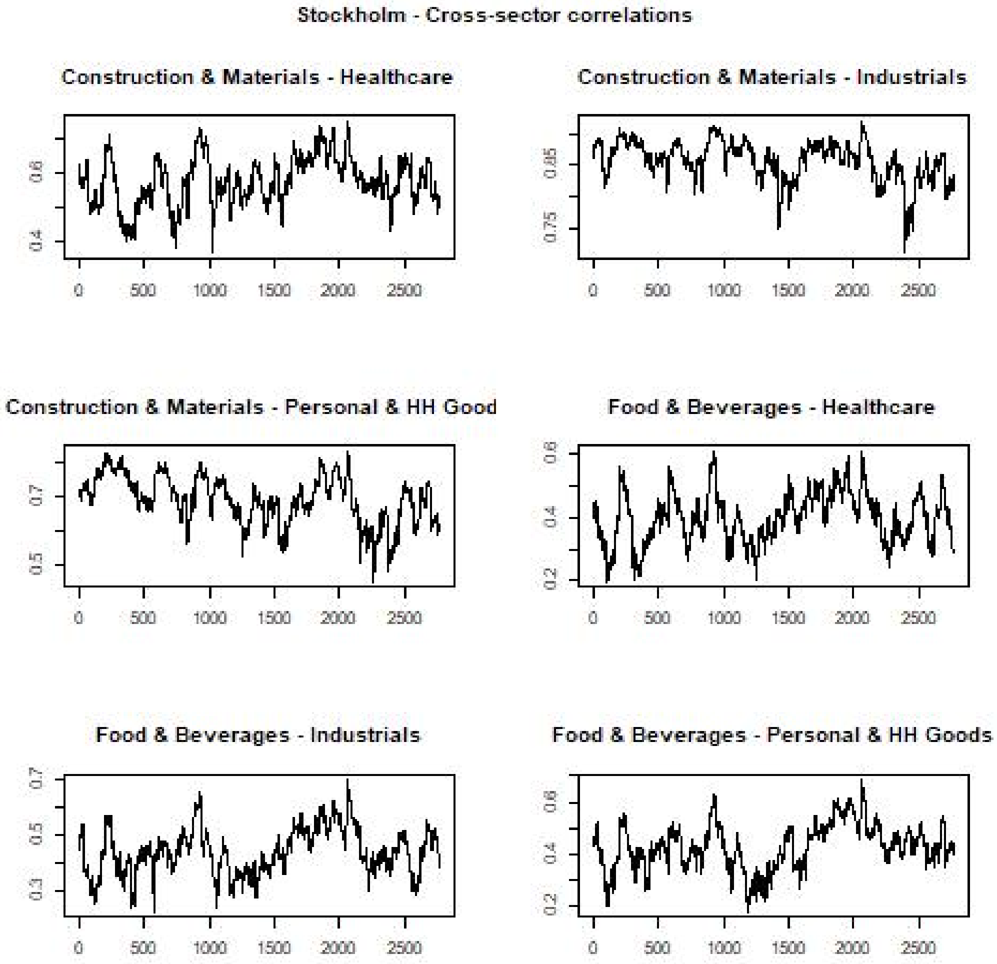

Figure 38: Cross-country conditional correlation of logarithmic returns from Stockholm sector indices - part 2