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Asset prices and macroeconomics: towards a unified macro-finance framework

Dissertation

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Abstract

The dissertation consists of three papers focused on fiscal policy and explaining what determines the dynamics of cross-sectional distribution of bond prices. The connecting factor of the thesis is however not just its main theme but also the used methodology. The valuation of bonds and effects of studied policies are endogenous outcome of the full-fledged macro-finance dynamic stochastic general equilibrium model.

The first chapter provides broader context and non-technical summary of the three papers in following chapters. The first paper studies the role of trend inflation in bond pricing. Motivated by recent empirical findings that emphasize low-frequency movements in inflation as a key determinant of term structure, we introduce trend inflation into the workhorse macro-finance model. We show that this compromises the earlier model success and delivers implausible business cycle and bond price dynamics. We document that this result applies more generally to non-linearly solved models with Calvo pricing and trend inflation and is driven by the behavior of price dispersion, which is *i*) counterfactually high and *ii*) highly inaccurately approximated. We highlight the channels behind the undesired performance under the trend inflation and show that several modeling features like price indexation or Rotemberg pricing can restore the model performance.

The second paper highlights how different types of government expenditures affect term structure of interest rates. We explore asset pricing implications of productive, wasteful and utility enhancing government expenditures in a New Keynesian macro-finance model with Epstein-Zin preferences. We decompose the pricing kernel into four underlying macroeconomic factors (consumption growth, inflation, time preference shocks, long run risks for consumption and leisure) and design novel method to quantify the contribution of each factor to bond prices. Our methodology extends the performance attribution analysis typically used in finance literature on portfolio analysis. Using this framework, we show that the property of bonds to serve as an insurance vehicle against the fluctuations in investors wealth induced by government spending is the main component in bond valuation. Increase in uncertainty surrounding government spending rises the demand for bonds leading to decrease in yields over the whole maturity profile. Bonds insure investors by *i*) providing buffer against bad times, *ii*) hedging inflation risk and *iii*) hedging real risks by putting current consumption gains against future losses. We also document that the

structure of government spending and related response of monetary policy is consequential for compensation investors require for holding bonds.

In the third paper we generalize a simple New Keynesian model and show that a flattening of the Phillips curve reduces the size of fiscal multipliers at the zero lower bound (ZLB) on the nominal interest rate. The factors behind the flatting are consistent with micro and macroeconomic empirical evidence: it is a result of, not a higher level of price rigidity, but an increase in the degree of strategic complementarity in price-setting – invoked by the assumption of a specific instead of an economy-wide labour market, and decreasing instead of constant-returns-to-scale. In normal times, the efficacy of fiscal policy and resulting multipliers tends to be small because negative wealth effects crowd out consumption, and because monetary policy endogenously reacts to fiscally-driven increases in inflation and output by raising rates, offsetting part of the stimulus. In times of a binding ZLB and a fixed nominal rate, an increase in (expected) inflation instead lowers the real rate, leading to larger fiscal multipliers. Conditional on being in a ZLB-environment, under a flatter Phillips curve, increases in expected inflation are lower, so that fiscal multipliers at the ZLB tend to be lower. Finally, we also discuss the role of solution methods in determining the size of fiscal multipliers.

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Chapter 1

Introduction: Three Essays on Understanding Bond Price Valuation and Fiscal Policy

1.1 Introduction

There has been enormous effort in many fields in economics and finance to understand and model the cross-sectional bond price observations jointly with its intertemporal dynamics. This effort has been largely motivated from macro perspective by the fact that the term structure of interest rate predicts surprisingly well recessions and it stands at the heart of monetary policy transmission. From finance perspective yield curve serves as a fundamental input in pricing of both financial and non-financial assets.

Term-structure modeling refers to two distinct, albeit related, problems (see figure 1.1). The first problem involves fitting a zero-coupon interest rate curve to a set of cross-sectional bond price observations. The second problem relates to the specification of the intertemporal dynamics of the entire term structure of interest rates. These two dimensions are not independent which significantly increases the complexity of both modeling and understanding its shape and dynamics. The figure 1.1 highlights three important stylized facts motivating much of the research in this dissertation. First, the term structure of interest rates is on average upward sloping and thus holing long term bonds pays term premium. Second, the term structure of interest rates features time varying trend component. Third, most of variation across time happens at business cycle frequency and thus is likely related to the business cycle fluctuations of macro-economy. However, despite both theoretical and empirical progress, there is no clear consensus about how macroeconomic information should be incorporated. The commonly used workhorse macro modeling framework (DSGE models) to provide policy guidance largely fails to explain bond yields in both dimensions: across time and section.

The inability of consumption based asset pricing models to explain why is the yield curve upward sloping was first documented by Backus *et al.* (1989) in the endowment economy and termed as "bond premium puzzle". Consequently, Den Haan (2005) confirms the presence of bond premium puzzle in the state of the art real business cycle model and Rudenbush & Swanson (2008) in the New Keynesian model. For instance, the celebrated models of Christiano *et al.* (2005) and Smets & Wouters (2007) imply term premia close to zero or negative as opposed to the empirically observed one which lies above 100 basis points. The failure of macroeconomic models to match even basic asset pricing stylized facts gave rise to famous critique by Cochrane (2005) who sees this as a sign of fundamental flaws of these models.

Cochrane (2005) argues that asset markets are the mechanism by which consumption and investment are allocated across time and states of nature by equating marginal rates of substitution and transformation. Asset prices are thus a key element in determining the state and dynamics of real economy. The existing models utilized for policy decision making can match the dynamics of the economy sufficiently well but fail in terms of asset prices. This is not just a sign that the model itself is flawed or at least incomplete but it brings important disadvantage from the practical point of view. Many interesting and for policy crucial questions come from the interaction between macroeconomic variables and asset prices - both the effects of asset prices on macro variables and the effects of interest rates and other macro variables on asset prices. For example, a recent rise in the importance of stress testing exercises is difficult to model without having unified framework for macroeconomic quantities and asset prices. In addition, there are many questions like understanding how the bond yield "conundrum" affects the economy or questions related to sovereign debt management which are likewise impossible to analyze in the framework which cannot explain jointly macro an finance stylized facts.

Recent advancements in literature have shown that by adding new modeling features to standard New Keynesian model some of this criticism can be be remedied. These modeling features lie in the first place on the modifications to consumer preferences either in the form of habits in consumption or time non-separable preferences as in Epstein & Zin (1989). In addition, time varying inflation target or preference shocks has provided to be useful in generating positive nominal term premium without distorting the model performance in matching marco stylized facts (Rudebusch & Swanson (2012), Andreasen *et al.* (2018)).

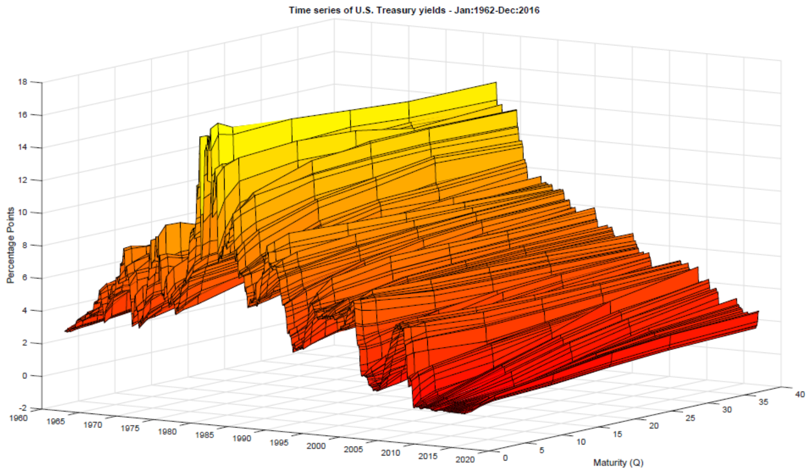
This dissertation collects three papers which extends the previous literature by aiming to improve current DSGE modeling framework along both dimensions - across time and maturity. This section provides short summary of these papers. The first paper was published as National Bank of Slovakia and University of Vienna Working Paper and received positive detailed referee reports from the two leading figures in the related field[1]. The second paper was published as National Bank of Slovakia Working Paper and was presented (among many others) at the Society for Economic Dynamics conference. The third paper was published at Journal of Macroeconomics and presented at European Economic Association and Royal Economic Society conference.

This dissertation is rather a selection than collection of my research. In the last decade I have developed with my coauthors baseline modeling framework which allows us to study the interaction between the real economy and asset prices. Consequently, we continued in more independent way and each of us has focused more attention on individual research studies. The selected papers in this dissertation are based primary on my contribution to their accomplishment and less on the topical integrity. My contribution to all three paper included in this dissertation has been substantial and dominant.

The first paper is motivated by the application of macro-finance model to generate stress test scenarios for macroprudential policy. The yield curve plays a crucial role in financial stability assessment. In recent years, stress testing has been established as an essential first step in designing macroprudential policy that ensures the stability and robustness of the banking and financial system (see Constancio (2015)). The stress scenarios are defined in terms of changes in asset prices and typically reflect projections from a separate workhorse macro model about the future states of the economy. However, the mapping from the macro to asset prices is "ad hoc", and not internally consistent with the scenario-generating macro model. To overcome this dichotomy, we make in this paper the crucial first step in extending the stylized macro model used by monetary policy authorities to include the yield curve in the internally consistent way.

[1] Guido Ascari whose research is focused on trend inflation and Martin Andreasen who has published dozens of papers on term structure modeling.

Figure 1.1: Time series of U.S. Treasury yields from 1962 till 2016



To do so we start by reflecting the recent consensus in asset pricing literature which shows that a trend in inflation is the key element in explaining much of the bond price dynamics. For instance, Cieslak & Povala (2015) document that a large portion of movements in U.S. Treasury bond risk premia at business cycle frequencies can be attributed to low-frequency movements in inflation, i.e. trend inflation. Bauer & Rudebusch (2017) show that accounting for time-varying trend inflation (rather than variation in the cyclical component of inflation) stands as the key element in understanding the empirical dynamics of U.S. treasury yields.

We incorporate trend inflation into the macro-finance model of Rudebusch & Swanson (2012) (henceforth, RS) which is a standard New Keynesian model with Calvo pricing and recursive preferences. The framework by RS has evolved as a workhorse model, being the first to offer a resolution to a long-lasting struggle of the macro and finance literature to explain why the term structure of interest rates is upward-sloping, so called "bond premium puzzle" (cf. Backus *et al.* (1989), and Den-Haan (1995)).

Nevertheless, instead of improving the model performance the introduction of trend inflation into the baseline RS model generates unrealistic business cycle and bond price dynamics. Moments from model simulated data become implausible.

We show that this happens because the distribution of prices in the economy spreads out drastically under trend inflation (see figure 1.2) and amplifies the inefficiency of price rigidities. The wide dispersion of prices leads to counterfactually high output losses, low bond yields and high required compensation for risk by bond holders. Further, we point to the fact that the measure of distribution of prices is not correctly captured by the standard approximation methods based on perturbation (see figure 1.2). Alongside Andreasen *et al.* (2018) we conjecture that that the polynomial approximation does not capture the upper bound on inflation imposed by the model which leads to unanchored inflation expectations and price inflation spirals.

The key contribution of the paper lies in the detailed formal explanation of the transmission and amplification mechanism between trend inflation, the distribution of prices, the real economy and bond prices. The intuition is as follows. Trend inflation drives the distribution of prices further apart from the average price index when only fraction of firms can change its price in each period[2]. As the price increases every period due to trend inflation some firms will be caught in the left tail of the price distribution, with a too low price. The price changing firms on the other hand are forward-looking and incorporate trend inflation into their price setting decision and thus set prices higher than the average price index. This spreads out prices relative to the average price index. The fact that with trend inflation there will be firms which charge very different prices has a strong impact on the real economy and asset prices. There two reasons for this. First, firms who cannot change its price due to the Calvo contracts will be stuck with too low prices. Due to low prices they face rapid rise in the demand for their products. However, increasing the production comes at the cost. The firm will have to hire more production inputs in an economy with decreasing returns to scale. By producing more the firm moves along the concave production function to the right and the firm marginal costs rise above the economy's optimal marginal costs. On the other side of the price distribution firms who set prices higher than optimal in the anticipation of the future price increase will produce too little. It would be optimal in terms of uses of resources if the low price firms decrease the use of their production capacity in favor of the high price firms. We show formally that in the presence of the trend inflation the economy on average produces more than optimal. The overproduction implied by the inefficient allocation of resources among firms leads to aggregate output losses as the same amount

[2]The difficulties related to the ability of Calvo pricing mechanism to match the empirical distribution of price changes has been documented in the literature, see for instance Gagnon (2009) or Nakamura *et al.* (2018).

of goods could be produced more efficiently (with less inputs) if the low price firms decrease production and high price firm increase .

A second inefficiency from trend inflation is generated by the fact that firms when setting their prices foresee trend inflation. Firms anticipate the growth in their future revenues generated by the future price increase. Nevertheless, we show that the growth of anticipated costs actually rises faster on average than the growth of revenues. This is why firms set their prices even higher than required by a simple compensation for the trend growth of the price level. This additional price markup thus amplifies the costs of monopolistic competition.

A third channel amplifying the effect of trend inflation on real economy and asset prices is the price-inflation spiral which is generated by the fact that the model policy functions are poorly represented by the polynomial function which approximates the true model solution. The model contains upper bound on inflation. If the upper bound is binding it is profit maximizing for firms to set prices so high that they effectively stop producing. The polynomial approximation does not capture the king in the policy function caused by the upper bound on inflation and leads to price-inflation spiral.

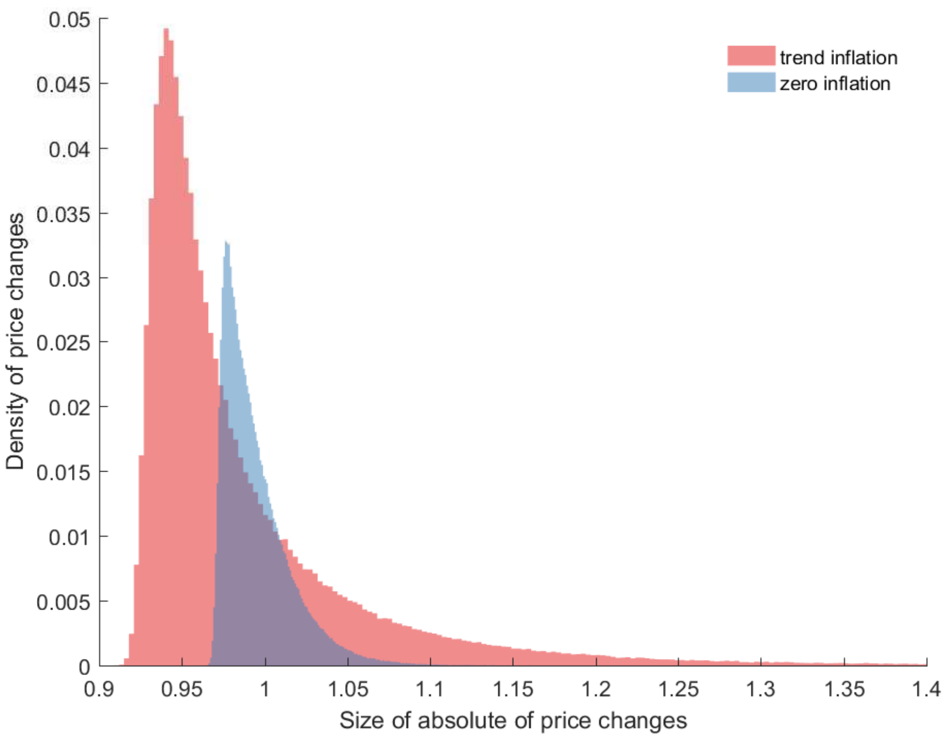
We present numerical results that this amplification mechanism can be attenuated by price indexation or by the assumption of a constant return to scale production function. The effect of dispersed prices is completely avoided when assuming Rotemberg pricing.

The focus of second paper lies in highlighting the link between government spending and the term structure of interest rates of default-free bonds[3]. Monetary policy has been for long recognized as an important factor driving the dynamics and term premia of bond prices. The impact of changes in government expenditures received however much less attention and it has been believed that government spending has marginal impact on the yield curve (see for instance, Evans & Marshall (2007)). This is a surprising result as the textbook economic theory predicts that an exogenous increase in public spending should lead to a rise in aggregate demand (i.e. Baxter & King (1993)) driving interest rates up[4] (i.e. Fisher & Turnovsky (1992)). In addition, if the rise in expenditures is financed by debt the bond supply literature documents the positive relationship between supply of outstanding government bonds and interest rates (see Krishnamurthy & Vissing-Jorgensen (2007) for literature review).

[3]U.S. Treasury bonds are typically considered to bear near zero default premium.

[4]We acknowledge that this line of argument holds only under the premise of closed economy which is commonly assumed for U.S..

Figure 1.2: Simulated Distribution of Price Changes



Note: The shaded areas plot the simulated distributions of the size of absolute price changes, $\frac{P_t^*}{P_t}$*, implied by the Calvo pricing model with (red) and without (blue) trend inflation.*

Nevertheless, the existing literature is either predominantly focused on studying implications of fiscal policy for the real economy or on the interactions between changes in bond prices and the real economy[5]. Existing, empirical research studying directly the link between fiscal policy and the yield relies mainly on the simple least-squares estimates reduced to single bond maturity (see Evans & Marshall (2007) or Laubach (2009) Gale & Orszag (2003)). We bridge the literature streams and study the effect of government spending on the yield curve in a unified framework where the relationship between the real economy and the whole maturity spectrum of bond prices is general equilibrium outcome.

In the public debate prevails the opinion that certain type of government expenditures like the infrastructure or education spending boost the economic growth while many other spending are purely wasted. From the point of view

[5]Impact of fiscal policy for the real economy has been extensively covered in the literature on fiscal multipliers (see e.g. Christiano *et al.* 2011 for a survey). Research studying impact of bond prices for real economy is dominated by financial frictions literature see e.g. Brunnermeier *et al.* 2013 for a survey).

of fixed income investor the way government spends its resources affects the compensation investor requires for holding the bonds. This is because the realized return depends on the state of the economy which is affected differently by each component of government spending. In addition, the structure of government expenditures matters for the inflation and thus for the real return on bonds. Also, for investors the relative return on bonds with respect to other assets matters. For instance, stocks tends to perform poorly in recession when savings are needed the most to smooth the consumption. In order to understand the impact of government expenditure policies on bond yields it is necessary to take into account the structural composition of the total expenditures.

In our study we contribute to the literature by separate government spending into four types: i) wasteful, ii) productive, iii) substitutable to private consumption, iv) non-substitutable. Most of the prevailing macroeconomic literature considers governments spending to be fully wasted. However, the theoretical and empirical literature on public capital (i.e. Agenor (2013), Barro (1990)) stresses the different impact of each type of government expenditures on the real economy both in terms of long run growth and business cycle. For instance, Albertini *et al.* (2014) estimates that public expenditures affects households' marginal utility of consumption. This happens when some of the government expenditures work as a substitution to private consumption as for instance public versus private health. Linnemann & Schabert (2006), Turnovsky (1997) and others show that output elasticity of public capital is significantly bigger than zero and thus that public capital plays substantial role in the productivity growth. This is because for example infrastructure spending increase the rate of return on private capital which further stimulates private investment and growth. Baxter & King (1993) among others argue that defense spending and increased safety of the nation contribute to country welfare and should be included in the utility function.

In our paper we show that the composition of government expenditures matters for how the overall expenditures affect the term structure of interest rates. From the modeling point of view this is because the bond pricing equation comes from the household optimization problem and as health and defense spending enter directly the utility function. Infrastructure spending on the other hand affects consumption and inflation which directly determine bond prices.

Understanding how changes in level and volatility of government spending impact the term structure of interest rates is crucial for the complex understanding of the impacts of government stimulus packages on the economy. Stimulus packages are most often financed by issuing government debt. If the increase in government spending leads to an increase of the level and slope of the term structure of interest rates it has direct adverse consequences for the current and future costs of government debt financing. Traditional cost benefit analysis of the stimulus packages may thus underestimate the true costs.

Our contribution lies also in incorporating the insight from fiscal foresight literature into our model. Fiscal expenditures are usually known well in advance before their are implemented. Ramey (2011) is among the first to forcefully document in empirical study the importance of the timing in the response of the economy to rises in public expenditures. Economy tends to react differently to the anouncement and actual realization of the expenditure. The lags in decision and implementation can be demonstrated by many examples. Trump's fiscal package to boost infrastructure spending has been debated since he won the election. Obamacare[6] was discussed for more than a year before coming into force and the implementation was only gradual. Ramey (2011) lists other examples related to defense spending as the aftermath of 9/11 or Soviet invasion of Afghanistan, where the rise in defense spending was anticipated in advance. On the other hand, Leeper *et al.* (2012) documents that non-negligible portion of expenditures are only foreseen imperfectly and therefore there is some surprise element even at the expenditures realization.

Building on the fiscal foresight literature, we distinguish between the: *i)* news shock, which is the shocks to agents' expectations about future policy, *ii)* the standard surprise shock, which results from the revision in expectations at the time the expenditures are realized, and the *iii)* changes in uncertainty which reflects the changes in the level of insecurity in forming the expectations about future public expenditures. Accounting for timing allows us to better disentangle the channels of transmission between fiscal policy shocks and yields.

We build our analysis on a variant of the standard New Keynesian (NK) DSGE model (e.g. Galí (2002), De Paoli *et al.* (2010) or Erceg *et al.* (1999)) which we augment by Epstein Zin (EZ henceforth) preferences as in Rudebusch & Swanson (2012), Andreasen (2012), Ferman (2011), Li & Palomino (2014) and commitment to fiscal consolidation as in Corsetti *et al.* (2009). This class of models have been argued to be able to match both macro and finance stylized facts but are still simple enough to allow us to fully understand the asset pricing

[6]Patient Protection and Affordable Care Act.

implications of fiscal policy. As an extension we build a mid-scale DSGE model following Andreasen *et al.* (2018) and Altig *et al.* (2011).

The other novelty of our study lies in introducing what we call attribution analysis. To get a intuition on what prices bonds one can derive second order approximation of the bond pricing equation and write it in terms of conditional second moments of underlying macro variables. Our method proposes a way how to calculate these conditional moments.

We document that government expenditures on one hand increase the risks for investors wealth which come from the real economy but on the other hand government sells the only instrument available to provide insurance against these risks. These two effects impact bond prices in the opposite direction. More specifically, we show that the fiscal risk premia in bond prices are driven by two incentives, *i)* the precautionary savings motives and *ii)* hedging property of bonds against the fluctuations in investor real value of wealth. In general, government spending directly affects households' consumption through intertemporal budget constrain. The rise in uncertainty about the size of future government spending decreases the predictability of future consumption. Even moderate fluctuations in consumption are in terms of utility costly for households. Bonds represent useful instrument providing buffer compensating the unexpected drops in consumption. Households thus invests into bonds for precautionary reasons. In addition, bonds provide great insurance if the real value of bonds rises in bad times when consumption drops. We show that the interaction of monetary policy with fiscal policy is the crucial element determining if bonds act as leverage or hedge for fluctuations in investors real wealth. Further, we explain that the predictability of future government spending policy is important for the precautionary buffer investors create. If the volatility of government spending is large and thus the size of future spending is uncertain, investors will increase their positions in bond portfolios to hedge stable level of consumption over time. If the unexpected changes in government spending have high persistence investors want to hedge their level of consumption for the whole period of the (above or under average) government spending. In other words, the duration of the portfolio matters. For this reason, long-run risks for future consumption and leisure are important factor determining the size of nominal term premium. Furthermore, we argue that fiscal policy that becomes more predictable on the evolution of debt and taxes (e.g. commitment to spending reversals) mitigates the impact of uncertainty on bond prices.

Wasteful government expenditures rises the term structure of interest rates up at the impact because they generate future inflation and undermine real value of bonds. On the other hand, if the predictability (represented by the volatility of the shock) of wasteful government expenditures decreases, the term structure shifts down because investors buy bonds to build up buffer against uncertain future. In case of productive government expenditures the lower inflation risk shifts the yield curve down. The impact of changes in the predictability depends on the monetary policy response both in terms of direction and quantity of the response.

Our study implies that fiscal policy contributes to yield curve flattening by decreasing the nominal term premia, however purely wasteful expenditures increase the nominal term premium as they generate inflation risk and bonds serve as an imperfect saving instrument.

In the third paper we focus on the wasteful government expenditures and how they affect the output. In addition, we also study the income side of government balance sheet and calculate a payroll tax, a sales tax and a financial asset tax multipliers both when the economy is at zero lower bound (ZLB) and in normal times with positive interest rates. We differ from the rest of the literature by taking into account the recently documented flattening of Phillips curve in our analysis. The flattening of the Phillips curve means that the empirically documented trade off between economic growth and inflation is decreasing (even in the short run) and thus government cannot buy extra growth by increasing inflation. Consistently with the empirical microeconomics the flattening of the Phillips curve is an endogenous outcome of strategic complementarity in price setting as opposed to more common (in theoretical macroeconomics) counterfactual increase in the price stickiness (e.g. Boneva *et al.* (2016) and Ngo (2019)). We show that the size of fiscal multipliers depends on how the flattening of Phillips curve is introduced into the modeling framework.

There is a general agreement that the fiscal multipliers tend to be small in normal times. This is for two reasons: one, increases in government expenditure need to be financed by either higher taxes or debt, and thus come with a negative wealth effect, which crowds out consumption and decreases demand; two, a fiscal expansion which increases inflation and output, triggers an endogenous response of the monetary authority, which raises interest rates, offsetting some of the expansionary effect of fiscal policy. In times when the economy is at the zero lower bound, such endogenous dampening response of monetary policy is absent, as the nominal interest rate stuck at the lower bound and thus constant; in such case, an increase in (expected) inflation, resulting from a fiscal expansion, leads to a drop in the real interest rate, which further stimulates demand and thus increases fiscal multipliers.

However, when the Phillips curve is flat the rise in output is accompanied only by mild increase in inflation which is associated with only moderate drop in real interest rate and thus the wealth effect dominates the determination of fiscal multipliers. This is the reason why fiscal multipliers are bellow one even at the ZLB. On the other hand in the normal times the flat Phillips curve implies larger fiscal multipliers as the monetary policy effect is attenuated and the determination of fiscal multipliers is dominated by the wealth effect. In case of very persistent government spending shocks agents want to compensate their loss in income due to the current and future taxes and therefore increase their labor supply. Higher supply of labor puts downward pressure on wages and consequently on inflation. This effect is particularly pronounced in case of steep Phillips curve. For this reason, the monetary policy effect actually increases the fiscal multipliers in normal times with steep Phillips curve as the increased and cheaper labor supply boosts the production.

Most of the related literature on fiscal multipliers abstract from the positive level (long run average) of government spending. We show that when the government spending to output ratio is positive it leads to higher fiscal multipliers. This is because the ratio decreases the intertemporal elasticity of consumption. Agents are less willing to substitute consumption in time after government spending shock even if the negative wealth effect incentivises them to do so. Thus, the drop in consumption due to the wealth effect is smaller and rise in output larger.

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Chapter 2

Trend Inflation Meets Macro-Finance: the Puzzling Behavior of Price Dispersion

2.1 Introduction

In recent years the macroeconomic profession has become increasingly aware of the importance of real-financial interactions and the need to explicitly incorporate a financial side into models used for economic policy analysis, such as the ones used at central banks. Of fundamental interest in this respect is the yield curve, which plays a crucial role for both monetary policy and financial stability assessment.

A growing body of asset pricing literature highlights the importance of trend inflation in explaining the behavior of U.S. treasury yield curve. For instance, Cieslak & Povala (2015) document that a large portion of movements in Treasury bond risk premia at business cycle frequencies can be attributed to low-frequency movements in inflation, i.e. trend inflation. Bauer & Rudebusch (2017) show that accounting for time-varying trend inflation (rather than variation in the cyclical component of inflation) stands as the key element in

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understanding the empirical dynamics of U.S. Treasury yields[1]. In this paper we ask what is the economics behind this empirical finding and if can we confirm this finding in the theoretical dynamic general equilibrium asset pricing model?

The macroeconomic models (DSGE framework) have for long struggled to explain why is the yield curve upward sloping and this puzzle has been labeled as "Bond Premium Puzzle" (cf. Backus *et al.* (1989), and Den-Haan (1995)). Although workhorse structural macro models have improved much in recent years in their ability to price financial assets (bond yields) these models still provide limited explanation of dynamics of prices and drivers of risk premia. For instance, the most successful state of the art model which is able to match both the basic bond pricing and macro stylized facts is the Rudebusch & Swanson (2012) (henceforth, RS). However, even this model still needs risk aversion of 110 (empirically justified levels are around 5) to match the level of term premia. The RS model as most of the models in DSGE literature treats the inflation trend as constant. It seems therefore natural to assume that the missing element to explain the yield curve could be the trend inflation.

Motivated by these crucial empirical findings and with the intent to improve our modeling frameworks that jointly address a macro and a finance side, we incorporate trend inflation into the macro-finance model of Rudebusch & Swanson (2012). Contrary to what the empirical literature suggests we find out that trend inflation compromises the model performance in matching macro and bond prices stylized facts. For instance, volatility of inflation increases from the original 3 percent to almost 40 percents and consumption volatility is about 16 times higher. Our paper explains why is the model fit undermined by trend inflation and proposes how to restore the original performance.

The reason for this are drastically amplified inefficiencies from price rigidity as well as numerical inaccuracies that arise when the Calvo pricing mechanism meets trend inflation – in particular when using higher order approximations. Our paper offers an understanding of the channels behind these results and provides possible remedies.

The empirical literature, in both macro and finance, has long treated the inflation trend as constant. Stock & Watson (2007) provide strong evidence that the dynamics of inflation have been largely dominated by the trend component.

[1] Figure 2.5, which reproduces Figure 1 of Rudebusch and Bauer (2017), summarizes these findings visually, by plotting time series for the ten-year yield, an estimate of trend inflation and the equilibrium nominal and real short rate.

Further, Cogley *et al.* (2010) and Ascari & Sbordone (2014) demonstrate that inflation innovations account for a small fraction of the unconditional variance of inflation, implying that most of the volatility stems from the trend component of inflation. Similarly, also the theoretical literature accounts for trend inflation explicitly only relatively recently. Noteworthy examples are Ascari & Rossi (2012), Ascari *et al.* (2011), Ascari & Ropele (2009), who show that trend inflation represents an important factor to consider in the design of monetary policy. As argued above, macro-finance models that are solved around a zero-inflation steady state may stand in contrast with the recent empirical evidence and it appears particularly important to realign current model frameworks with the empirical findings and incorporate positive trend inflation as a firm model element.[2]

We find that introducing trend inflation into the baseline RS model generates unrealistic business cycle and bond price dynamics. Moments from model simulated data become implausible, price dispersion and the implied output losses of price dispersion rise to unrealistic values, and price dispersion itself is inaccurately approximated. We document that the nonlinear behavior of price dispersion is at the core of the poor model performance when positive trend inflation meets Calvo pricing, and that problems are aggravated under decreasing returns to scale in the production function. It is important to emphasize that the encountered problems are not specific to the asset pricing related features of the RS model, but, more generally, apply to the class of macro models with a Calvo pricing mechanism and positive trend inflation, when solved nonlinearly (under both second or third order approximations).[3] In fact, as we show throughout the paper, similar results can be obtained from a standard New Keynesian (NK) model of Clarida, Galí and Gertler (1999, hereafter CGG) under certain specifications and parameterizations, albeit generally to a lesser degree. The findings of this paper are, thus, relevant to more than just the

[2]It should be noted that also in the domain of macro-finance models some, few contributions have incorporated trend inflation, e.g., Andreasen *et al.* (2018), Andreasen & Kronborg (2017), Kliem & Meyer-Gohde (2017) – typically, under the additional assumption of inflation indexation, which, as we will show, serves as one of our proposed remedies.

[3]However, while the problem is not specific to the macro-finance literature, the use of decreasing returns to scale production function (or, to be precise, a production function with fixed capital) in the context of a macro-finance model is no coincidence, as it contributes strongly to being able to match term premia; relaxing this assumption quickly leads to much smoother stochastic discount factors, and lower generated term premia.

macro-finance asset pricing literature and have broader applicability.[4,5]

The key contribution of our paper lies in the detailed formal explanation of the transmission and amplification mechanism between trend inflation, the distribution of prices, the real economy and bond prices. We emphasize two channels through which trend inflation has a key influence on model dynamics and drives up the model-implied levels of price dispersion: i) the marginal-cost channel, and ii) a trend-inflation-markup channel. Under the marginal-cost channel we understand the fact that firms that could not reset their prices recently and, because of trend inflation, are stuck with a (too) low price will employ an inefficiently large amount of labor and produce an inefficiently large amount of output; as these firms move along the concave production function to the right, their marginal cost will increase compared to aggregate marginal costs. Under the trend-inflation-markup channel we understand that firms incorporate trend inflation into their forward-looking pricing decision. They know that prices will go up and that they will not be able to change current prices for some period, so they set their optimal prices higher (at a markup in addition to the one from monopolistic competition) in the case of positive trend inflation compared to the case when trend inflation is zero. We demonstrate that the above channels lead to levels of price dispersion and implied output-losses from dispersion that lie significantly above values typically obtained in the case of zero trend inflation, and become counterfactually high. In addition, we show that the Calvo price dispersion equation also becomes poorly approximated by local perturbation methods.[6]

We propose several modeling devices that provide a remedy to the unrealistic business cycle and bond price moments, and to the inaccuracies to the price dispersion equation. In particular, an otherwise equivalent setup with Rotemberg price adjustment costs instead of Calvo pricing, a linear-in-labor production function, or the introduction of inflation indexation[7] can to a large degree

[4]E.g., the literature on globally solved ZLB-models under Rotemberg or Calvo, such as in Boneva *et al.* (2016) and Miao & Ngo (2019).

[5]There are arguably more realistic setups than the Calvo mechanism to capture nominal rigidities, such as discussed by the on state-dependent pricing (see, e.g., among others Golosov & Lucas (2007), Midrigan (2011), or Costain & Nakov (2011). Nonetheless, the Calvo mechanism, which belongs to the class of time-dependent pricing mechanisms, continues to remain the most widely used device to introduce nominal rigidities.

[6]We should note, that the problem of counterfactually price dispersion and its poor approximation, is present already in the original RS specification, without trend inflation. Positive steady state inflation, however, aggravates the problem substantially, up to the point that simulated model moments stop making sense. This issue of poor approximation has been raised also by Andreasen & Kronborg (2017).

[7]Andreasen *et al.* (2018) also points to the increased volatility of price dispersion with

restore the performance of the RS model (or more generally, a trend-inflation-augmented Calvo pricing model) in matching the data. The key contributions of this article are thus, to offer both a warning about potential pitfalls of the Calvo setting and some guidance to the macroeconomic modelers for avoiding these pitfalls.

The rest of the paper proceeds as follows. Section 2.2 develops the main body of the paper. Section 2.2.2 documents in detail how simulated model moments are affected by the incorporation of trend inflation and discusses model devices that help remedy this situation. Section 2.2.3 is dedicated to a discussion of the channels that lead to high levels of price dispersion under the presence of trend inflation, and to the large inefficiencies they create. Section 2.2.4 uses model simulations to further develop an understanding of the behavior of price dispersion and studies its numerical properties. Section 3 concludes.

2.2 The Baseline Rudebusch and Swanson Model with Trend Inflation

Our example model, the Rudebusch and Swanson model with trend inflation, is in many aspects a standard model in the New Keynesian tradition. A continuum of firms operate under monopolistic competition and are subject to nominal rigidities à la Calvo. Households have preferences over consumption and labor –albeit in the form of Epstein-Zin preferences instead of the more conventional CRRA preferences. The central bank follows a Taylor rule, with a time-varying inflation target that is centered around a positive steady-state inflation level, instead of a zero trend inflation as in the original article.[8] Throughout the paper, Π_t denotes the gross inflation rate, defined as $\Pi_t = P_t/P_{t-1}$; lower case variable π_t instead denotes the (annualized) net inflation rate in percent, $\pi_t = 100\log(\Pi_t^4)$.

steady state inflation and they use this fact together with price indexation to match the volatility of nominal term premium.

[8]For more detailed exposition on the model we refer the reader to the original article of Rudebusch & Swanson (2012). Appendix 2.A provides summary of the model equations.

2.2.1 Model Sketch, RS Model

Households

The description of the households and firms' problems below closely follows RS.

The household maximizes the continuation value of its utility (V), which is of the Epstein-Zin form and follows the specification of RS:

$$V_t = \begin{cases} U(C_t, N_t) + \beta \left[E_t V_{t+1}^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \text{ if } U(C_t, N_t) \geq 0, \\ U(C_t, N_t) - \beta \left[E_t (-V_{t+1})^{1-\alpha} \right]^{\frac{1}{1-\alpha}} \text{ if } U(C_t, N_t) < 0. \end{cases} \tag{2.1}$$

The households' problem is subject to the flow budget constraint:

$$B_t + P_t C_t = W_t N_t + D_t + R_{t-1} B_{t-1}. \tag{2.2}$$

In equation (2.1), β is the discount factor. Utility (U) at period t is derived from consumption (C_t) and leisure ($1 - N_t$). E_t denotes expectations conditional on information available at time t. As the time endowment is normalized to one, leisure time ($1 - N_t$) is what remains after spending some time working (N_t). $W_t N_t$ is labor income, R_t is the return on the one-period nominal bond, B_t, D_t is dividend income.

To be consistent with balanced growth, RS impose the following functional form on U:

$$U(C_t, N_t) = \frac{C_t^{1-\varphi}}{1-\varphi} + \chi_0 Z_t^{1-\varphi} \frac{(1-N_t)^{1-\chi}}{1-\chi}, \quad \varphi, \chi > 0, \tag{2.3}$$

where Z_t is an aggregate productivity trend, and φ, χ, $\chi_0 > 0$. The intertemporal elasticity of substitution (IES) is $1/\varphi$, and the Frisch labor supply elasticity is given by $(1 - \overline{N})/\chi\overline{N}$, where $\overline{N}$ is the steady state level of hours worked.

Firms

Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)(di) \right]^{\frac{1}{1-\epsilon}}$, where $P_t(i)$ is the price of intermediate good produced by firm i, using the technology $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(i) di \right]^{\frac{\epsilon}{\epsilon-1}}$. Final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0, 1]$ into a single final good. Parameter ϵ determines the elasticity of substitution between goods variety. The cost-minimisation problem of final good firms delivers demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t. \tag{2.4}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i produces according to a Cobb-Douglas production function, where θ denotes the capital share. Aggregation across firms, yields:

$$S_t Y_t = A_t \bar{K}^{\theta} (Z_t N_t)^{1-\theta}. \tag{2.5}$$

$\bar{K}$ refers to the fact that firms have fixed capital[9] and S_t is the cross-sectional price dispersion. Technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A, \tag{2.6}$$

where ϵ_t^A is an independently and identically distributed (i.i.d.) shock with zero mean and constant variance.

Intermediate firms maximize the present value of future profits facing Calvo contracts by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} \left[P_t(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right] \right\}, \tag{2.7}$$

where $Q_{t,t+j}$ is the real stochastic discount factor from period t to $t+k$. The term $W_{t+j}N_{t+j}(i)$ represents the cost of labor. The optimal price is a weighted average of current and future expected nominal marginal costs,

$$P_t(i) = \frac{\epsilon}{\epsilon - 1} \sum_{k=0}^{\infty} \Upsilon_{t+k} MC_{t+k}(i), \tag{2.8}$$

Where $\Upsilon_{t+k} = \frac{E_t \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} Y_{t+k}(i)}{E_t \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \frac{P_t}{P_{t+k}} Y_{t+k}(i)}$ is the time varying mark-up implied by price rigidity and $\frac{\epsilon}{\epsilon-1}$ is the mark-up implied by monopolistic competition.

Average real marginal cost is defined as

$$MC_t = \frac{1}{1-\theta} \left(\frac{W_t}{A_t}\right) \left(\frac{Y_t}{\bar{K} A_t}\right)^{\frac{\theta}{1-\theta}}. \tag{2.9}$$

[9] Firm-specific capital can be interpreted as a model with endogenous investment that features high adjustment costs in investment.

Fiscal Policy and Monetary Policy.

Government spending follows the process:

$$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G, \quad 0 < \rho_G < 1, \tag{2.10}$$

where $\bar{g}$ is the steady-state level of $g_t \equiv G_t/Z_t$, and ε_t^G is an i.i.d. shock with mean zero and variance σ_G^2.

The model is closed by a monetary policy rule:

$$4i_t = 4\rho_i i_{t-1} + (1-\rho_i)\left[4(\bar{i} - \bar{\pi}) + (\pi_t^{avg}) + \phi_\pi(4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y\left(\frac{\mu_t Y_t}{\bar{\mu}\bar{Y}} - 1\right)\right], \tag{2.11}$$

where i_t is the (net) policy rate, $i_t = log(1 + i_t)$, π_t^{avg} is a four-quarter moving average of (net) inflation (defined below), and Y_t^* is the trend level of output $\bar{Y} Z_t$ (where $\bar{Y}$ denotes the steady-state level of Y_t/Z_t). π_t^* is the target rate of inflation, and ε_t^i is an i.i.d. shock with mean zero and variance σ_i^2. ρ_i captures the motive for interest rate smoothing. The four-quarter moving average of inflation (π_t^{avg}) can be approximated by a geometric moving average of inflation:

$$\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1 - \theta_{\pi^{avg}})\, \pi_t, \tag{2.12}$$

where $\theta_{\pi^{avg}} = 0.7$ ensures that the geometric average of inflation has an effective duration of approximately four quarters. The inflation target π_t^* is time varying and driven by following process,

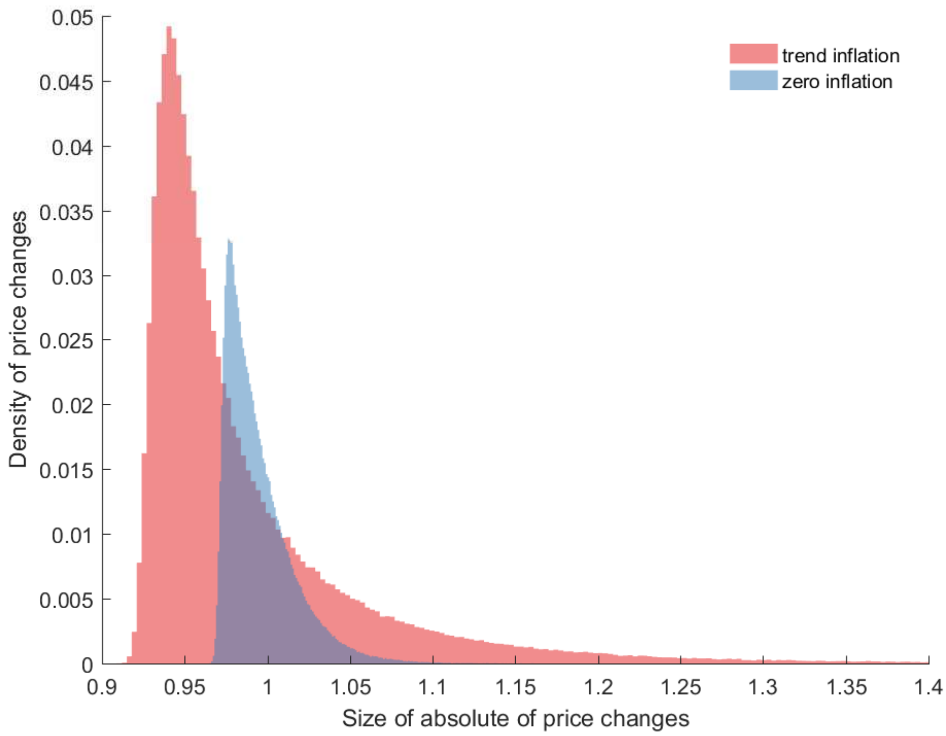
$$\pi_t^* = (1 - \rho_{\pi^*})\, 4\pi_t^{avg} + \rho_{\pi^*}\pi_{t-1}^* + \zeta_{\pi^*}\left(4\pi_t^{avg} - \pi_t^*\right) + \sigma_{\pi^*}\varepsilon_{\pi^*,t}. \tag{2.13}$$

The detailed explanation of how long-term bonds are priced in a model is described in the next chapter, section 3.3

2.2.2 Model Moments

Table 2.1 illustrates that relaxing the assumption of zero trend inflation ($\bar{\pi} = 0$), and, instead, allowing for positive trend inflation ($\bar{\pi} > 0$) in the RS model, produces unreasonable, largely inflated macro and finance unconditional second moments. The mechanism that accelerates the model dynamics is closely linked to the distribution of prices in the model economy. Figure 2.1 shows the simulated distribution of price changes. The main effect of trend inflation is to make large price changes more likely because the subset of firms that can change the price needs to react to the positive trend in inflation when changing the price. Some firms with relatively low prices fixed far in the past will have to make large price changes to compensate for the rise in the price level that took place over time due to trend inflation. The distribution of prices can be described succinctly by the measure of the price dispersion, S_t, defined as

Figure 2.1: Simulated Distribution of Price Changes



Note: The shaded areas plot the simulated distributions of the size of absolute price changes, $\frac{P_t^*}{P_t}$, implied by the Calvo pricing model with (red) and without (blue) trend inflation.

$$\begin{aligned} S_t^{\frac{1}{1-\theta}} &\equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\varepsilon}{1-\theta}} di \\ &= (1-\zeta)\,(p_t^*)^{\frac{-\varepsilon}{1-\theta}} + \zeta\,(\Pi_t)^{\frac{\varepsilon}{1-\theta}}\, S_{t-1}^{\frac{1}{1-\theta}}; \end{aligned} \tag{2.14}$$

where ζ is the Calvo parameter (the per-period probability that the price cannot be changed), $1-\theta$ is the labor income share, and ε is the elasticity of substitution between varieties. Given the definition in equation (2.14), price dispersion, S_t, is bounded by 1 from below, or, equivalently, S_t^{-1} is bounded by

1 from above which means that when $S_t = 1$ all firms have the same prices in the economy.

Table 2.1: Empirical and Model-Based Unconditional Moments

Unconditional Moment	US data 1961-2007	RS1 $\bar{\pi} = 0\%$	RS2 $\bar{\pi} = 1.6\%$	RS3 $A_t N_t^{1-\theta}$	RS4 $A_t K_t^{\theta} N_t^{1-\theta}$	RS5 $\pi_t^* = \bar{\pi}^*\%$
SD(dC)	2.69	0.72	8.29	7.83	5.89	7.42
SD(C)	0.83	0.88	12.88	11.13	447.44	9.60
SD(N)	1.71	2.51	38.16	30.99	482.68	27.27
Mean(i)	5.72	3.06	0.46	4.12	-787.40	1.98
SD(i)	2.71	3.41	49.39	41.39	2212.94	34.86
Mean(π)	3.50	-0.54	-2.12	1.42	-791.14	-0.66
SD(π)	2.52	3.01	40.84	36.47	2211.99	29.83
SD($i^{(40)}$)	2.41	2.33	31.25	29.62	2215.71	23.67
Mean($NTP^{(40)}$)	1.06	0.91	2.50	3.41	0.55	3.23
SD($NTP^{(40)}$)	0.54	0.42	7.21	6.63	5.94	6.27
Mean($R^{(40)} - R$)	1.43	0.88	2.72	3.24	0.98	2.90
SD($R^{(40)} - R$)	1.33	1.59	26.57	21.95	36.31	19.98
Mean(S^{-1})	< 1.00	0.99 [0.83,1.07]	1.05 [0,56]	1.01 [0.83,1.07]	1014.74 [0,7e^5]	1.01 [0,47]

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. The red colored numbers represent values of the inverse price dispersion that violate the economically feasible range, as S^{-1} is bounded from above by one. The interval indicated below row 'Mean(S^{-1})' reports the range (minimum and maximum values) of S^{-1} observed over the simulation. ***RS1*** *is the original RS model which has following features: fixed capital $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$, time-varying inflation target, π_t^*, zero trend inflation, $\bar{\pi} = 0\%$.* ***RS2*** *is RS1 with positive trend inflation $\bar{\pi} = 1.6$.* ***RS3*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and a labor-only-DRS production function, $Y_t = A_t N_t^{1-\theta}$.* ***RS4*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and variable capital $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$.* ***RS5*** *is RS1 with trend inflation $\bar{\pi} = 1.6$ and a constant inflation target in Taylor rule, $\pi_t^* = \bar{\pi}^*$.*

The first column of table 2.1 reports targeted empirical moments. The subsequent columns are model-based unconditional moments, calculated from third-order approximated and pruned model simulations of several model versions of the RS model. Column RS1 reports simulated moments from the original baseline RS model with zero trend inflation, using the RS best fit calibration from Table 3 of their paper[10]. Column RS2 reports results for the RS model with an annualized steady-state inflation of 1.6%.[11]

[10]The model calibration is summarized in Table 2.4.

[11]Note that this is an only very modest assumed level of annualized trend inflation. Empirically, the observed value of annualized trend inflation lies well above 2% for most of the sample periods since the second world war. However, we confirm Ascari and Ropele's (2009) result that at a rate higher than 1.6% the model solution becomes indeterminate for the empirically relevant calibration of the Taylor rule. Note that the Calvo pricing mechanism imposes an upper bound on inflation, which in our model setup is above 16% of annualized

Even this very moderate level of trend inflation inflates the model moments, both macro and finance, to unrealistic values. Whereas, under the assumption of zero trend inflation, the mean and standard deviation of S_t^{-1} stays in the economically justifiable range (a value of 0.99 can be interpreted as an output loss of 1 percent due to price dispersion), with trend inflation (column RS2) the mean of the inverse price dispersion becomes economically unfeasible. Also a large standard deviation and the wide range over which values of S_t^{-1} are observed in a simulation (reported in the squared brackets below the values of column 'Mean(S^{-1})') documents that periods where almost all output is lost due to price dispersion are frequent. Column RS3 reports moments for a model version where the feature of fixed capital is removed and replaced by a labor-only DRS production function; column RS4 is a version when capital is allowed to be variable, as in a standard Cobb-Douglas production function. Column RS5 relaxes RS's assumption of a time-varying inflation target and replaces it with a fixed target (as is more common in standard New Keynesian (NK) models). In all cases we observe that the problems of counterfactually high levels of price dispersion persist. In Appendix 2.B we develop a set of results for a version of the standard CGG NK model that mirrors the findings just described, documenting that poor model performance is not specific to the asset pricing features of our example model. [12] Another important issue typical for non-linearly solved NK models which violates the empirical observations is the negative mean of inflation. In case of non-linearly solved models variance of the shocks is reflected in the expectation of agents which increases equilibrium level of savings, lowers average yields and inflation through Fisher equation. The precautionary saving effect thus pushes the stochastic steady state of the model bellow the zero inflation deterministic steady state [13].

inflation. Nevertheless, much of the explosive dynamics we observe after introducing positive trend inflation can be attributed to the fact that our approximation method does not accurately capture this upper bound on inflation as conjectured by Andreasen & Kronborg (2017).

[12]To be precise, while the channels (to be described in detail in section 2.2.3 below) that drive up price dispersion are present at all times, whether or not they lead to the aforementioned problems also in the NK model is a quantitative matter. For example, we describe a model version of the NK model with difference-stationary technology shocks, i.e. shocks to the economy's growth rate. With moderate but persistent shocks, we can demonstrate that the same set of problems as in RS also arises in the NK model. The intuition is clear: with persistent shocks the dispersion of prices across the economy will increase because firms which set their prices infrequently face very different economic conditions. When we employ a parameterization in which shocks are less persistent and have a milder impact on the real economy, as well as for a CGG version with trend-stationary shocks, we find that model dynamics stay within the standard range.

[13]see Andreasen *et al.* (2018) for ways how to tackle this issue

After we identified an unreasonable behavior of price dispersion as the main culprit in the poor performance of models with a Calvo pricing mechanism with trend inflation, we now turn to a number of candidates of model specifications that restore the model's moments fit, which are reported in Table 2.2. Table 2.2 presents simulated moments from versions of the trend-inflation-augmented-RS model, where *i*) the assumption about how prices are set in the economy is modified to a setting with Rotemberg adjustment costs (instead of Calvo pricing), where *ii*) we use a linear production function instead of decreasing return to scale (DRS), or where *iii*) we remove the effects of trend inflation by inflation indexation to either steady-state inflation or last period inflation[14]. (As before, Appendix 2.B shows a set of parallel results for the CGG New Keynesian model.) The model features just discussed, which fix the problems with exploding moments documented in Table 2.1, have a common mechanism: they mitigate the dispersion of prices in the economy. The following subsection describes the mechanisms at play in more depth.

2.2.3 Trend Inflation and Price Dispersion – Channels

A well-known feature of the Calvo assumption that in each period only a fraction of firms is allowed to re-set their prices optimally which means that firms with many different prices co-exist in the economy, captured by the measure of price dispersion, S_t, equation ((2.14)). As first brought to light in a paper by Ascari (2004), and further contributions by the same author that are summarized in Ascari & Sbordone (2014), price dispersion raises the resource cost of production by introducing a wedge between aggregate output and the amount of inputs[15] needed to produce this level of output, $Y_t = S_t^{-1} A_t K_t^{\theta} N_t^{1-\theta}$. This wedge becomes significantly amplified in the case of trend inflation. Trend inflation adds a drift into the evolution of prices and, thus, drives the distribution of prices further apart from the average price index P_t. To better understand the mechanism at play, we lay out three channels through which trend inflation has a key influence on price dispersion and the dynamics of real economic vari-

[14] There is little empirical support for firm price indexation as well as for Calvo pricing mechanism. We show that in the presence of Calvo pricing the economic costs of positive trend inflation can be largely mitigated by price indexation thus it is optimal for firms to index their prices. In other words, the little evidence for price indexation is implied by limited evidence for Calvo pricing.

[15] Ascari & Sbordone (2014) discuss the steady-state implications of trend inflation, whereas our focus is more on the dynamics, which is crucial for asset pricing.

Table 2.2: Empirical and Model-Based Unconditional Moments

Unconditional Moment	RS2 $\bar{\pi} = 1.6\%$	RS6 Rotemberg	RS7 $Y_t = A_t N_t$	RS8 $\iota = 0$	RS9 $\iota = 1$
SD(dC)	8.29	0.43	0.45	0.71	0.49
SD(C)	12.88	0.49	0.53	0.89	0.68
SD(N)	38.16	1.45	1.39	2.50	1.85
Mean(i)	0.46	3.16	4.80	5.73	4.83
SD(i)	49.39	2.09	2.46	3.43	3.07
Mean(π)	-2.12	-0.48	1.05	2.22	1.42
SD(π)	40.84	2.14	2.33	2.98	2.58
SD($i^{(40)}$)	31.25	1.54	1.54	2.37	1.84
Mean($NTP^{(40)}$)	2.50	0.83	0.64	1.08	1.23
SD($NTP^{(40)}$)	7.21	0.36	0.10	0.55	0.03
Mean($R^{(40)} - R$)	2.72	0.84	0.61	1.11	1.27
SD($R^{(40)} - R$)	26.57	1.03	1.13	1.61	1.59
Mean(S^{-1})	1.05	0.00	1.00	0.99	1.00
SD(S^{-1})	0.82	0.00	0.00	0.01	0.00

Note: All variables are quarterly values expressed in percent. Inflation, interest rates and the term premium are expressed at an annual rate. Unlike in Table 2.1, there are no observations of the inverse price dispersion in violation of the economically feasible range. ***RS2****: equal to RS2 from Table 2.1.* ***RS6****: as in RS2, but with Rotemberg adjustment costs instead of Calvo pricing.* ***RS7****: as in RS2, but with a labor-only-CRS production function, $Y_t = A_t N_t$.* ***RS8****: as in RS2, but with indexation to steady-state inflation ($\iota = 0$).* ***RS9****: as in RS2, but with indexation to last-period inflation ($\iota = 1$).*

ables: i) the marginal-cost channel and ii) a trend-inflation markup channel[16] and iii) price-inflation spiral .

The Marginal-Cost Channel

Let, in the following, variables carrying an asterisk denote prices and quantities of a firm that, in period t, is allowed to re-set its price optimally . Let variables without asterisk denote aggregate economy-wide variables, that include firms that are not allowed to re-set their price in the current period and are stuck with prices from the past.

Lemma 2.1. *In the economy with trend inflation and Calvo contracts firms which cannot change its price produce more output than it is optimal under*

[16] Our decomposition is somewhat different compared to other contributions in the literature, where the focus is on a trend-inflation markup channel. For example, Ascari & Sbordone (2014) decompose the markup, ϕ_t, into a price adjustment gap, P_t^*/P_t and P_t^*/MC_t to study the implications of trend inflation for the model's deterministic steady-state. Our discussion of the marginal-cost channel is in this sense novel.

the flexible prices.

$$Y^*_{t+k} = \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}} Y_{t+k}, \tag{2.15}$$

Proof. The demand function for the firm resetting its price at time t, is given by the relationship, $Y^*_{t+k} = A_{t+k} K^{\theta}_{t+k|t} N^{1-\theta}_{t+k|t} = \left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$ which pins down the optimal level labor input hired by price-setter,

$$N^*_{t+k} = \left(\left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k} K^{\theta}_{t+k|t}}\right)^{\frac{1}{1-\theta}} \tag{2.16}$$

The fraction of aggregate labor input and the labor input by price-setting firm, N^*_{t+k}/N_{t+k} gives,

$$N^*_{t+k} = \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \left(\frac{K_{t+k}}{K_{t+k|t}}\right)^{\frac{\theta}{1-\theta}} N_{t+k}. \tag{2.17}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_{t+k}}{K^*_{t+k}} = \frac{Y_{t+k}}{Y^*_{t+k}}, \tag{2.18}$$

Using the labor demand equations of price resetting and aggregate firms, together with equation (2.18), we get,

$$Y^*_{t+k} = \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}} Y_{t+k}, \tag{2.19}$$

Using the result from Proposition 2.1 in equation 2.19 implies that in case of positive inflation together with CRS, $Y^*_{t+k} \leq Y_{t+k}$. Thus, as from above $\frac{K_{t+k}}{K^*_{t+k}} = \frac{Y_{t+k}}{Y^*_{t+k}}$, then $K^*_{t+k} \leq K_{t+k}$ and thus $N_{t+k^*} \leq N_{t+k}$. □

The Lemma 2.1 shows formally that the dispersion of prices in the economy with positive trend inflation leads to a situation where the firm that, at t, is allowed to re-set its price optimally chooses to produce less than the average firm, so that $Y^*_t < Y_t$ (equation (2.19)). The average firm will therefore hire more labor units, $N_t > N^*_t$, and under DRS face higher marginal costs, $MC_t >$

MC_t^*. The wedge between the quantities of the price re-setting and the average firm (denoted $\phi_{\{n,mc,y\},t}$) generally depends on the ratio of two price indexes: the price adjustment gap, P_t^*/P_t, and price dispersion, S_t, (as defined in equation (2.14)).

For the DRS case (as in the RS model) the ratio of labor demands between the firm re-setting its price at time t, N_t^*, and the average firm, N_t, is given by,

$$N_t^* = \phi_{n,t} N_t \qquad \text{where} \qquad \phi_{n,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \tag{2.20}$$

Firms that are not able to reset their prices hire an inefficient amount of labor where $\phi_{n,t}$ can be interpreted as a measure of labor market inefficiency[17].

Proposition 2.1. *The ratio of price indexes,* $\phi_n = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \gtreqless 1$, *for*

$$\phi_n \begin{cases} < 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \textit{for } \bar{P} = P_t^* = P_t(j) = P_t, \\ > 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{2.21}$$

where $\bar{P}$ is the deterministic steady state of price and $\hat{\pi}_t$ is deviation of inflation from its steady state.

Proof. The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From the Proposition 2.2 in Appendix A, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. Because $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$ (equation (A.1)) for $\Pi_t \geq 1$ it holds that $P_t^* \geq P_t$ In case of positive steady state inflation, $\bar{\pi}_t > 0$ the inflation deviation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. □

Proposition 2.1 shows that in a setting *without trend inflation* ($\bar{\pi} = 0$), the ratio of the price adjustment gap to price dispersion will be smaller than one,

[17] It should be noted that already Ascari (2004) discusses a related effect by looking at the production function, and pointing to the fact that the relationship between employment and output is proportional to the price adjustment gap.

so that $\phi_{n,t} < 1$ in states of the economy with positive inflation realizations, $\pi_t > 0$, and bigger than one, so that $\phi_{n,t} > 1$, in states in which $\pi_t < 0$. This means that unless for the case of $\pi_t = 0$, and where all firms charge the same price, the average firm that cannot adjust its price will hire more (less) labor than optimal, $N_t > N_t^*$ ($N_t < N_t^*$), depending on states of nature with $\pi_t > 0$ ($\pi_t < 0$). In the case of *positive trend inflation*, ($\bar{\pi} > 0$), the probability of realizations of deflationary states of the world decreases, because current inflation needs to fall not only below its steady-state value of $\bar{\pi} > 0$ but below zero. Equivalently, the likelihood of observing states of nature where $\pi_t > 0$ increases. For this reason, the value of $\phi_{n,t}$ will be less than one, $\phi_{n,t} < 1$ for most of the states of the world and accordingly also moves the average value $\phi_{n,t}$ below one. Positive trend inflation thus amplifies the inefficiency of the labor market.

In the DRS case, the effect of price dispersion on the real economy is magnified by the fact that the marginal costs of the average firm will be higher than the marginal costs of the price re-setting firm. Lemma 2.2 shows that average marginal costs are proportional to the marginal costs of price re-setting firm and this proportion is determined by the ratio of two price indexes, price dispersion and price adjustment gap.

Lemma 2.2.

$$MC_t^* = \phi_{mc,t} MC_t \qquad \textit{where} \qquad \phi_{mc,t} = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_t^{\frac{1}{1-\theta}}}. \tag{2.22}$$

Proof. The demand function for the firm resetting its price at time t, for the horizon k is given by,

$$Y_{t+k}^* = A_{t+k}\bar{K}^{\theta} N_{t+k|t}^{1-\theta} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{2.23}$$

where P_t^* is the optimal price of firm resetting its price at time t for the horizon k. The factor demand of the price re-setting firm, N_{t+k}^* is,

$$N_{t+k}^* = \left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^{\theta}}\right)^{\frac{1}{1-\theta}}. \tag{2.24}$$

The ratio of the price re-setting (equation (2.24)) and the aggregate firm's factor demands $N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di$ is given by

$$\frac{N^*_{t+k}}{N_{t+k}} = \frac{\left(\left(\frac{P^*_t}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^\theta}\right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_{t+k}S_{t+k}}{A_{t+k}K^\theta}\right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \tag{2.25}$$

Marginal costs for the price resetting firm are,

$$MC^*_{t+k} = \frac{W_{t+k}}{(1-\theta)A_{t+k}K^\theta N_{t+k}^{-\theta}} \frac{N_{t+k}^{-\theta}}{\left(N^*_{t+k}\right)^{-\theta}}, \tag{2.26}$$

Aggregate marginal cost come from, $\frac{\partial W_t N_t}{\partial Y_t}$ and using $N_{t+k} = \left[\frac{Y_{t+k}}{A_{t+k}K^\theta}\right]^{\frac{1}{1-\theta}} S_{t+k}^{\frac{1}{1-\theta}}$. delivers,

$$\frac{MC_{t+k}}{S_{t+k}} = \frac{W_{t+k}}{(1-\theta)\left(A_{t+k}K^\theta N_{t+k}^{-\theta}\right)}. \tag{2.27}$$

Plugging equation (2.25) into (2.26) and rearranging delivers,

$$MC^*_{t+k} = MC_{t+k} \frac{\left(\frac{P^*_t}{P_{t+k}}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} \tag{2.28}$$

□

Proposition 2.1 also implies that the mean of $\phi_{mc,t}$ will be less than one with trend inflation, $\phi_{mc,t} < 1$. The quantitative impact of the higher cost of production in a stochastic steady-state on the economic dynamics is substantial. The explanation is straightforward. The average firm needs to employ more factor inputs to meet the higher demand for its goods (given by its lower prices), and, as its moves along the concave production function to the right, the marginal costs rise with the level of production. The fact that the average firm will produce at a higher marginal cost than optimal at time t adds an additional inefficiency in the production and amplifies the real costs of price dispersion in the economy.

In the case of constant returns to scale[18], all firms face the same marginal costs (equation (A.15)), and this channel is muted. However, in case of CRS,

[18]Especially in case of a linear production function, $\theta = 0$.

the average firm will still produce more output and thus employ more factor inputs[19] (equation (2.19) and Proposition 2.1).

Trend-Inflation Markup Channel

The presence of trend inflation leads firms to set their price at an additional markup over (current and future expected) marginal costs, which we call the trend-inflation markup: a markup implied by sticky prices and elevated by trend inflation that occurs over and above the traditional markup from monopolistic competition. Trend inflation enters the firm price decision problem, and therefore the first order condition for the optimal price represents another important channel. The price re-setting firm is forward-looking, it can foresee trend inflation and will therefore, on average, set its price above the aggregate price level (which includes non-resetting firms' prices from the past), $P_t^* > P_t$. It is because the optimal price has to equate the present value of future marginal revenues with marginal costs,

$$\sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\epsilon-1} Y_{t+k} \left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}} Y_{t+k} MC_{t+k}^r(j), \tag{2.29}$$

The trend growth in prices increases both the firms' costs of production and the revenues from the sold output. Nevertheless, nominal marginal costs (the expression in the infinite sum on the right hand side of equation (2.29)) grow at a faster rate than nominal revenues (the left hand side of equation (2.29)). So, to keep the equality of marginal revenues with marginal cost in present value terms, the price setting firm must set P_t^* above P_t.[20] The difference between the rate of growth in marginal cost and marginal revenue shapes the firm's markup over (present and future) marginal costs. Equation (2.30) defines the price adjustment gap that depends on the weighted average of the firm's current and expected future real marginal costs.

$$\left(\frac{P_t^*}{P_t}\right)^{1+\frac{\varepsilon\theta}{1-\theta}} = \frac{\epsilon}{\epsilon-1} E_t \sum_{k=0}^{\infty} \phi_{t+k} MC_{t+k}(j) \qquad \text{where} \qquad \phi_{t+k} = \frac{m_{t+k} \Pi_{t+k}^{\frac{\epsilon}{1-\theta}}}{\sum_{k=0}^{\infty} m_{t+k} \Pi_{t+k}^{\epsilon-1}}, \tag{2.30}$$

[19]In the model with capital the wedge between capital hired by the average and the price re-setting firm will further amplify the effects of price dispersion.

[20]Note that trend-inflation markup channel is enforced by the markup channel through the parameter θ which further widens the gap between costs and revenues. Thus, strictly speaking there is a third interaction channel.

where $m_{t+k} = \zeta^k E_t Q_{t,t+k} Y_{t+k}$. Ascari & Sbordone (2014) show that the mark-up, ϕ_t, increases with inflation[21]– and, thus, as trend inflation increases, the firm's trend-inflation markup amplifies the distortion implied by monopolistic competition.

The rise in ϕ_t means that firms put more weight on marginal costs far in the future compared to current marginal costs[22]. Future marginal costs are discounted by the model's implied yield curve with maturity k, where $Q_{t,t+k}(1/\Pi_{t+k})$ is the nominal price of the bond with maturity k. In the model with an upward sloping yield curve and high inflation risks, firms will discount the future relatively more.

A decrease in the wedge between marginal costs can mitigate these channels, which can be done by introducing a (full) inflation indexation. Another option is to increase the monopolistic mark-up (decrease ϵ): having a larger mark-up allows the firm to accommodate bigger deviations from the optimal price. Note, also, that θ and ϵ increase the non-linearity of model equilibrium conditions, which, as we later show, substantially increases approximation errors.

2.2.4 Price-Inflation Spiral and Approximation Accuracy of Price Dispersion

Less known feature implied by the Calvo pricing is an endogenous upper bound on inflation. This upper bound on inflation is effective only under the trend inflation assumption or higher order of approximation. Most models in the field are linearized up to the first order around zero inflation steady state this issues have never been of much of concern. Lemma 2.3 demonstrates that the upper bound on inflation says that given a concave profit function,a firm could maximise profits by not producing at all whenever trend inflation is beyond the upper bound. As trend inflation approaches the upper bound, firms profit maximising output levels fall, and consequently, the aggregate steady-state output falls.

Lemma 2.3. *The steady state of the model and measure of price dispersion is*

[21] As Π goes up, the numerator grows faster – at rate $\Pi_{t+k}^{\frac{\epsilon}{1-\theta}}$ – than the denominator –which grows by $\Pi_t^{\epsilon-1}$

[22] Ascari & Sbordone (2014) shows that overly forward looking agents de-anchor inflation expectations and decrease the determinacy region. This fact also applies to our model as the model solution is indeterminate for $\bar{\pi} > 1.6\%$.

defined only when

$$\pi < \pi^{upper} \qquad \pi_t < \pi^{upper}. \tag{2.31}$$

Proof. The model with fixed capital and production function $Y_t(i) = A_t \bar{K}^{\theta} N_t^{1-\theta}(i)$ delivers following equation for price dispersions:

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left(p_t^*\right)^{-\frac{1+\lambda}{(1-\theta)\lambda}} + \zeta\left(S_{t-1}\right)^{\frac{1}{1-\theta}} \pi_t^{\frac{1+\lambda}{(1-\theta)\lambda}} \tag{2.32}$$

where S_t is the price dispersion and p_t^* is the ratio of re-set price to CPI index. The reset price, p_t^*, can be written in terms of CPI inflation, π_t by using the definition of aggregate price index and elasticity of substitution between differentiated goods, $\frac{1+\lambda}{\lambda} = \epsilon$.

$$S_t^{\frac{1}{1-\theta}} = (1-\theta)\left[\frac{1-\zeta\left(\pi_t\right)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \theta(\pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}} \tag{2.33}$$

which implies

$$\left(\left[\frac{1-\zeta\left(\pi_t\right)^{\epsilon-1}}{1-\zeta}\right]\right) > 0 \tag{2.34}$$

and thus,

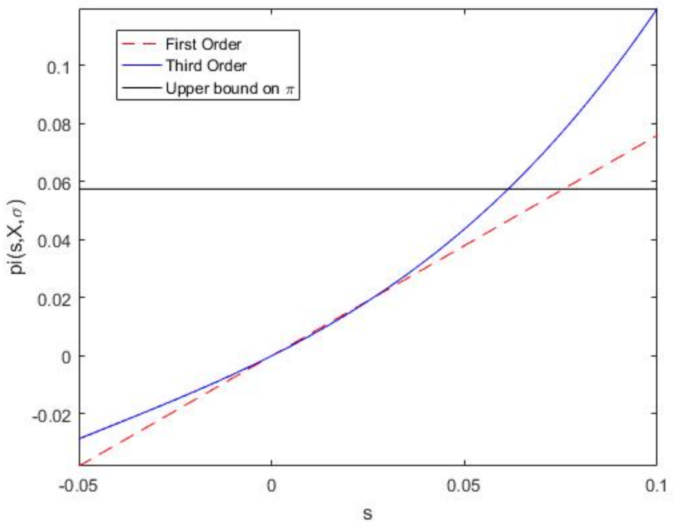
$$\pi_t < \left(\frac{1}{\zeta}\right)^{\frac{1}{\epsilon-1}} \tag{2.35}$$

□

The upper bound on inflation implies a king in the policy function which is poorly approximated by the standard perturbation methods. This fact has been already pointed out by Andreasen & Kronborg (2017). The fact that the approximation of the policy function does not reflect that the upper bound on inflation leads to price-inflation spiral. Agents in the model does not reflect the maximum inflation in their expectations when setting their prices. Figure 2.2 shows the policy function in terms of percentage deviations from steady state for inflation as a function of the state variable capturing the dispersion of prices in the economy. The other state variables are assumed to be at their respective steady state values. It can be seen that neither linearized not even third order approximation can account for the king in policy function.

In what follows, we focus our analysis further on the dispersion of prices in the economy. As the model parameters of RS were calibrated to match moments for the case of $\bar{\pi} = 0$, it may be argued that the model might be not

Figure 2.2: Upper bound and policy function for Price dispersion



well calibrated. We first confirm that the patterns documented in Table 2.1 hold across a wide range of parameter values.

Figure 2.3 shows how mean of (the inverse) price dispersion changes over different ranges of parameter values and orders of approximation. The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second, third-order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. Whereas the mean simulated price dispersion is affected strongly by varying the trend inflation (panel 1), including pushing S^{-1} to an infeasible region[23], varying other model parameters does not affect the simulated mean price dispersion drastically (and never pushes S^{-1} to an infeasible region). Other than variations in trend inflation, only regions of relatively high elasticities of substitution or high price rigidities lead to large costs from price dispersion (of, e.g. more than 1%, reflected in S^{-1} falling below 0.99). The second set of panels presents comparable figures for the case of positive trend inflation. Pink diamonds reflect the 'RS Table 3'-baseline parameterization, apart for steady-state inflation, which now is $\bar{\pi} = 1\%$. Since the accuracy of the mean price dispersion is already somewhat compromised at $\bar{\pi} = 1\%$, regions of relatively high elasticities of substitution or high price rigidities quickly lead to problems (flat lines in the last two reported panels represent cases with indeterminate solutions). Varia-

[23] S^{-1} is bounded from above by one. See Proposition 2.2

tions in other key parameters continue to leave mean price dispersion mostly unaffected.

To further examine the role of price dispersion in generating explosive dynamics we look into the numerical accuracy of the approximation to the price dispersion equation (2.14). Alongside the more rigorous study of Andreasen & Kronborg (2017) on numerical accuracy of approximation methods we calculate a more accurate measure of price dispersion, noting that price dispersion can be written recursively as

$$S_t^{\frac{1}{1-\theta}} = (1-\zeta)\left[\frac{1-\zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta}\right]^{\frac{\epsilon}{(\epsilon-1)(1-\theta)}} + \zeta(\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}. \tag{2.36}$$

We proceed as follows.[24] First, we use an initial value S_{t-1} from the approximated model as a starting point. Second, we iterate the equation forward to get an exact solution conditional on the model-approximated time path of Π_t. Third, we compare this more exact measure of price dispersion with its counterpart from the third- order approximation[25].

The panels in Figure 2.4 contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (2.36) of the main text, for several model versionS. As can be seen, the third order approximation a) deviates sharply from the path of price dispersion using the exact formula, and b) includes many infeasible realizations of $S^{-1} > 1$. The problems diminish or disappear when adopting one of the proposed fixes documented in section 2.2.2.

Subpanel 'RS1' of Figure 2.4 stresses this finding by showing that the approximation of price dispersion is poor even for the original RS model[26] as the deviations between the third-order and 'exact' solution are large. Perturbation methods do an even poorer job in the case of positive trend inflation. In addition, in the case of positive inflation the third order approximation generates state of the worlds which are economically infeasible as S_t^{-1} exceeds one, which means that more resources are spent than produced, $Y_t < C_t + I_t + G_t$. Subpanel 'RS1*' in the second row shows that a first-order approximation delivers smaller approximation errors.

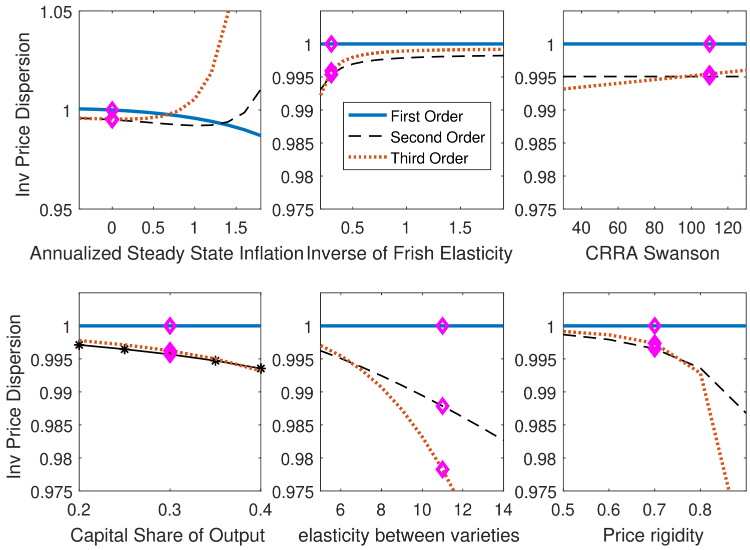
[24] We very much thank Larry Christiano for suggesting to look at the problem in this way.

[25] Andreasen & Kronborg (2017) shows that although the conditioning on inflation delivers somewhat different solution compared to the use of more accurate projection methods, the approximation errors of our more exact measure should be small. For this reason, the conditioning on inflation should not harm our argument.

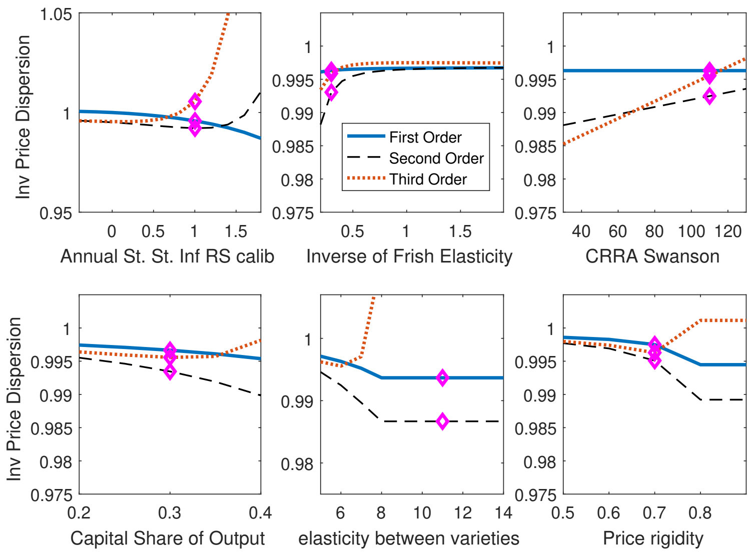
[26] Note that the original Rudebusch & Swanson (2012) results are sensitive to the seed of random number generator even for very long simulations.

Figure 2.3: Parameters sensitivity in the RS (2012) model

Parameter sensitivity of RS (2012) model, case of zero trend inflation, $\bar{\pi} = 0\%$

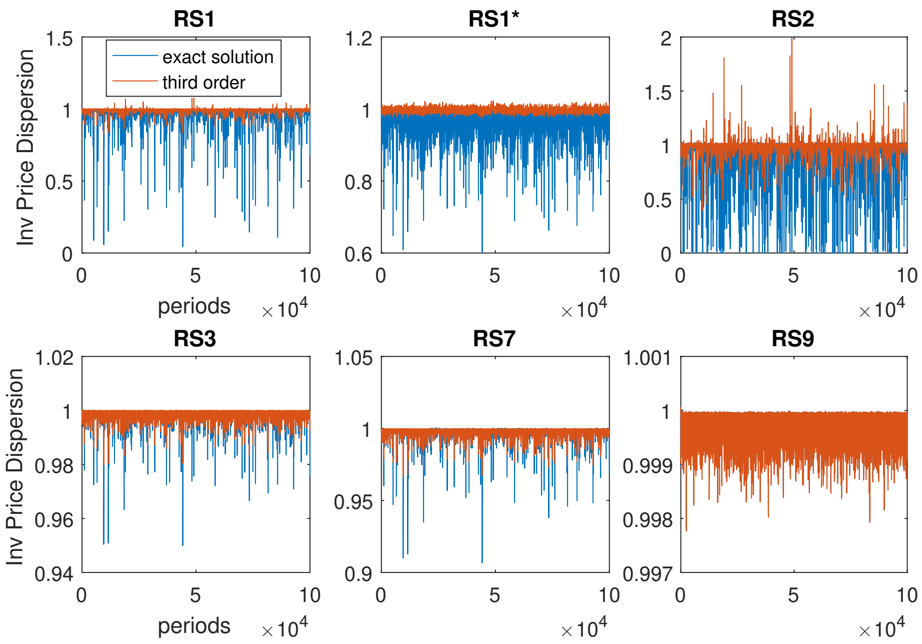


Parameter sensitivity of RS (2012) model, case of positive trend inflation, $\bar{\pi} = 1\%$



Note: The first set of panels shows the sensitivity of the mean simulated price dispersion to changes in key model parameters for the case of zero trend inflation, for different orders of approximation (first, second and third order approximations). Pink diamonds reflect the case of the 'RS Table 3'-baseline parameterization. The second set of panels presents analogous figures for the case of positive trend inflation. Flat lines in the last two reported panels represent cases with indeterminate solutions.

Figure 2.4: Approximation Errors for Price Dispersion



Note: The panels contrast simulated paths for price dispersion, as computed from a third-order approximation of the model with the 'exact' behavior for price dispersion, using equation (2.36), conditioning on the simulated time path of Π *from the third-order-approximated model.* ***RS1****: original RS model, with the following features: fixed capital* $Y_t = A_t \bar{K}^{\theta} N_t^{1-\theta}$*, time-varying inflation target,* π_t^**, zero trend inflation,* $\bar{\pi} = 0\%$*.* ***RS1*****: as in RS1, but approximated only up to the first order.* ***RS2****: as in RS1, but with positive trend inflation of* $\bar{\pi} = 1\%$*.* ***RS3****: as in RS1, but with trend inflation of* $\bar{\pi} = 1\%$ *and a labor-only-DRS production function,* $Y_t = A_t N_t^{1-\theta}$*.* ***RS7****: as in RS1, but with trend inflation of* $\bar{\pi} = 1\%$ *and with a labor-only-CRS production function,* $Y_t = A_t N_t$*.* ***RS9****: as in RS1, but with trend inflation of* $\bar{\pi} = 1\%$ *and with indexation to last-period inflation (*$\iota = 1$*).*

The approximation errors for the cases of indexation to past inflation and constant return to scale in labor are negligible. The RS model with positive steady-state inflation and indexation delivers both small price dispersion and negligible approximation errors, as can be observed by the almost complete overlay of the two simulated series. However, it should be noted that in the case of the linear production function, the more exact measure of price distortion is still large. There are states of the world when the price dispersion implies an almost 10% quarterly output loss, which is at odds with empirical evidence (see, for example, Nakamura *et al.* (2018)). Andreasen & Kronborg (2017) conjectures that this explosive dynamics in price dispersion come from the price-inflation spiral generated by the fact that the perturbation methods up to third order fails to account for an upper bound on inflation.

2.3 Conclusion

This note emphasizes that an attempt to realign the current macro-finance workhorse modeling framework of RS with recent empirical evidence should include incorporating positive trend inflation into such a framework. We document that pricing assets in models that are based on the Calvo price mechanism can lead to extremely counterfactual model dynamics, once trend inflation is present; we then propose a number of directions to overcome such complications. This way, we contribute to providing guidance along the path of finding a new, empirically well-motivated and consistent modeling framework.

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2.A Rudebusch and Swanson (RS) Model

This appendix gives a summary of the Rudebusch & Swanson (2012) model equilibrium.

System of Model Equations

Table 2.3 summarizes the system of equations of the Rudebusch Swanson model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables) defined as $c_t = \frac{C_t}{Z_t}$, $y_t = \frac{Y_t}{Z_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t Z_t}$, $p_t^* = \frac{P_t^*(j)}{P_t}$, $mc_t(i) = \frac{MC_t(i)}{P_t}$, $y_t = \frac{Y_t}{Z_t}$, $\mu_t = \frac{Z_t}{Z_{t-1}}$. The best fit calibration of the RS model based on their Table 3 is summarized in Table 2.4. In this setting, model dynamics are driven by three types of shocks, stationary technology shocks, government spending shocks, and inflation target shocks (in particular, there are no trend productivity shocks, so that $\mu_t = \frac{Z_t}{Z_{t-1}} = \mu$ is constant).

Table 2.3: System of model equations, Rudebusch Swanson model

(RS1):	$V_t = \frac{c_t^{1-\varphi}}{1-\varphi} + \chi_0 \frac{(1-N_t)^{1-\chi}}{1-\chi} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$
(RS2):	$Q_{t-1,t} = \mu_t^{-\gamma} \left(\frac{(V_t \mu_t^{1-\gamma})}{[E_{t-1}(V_t \mu_t^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \left(\frac{c_t}{c_{t-1}} \right)^{-\varphi}$
(RS3):	$\chi_0 (1-N_t)^{-\chi} c_t^{\varphi} = w_t$
(RS4):	$1 = \beta E_t \left\{ Q_{t,t+1} \frac{(1+i_t)}{\Pi_{t+1}} \right\}$
(RS5):	$(p_t^*)^{1+\frac{\theta\epsilon}{1-\theta}} = \frac{aux_{1t}}{aux_{2t}}$
(RS6):	$aux_{1t} = \frac{\epsilon}{\epsilon-1} mc_t y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\frac{\epsilon}{1-\theta}} aux_{1t+1}$
(RS7):	$aux_{2t} = y_t + \beta\zeta Q_{t,t+1} \Pi_{t+1}^{\epsilon-1} aux_{2t+1}$
(RS8):	$S_t Y_t = A_t \bar{K}^{\theta} (N_t)^{1-\theta}$
(RS9):	$S_t^{\frac{1}{1-\theta}} = (1-\zeta)(p_t^*)^{\frac{-\epsilon}{1-\theta}} + \zeta (\Pi_t)^{\frac{\epsilon}{1-\theta}} S_{t-1}^{\frac{1}{1-\theta}}$
(RS10):	$\Pi_t^{1-\epsilon} = (1-\zeta)(p_t^* \Pi_t)^{1-\epsilon} + \zeta$
(RS11):	$MC_t = \frac{1}{1-\theta} \overline{K}^{\frac{\theta}{1-\theta}} \frac{W_t}{A_t} \left(\frac{y_t}{A_t} \right)^{\frac{\theta}{1-\theta}}$
(RS12):	$y_t = c_t + \overline{I} + g_t$
(RS13):	$4i_t = 4\rho_i i_{t-1} + (1-\rho_i) \left[4(\bar{i} - \bar{\pi}) + (\pi_t^{avg}) + \phi_\pi (4(\pi_t^{avg}) - (\pi_t^*)) + \phi_Y \left(\frac{\mu_t Y_t}{\bar{\mu} \bar{Y}} - 1 \right) \right]$
(RS14):	$\pi_t^* = (1-\rho_{\pi^*}) 4\pi_t^{avg} + \rho_{\pi^*} \pi_{t-1}^* + \zeta_{\pi^*} (4\pi_t^{avg} - \pi_t^*) + \sigma_{\pi^*} \varepsilon_{\pi^*,t}$
(RS15):	$\pi_t^{avg} = \theta_{\pi^{avg}} \pi_{t-1}^{avg} + (1-\theta_{\pi^{avg}}) \pi_t$
(RS16):	$\log A_t = \rho_A \log A_{t-1} + \sigma_A \varepsilon_{A,t}$
(RS17):	$\log(g_t/\bar{g}) = \rho_G \log(g_{t-1}/\bar{g}) + \varepsilon_t^G$

2.A.1 Aggregation

Here we describe in detail the aggregation across the i-firms in case of decreasing return to scale and constant return to scale production function.

Aggregate Price Index

The aggregate price index $P_t = \left[\int_0^1 P_t^{1-\epsilon}(j)dj\right]^{\frac{1}{1-\epsilon}}$ can be written using the Calvo result as,

$$\frac{P_t^*}{P_t} = \left[\frac{1-\zeta\left(\Pi_t\right)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}, \tag{A.1}$$

Aggregation for DRS

The production function of intermediate firm i is given by $Y_t(i) = A_t K^\theta N_t^{1-\theta}(i)$. Using this, plug in for $Y_t(i)$ into the demand for variety i, equation 2.4, solve for $N_t(i)$ and integrate over all varieties i. Since the workers are all the same the aggregation of hours worked is $N_t = \int_0^1 N_t(i)di$. The aggregation delivers,

$$N_t = \left(\frac{Y_t}{A_t K^\theta}\right)^{\frac{1}{1-\theta}} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} di, \tag{A.2}$$

which can be re-written as

$$Y_t = S_t^{-1} A_t K_t^\theta N_t^{1-\theta}, \tag{A.3}$$

where the variable $S_t^{\frac{1}{1-\theta}} = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$ defines the price dispersion.

Resetting Firm vs. Aggregate Quantities for DRS

The demand function for the firm resetting its price at time t, for the horizon k is given by,

$$Y_{t+k|t} = A_{t+k}\bar{K}^\theta N_{t+k|t}^{1-\theta} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}, \tag{A.4}$$

where P_t^* is the optimal price of firm resetting its price at time t for the horizon k. The factor demand of the price re-setting firm, $N_{t+k|t}$ is,

$$N_{t+k|t} = \left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^\theta}\right)^{\frac{1}{1-\theta}}. \tag{A.5}$$

The ratio of the price re-setting (equation (A.5)) and the aggregate firm's factor demands (A.2)) is given by

$$\frac{N_{t+k|t}}{N_{t+k}} = \frac{\left(\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} \frac{Y_{t+k}}{A_{t+k}\bar{K}^{\theta}}\right)^{\frac{1}{1-\theta}}}{\left[\frac{Y_{t+k}S_{t+k}}{A_{t+k}K^{\theta}}\right]^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]}. \quad (A.6)$$

An analogous ratio can be derived for aggregate marginal cost and marginal costs of the price resetting firm. Marginal costs for the price resetting firm are,

$$MC_{t+k|t} = \frac{W_{t+k}}{(1-\theta)A_{t+k}K^{\theta}N_{t+k}^{-\theta}} \frac{N_{t+k}^{-\theta}}{N_{t+k|t}^{-\theta}}, \quad (A.7)$$

Aggregate marginal cost come from, $\frac{\partial W_t N_t}{\partial Y_t}$ and using $N_{t+k} = \left[\frac{Y_{t+k}}{A_{t+k}K^{\theta}}\right]^{\frac{1}{1-\theta}} S_{t+k}^{\frac{1}{1-\theta}}$. delivers,

$$\frac{MC_{t+k}}{S_{t+k}} = \frac{W_{t+k}}{(1-\theta)\left(A_{t+k}K^{\theta}N_{t+k}^{-\theta}\right)}. \quad (A.8)$$

Plugging equation (A.6) into (A.7) and rearranging delivers,

$$MC_{t+k|t} = MC_{t+k}\frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\theta\epsilon}{1-\theta}}}{S_{t+k}^{\frac{1}{1-\theta}}} \quad (A.9)$$

Aggregation for CRS

The cost minimization problem is given by

$$\min_{N_t(i)} W_t N_t(i) + R_t^k K_t + MC_t^r(i)\left[Y_t(i) - A_t K_t(i)^{\theta} N_t^{1-\theta}(j)\right], \quad (A.10)$$

subject to the production function, $Y_t(i) = A_t K_t(i)^{\theta} N_t^{1-\theta}(i)$, where $MC_t(i)$ is the multiplier associated with the constraint.

The firm's demand for labor,

$$W_t = MC_t^r(i)(1-\theta)A_t K_t(i)^{\theta} N_t^{-\theta}, \quad (A.11)$$

The firm's demand for capital,

$$R_t^k = MC_t^r(i)A_t\theta K_t(i)^{\theta-1}N_t^{1-\theta}(i), \tag{A.12}$$

Plugging the factor demands into the definition of total costs, $TC_t(i) = W_tN_t(i) + R_t^kK_t(i)$ delivers,

$$TC_t(i) = [MC_t^r(i)]\, Y_t(i). \tag{A.13}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(i)}{dY_t(i)} = MC_t^r(i)$, which shows that the Lagrange multiplier equals real marginal costs.

From equation (A.11) and equation (A.12) we get that,

$$\frac{1-\theta}{\theta} = \frac{W_tN_t(i)}{R_t^kK_t(i)}. \tag{A.14}$$

Since factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(i)}{K_t(i)}$ is the same for all firms. Plugging factor demands into production function delivers,

$$MC_t^r = \int_0^1 MC_t^r(i)di = \frac{\left(R_t^k\right)^\theta W_t^{1-\theta}}{A_t\theta^\theta(1-\theta)^{1-\theta}}, \tag{A.15}$$

Resetting Firm vs. Aggregate Quantities for CRS

From the relationship $Y_{t+k|t} = A_{t+k}K_{t+k|t}^\theta N_{t+k|t}^{1-\theta} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$ we can derive,

$$N_{t+k|t} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \left(\frac{K_{t+k}}{K_{t+k|t}}\right)^{\frac{\theta}{1-\theta}} N_{t+k}. \tag{A.16}$$

The ratio of capital demand equations for the price resetting firm and the aggregate firm delivers,

$$\frac{K_{t+k}}{K_{t+k|t}} = \frac{Y_{t+k}}{Y_{t+k|t}}, \tag{A.17}$$

Using the labor demand equations of price resetting and aggregate firms, together with equation (A.17), we get the relationship,

$$Y_{t+k|t} = \frac{\left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon}}{\left[\int_0^1 \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}} Y_{t+k}, \tag{A.18}$$

Later we show that in case of positive inflation, $Y_{t+k|t} \leq Y_{t+k}$ in the case of CRS. Thus, as from above $\frac{K_{t+k}}{K_{t+k|t}} = \frac{Y_{t+k}}{Y_{t+k|t}}$, then $K_{t+k|t} \leq K_{t+k}$ and thus $N_{t+k|t} \leq N_{t+k}$.

2.A.2 Proofs and Propositions

Proposition 2.2. *Price dispersion is bounded by one, $S_t \geq 1$.*

Proof. The aggregate price index, $P_t = \left[\int_0^1 P_t^{1-\epsilon}(i)\right]^{\frac{1}{1-\epsilon}}$ divide by P_t is $1 = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ Defining $v_{i,t} = \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}$ we get that $\left[\int_0^1 v_{i,t}\right]^{\frac{1}{1-\epsilon}} = 1$. Writing price dispersion, $S_t = \left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]^{1-\theta}$, in terms of $v_{i,t}$, $v_{i,}^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}} = \left[\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}\right]^{\frac{-\epsilon}{1-\epsilon}\frac{1}{1-\theta}}$ Thus, price dispersion can be written in terms of variable v as, $S_t^{\frac{1}{1-\theta}} = \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}}$ And as $\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta} > 1$, Jensen's inequality implies that

$$1 = \left[\int_0^1 v_{i,t}\right]^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} \leq \int_0^1 v_{i,t}^{\frac{\epsilon}{\epsilon-1}\frac{1}{1-\theta}} = S_t^{\frac{1}{1-\theta}}. \tag{A.19}$$

□

Proposition 2.3. *The ratio of price indexes, $\phi_n = \frac{\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}}}{\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} dj\right]} \gtreqless 1$, for*

$$\phi_n \begin{cases} < 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t > 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t > -\bar{\pi}, \\ = 1 & \textit{for } \bar{P} = P_t^* = P_t(j) = P_t, \\ > 1 & \textit{for } \bar{\pi} = 0 \quad \& \quad \hat{\pi}_t < 0, \\ & \textit{for } \bar{\pi} > 0 \quad \& \quad \hat{\pi}_t < -\bar{\pi}, \end{cases} \tag{A.20}$$

where $\bar{P}$ is the deterministic steady state of price and $\hat{\pi}_t$ is deviation of inflation from its steady state.

Proof. The ratio $\phi_n < 1$ if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} < \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di$. From the Proposition 2.2, $\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{\frac{-\epsilon}{1-\theta}} di \geq 1$. Thus, it must be true that if $\left(\frac{P_t^*}{P_t}\right)^{-\frac{\epsilon}{1-\theta}} \leq 1$ then $\phi_n \leq 1$. This will hold for all cases when $P_t^* \geq P_t$. Because $\frac{P_t^*}{P_t} = \left[\frac{1-\zeta(\Pi_t)^{\epsilon-1}}{1-\zeta}\right]^{\frac{1}{1-\epsilon}}$ (equation (A.1)) for $\Pi_t \geq 1$ it holds that $P_t^* \geq P_t$ In case of positive steady state inflation, $\bar{\pi}_t > 0$ the inflation deviation from its steady state can reach $\hat{\pi}_t > -\bar{\pi}$ for $\phi_n \leq 1$. □

2.A.3 Calibration

Table 2.4: Calibration of the RS table 3 (best fit) model

Symbol	Variable	Value
β	Discount factor	0.99
$CRRA$	Risk aversion	110
IES	Intertemporal elasticity	0.09
ϵ	Elasticity of substitution	6
$Frisch$	Frisch elasticity	0.28
ϕ_π	Response to inflation	0.53
ϕ_y	Response to output	0.93
ρ_i	i_t smoothing	0.73
ζ	Price adjustment	0.76
$\bar{G}/\bar{Y}$	Government spending on output	0.17
ρ_G	Autocorrelation Government spending shock	0.95
σ_G	Volatility of Government spending shock	0.004
ρ_A	Autocorrelation of TFP shock	0.95
σ_A	Volatility of TFP shock	0.005
$\theta_{\rho_{pi^*}}$	Inflation target shock persistence	0.995
σ_{π^*}	Volatility of inflation target shock	0.0007
ζ_{π^*}	Inflation target adjustment	0.003
θ	Capital share of output	1/3
$\bar{\Pi}$	Steady state inflation	1.004
δ	Capital depreciation	0.02

2.B Basic New Keynesian (CGG) Model

This section of the appendix outlines the basic New Keynesian model and presents results analogous to the one in the main text. The model closely follows the sticky price model of Clarida *et al.* (1999), with two exceptions: one, we use a production function that is assumed to be of the DRS-labor-only type as our baseline, as in the RS model. Two, we assume that productivity shocks are difference-stationary (in the case of trend-stationary shocks the channels leading to high levels and poor approximation of price dispersion are quantitatively inconsequential).

Otherwise, the model features are standard, firms are monopolistically competitive, face nominal rigidities à la Calvo, and the monetary authority follows a standard Taylor rule. Below we provide a sketch of the model and a list of first order and equilibrium conditions.

2.B.1 Model Sketch, CGG model

Households

A representative household has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\left(\frac{C_t}{A_t}\right)^{1-\tau}}{1-\tau} - \xi_t \frac{N_t^{1+\varphi}}{1+\varphi} \right\}, \tag{A.21}$$

where utility from consumption is divided by the (growing) level of technology, such as to have a well-defined balanced growth path. The household maximizes the above preferences subject to its budget:

$$P_t C_t + B_t \leq B_{t-1} R_{t-1} + W_t N_t + T_t. \tag{A.22}$$

Final Good Firms

Final good firms have production technology

$$Y_t = \left[\int_0^1 Y_t\left(i\right)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.23}$$

where $Y_t\left(i\right)$ are differentiated types of intermediate goods used as production inputs. The final good firm maximizes profits by selling Y_t at P_t and buying $Y_t\left(i\right)$ at prices $P_t\left(i\right)$.

Intermediate Goods Firms

An intermediate good firm's problem can be split into a (static) cost minimization and a (dynamic) profit maximization problem. The cost minimization problem reads

$$\min_{N_t(j)} \left\{ W_t N_t\left(j\right) + MC_t\left(j\right) \left[Y_t\left(j\right) - A_t N_t\left(j\right)^{1-\alpha} \right] \right\},$$

Table 2.5: System of model equations, New Keynesian model

(NK1):	$\xi_t N_t^{\varphi} c_t^{\tau} = w_t$
(NK2):	$c_t^{-\tau} = \beta E_t c_{t+1}^{-\tau} \frac{1}{dA_{t+1}} \frac{R_t}{\Pi_{t+1}}$
(NK3):	$p_t^{*\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right)} = (1-\nu) \frac{\varepsilon}{(\varepsilon-1)} \frac{aux_{1,t}}{aux_{2,t}}$
(NK4):	$aux_{1t} = mc_t y_t + E_t \beta\theta \Pi_{t+1}{}^{\frac{\varepsilon}{1-\alpha}} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{1t+1}$
(NK5):	$aux_{2t} = y_t + E_t \beta\theta \Pi_{t+1}^{\varepsilon-1} \frac{c_{t+1}^{-\tau}}{c_t^{-\tau}} aux_{2t+1}$
(NK6):	$S_t y_t = N_t{}^{1-\alpha}$
(NK7):	$S_t^{\frac{1}{1-\alpha}} \equiv (1-\theta)(p_t^*)^{\frac{-\varepsilon}{1-\alpha}} + \theta (\Pi_t)^{\frac{\varepsilon}{1-\alpha}} \Delta_{t-1}^{\frac{1}{1-\alpha}}$
(NK8):	$p_t^* = \left[\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\right]^{\frac{1}{1-\varepsilon}}$
(NK9):	$mc_t = \frac{1}{1-\alpha} w_t y_t^{\frac{\alpha}{1-\alpha}}$
(NK10):	$c_t = y_t$
(NK11):	$\frac{R_t}{R} = \left(\frac{R_t}{R}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi} \left(\frac{y_t}{y_t^{flex}}\right)^{\rho_y}\right]^{1-\rho_R} e^{\varepsilon_{R,t}}$
(NK12):	$y_t^{flex} = \left[(1-\nu)\frac{\varepsilon}{(\varepsilon-1)}\frac{1}{\xi_t}\right]^{\frac{1-\alpha}{\varphi+\tau(1-\alpha)}}$
(NK13):	$\log(dA_t) = \rho_A \log(dA_{t-1}) + (1-\rho_A) dA + \varepsilon_{dA,t}$
(NK14):	$\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + (1-\rho_\xi)\xi + \varepsilon_{\xi,t}$

from which an expression for the firm's marginal cost $MC_t(j)$ can be derived. The firm's profit maximization problem, taking as given the demand function the firm faces for its product, is then given by:

$$\max_{P_t(j)} E_t \sum_{k=0}^{\infty} \theta^k \Omega_{t,t+k} \left\{ \left[P_t(j) - MC_t(j)\right] Y_t(j) \right\}. \tag{A.24}$$

System of Model Equations

Table 2.5 summarizes the system of equations of the New Keynesian model in terms of stationary allocations and real (relative) prices (i.e., in term of detrended and deflated variables, denoted by lowercase variables), defined as $c_t = \frac{C_t}{A_t}$, $y_t = \frac{Y_t}{A_t}$, $\Pi_t = \frac{P_t}{P_{t-1}}$, $w_t = \frac{W_t}{P_t A_t}$, $b_t = \frac{B_t}{P_t A_t}$, $t_t = \frac{T_t}{P_t A_t}$, $p_t^* = \frac{P_t^*(j)}{P_t}$, $mc_t(j) = \frac{MC_t(j)}{P_t}$, $y_t = \frac{Y_t}{A_t}$, $dA_t = \frac{A_t}{A_{t-1}}$, and where price dispersion is defined as $S_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{\frac{-\varepsilon}{1-\alpha}} dj$. Table 2.6 summarizes parameter values used in model simulations.

Table 2.6: Calibration of the CGG New Keynesian model

Symbol	Variable	Value
β	Discount factor	0.99
τ	Coefficient of relative risk aversion	1
ϵ	Elasticity of substitution betw. varieties	9
φ	Inverse Frisch elasticity	5
ρ_π	Coefficient on inflation, Taylor rule	1.5
ϕ_y	Coefficient on output gap, Taylor rule	0
ρ_R	Interest rate smoothing, Taylor rule	0.75
θ	Calvo parameter	0.75
α	1-alfa is the weight on labor in prod. fct.	1/4
ρ_ξ	Autocorrelation, preference shock	0.95
σ_ξ	Volatility, preference shock	0.01
ρ_{dA}	Autocorrelation, TFP growth shock	0.85
σ_{dA}	Volatility, TFP growth shock	0.005

2.B.2 Results

This section of the appendix lays out results from model simulations for the New Keynesian model. Table 2.7 and 2.8 mirror the model versions and results for the RS model in the main text. Table 2.7 reports model moments for the baseline model with zero trend inflation (NK1), the version with positive trend inflation (NK2) and the version with positive trend inflation and variable capital (NK3). As with the RS model, the trend-inflation augmented model version gives rise to problems of inflated model moments and counterfactual regions over which price dispersion travels, as witnessed in particular by the maximum values of S^{-1} observed over the simulation. As stressed already in the main text, whether or not the NK model is susceptible to counterfactual levels of price dispersion and the resulting problems of unreasonable model moments is ultimately a quantitative question. Simply changing the persistence parameter ρ_{dA} from the reported value in table 2.6 to 0.5 implies that none of the model versions, also not NK2 or NK3, give rise to any problems and display well-behaved regions for price dispersion, with MAX(S^{-1}) strictly smaller than one and MIN(S^{-1}) not lower than 0.98. Similarly, we never encounter any sign of elevated levels of price dispersion in a model version with trend-stationary shocks.

Table 2.8 reports model moments for the model versions that feature one of the modeling devices that keep the behavior of price dispersion contained and therefore provide a fix to the problems of inflated moments, paralleling

Table 2.7: Empirical and Model-Based Unconditional Moments

Unconditional Moment	USdata 1961-2007	NK1 $\bar{\pi} = 0$	NK2 $\bar{\pi} = 2.0\%$	NK3 $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$
SD(C)	0.83	1.59	11.31	6.30
SD(N)	1.71	2.92	4.62	0.97
Mean(π)	3.50	-0.38	-0.41	-0.57
SD(π)	2.52	3.25	6.11	2.23
MEAN(i)	5.72	-0.57	-0.63	-0.87
SD(i)	2.71	3.40	7.99	2.80
MEAN(S^{-1})	0.00	0.98	0.96	1.00
SD(S^{-1})	0.00	0.02	0.13	0.00
MIN(S^{-1})	0.00	0.88	0.21	0.97
MAX(S^{-1})	0.00	1.00	1.35	1.00

Note: Model moments are calculated from the simulated series. ***NK1****: model with labor-only-DRS production function $Y_t = A_t N_t^{1-\alpha}$, zero trend inflation, $\bar{\pi} = 0\%$.* ***NK2****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$.* ***NK3****: as in NK1, but: with positive trend inflation $\bar{\pi} = 2\%$, with variable capital $Y_t = A_t K_t^{\theta} N_t^{1-\theta}$.*

table 4.3 of the main text. In particular, NK4 considers the case of Rotemberg adjustment costs, NK5 is the model version with a linear-in-labor production function, and NK6 and NK7 are the model versions with inflation indexation, either with respect to steady state inflation or with respect to past quarter inflation.

2.C Additional Figures

Table 2.8: Empirical and Model-Based Unconditional Moments

Unconditional Moment	NK2 $\pi = 2.0$	NK4 Rotemberg	NK5 $Y_t = A_t N_t$	NK6 $\iota = 0$	NK7 $\iota = 1$
SD(C)	1.59	1.51	1.18	1.59	1.85
SD(N)	2.92	4.42	1.40	2.92	2.82
Mean(π)	-0.38	-0.38	-0.36	-0.38	-0.38
SD(π)	3.25	3.62	4.06	3.25	2.92
MEAN(i)	-0.57	-0.57	-0.54	-0.57	-0.58
SD(i)	3.40	3.40	3.88	3.40	3.27
MEAN(S^{-1})	0.98	1.00	0.99	0.98	1.00
SD(S^{-1})	0.02	0.00	0.01	0.02	0.00
MIN(S^{-1})	0.88	0.98	0.94	0.88	0.98
MAX(S^{-1})	1.00	1.00	1.00	1.00	1.00

Note: Model moments are calculated from the simulated series. ***NK2****: equal to NK2 from Table 2.7.* ***NK4****: as in NK2, but: with Rotemberg adjustment costs instead of Calvo pricing.* ***NK5****: as in NK2, but: with labor-only-CRS* $Y_t = A_t N_t$. ***NK6****: as in NK2, but: with indexation to steady state inflation.* ***NK7****: as in NK2, but: with indexation to last-period inflation.*

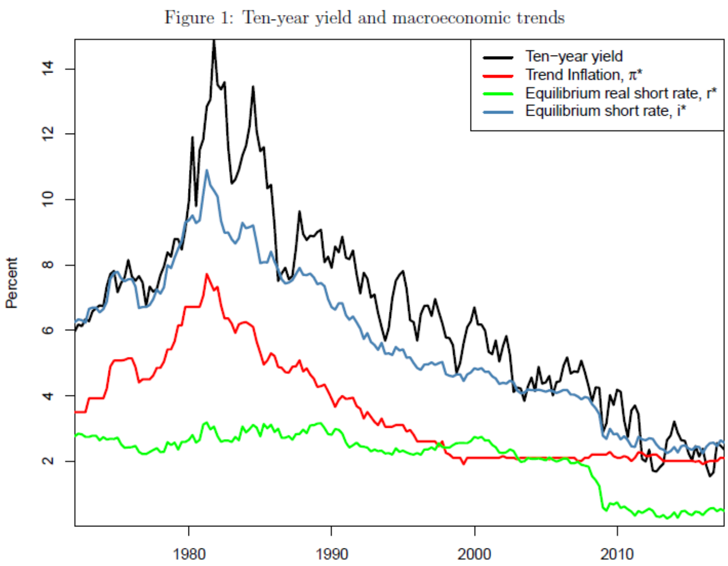

Figure 2.5: 10Y T-yield,estimates of π^*, the equilibrium real rate, r^*,equilibrium short rate, $i^* = \pi^* + r^*$. The data are quarterly from 1971:Q4 to 2017:Q2. Source: Bauer and Rudebusch (2017)

Chapter 3

Government Spending and the Term Structure of Interest Rates in a DSGE Model

3.1 Introduction

How does the term structure of default-free bond prices respond to a rise in government expenditures? Is the uncertainty about the size of government spending important for bond prices? Does the impact of uncertainty depends on monetary policy conduct? Can fiscal policy immunize its impact on the term structure of interest rates? The literature provides little guidance. The impact of government expenditures on real economy has been for long studied in literature on fiscal multipliers.[1] Impact on the term structure of interest rates has been however mostly neglected. Recent development in financial friction literature[2] stressed the impact of bond prices on real economy and the term

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[1] See for a survey Christiano *et al.* 2011.

[2] See Brunnermeier *et al.* 2013 or Brazdik *et al.* 2012 for a survey.

structure of interest rates become one of the instrument of monetary policy. We contribute to the debate on implications of fiscal policy by studying the link between government expenditures (productive, wasteful and substitutable and non-substitutable utility enhancing) and bond prices. We build our analysis on a variant of macro-finance DSGE model[3] matching both macro and finance stylized facts. We show that investors use default free bonds to protect their wealth against economic uncertainty caused by unpredictability of future government spending. As increasing uncertainty about future wasteful government spending rises the price of issued Treasuries, fiscal authority can decrease the cost of its debt by increasing the unpredictability of future expenditures. This is because bonds serves as the only insurance vehicle[4] for investors against the fluctuations in their wealth. We stress that the focus here is on U.S. Treasuries which bear very low default premium and this is why our model implied bond prices are default-free.

We extend the existing literature in several directions. First, we augment the method of Hordahl *et al.* (2008) and show how to derive the second order approximation of the pricing kernel in the model with Epstein Zin (EZ) preferences. Deriving the pricing kernel in terms of conditional second moments of underlying macro variables provides us with an analytical explanation of how exogenous shocks translate into bond prices. It allows us to interpret the bond risk premium in terms of macroeconomic factors and therefore we can provide richer structural decomposition than Andreasen (2012) who decomposes term premia only into real and nominal part.

Second, we propose a new method to explicitly calculate the model implied conditional second moments of underlying macro variables which we refer to as macroeconomic factors. The idea rests on a simple factor model used in asset management for performance attribution.[5] The linear property of the approximated pricing kernel allows us recursively calculate the conditional moments in

[3]We consider several modeling set-ups. We started with augmented model of Andreasen (2012) and Ferman (2011) which are based on Rotemberg pricing and contain non-zero steady state inflation. Later on we moved to modeling framework of Rudebusch & Swanson (2012) which become the state of the art model in the literature. Rudebusch & Swanson (2012) model is based on Calvo pricing and zero-trend inflation.

[4]The insurance property of bonds has been empirically documented by Barsky (1989), Fama & French (1989), Ilmanen (2003), Shiller & Beltratti (1992) among others. Gulko (2002) calls this phenomenon decoupling, meaning that U.S. Treasury bonds offer effective diversification during financial crises, at the time diversification is needed most.

[5]Performance attribution often called as the Brinson model (Brinson & Fachler 1985). The model is widely used by the investment management community to attribute portfolio returns by using a simple sector-based investment process of sector allocation and stock selection.

terms of factors directly entering the bond price equation (consumption growth, inflation, time preference shocks and long-run consumption and leisure risks). To our knowledge this is the first attempt to quantify the conditional second moments which allows us to price individual macro risks.

Third, the existing macro finance literature studies exclusively the determinants of the nominal term premium. Motivated by Krishnamurthy & Vissing-Jorgensen (2012) we extend the analysis to stochastic steady state of bond prices. Krishnamurthy & Vissing-Jorgensen (2012) shows empirically that liquidity and safety drive investors' high valuation of US Treasuries. They show that Treasury yields are reduced by 73 basis point, on average, compared to "riskless" rate. By studying the spread between stochastic and deterministic ("riskless") steady state of bond prices we can ask, as Krishnamurthy & Vissing-Jorgensen (2012), what determines the valuation of bonds. We show that bonds contain (apart from the safety and liquidity[6]) also premium for its diversification attributes against business cycle risks. We calculate that bond yields are reduced by 96 basis point, on average[7], because they contain premium for *i*) providing buffer against bad times, *ii*) hedging inflation risk and *iii*) hedging real risks by putting current consumption gains against future losses. The importance of these three determinants of risk premia were stressed separately in the literature before. The precautionary saving was studied for one period bond price in NK DSGE model by De Paoli *et al.* (2010). The inflation risk was shown by Rudebusch & Swanson (2012) to be a way how to generate high nominal term premium in NK DSGE model. Kaltenbrunner & Lochstoer (2010) shows in RBC model how the hedging property of bonds against real risk contributes to low nominal term premium. Our novel decomposition allows us to decouple and quantify the effect of this three channels on both risk and term premia in a single model.

Finally, when describing the yield curve we study separately the impact of transitory changes in government spending and the impact of changes in volatility of government spending.

We find that the character of government expenditures affects both term and risk premia. Bonds carry large insurance premium if government expenditures are wasteful (defense expenditure). Bonds serve as poor hedging instrument against productive (infrastructure expenditures) and utility enhancing (health,

[6] We approximate safety and liquidity by preference shocks as suggested by Fisher (2015)

[7] Average over the year 1969 to 2009 in our baseline calibration and also averaged over the maturities.

education expenditures) risk in government expenditures. Spending reversals reduce the insurance premium in bond prices if government expenditures are wasteful by improving the predictability of future path of government expenditures. Further, using the structural decomposition into macroeconomic factors we show that the ability of bonds to hedge investors against inflation risk is the most important driver for high valuation of bonds. However, we emphasize, that the size of inflation risks is highly sensitive to monetary policy conduct and type of exogenous shock.

Wasteful government expenditures increase nominal term premium by 2.2 basis point and only about 1 basis point if the monetary policy puts zero weight on output gap. Productive expenditures increase term premium only for high production elasticity of this expenditures. Both substitutable and non-substitutable to consumption utility enhancing expenditures decrease significantly the term premium.

The term structure rises on the impact of government spending shocks due to higher expected future short term interest rates. The uncertainty related to wasteful government spending on the other hand shifts the whole term structure down. This can be attributed partly to precautionary saving motives and partly to the hedging property of bonds[8]. High volatility in government spending motivates households to insure themselves against a fluctuations in their wealth. The precautionary saving motive grows with the size of uncertainty. The decomposition of the pricing equation further shows that a rise in fiscal uncertainty amplifies the hedging property of bonds against consumption risks given by the negative covariance between expected future consumption and leisure with realized consumption growth and is independent of monetary policy.

We illustrate the intuition on a negative shock to wastful government purchases. The representative household associates lower government spending with lower current and future taxes which have a positive wealth effect stimulating current private consumption and implying higher realized consumption growth. Due to the transitory nature of the shock the government spending reverts back to its long-run mean. Hence, a negative transitory shock to government spending generates a positive shock for realized consumption growth (crowding-in) but negative shocks to future consumption as government spending increases back to its long-run mean. This is salient feature for models with

[8]The hedging property comes from negative correlation between macro variables, thus it can be understood as gains from diversification.

EZ preferences and transitory shocks (see Kaltenbrunner & Lochstoer 2010). The impact on short bonds is stronger however, so the influence of the hedging property decays over the maturity profile. This fact allows to generate a sizable nominal term premium within our New Keynesian DSGE model.

The response of monetary policy to government spending determines the degree of diversification of bonds to inflation risks. If monetary policy responds to output gap in the Taylor rule (given the response to current inflation), inflation works as a hedge against the long-run consumption and leisure risks and bonds serve as a form of insurance. This is in striking contrast to the way productivity (TFP) shocks impact the risk premium. In case of TPF shocks, when monetary policy responds more to inflation (coefficient on inflation in the Taylor rule is larger) inflation risks are smaller. The higher is the coefficient on output gap in the Taylor rule the lower are real risks but the higher are inflation risks. This is due to the tradeoff between inflation and output gap stabilization.

We also find that spending reversals break the link between the uncertainty about government spending and risk premium. The increase in predictability of the evolution of debt and taxes mitigates the impact of uncertainty (through second order terms) on macroeconomic variables. This fact helps the investor to form a more accurate expectation. Larger time t conditional information set decreases the risk of bond mispricing, therefore the risk premiums are lower.

The remainder of the paper is structured as follows. The model is lay out in section 3.2. The section 3.3 details how the bonds are defined and priced in the model. In section 3.4 we discuss in detail the decomposition of government spending and our empirical approach. Section 3.5 explains the methodology used for the decomposition of the bond pricing equation. Our results are discussed in section 3.6.

3.2 The Model

We choose as an base line model for our analysis a general equilibrium model of Rudebusch & Swanson (2012) to quantitatively examine the links between government spending and dynamics of the term structure of interest rates. In the second step we recalculate our results in the fully fledged macro-finance DSGE model of Andreasen *et al.* (2018) .

Our economy is populated by: *i*) a representative household who has recursive preferences, supplies labor and buys public bonds, *ii*) firms operating on the final and intermediate goods market with the latter facing Calvo style nominal rigidities, *iii*) a monetary policy following a Taylor rule and *iv*) a government which funds its expenditures by by lump-sum taxes and by issuing government bonds. Government expenditures can be either wasteful, productive (G_t^p) or utility enhancing (G_t^u). The utility enhancing expenditures are divided into non-separable, G_t^n, from private consumption government expenditures (we assume that G_t^n are perfectly substitutable to C_t), and separable, G_t^s.

3.2.1 Households

The economy is inhabited by a continuum of households. The representative household chooses paths for consumption C_t and leisure, L_t to maximize expected utility, $E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, L_t, G_t^u)$, where β is the subjective discount factor of future stream of utilities, subject to a budget constraint:

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + W_t N_t + T_t + \Pi_t \tag{3.1}$$

where $E_t Q_{t,t+1} B_{t+1}$ is the present value of a portfolio of risk free bonds. $Q_{t,t+1}$ is the stochastic discount factor, $W_t N_t$ is the household labor income, time constrain is normalized to one, $N_t + L_t = 1$ and P_t is the aggregate price level. T_t summarizes all lump-sum transfers to the household and Π_t are firms' profits.

We follow Rudebusch & Swanson (2012) we write the value function of the household as[9]

$$V_t = u(C_t, L_t) + \beta (E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{3.2}$$

The period utility is represented by

$$u(C_t, L_t) = e^{b_t} \left(\frac{X_t^{1-\gamma}}{1-\gamma} + \chi Z_t^{1-\gamma} \frac{(1-N_t)^{1-\eta}}{1-\eta} + \Omega(G_t^n) \right) \tag{3.3}$$

where parameters γ and η pin down intertemporal elasticity of substitution of the consumption bundle and Frish elasticity. $X_t = \tilde{C}_t - b \left(\mu_{z,t}^* \right)^{-1} \tilde{C}_{t-1}$ defines the habit in composite consumption index, $\tilde{C}_t$. The composite index aggregates private consumption, C_t, and government services, G_t, which are

[9]Household objective function can be written in recursive form as $V_t = u(C_t, L_t) + \beta E_t V_{t+1}$. We follow Rudebusch & Swanson (2012) and use the transform of Epstein & Zin (1989) preferences, $V_t = u(C_t, L_t) + \beta (E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}}$ when $u(C_t, L_t) > 0$ and $V_t = u(C_t, L_t) - \beta (E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}}$ when If $u(C_t, L_t) < 0$.

either substitutable or complementary to private consumption,

$$\tilde{C}_t = \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_t^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}} \tag{3.4}$$

We study the cases when public expenditures G_t^s are either substitutable or complementary to private consumption[10]

Swanson (2012) shows the relationship of parameter α to the relative risk-aversion[11].

The preference shock, b_t follows the autoregressive process:

$$b_t = \rho_b b_{t-1} + \sigma_b \epsilon_t^b \tag{3.5}$$

where $\epsilon_t^b \in N(0,1)$, σ_b controls the volatility of the preference shocks and ρ_b sets the persistence. Fisher (2015) shows that preference shocks can be understood as shock to demand for safe assets. This shock is switched off in our baseline model.

The household optimization exercise delivers an Euler equation which allows us to price a bond of any maturity:

$$Q_{t,t+1} = e^{b_{t+1}-b_t}\beta\left(\frac{\Lambda_{t+1}}{\Lambda_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}\left[\frac{V_{t+1}}{[E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right] \tag{3.6}$$

where

$$\Lambda_t = X_t^{-\gamma}\tilde{C}_t^{\frac{1}{\omega}}\phi_c C_t^{\frac{-1}{\omega}} - \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} X_{t+1}^{-\gamma} b\left(\mu_{z,t+1}^*\right)^{-\gamma}\tilde{C}_{t+1}^{\frac{1}{\omega}}\phi_c C_{t+1}^{\frac{-1}{\omega}} \tag{3.7}$$

and for $b=0$, $\Lambda_t = \tilde{C}_t^{-\gamma+\frac{1}{\omega}}\phi_c C_t^{\frac{-1}{\omega}}$

3.2.2 Firms

Final good firms operate under perfect competition with the objective to minimize expenditures subject to the aggregate price level $P_t = \left(\int_0^1 P_t(i)^{\frac{-1}{\lambda_t}} di\right)^{-\lambda_t}$,

[10] Notice, that with recursive preferences non-substitutable public expenditures also enters the households first order conditions (through V_t) even if these expenditures are separable from leisure and consumption.

[11] The connection between the coefficient of relative risk-aversion (CRRA) and parameter α in the recursive formulation for the particular form of the period utility in equation (3.3) is given by $CRRA = \frac{\gamma}{1+\frac{\gamma}{\eta}} + \frac{\alpha(1-\gamma)}{1+\frac{\gamma-1}{1+\eta}}$.

where $P_t(i)$ is the price of intermediate good produced by firm i, using the technology $Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_t}} di \right)^{1+\lambda_t}$. The final good firms aggregate the continuum of intermediate goods i on the interval $i \in [0, 1]$ into a single final good. Here λ_t means that the markup is time-varying (see equation (3.15)).

The cost-minimisation problem of final good firms deliver demand schedules for intermediary goods of the form:

$$Y_t(i) = \left(\frac{P_t}{P_t(i)} \right)^{\frac{1+\lambda_t}{\lambda_t}} Y_t \tag{3.8}$$

A continuum of intermediate firms operates in the economy. Intermediate firm i uses the Cobb-Douglas technology The capital share of output is controlled by θ and we assume that there are constant returns to scale over privately provided inputs. The output elasticity with respect to productive government expenditures is determined by θ_g.

$$Y_t(i) = A_t K_t(i)^{\theta} (Z_t N_t(i))^{1-\theta} (G_t^p)^{\theta_g} \tag{3.9}$$

where $K(i)$ and $N_t(i)$ is the amount of capital and labor employed. The aggregation across firms, yields:

$$S_t Y_t = A_t \bar{K}^{\theta} (Z_t N_t)^{1-\theta} (G_t^p)^{\theta_g} \tag{3.10}$$

$\bar{K}$ refers to the fact that firms have fixed capital and S_t is the cross-sectional price dispersion. The production function thus features decreasing return to scale in hours worked.

In equation (3.10) technology follows the autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_A \epsilon_t^A \tag{3.11}$$

where ϵ_t^A is an independently and identically distributed (iid) shock with zero mean and constant variance. The mark-up shock is however muted in our baseline model.

Intermediate firms maximize the present value of future profits facing Calvo contracts by choosing price, $P_t(i)$,

$$E_t \left\{ \sum_{k=0}^{\infty} \zeta^k Q_{t,t+k} \left[P_t(i) Y_{t+k}(i) - W_{t+k} N_{t+k}(i) \right] \right\} \tag{3.12}$$

where $Q_{t,t+j}$ is the stochastic discount factor from period t to $t+k$ which is given by equation (3.6). The term $W_{t+j}N_{t+j}(i)$ represents the cost of labor. The optimal price is a weighted average of current and future expected nominal marginal costs,

$$P_t(i) = (1+\lambda_t)\sum_{k=0}^{\infty}\mu_{t+k}MC_{t+k}(i) \tag{3.13}$$

Where $\mu_{t+k} = \frac{E_t\zeta^k Q_{t,t+k}Y_{t+k}(i)}{E_t\sum_{k=0}^{\infty}\zeta^k Q_{t,t+k}Y_{t+k}(i)}$ is the time varying mark-up implied by price rigidity and $1+\lambda_{t+k}$ is the mark-up implied by monopolistic competition.

The average real marginal cost is defined as

$$MC_t = \left[(G_t^p)^{\theta_g}\bar{K}^{\theta}\right]^{\frac{1}{\theta-1}}\frac{1}{1-\theta}\left(\frac{W_t}{A_t}\right)\left(\frac{Y_t}{A_t}\right)^{\frac{\theta}{1-\theta}} \tag{3.14}$$

The markup (or cost-push) shock is given by:

$$\log(1+\lambda_t) = (1-\rho_\lambda)\log(1+\bar{\lambda}) + \rho_\lambda\log(1+\lambda_{t-1}) + \sigma_\lambda\epsilon_t^\lambda \tag{3.15}$$

3.2.3 Government

The model is closed with a monetary policy rule assuming that monetary authority sets the short-term nominal interest rate i_t based on a Taylor rule as in Rudebusch & Swanson (2012).

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\left[\frac{\bar{i}}{\bar{\pi}} + \log\Pi_t + \phi_y\left(\frac{\mu_{z,t}Y_t}{\bar{\mu}_z\bar{Y}} - 1\right)\right] + \phi_\pi\log\left(\frac{\Pi_t}{\pi^*}\right) + \varepsilon_t^i \tag{3.16}$$

where $\frac{\bar{i}}{\bar{\pi}} = \bar{\pi}/(\beta\bar{\mu}_z^{-\gamma})$ is the steady-state real interest rate. We assume the productivity growth is deterministic and $\mu_{z,t} = \frac{Z_t}{Z_{t-1}}$. ε_t^i is an iid shock with mean zero and variance σ_i^2.

The four-quarter moving average of inflation (Π_t) can be approximated by a geometric moving average of inflation:

$$\log\Pi_t = \theta_\pi\log\Pi_{t-1} + (1-\theta_\pi)\log\pi_t \tag{3.17}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ and $\theta_\pi = 0.7$ ensures that the geometric average in equation ((3.17)) has an effective duration of approximately four quarters. Parameters

ϕ_π and ϕ_y determine the weight monetary policy authority puts on stabilizing the deviations of inflation and output from their steady state values.

The target rate of inflation, π^*_t, is given by ,

$$\pi^*_t = \rho_{\pi^*}\pi^*_{t-1} + \zeta_{\pi^*}(\Pi_t - \pi^*_t) + \varepsilon^{\pi^*}_t \tag{3.18}$$

The fact that inflation target is time varying introduces into the model sizable long-run inflation risk.

The total government expenditures are shared as follows: $\bar{G}^p = \tau_p\bar{G}$, $\bar{G}^{un}_t = \tau_{un}\bar{G}_t$, $\bar{G}^{us} = \tau_{us}\bar{G}$ and wasteful government expenditures $\bar{G}^w = (\tau_p - \tau_{un} - \tau_{us})\bar{G}_t$. Government expenditures are assumed to follow an exogenous autoregressive processes of the form:

$$\log G^w_t = (1-\rho_G)\log\bar{G^w} + \rho_G \log G^w_{t-1} + \sigma^w_G\epsilon^{GW}_t + \sigma^w_{news}\epsilon^{GW}_{t-4} \tag{3.19}$$
$$\log G^p_t = (1-\rho^p_G)\log\bar{G}^p + \rho^p_G \log G^p_{t-1} + \sigma^p_G\epsilon^{GP}_t + \sigma^p_{news}\epsilon^{p}_{t-4} \tag{3.20}$$
$$\log G^{un}_t = (1-\rho^{un}_G)\log\bar{G}^{un} + \rho^{un}_G \log G^{un}_{t-1} + \sigma^{un}_G\epsilon^{GUN}_t + \sigma^{un}_{news}\epsilon^{un}_{t-4} \tag{3.21}$$
$$\log G^{us}_t = (1-\rho^{us}_G)\log\bar{G}^{us} + \rho^{us}_G \log G^{us}_{t-1} + \sigma^{us}_G\epsilon^{GUS}_t + \sigma^{us}_{news}\epsilon^{us}_{t-4} \tag{3.22}$$

where ϵ^G_t, ϵ^{GUN}_t and ϵ^{GUS}_t are iid shocks with zero mean and unit variance. Parameters σ_G's scales the standard deviation of the shock. We assume in our benchmark model that government runs a balanced budget financed through lump-sum taxes obtained from the household sector. We relax the assumption about lump-sum taxes only when we study spending reversals as in Corsetti *et al.* (2009). With spending reversal the reduction in debt is aided by restraint on government purchases in the future. Corsetti *et al.* (2009) show that spending reversals and, hence, higher savings of the government in the future generate crowding-in effects of government spending.

We utilize the framework introduced by Corsetti, Meier, & Müller (2009) to study the effects of fiscal consolidation on the term structure. Government consumption is financed through either lump-sum taxes, T_t (taxes are in nominal terms) or the issuance of nominal debt, D_t,which can alternatively be expressed in real terms after dividing by the price level:

$$T_{Rt} + Q_{t,t+1}D_{Rt+1} = \frac{D_{Rt}}{\pi_t} + G_t \tag{3.23}$$

where $T_{Rt} = \frac{T_t}{P_t}$ are taxes in real terms and $D_{Rt} = \frac{D_t}{P_t}$ is a measure for real beginning-of-period debt. Corsetti, Meier, & Müller (2009) use a fiscal rule of the form, $T_{Rt} = \Psi_t D_{Rt}$

Spending reversals are captured by the following process for government purchases:

$$\log G_t = (1-\rho)\log \bar{G} + \rho \log G_{t-1} - \Psi_G \log D_{Rt} + \eta_t \tag{3.24}$$

where $\bar{G}$ is the steady state level of government spending and ρ controls the persistence. The Ψ-parameters capture a systematic feedback effect of public debt on government spending (negative) and taxes (positive).

The idea of Corsetti *et al.* (2009) is that it is not necessary to increase taxes in response to higher government debt because government expenditures can be reduced to help settle debt. Increase in government spending will subsequently cause spending to fall below trend level for some time. The anticipated spending reversal does not crowd out private consumption and boosts the expansionary effect of G on output at the impact.

The fixed nature of capital implies fixed investment that is used to replace depreciated capital: $I_t = \bar{I} = \delta \bar{K}$, where δ is the depreciation rate. In equilibrium firms and households optimally choose prices with respect to their constraints and each market clears. The market clearing in the goods market requires that the aggregate demand equals to aggregate output in the economy, $Y_t = C_t + G_t + \bar{I}$.

3.3 Bond Pricing

The price of a default-free n-period zero coupon bond that pays \$1 at maturity can be described recursively as:

$$p_t^{(n)} = E_t\{Q_{t,t+1} p_{t+1}^{(n-1)}\}$$

where $Q_{t,t+1}$ is the stochastic discount factor; $p_t^{(n)}$ denotes the price of the bond at time t with maturity n, and $p_t^{(0)} \equiv 1$, i.e. the time-t price of \$1 delivered at time t is \$1.

The price of bond can be decomposed into the risk neutral price and a term premium. The risk neutral bond price, $\hat{p}_t^{(n)}$, is defined through the expectations hypothesis of the term structure:

$$\hat{p}_t^{(n)} = e^{-i_t} E_t \hat{p}_{t+1}^{(n-1)} \qquad NTP_{n,t} = i_t^{(n)} - \frac{1}{n}\sum_{j=0}^{n-1} E_t[i_{t+j}] \tag{3.25}$$

where the bond price is discounted by one period rate, i_t. The price of bond reflects in this case expectations about the inflation and economic activity but abstracts from the uncertainty surrounding the expectations[12]. The continuously compounded yield to maturity of the n-period zero-coupon bond can be written as $i_t^{(n)} = -\frac{1}{n}\log p_t^{(n)}$, (see for instance Cochrane (2001)). The term premium, $NTP_{n,t}$ is defined as the difference between the yield expected by the risk-averse investor ($i_t^{(n)}$) minus the yield awaited by the risk-neutral investor ($\hat{\imath}_t^{(n)}$). We also report slope of the the term structure, $Slope_{t,n} = i_t^{(n)} - i_t^{(1)}$, which is imperfect but frequently used measure of the term premium of nominal bonds.

Further, we define risk-less price of bond, $\tilde{p}_t^{(n)}$ which represents the price of bond with deterministic cashflow.

$$\tilde{p}_t^{(n)} = e^{-\beta} E_t \tilde{p}_{t+1}^{(n-1)} \qquad RP_{n,t} = i_t^{(n)} - \frac{1}{n}\log\tilde{p}^{(n)} \tag{3.26}$$

Risk premium of bond, $RP_{n,t}$, is represented by the difference between yield expected by the risk-averse investor ($i_t^{(n)}$) and risk-less rate which would hold under perfect foresight $\bar{i}_t^{(n)} = \frac{1}{n}\log\tilde{p}^{(n)}$. If risk premium turns negative we call it insurance premium. The risk-less price of bond reflects value of bond in absence of uncertainty and implied risk premium helps to understand the bond valuation. The figure 3.1 illustrates graphically the difference between the risk and term premia.

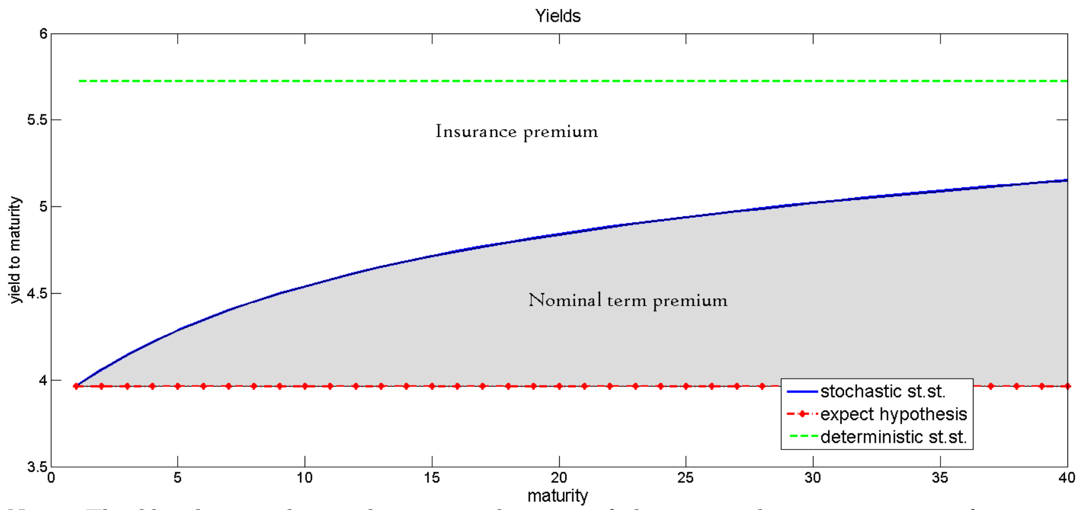
Up to the second order approximation the unconditional yield to maturity can be written,

$$-i_t^{(n)} = E_t q_{t,t+n} + \frac{1}{2}\mathrm{Var}_t(q_{t,t+n}) \tag{3.27}$$

where $q_{t,t+1}$ is the log deviation of one period stochastic discount factor from its deterministic steady state. The expectation term represents the intertemporal substitution effect and compensation for risk is given by the variance term (see Paoli & Zabczyk (2012a)). Alvarez & Jermann (2005) show that if $Q_{t,t+1}$ is log-normal, the entropy of the stochastic discount factor, $logE_tQ_{t,t+1} - E_tlogQ_{t,t+1}$, represents measure of market price of risk which is given by second moment, $\mathrm{Var}_t(q_{t,t+n})$. The effect of uncertainty on bond valuation is solely explained by the conditional variance of stochastic discount factor.

[12]This can be understood as bond-price of 10-year bond expected by the so-called risk-neutral investor who is rolling over a one-period investment for 10 years.

Figure 3.1: Yield curve and risk premia



Note: The blue line is the stochastic steady state of the nominal term structure of interest rates. The space between the yield curve under perfect foresight (green line) and yield curve (blue line) represents the insurance premium investors are willing to pay to generate buffer savings. The difference between the yield curve and and yields consistent with expectations hypothesis (red line) constitute the nominal term premium. The intersection of the blue and red line is the one period nominal interest rate

Lemma 3.1. *We can decompose the stochastic discount factor in equation* (3.27) *into underlying macro factors*[13],

$$
\begin{aligned}
i_t^{(n)} &= -\frac{1}{2n}E\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\zeta}_{t+j}\right)\right]-\frac{\gamma^2}{2n}E\left[\mathrm{Var}_t\left(\Delta^n\hat{c}_{t+n}\right)\right]-\frac{1}{2n}E\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)\right]\\
&+ -\frac{\alpha^2}{2n}E\left[\mathrm{Var}_t S_{t+n}\left(\cdot\right)\right]+\frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\Delta^n\hat{c}_{t+n}\right)\right]\\
&+ \frac{1}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right]-\frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{c}_{t+n},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right]\\
&+ \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},S_{t+n}\left(\cdot\right)\right)\right]-\frac{\gamma\alpha}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{c}_{t+n},S_{t+n}\right)\right]\\
&- \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+n},S_{t+n}\right)\right] \qquad (3.28)
\end{aligned}
$$

where $\hat{\zeta}=e^{b_{t+1}-b_t}$ is the preference shock and $S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j}+a\hat{c}_{t+j}-b\hat{n}_{t+j}\right]\right)$ surprise operator filtering out conditional expectations (see Uhlig (2010), Piazzesi & Schneider (2007)) and can be interpreted as the revaluation in the

[13] appendix provide full derivation of equation (3.28)

expectations about the future path of consumption and leisure. Dew-Becker & Giglio (2016) shows that the surprise operator can be also interpreted as a price investors are willing to pay to hedge against low frequency (permanent) fluctuations in consumption and leisure. The parameter α determines if agents prefer early or late resolution of uncertainty. Early resolution of uncertainty means that agents wish to smooth consumption over the state of nature rather than over time[14].

Proof. The price of bond with maturity n is defined $P_t^{(n)} = E_t[Q_{t,t+n}]$; in the non-stochastic steady state $\bar{P} = \bar{Q}$. Lower case letters define logarithm of their upper case counterparts.

$$\begin{aligned} \bar{p}(1 + \hat{p}_{t,n} + \frac{1}{2}\hat{p}_{t,i}^2) &= E_t\left[\bar{q}(1 + \hat{q}_{t+n} + \frac{1}{2}\hat{q}_{t+n}^2)\right] \\ &= \bar{q}_t E_t\left[1 + \hat{q}_{t+n} + \frac{1}{2}\hat{q}_{t+n}^2\right] \end{aligned}$$

After canceling out steady state, we get:

$$\hat{p}_{t,n} = E_t[\hat{q}_{t,t+n} + \frac{1}{2}\hat{q}_{t,t+n}^2] - \frac{1}{2}\hat{p}_{t,n}^2$$

Up to the first order $\hat{p}_{t,n} = E_t\{\hat{q}_{t,t+n}\}$, thus we can substitute for the quadratic term $\hat{p}_{t,n}^2 = (E_t\{\hat{q}_{t,t+n}\})^2$. It follows that:

$$\hat{p}_{t,n} = \mathrm{E}_t\left[\hat{q}_{t,t+n} + \frac{1}{2}\hat{q}_{t,t+n}^2\right] - \frac{1}{2}(\mathrm{E}_t\hat{q}_{t,t+n})^2$$

From the last equation using the definition of variance [15] we can define price of one period bond.

$$\hat{p}_{t,n} = E_t[\hat{q}_{t,t+n}] + \frac{1}{2}\mathrm{Var}_t[\hat{q}_{t,t+n}] \tag{3.29}$$

using the definition of yield to maturity, $\widehat{ytm_t} = -(1/n)\hat{q}_{t,n}$ we can write equation (3.29)

$$\widehat{ytm}_t^n = -\frac{1}{n}E_t q_{t,t+n} - \frac{1}{2n}\mathrm{Var}_t(\hat{q}_{t,t+n}) \tag{3.30}$$

and use equation (A.27) and plug it into 3.30 to get

[14] see 3.A.6 for more details

[15] $Var(x) = E[x^2] - (E[x])^2$

$$\widehat{ytm}_t^n = -\frac{1}{n}\mathrm{E}_t\left\{\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right] - \gamma\Delta^n\hat{c}_{t+n} - \sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right] - \alpha S_{t+n}\left(\cdot\right)\right\}$$
$$-\frac{1}{2n}\mathrm{Var}_t\left(\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right] - \gamma\Delta^n\hat{c}_{t+n} - \sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right] - \alpha S_{t+n}\left(\cdot\right)\right) \qquad (3.31)$$

The term under the expectations in the equation (3.31) is on average zero. The term thus corresponds to the deterministic steady state. The variance components represent the Jensen's inequality term and arise from the relative convexity of nominal bonds. Unconditional mean of the term structure in the equation (3.31) thus delivers equation (3.28)

□

To understand how the change in uncertainty of government expenditures impacts the term structure of interest rates, it is useful to further split the covariance terms into correlations and standard deviations.

$$\begin{aligned} E_t\left[\frac{\partial\widehat{ytm}_t^n}{\partial\sigma_g}\right] &= -\frac{1}{2n}\left\{\gamma^2\frac{\partial}{\partial\sigma_g}\mathrm{Var}\left(\Delta^n\hat{c}_{t+n}\right) + \frac{\partial}{\partial\sigma_g}\mathrm{Var}\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) + \alpha^2\frac{\partial}{\partial\sigma_g}\mathrm{Var}S_{t+n}\left(\cdot\right)\right\} \\ &- \frac{\gamma}{n}\frac{\partial(\sigma_{\hat{c}}\sigma_{\hat{\pi}})}{\partial\sigma_g}\mathrm{Corr}\left(\Delta^n\hat{c}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) - \frac{\gamma\alpha}{n}\frac{\partial(\sigma_{\hat{c}}\sigma_S)}{\partial\sigma_g}\mathrm{Corr}\left(\Delta^n\hat{c}_{t+n}, S_{t+n}\right) \\ &- \frac{\alpha}{n}\frac{\partial(\sigma_{\hat{\pi}}\sigma_S)}{\partial\sigma_g}\mathrm{Corr}\left(\sum_{j=1}^{n}\hat{\pi}_{t+n}, S_{t+n}\right) \end{aligned} \qquad (3.32)$$

Equation (3.32) highlights the fact that since the parameters and correlations are constant the change in the level of the yield curve is coming from the change in volatility of macroeconomic factors. The correlation terms are model specific and defines the price of risk whereas the change in the volatility determines the quantity of risk. The covariance terms represents the nominal amount of risk given by price times quantity of risk.

3.3.1 What Prices Risk?

Equation (3.28) explains what prices bonds in the model. Government bonds function in the model as an instrument to smooth agents wealth over time. Thus, the price of bonds is determined by the way how well can bonds mitigate or amplify the fluctuations in agents wealth. The factors directly affecting the wealth agents in the model are implied by the pricing kernel in equation (3.28) which features four factors determining bond price: i) changes in the preferences ii) consumption growth, iii) inflation, iv) time duration of risk for investors wealth.

The ability of bonds to transfer wealth over time and smooth its fluctuation is priced by the conditional variance terms in the equation (3.28). Bonds can however also provide protection against unfavorable states of nature. This hedging or leveraging properties of bonds are priced by the covariance terms. For instance, inflation can leverage drop in economic activity by decreasing the real value of bonds[16] in times when it hurts most when savings are most needed to smooth consumption.

What role plays the long-run risk entering the pricing equation through the S_{t+n} term? In general, due to recursive preferences the timing of the resolution of uncertainty matters for households. The price of a 10-year bond is determined by a set of expectations conditional on time t,

$$E_t[P_t^{10}] = Q_{t,t+1}Q_{t+1,t+2}Q_{t+2,t+3}\dots Q_{t+9,t+10}. \tag{3.33}$$

The nominal term premium comes from the stochastic character of $Q_{t,t+n}$, the volatility of underlying macro series increases the chance of forming wrong expectations about future realized bond prices. The nominal term premium represents the compensation for revaluation in expectations as the new information arrives. The predictability (persistence) in consumption and leisure means that the arrival of new information creates an impact lasting for several quarters. Agents with Epstein Zin preferences dislike surprises with long-run effects which makes the reevaluation in expectations especially costly.

To get a required pay off in ten years agents have two options: *i*) to take a position in a long-term bond, *ii*) to buy a one year bond and roll it forward each period.[17] Which variant the household chooses depends on the intensity of households' risk aversion and inter-temporal smoothing motives. Households with low inter-temporal elasticity of substitution (IES) have a strong desire to smooth utility over time and will choose to buy bonds with long maturity.

[16] Piazzesi & Schneider (2007), Wright (2011) study these risks empirically)

[17] In the model households can buy only one period bonds. The decision between taking position in 10-year bonds and rolling forward one quarter bonds is only illustrative. The set of expectations households formed at time t is the analogy of being locked in bonds with long maturity.

Risk-averse households smoothing utility across the state of the world will prefer to roll over one year bonds. To purchase long-term bonds risk averse agents require a discount to compensate them for the uncertainty related to future shock realization. In our model the conditional volatility of the shocks is constant over time thus the precautionary saving motive plays a negligible role as the risks are the same for all maturities. What determines the NTP is the difference in how much the macro variables co-vary (leverage and hedging property of bonds) across the maturity. Long-run risk has little impact on short maturity bonds. It has nevertheless significant impact on the long maturity bonds because of the persistence.

3.3.2 Calibration and Solution Method

Our baseline calibration of the core model parameters follows the one of Rudebusch & Swanson (2012). The main parameter values are summarized in Table 3.1. Under Rotemberg price setting $\zeta = \frac{\varphi\left(1-\theta+\theta\frac{1+\bar{\lambda}}{\bar{\lambda}}\right)}{(1-\varphi)(1-\varphi\beta)(1-\theta)}$ is set such that the slope of New Keynesian Phillips curve corresponds to the Calvo case with an average duration of price stickiness equal to $\frac{1}{1-\varphi} = 4$ quarters.

Table 3.1: Calibration of the model

Symbol	Variable	Value
β	Discount factor	0.99
$CRRA$	Risk aversion	110
IES	Intertemporal elasticity	0.09
$1/\eta$	Frisch elasticity	0.28
η_g	G_n elasticity	1
ϕ_π	Response to inflation	0.53
ϕ_y	Response to output	0.93
ζ	Price adjustment	4Q
$\bar{G}/\bar{Y}$	Government spending	0.2
ρ_G	Autocorr. coeff. G	0.94
σ_g	Std. of G	0.008
θ_g	Output elasticity of G^p	0.1
θ	Capital share of output	1/3
$\bar{pi}$	Steady state inflation	0%

The model is approximated to the third-order using Dynare routines.For the attribution analysis we limit the solution to the second order to avoid extra interaction terms (which are very small) to simplify the exposition.

3.4 Components of Government Expenditures

This section discusses the split of government expenditures into components and reviews their fluctuations. Government expenditures are often disentangled in the theoretical literature on public capital to purely wasteful, productive and utility enhancing (e.g. Albertini *et al.* (2014), Getachew & Turnovsky (2015), Barro (1990)). This separation is imposed by the theory (model) and thus is not exactly in line with the national accounting view. In the empirical literature government spending are usually either not decomposed or the focus is on defense spending[18]. Since the seminal papers by Aschauer (1985) and Aschauer (1989) increased attention is also given to infrastructure government expenditures and the estimation to what extent does government spending directly substitute for private consumer expenditure. From the modeling point of view, infrastructure government expenditures are included as productive input to firms' production[19]. The substitutability of government expenditures to private consumption is captured by introducing these expenditures into utility function. e.g.Aschauer (1985)).

The impact on the economy of this type of government expenditures is interesting because infrastructure and utility enhancing spending provide positive externality compared to purely wasteful expenditures and thus affect welfare and asset prices of agents through the impact on marginal utility of consumption. However, linking the theoretical concept of government spending with the observations is only approximate. In what follows we discuss the components in more detail.

3.4.1 Infrastructure Spending

We follow the the literature on public capital (e.g. Agenor (2013)) and associate productive expenditures with spending on transportation and water infrastructure[20].

The literature studying relationship between public investment into infrastructure and economic activity is extensive (For the review of theoretical literature see Turnovsky (1997) or Agenor (2013). Bom & Ligthart (2014) provided detailed review of empirical literature). The empirical literature has devoted a

[18] for detailed review on empirical literature see Gramlich (1994)

[19] see Gramlich (1989) for review of the challenges related to measurement and estimation

[20] Highways, Mass Transit and Rail, Aviation, Water Transportation, Water Resources and Water Utilities.

great deal of effort to measuring the output elasticity of public capital. On one side, structural macroeconomic models contain the output elasticity of public capital as a fundamental parameter and the knowledge of its exact value is thus required to numerically analyze the output effects of fiscal policy shocks. Moreover, many of theoretical papers (e.g. Linnemann & Schabert (2006)) highlight the high sensitivity of the fiscal multipliers to the parameter driving the elasticity of productive government spending in producing output. On the other side, the relationship is of vital policy importance. It gives answer to questions as: How should policy makers construct composition of austerity and stimulus packages? How should these policies be sequenced and implemented to smooth business cycle or support long run growth?

This is why most of the empirical literature focuses on estimation of the elasticity. The range of estimates is relatively wide, Aschauer's (1989) original estimate of 0.39 to 0.56 makes the upper bound. The consensus in recent literature estimates the elasticity between 0.1 and 0.2 (see Bom and Ligthart (2014) for meta-study or Irmen & Kuehnel (2009)). We calibrate the elasticity to $\theta_G = 0.1$ and pursue sensitivity analysis.

When we talk about infrastructure spending we follow Linnemann & Schabert (2006) and assume the flow of expenditures, and not the stock of accumulated government capital as common in some papers (e.g. Baxter & King (1993)). We make this assumption purely for analytical convenience[21]. As Linnemann & Schabert (2006), we argue the stock-flow dynamics affects the timing without opening different transmission channels. Barro (1990) makes similar argument stating that the distinction matters for empirical analysis but is irrelevant in the theoretical model as conceptually we can think about the government as doing no production or owing capital. Then government just buys flow of output (highways, water reservoir etc.) from private companies. In the review paper, Irmen & Kuehnel (2009) claim that the stock case confirms most results that are obtained in the flow case.

What is the typical transmission of the shock in infrastructure spending? Investment in infrastructure impacts the economy through the familiar direct productivity effect (see Barro (1990)). Government expenditures are introduced into the production function of individual firms which increases the rate of return to private capital increase. This means that we can expect qualita-

[21]In this way we cannot separate the expenditures into maintenance and capital investment but this is not focus of our analysis.

tively similar effect on the term structure of interest rate as from productivity shocks.

3.4.2 Utility Enhancing

Utility enhancing government expenditures can be formulated as being *i*) substitutable or *ii*) non-substitutable (independent) to private consumption.

We interpret the substitutable spending as expenditures for health, culture and education[22]. Substitutable government expenditures can work either as substitute or complement to private consumption. For example, public health or education may decrease demand for private health and education. Higher expenditure for public health may on the other hand increase consumption of drugs produced by private companies and thus work as a complement to private consumption. In our analysis, we follow the weak consensus in empirical literature (e.g.Fiorito & Kollintzas (2004) or Ercolani & Valle e Azevedo (2014)) and consider this expenditures to be substitutes to private consumption. In this case, rise in public health, culture and education expenditures decreases the utility from private consumption which leads agents to substitute part of their private consumption with the new public services. The idea of utility enhancing government expenditures is not new in theoretical literature and goes back at least to Barro (1981) and Baxter & King (1993). The review of the empirical literature can be find for example in Fiorito & Kollintzas (2004).

Most of the state of the art macro models (e.g. Smets & Wouters (2007) or Rudebusch & Swanson (2012)) calibrate/estimate government spending shock by assuming that all government expenditures are purely wasted and act only as a shock to market clearing condition. Ramey (2011) argues that since only defense spending are likely to be independent from the business cycle, the modeler should base his estimate about the shock process using defense spending only. We depart from this interpretation and associate defense spending[23] with utility enhancing government expenditures which are independent of private consumption (as in Baxter & King (1993) or Fiorito & Kollintzas (2004)). With additively separable preferences non-substitutable utility enhancing expenditures (defense spending) does not have any impact on the equilibrium conditions. Note, however that this is not the case with recursive preferences

[22]which are taken from NIPA 3.15.5 and 3.15.4. All the prices are chained 2009 dollars.
[23]taken from NIPA 3.15.5 and 3.15.4

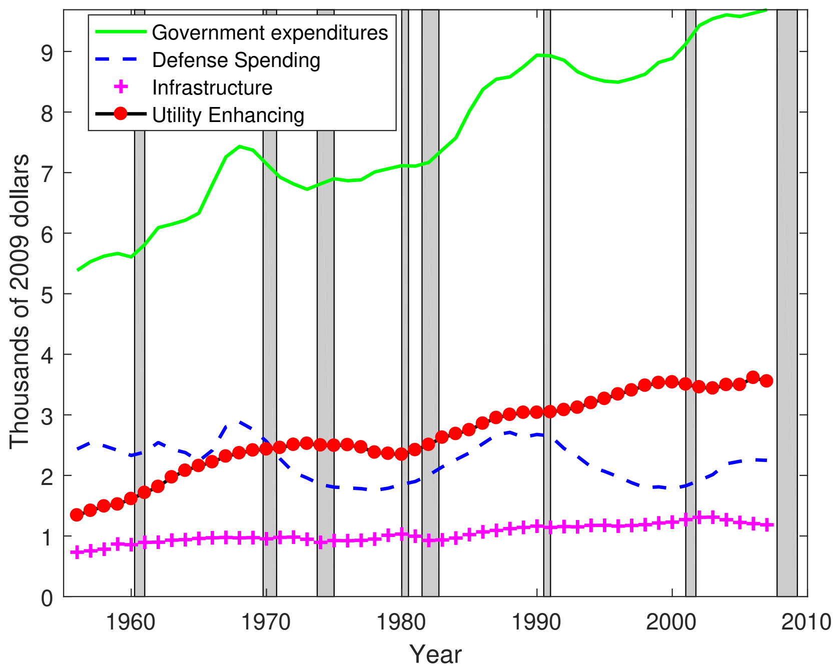


Figure 3.2: Real Government Spending Per Capita and its components (in thousands of chained dollars, 2009)

where both separable G_t^s and non-separable G_t^n enter first order conditions through the value function V_t.

The residual part of the government expenditures (e.g. welfare and social services, unemployment) are modeled as purely wasteful spending.

3.4.3 Fluctuations

Figure 3.2 shows the paths of government spending and its components since 1956. The gray bars shows the NBER peak to thought dates. We see that all expenditure types show a significant upward trend over time. Military build-up around the Vietnam war, impact of Soviet invasion of Afghanistan and the build up after 9/11 are visible in defense spending and drive its volatility.

Figure 3.3 shows the shares of the components of government in percent of total government spending. We use their arithmetic averages to calibrate how the model total government expenditures are shared across its components. Since we identify productive government spending with infrastructure spending we set $G_t^p = \tau_p G_t$ and $\tau_p = 0.14$. As an approximation to substitutable utility enhancing government spending we use government expenditure on public health care, public education and culture. We find $\tau_s = 0.32$ for

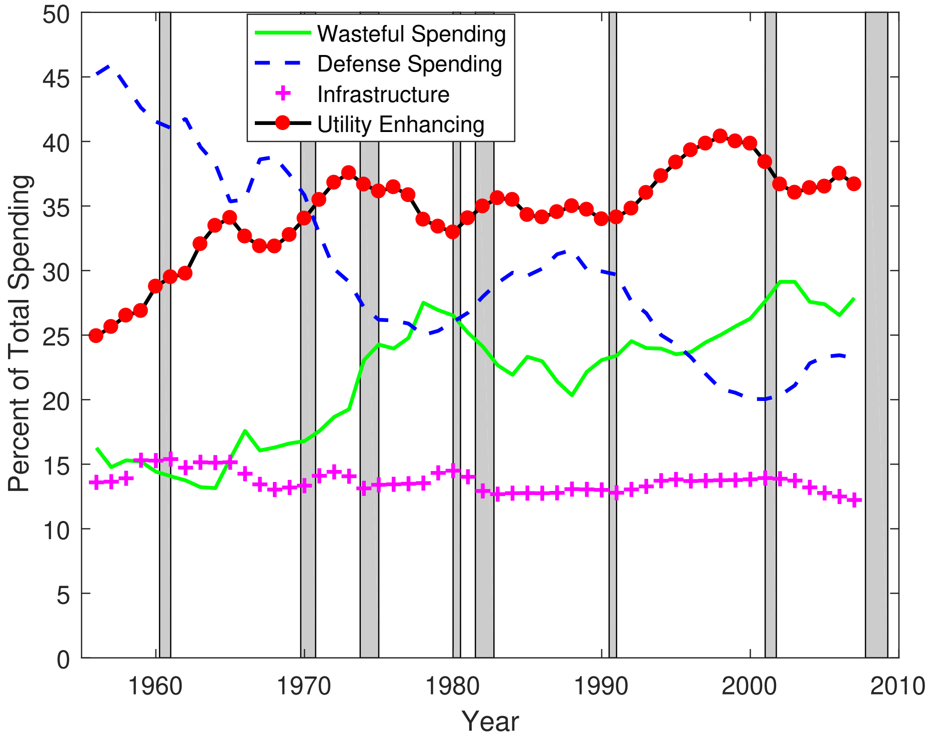


Figure 3.3: Components of Government Spending Fraction of Real Government Expenditures, 2009)

the share of substitutable utility enhancing expenditure. Non-substitutable to private consumption utility enhancing expenditures $\tau_n = 0.3$ are based on defense component of government spending. As the remaining expenditures are considered to be wasted with the share of $\tau_w = 0.24$ in total government expenditures.

Ramey (2011) highlights the fact that only defense spending are likely to be exogenous to the state of the economy and thus can be used to infer shock innovations of government spending. Expenditures like health or education are usually conditional on the state of the economy and thus it is not clear if they satisfy the requirement of shock orthogonality[24].Nevertheless, figure 3.4 shows that all the HP filtered cyclical component of government spending types are near acyclical (including utility enhancing expenditures) thus might well serve for inferring shocks.

Table 3.2 shows the volatility of the shock innovations, ϵ_g , and persistence, ρ_G, of government spending for various sub-samples. In our data sub-samples the volatility ranges approximately from 50 bps to 6% in case of defense spending. We consider this range to evaluate the impact of government spending volatility on on the term structure of interest rates. Our baseline calibration matches the long run average period between 1969 and 2009[25] for quarterly data.

	Wasteful 24%		Defense spending 30%		Infrastructure 14%		Educ.,Health 32%	
Period	ϵ_g	ρ_G	ϵ_g	ρ_G	ϵ_g	ρ_G	ϵ_g	ρ_G
1969 - 2009	0.69	0.91	1.4	0.98	0.99	0.93	0.58	0.98

Table 3.2: Quarterly standard deviation (ϵ_g) of innovations and persistence (ρ_G) to components of Government Expenditures. Results are in % deviations from the one-sided HP trend. Percentage in the headline are the component shares in the total government spending.

We infer the range for σ_G from the sub-samples of total government spending, $\sigma_G = 6$ implies volatility of G_t as in period 1947 to 1957. The low volatility $\sigma_G = 0.4$ corresponds to period from 1987 to 1997.

[24]She also argues that public expenditures are in general anticipated 3 to 4 quarters before they take place. In this paper we abstract from this finding.

[25]Details on the data source and calculations are provided in appendix.

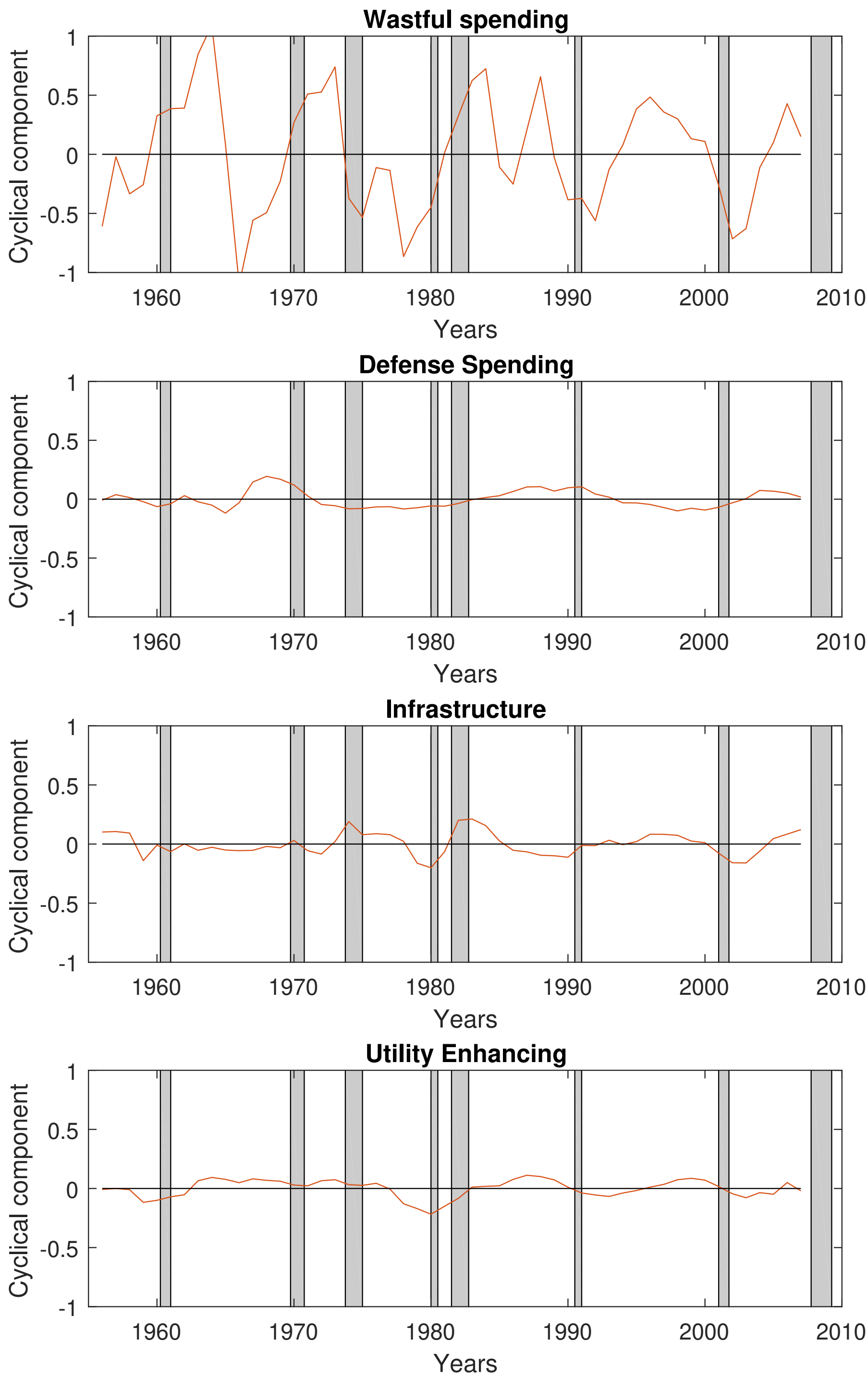


Figure 3.4: One-side-HP Filtered Cyclical Components of Government Spending Types

3.4.4 Data

Infrastructure spending (GI): Total Public Infrastructure Spending by Federal, State, and Local Governments, 1956-2007 (in millions of 2009 dollars), Congressional Budget Office Study, Public Spending on Transportation and Water Infrastructure.

Total government spending (G): Government consumption expenditures and gross investment, 1956-2007 (in billions of chained 2009 dollars), NIPA 1.1.6

Defense spending (Def) National defense expenditures, NIPA 3.9.5, 1956-2007 (in billions of dollars)

Price index: National defense: Price Indexes for Government Consumption Expenditures and Gross Investment, NIPA 3.9.4

Health, education, recreation expenditures (GU): NIPA 3.15.5, 1956-2007 (in billions of dollars)

Price index: Health, education, recreation: Price Indexes for Government Consumption Expenditures and Gross Investment by Function, NIPA 3.15.4

Population: Total population, Data and Programs for "Macroeconomic Shocks and Their Propagation" Handbook of Macroeconomics collected by Valerie A. Ramey, available at her webpage.

3.4.5 Data Transformation

Raw data are transformed as follows. All quantities are expressed in billions of 2009 dollars, divided by total population and normalized such that $1956 = 1$. Utility enhancing expenditures are calculated as a sum of health, education and recreation expenditures. The transformed data are filtered by the Kalman filter implementation of one sided HP filter using code written by Alexander Meyer-Gohde. Wasteful government expenditures, GW, are calculated as:

$$GW = G - GI - Def - GU \tag{3.34}$$

3.5 Factor attribution

In this section we propose a method to quantitatively evaluate the specific channels of the transmission mechanism as discussed in the section 3.3.1. The price of bonds contains compensation for risk related to macroeconomic fundamentals.

The second order approximation of the benchmark model pricing kernel points to four risk factors driving term structure, *i*) consumption growth, *ii*) inflation, *iii*) long-run risk [26], *iv*) preference shock. To track the propagation of exogenous shock to yields through macroeconomic factors is complicated by the fact that the effects of the factors are cross correlated. For instance, in the case of two factors, consumption growth and inflation, the term structure of interest rates can be written as a composite function $ytm(c(G, \pi(G)), \pi(G, c(G)))$. The yield curve moves in response to G shock because consumption directly adjusts to the new level of government expenditure and because consumption responds to the new inflation rate. Taking the derivative with respect to G delivers $\frac{\partial ytm}{\partial c}\left[\frac{\partial c}{\partial G} + \frac{\partial c}{\partial \pi}\frac{\partial \pi}{\partial G}\right] + \frac{\partial ytm}{\partial \pi}\left[\frac{\partial \pi}{\partial G} + \frac{\partial \pi}{\partial c}\frac{\partial c}{\partial G}\right]$. In the following analysis we quantify the change in yields driven separately by factor stand alone effects, consumption growth $\frac{\partial ytm}{\partial c}\frac{\partial c}{\partial G}$ and inflation $\frac{\partial ytm}{\partial \pi}\frac{\partial \pi}{\partial G}$ and the interaction effect coming from the factor cross derivatives, $\frac{\partial ytm}{\partial c}\frac{\partial c}{\partial \pi}\frac{\partial \pi}{\partial G} + \frac{\partial ytm}{\partial c}\frac{\partial \pi}{\partial G}\frac{\partial c}{\partial G}$. For n factors the derivative of composite function can be written

$$\frac{\partial ytm}{\partial G} = \sum_{i=1}^{n}\sum_{j=1}^{n-1}\frac{\partial ytm}{\partial F_i}\left[\frac{\partial F_i}{\partial G} + \frac{\partial F_i}{\partial F_j}\frac{\partial F_j}{\partial G}\right] \quad \text{for } i \neq j \tag{3.35}$$

where F stands for the macroeconomic factor driving the yield curve dynamics. To decompose the effects of changes in government spending on the yield curve we use the idea of Brinson multi-factor model[27] (Brinson & Fachler (1985)). Figure 3.5 illustrates the idea behind the decomposition. Without loss of generality let us abstract from the preference shock for now and consider only the remaining three factors. We start the analysis at the deterministic steady state where the term structure is just a flat line at $\frac{1}{\beta}$. Adding the stand alone risk factor increases the level of yield curve to the factor specific node. In terms of equation (3.35) we quantify the first term after multiplying the bracket. However, factors interactions contribute to the change in the yield curve as well. Thus, we need to calculate the factor cross derivatives as well. In figure 3.5 this is represented by the nodes at the dashed lines intersection. For example, the total effect of changes in consumption growth and inflation

[26]The long-run risk may be interpreted in several ways as highlighted in Epstein & Zin (1989). The crucial point is that time to resolve uncertainty matters thus shocks to continuation value matters

[27]this version of factor model is widely use in portfolio management for return attribution analysis

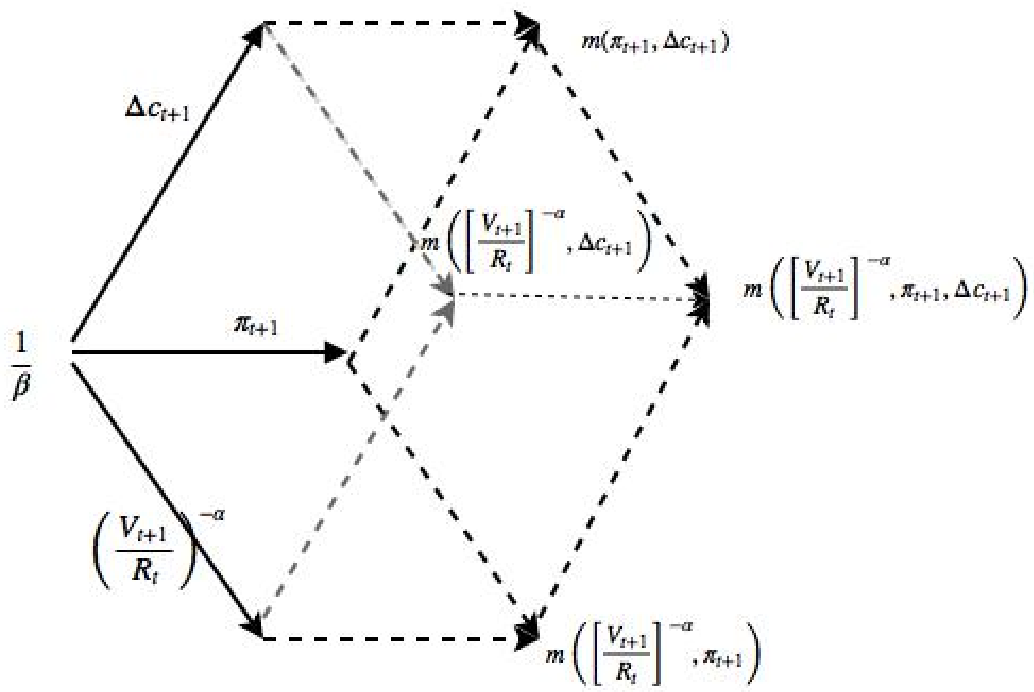


Figure 3.5: Intuition behind the decomposition

on the yield curve is the sum of the stand alone impacts, Δc_{t+1}, π_{t+1}, and their interaction, $m(\Delta c_{t+1}, \pi_{t+1})$. Considering all three factors in figure 3.5, the total change is the sum of risks attributed to the stand alone factors, interaction of two factors and interaction of all three factors together. In general, the total effect in the n-factor pricing equation can be decomposed into n groups of factor interactions and stand alone factor risks.

Figure 3.5 demonstrates how to calculate the risk groups within our macro model. Let's again focus only on two factors, consumption growth and inflation. First, calculate the yield curve within the macro model where the pricing equation contains only consumption growth or inflation. Second, subtract the determinist steady state. In this way we can isolate the individual contribution of inflation and consumption growth as a risk factor in pricing equation. Third, evaluate the model with both risk factors and subtract the stand alone risks factors calculated in the previous step and subtract again the determinist steady state to find the attribution of the factors interaction. More formally,

$$R_1 = \sum_{i}^{n}(M(F_i) - M(st.st.)) \tag{3.36}$$

$$R_{2,i} = \sum_{i}^{n}\sum_{j}^{n} M(F_i, F_j) - R_1 - M(st.st.)) \tag{3.37}$$

$$R_g = \sum_{g}^{n}\sum_{i}^{n}\sum_{j}^{n}(M(F_i, F_j, \dots F_g) - R_{n-1} - R_{n-2} \dots R_1) \tag{3.38}$$

where $M(F_i)$ is the model with the risk factor i and $M(st.st.)$ is the model at the steady state.

Up to the second order approximation the interaction terms of two factors correspond to the covariance terms in the equation (3.28) and stand alone factors represent the variances. Higher order interactions are non-zero only for higher order approximation. This can be very clearly seen from the second order approximation of the term structure. We define the purely risk-free rate as $ytm_t^{rf,n} = \frac{\beta}{\bar{\pi}}$ which serves as a benchmark. The pricing kernel reflecting the consumption growth risk is $ytm_t^{c,n} = \frac{\beta}{\bar{\pi}} - \frac{\gamma^2}{2n}\mathrm{Var}_t\left(\Delta^n \hat{c}_{t+n}\right)$. The risk premium attributed to the consumption growth, rp_c is then

$$E[ytm_t^{c,n} - ytm_t^{rf,n}] = -\frac{\gamma^2}{2n} E\mathrm{Var}_t\left(\Delta^n \hat{c}_{t+n}\right) \tag{3.39}$$

Up to the second order, the pricing kernel accounting both for nominal and real risk can be written as

$$Eytm_t^{c,\pi,n} = \frac{\beta}{\bar{\pi}} - \frac{\gamma^2}{2n} E\mathrm{Var}_t\left(\Delta^n \hat{c}_{t+n}\right) + \frac{1}{2n} E\mathrm{Var}_t \sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right) - \frac{\gamma}{n} E\mathrm{Cov}_t\left(\Delta^n \hat{c}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right) \tag{3.40}$$

thus we can calculate the covariance term as the difference between the total risk premium, $E[ytm_t^{c,\pi,n} - ytm_t^{rf,n}]$, and the risk premia of individual factors, rp_c, rp_π.

$$E[ytm_t^{c,\pi,n} - ytm_t^{rf,n} - RP_i] = \frac{\gamma}{n} E\left[\mathrm{Cov}_t\left(\Delta^n \hat{c}_{t+n}, \sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right] \tag{3.41}$$

where the sum of stand alone risk premiums is $RP_i = rp_c + rp_\pi$. Adding other factors and calculating the risk premiums follows the same pattern.

3.6 Transmission Mechanism: Insights from the Model

In this section, we analyze the effects of government expenditures and its components on bond prices by distinguishing between the impact of *i*) transitory changes and *ii*) the changes in uncertainty. Further, we stress the importance in distinguish between the realized and anticipated shocks. This is motivated by the seminal results coming from fiscal foresight literature. Ramey (2011) is among the first to forcefully document in empirical study the importance of fiscal foresight in the response of the economy to rises in public expenditures. She shows that different types of government spending are anticipated several quarters before they occur and how failing to account for the anticipation effect has crucial consequences for how the economy responds to a rise in government purchases. For instance, there has been long debate in economics if government spending triggers rise or drop in consumption (see Gali *et al.* (2007)). Accounting for the shock timing allow us to separate the rise and drop in consumption in response to rise in total government spending.

The importance of news shocks for yield curve has been established in Kurmann & Otrok (2013); they show that it is the news about future total factor productivity which explains more than 50% of the unpredictable movements in the slope of the yield curve. The effect of news about government spending on the yield curve has not however been studied in the literature. Yet intuitively many fiscal policy measures are known well in advance. The lags in decision and implementation can be demonstrated by many examples. Trump's fiscal package to boost infrastructure spending has been debated since he won the election. Obamacare[28] was discussed for more than a year before coming into force a the implementation was only gradual. Ramey (2011) lists other examples related to defense spending as the aftermath of 9/11 or Soviet invasion of Afghanistan, where the rise in defense spending was anticipated in advance. The fiscal foresight literature (see Leeper *et al.* (2012)) complements the 'news' literature (Beaudry & Portier (2006) and Barsky & Sims (2011)) which posits that business cycles arise on the basis of expectations of future fundamentals rather than on the impact of shock. Laubach (2009) shows the upward effect of fiscal expansion on the long-term yields by comparing the budget deficit forecasts with the long-horizon forward rates. Leeper *et al.* (2012) documents

[28]Patient Protection and Affordable Care Act

that non-negligible portion expenditures are only foreseen imperfectly

In the earlier literature Gale and Orsag (2003) provide extensive literature review on how the timing of fiscal policy in case of deficit and debt matters for the response of the yields. For instance, Barth (1991) surveys 42 studies and finds: from 19 studies with projected deficits 13 have positive, 5 mixed effects, 1 no effect. Gale and Orsag (2003) redo Barth (1991) and find: 18 studies have positive effect, 6 mixed effects, 19 not significant or negative. Similar conclusion found by Mankiw (1999). Often cited papers by Evans (1987) or Plosser (1982) find no effect. Ardagna *et al.* (2007) use both a simple static estimation and a vector autoregression model for a panel of countries and show that an increase in the primary government deficit increases the long-term yields. However, in the case of an increase of the government debt, the yields are affected only for the above-averagely indebted countries. Born *et al.* (2013)) studies anticipated capital and labor tax shocks to business cycle volatility in an estimated New Keynesian DSGE model.

Building on the fiscal foresight literature, we distinguish between the: *i)* news shock, which is the shocks to agents' expectations about future policy, *ii)* the surprise shock *iii)* change in uncertainty which reflects the changes in the level of insecurity in forming the expectations about future public expenditures.

We show that: *i*) the response of the term structure of interest rates depends on the type of government spending shocks *ii*) the traditional wasteful government spending shocks tend rise yield curve at the impact but lower as the uncertainty rises while the opposite holds for the productive expenditures. *iii*) the fiscal-monetary policy interaction are crucial for determining the quantitative response *iv*) fiscal authority committed to fiscal consolidation immunizes the effect of its spending on the term structure,

3.6.1 Transitory Response

We analyze the transmission of the government spending expenditures by examining the model's impulse responses to shocks. The first column of Figure 3.6 reports the responses of consumption, inflation, bond price with maturity 10 years and 3 month bond in the model to a positive 100 basis points shock to government expenditures which are fully wasted. The second, third and forth columns report analogous IRFs to productive, utility enhancing government expenditures which are perfectly substitutable to consumption or non-substitutable. As is standard in the literature the wasteful government spending shock operates through the wealth effect (see Baxter & King (1993)) and lowers consumptions and inflation in case of positive coefficient on output gap (see Linnemann & Schabert (2003) for discussion). The response of nominal term premium is about 100 times smaller than response of bond yields. This finding is in line with other structural models (see for instance Rudebusch & Swanson (2012)). The response of bond yields is due to the change in risk neutral rate and is driven mostly by the dynamics of policy rate. Consistently with Wright (2011) we find that term premia moves mainly due to changes in uncertainty rather than to level shocks (see next section for further discussion). Quantitatively, the less standard in macro literature government expenditures (productive, substitutable, non-substitutable) are more important in explaining the variation in long term bond prices.

Positive surprise shock in productive (infrastructure) government expenditures in column 2 increases the rate of return to private capital and labor which stimulates consumption through the increase in households capital and labor income. Analogously to shock in productivity[29] lower marginal costs lead to fall in inflation and nominal bond yields. Nominal term premium decreases because in the consequent periods after the shock consumption decreases and inflation increases as they return back to steady state. Higher inflation undermines the real value of bonds exactly in time of lower consumption.

Positive surprise shock in substitutable (education and health) government expenditures in column 3 lowers the marginal utility of composite consumption index which makes households to substitute part of their private consummation by the newly available government consumption. The co-movement of inflation and consumption makes nominal bonds good saving instrument hedging the fluctuations in consumption and thus lowers the nominal term premium. The drop in inflation triggers response by monetary policy authority which lowers its interest rate which partly transmits to long term bonds through the risk neutral rate.

Positive surprise shock non-substitutable to private consumption (defense) expenditures rises consumption due to long run risks. Surprise increase in non-substitutable spending is positive shock to continuation value which directly enters the pricing equation (see equation (3.28)) and decreases long run consumption risk. Nevertheless, the wealth effect is stronger and overall private consumption drops down. The response of bond yields is driven by decrease in

[29]see for instance Rudebusch & Swanson (2012) for discussion of the impact of productivity shocks on term structure of interest rates

nominal term premium and amplified by the risk neutral price which pushes the bond yield further down reflecting the decrease in policy rate. Our model with long run risk consistently with the empirical results of Ramey (2011)[30] and Blanchard & Perotti (2002)) predicts that consumption increases in response to unexpected rise in defense spending[31]. The overall effect on consumption is however negative due to wealth effect which comes through the increase in total government spending.

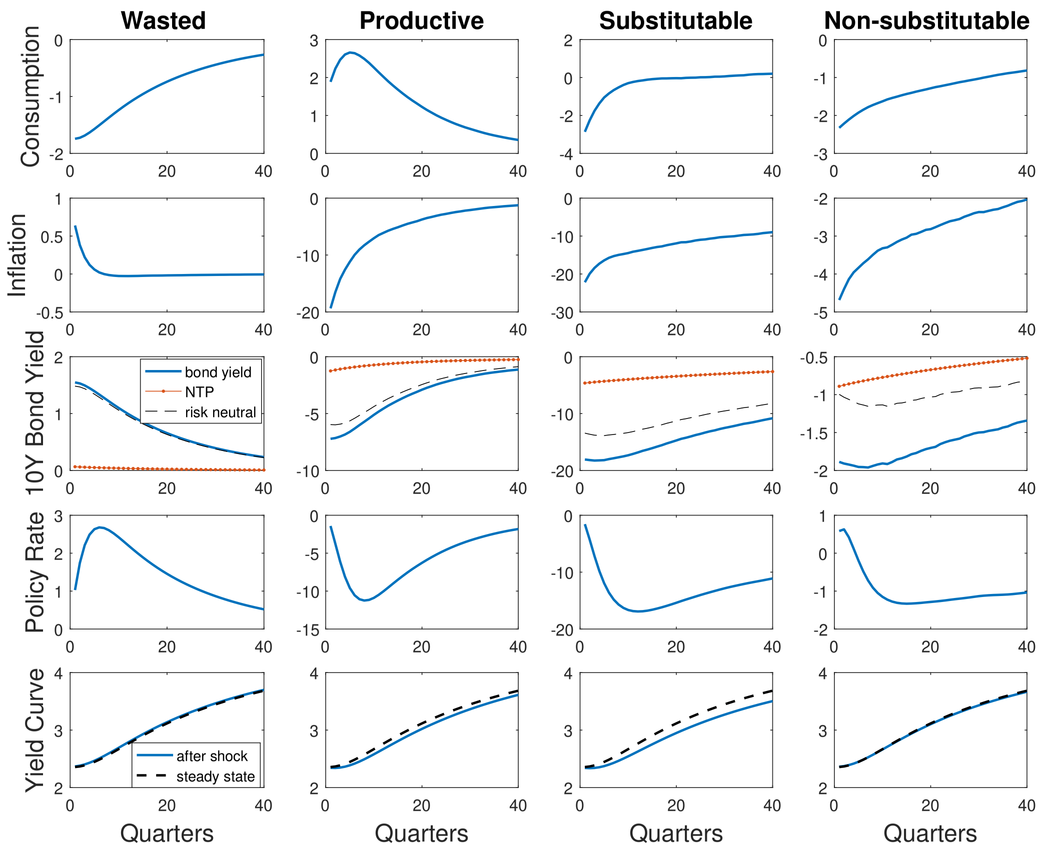
Figure 3.7 shows that news shock are quantitatively more pronounced and the response of the yield curve is stronger. The main difference lies in substitutable government expenditures which rise at impact both consumption and inflation.

We conclude that the overall impact of level shocks of government expenditures on term premia are quantitatively low. The response of consumption, inflation and policy rate to the shocks forms however conditional expectations in equation (3.28) and thus determine the level of term and risk premia.

[30]Ramey (2011) shows that consumption drops in response to anticipated consumption shocks.

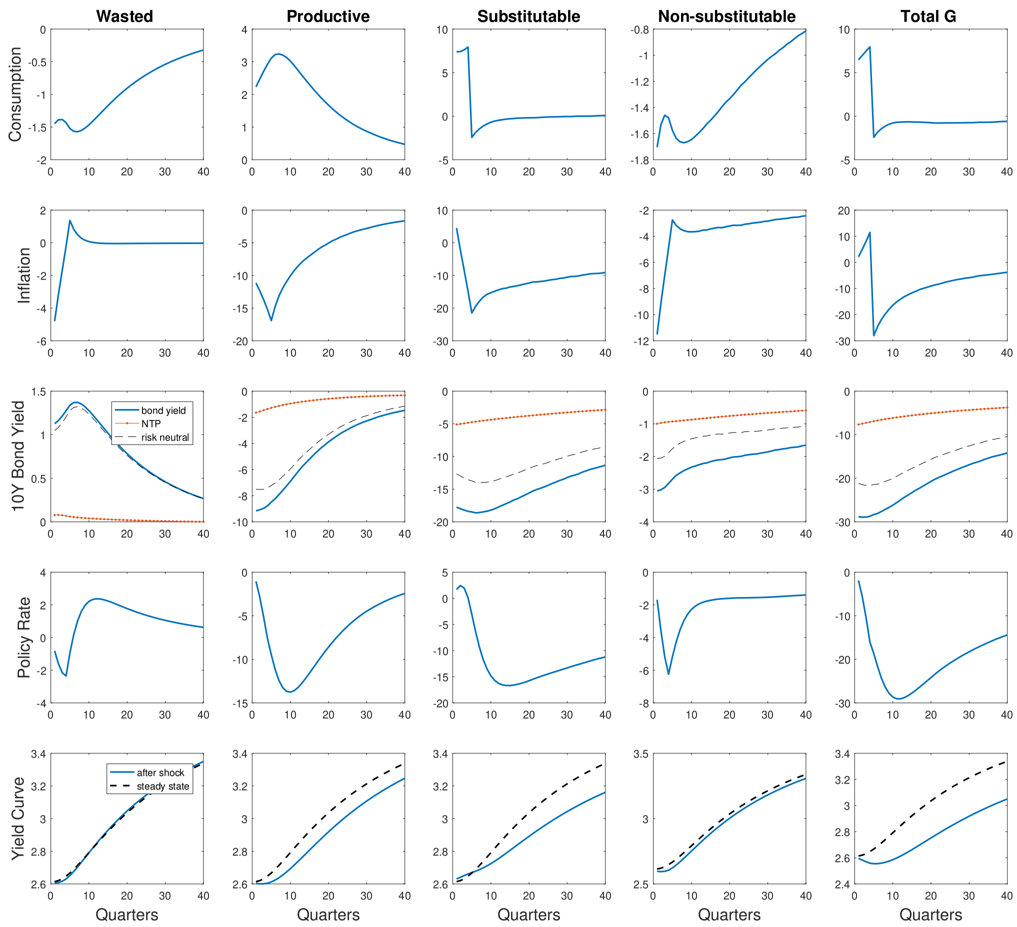
[31]To explain the rise in consumption has been long puzzle in DSGE literature. See Corsetti *et al.* (2009) and Gali *et al.* (2007)

Figure 3.6: Impulse responses to shock in G's



Note: Impulse responses of consumption, inflation, bond yield with maturity 10 years and 3 months to positive 100 *bps. shock in wasteful, productive, utility enhancing (substitutable and non-substitutable to private consumption) government expenditures. Responses are in basis points. Inflation and bond yields are annualized.*

Figure 3.7: Impulse responses to Anticipated shocks in in G's



Note: Impulse responses of consumption, inflation, bond yield with maturity 10 years and 3 months to positive 100 *bps. shock in **news** to wasteful, productive, utility enhancing (substitutable and non-substitutable to private consumption) government expenditures.*

Fiscal Policy Uncertainty -Wasteful Government Spending

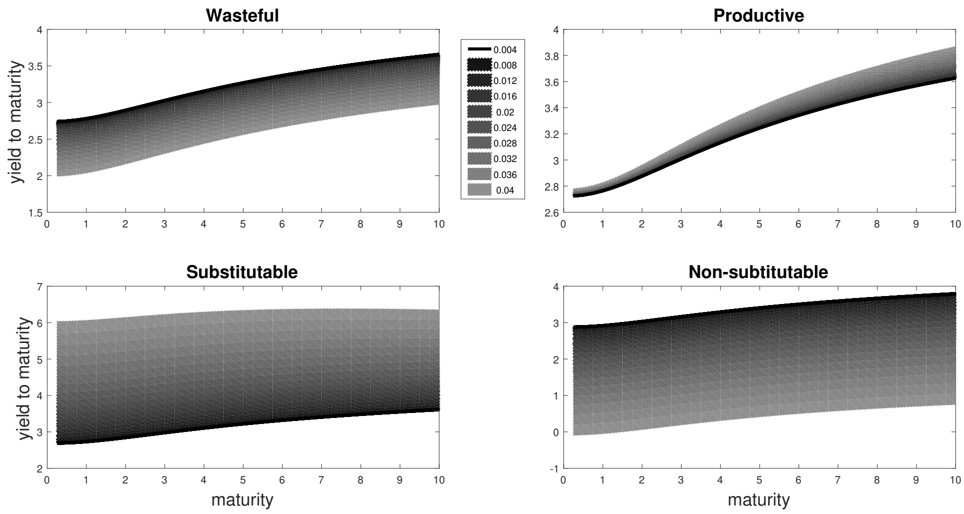
Figure 3.8 demonstrates that the rise in uncertainty related to government spending in our benchmark model decreases the level of the term structure of interest rates. The darker line corresponds to term structure of interest rates with lower volatility. The lowest volatility of government spending innovations we consider is 40 bps and the highest 4 percentage points.

The driving force behind the drop is the insurance property of bonds against uncertain future realization of fiscal policy. High volatility in government spending motivates consumption smoothing households to insure themselves against a drop in their wealth. The precautionary savings motive grows with the volatility. De Paoli *et al.* (2010) argues that productivity shocks in the log-linearized models abstracting from precautionary saving may give significantly biased policy implications. We extend in this sense the De Paoli *et al.* (2010) argument to government spending shocks. High volatility of fiscal policy increases the importance of the households risk aversion for the evolution of the interest rates throughout the whole maturity structure. In addition to the precautionary motive the drop in yields is driven by the hedging property of bonds. The value of bonds is negatively correlated with the long-run consumption and leisure risks.

Policy making authorities in a certainty equivalent world will underestimate the growth in the demand for government bonds and thus the consequent drop in stochastic steady state of consumption. The increase in uncertainty will make financing of the government debt cheaper in the default free world but at the cost of causing large demand shifts away from consumption to government bonds. By not taking these effects into account fiscal and monetary policy mix delivers suboptimal results from a welfare point of view.

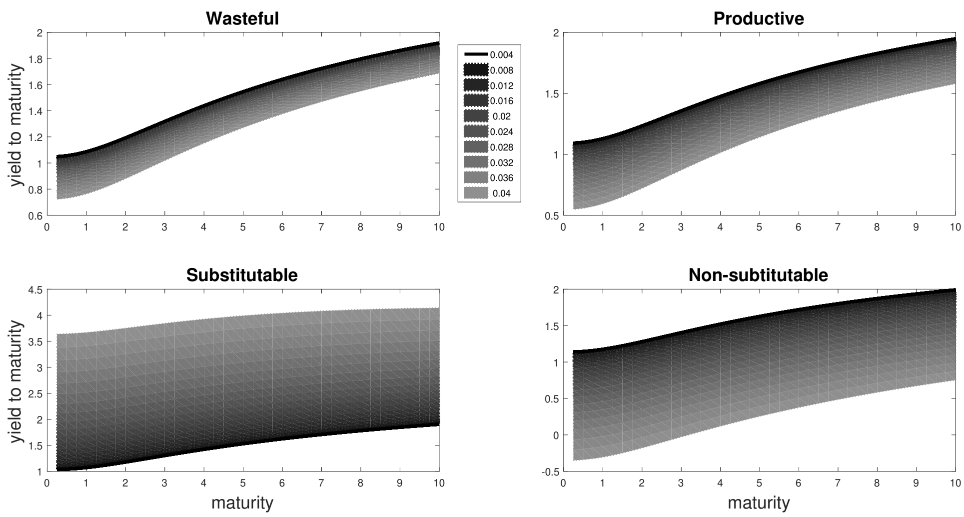
Figure 3.8 shows the impact of fiscal uncertainty on the term structure when monetary policy response to both inflation and output in its reaction function. Figure 3.9 shows the impact of uncertainty when monetary policy authority doesn't respond to fluctuations in aggregate output. Apart from some quantitative differences the direction of the yield curve response is the same for government spending types but productive expenditures. In the inflation targeting like regime government bonds provide good hedging instrument against the rise in uncertainty in infrastructure spending. As we later demonstrate using the attribution analysis, this is driven by increase in inflation together with worsening long-run consumption and leisure prospects.

Figure 3.8: Term structure response to volatility of G's shock



Note: The figure shows the mean of the term structure of interest rates as it varies under different different size of volatility shocks for wasteful, productive, substitutable and non-substitutable government expenditures.

Figure 3.9: Term structure response to volatility of G's shock for $\phi_y = 0$



Note: The figure shows the mean of the term structure of interest rates as it varies under different different size of volatility shocks for wasteful, productive, substitutable and non-substitutable government expenditures when the weight on output gap in the Taylor rule is set to zero.

Next we discuss the conditions under which positive government spending shocks generate inflation risks. The specification and the size of the coefficients on inflation and output gap in the interest rate rule have large impact on the effects of shocks in the model. In a framework similar to this paper Horvath, Kaszab and Marsal (2016) find that the form of the interest rate rule has large impact on the ability of the model to generate inflation risks. Here we briefly explain that persistent and large government spending shocks are less inflationary with an a positive output-gap coefficient of $\phi_y > 0$ (rather than zero) because a positive output-gap coefficient raises the real interest rate even more in response to positive spending shocks discouraging households from further spending in the present. Furthermore, they point out that the general equilibrium outcome of a positive government spending shock financed by lump-sum taxes is a fall in inflation and short-term nominal interest rate when $\phi_y > 0$. A rise in government purchases leads to higher future taxes (a negative wealth effect) inducing households to cut consumption expenditures and to have less leisure as long as both are normal goods. With a given time frame less leisure translates into higher hours worked (an outward shift in labor supply). The shrinkage in household spending causes firms to produce less and, therefore, demand less labor. The leftward movement of the labor demand curve pushes the real wages down which has downward pressure on inflation through the New Keynesian Phillips curve (see also Linnemann & Schabert (2003)).

Fiscal Policy Uncertainty - Productive Government Spending

The second quadrant in Figure 3.8 shows the response of the stochastic steady state of the term structure of interest rates to the rise in uncertainty related to productive government spending. Higher uncertanty about productive expenditures increases both the level and the slope of the term structure of interest rates. As the attribution analysis shows the rise is driven by the fact that government bonds serve as a poor instrument in protecting against wealth against the swings in this type of government expenditures.

The impact of productive government spending on the yield curve depends quantitatively on its share in output, θ_g. The sensitivity on θ_g of the economy response is also documented in Getachew & Turnovsky (2015). The size of insurance property of bonds rises with the share of government expenditures in production. The response also depends on the monetary policy conduct. If the weight on output is zero in the monetary policy rule bonds can protect its holders against the fluctuations in their wealth due to productive government spending. For this reason, the yield drops down as the demand for bonds rises.

Fiscal Policy Uncertainty - Utility Enhancing Government Spending

The third quadrant in Figure 3.8 shows the response of the stochastic steady state of the term structure of interest rates to the rise in uncertainty related to substitutable government spending. Uncertainty about government expenditures directed to goods which are imperfect substitutes to private consumption (i.e. health care, education) rises the term structure of interest rates. Bonds leverage the risks for investors wealth coming from this type of expenditure and thus need to carry premium for holding. The leverage property depends quantitatively on the monetary policy conduct (Figure 3.9). When central bank puts zero weight on output gap in Taylor rule, $\phi_y = 0$ bonds are on average more expensive (provide lower yield).

Baxter & King (1993) motivate defense expenditures by putting them into utility function in separable form. They argue that defense spending are not productive but they increase utility of households without affecting their consumption leisure decisions as G falls our from equilibrium conditions due to its exogeneity. Azevedo & Ercolani (2012) or Albertini *et al.* (2014) consider defense spending as a part of production function by arguing that defense spending increases the productivity of private capital and leisure. Barro (1990) and Turnovsky (1996) associate utility enhancing government expenditures with for example cultural and recreational public services such as museums, public parks or public social events like fireworks. The interpretation of government expenditures not substitutable to private consumption is in our case affected by the fact that we use recursive preferences. In this case, government spending enters equilibrium conditions through the continuation value, V_t and thus affects long run consumption and leisure decisions of households. The forth quadrant in Figure 3.8 shows the utility enhancing government expenditure which are not substitutable to private consumption. The rise in the volatility decreases the level of yield curve in both monetary policy regimes (figure 3.9). This is driven again by the wealth smoothing ability of bonds which serves as a hedging instrument.

Implications for Monetary Policy

Figure 3.10 shows the impulse responses to a positive government spending shock on impact for the whole term structure of interest rates. Each panel represents the impact of the shock with around 40 bps for the maturity of one quarter and 6 percentage points for bond with a maturity of 10 years. The red dashed line with dots is the stochastic steady state (unconditional mean) of the term structure assuming that the monetary policy authority adjusts its interest rate solely in response to inflation and there is no response to the output gap ($\phi_y = 0$). The red line is the term structure one period after the economy is hit by an increase in government spending. The blue line constitutes the analogy when the weight on output stabilization is positive ($\phi_y = 0.075$). The degree of uncertainty about government spending has important consequences for setting the monetary policy. In an economy with a low degree of uncertainty about government spending monetary policy targeting only inflation reduces the inflation risk premia more than monetary policy which also smooths deviations from the output gap. Nevertheless, when the degree of uncertainty about government spending increases above 3 percentage points, the inflation risk generated by fiscal policy overweighs the stabilizing effect of strictly inflation targeting policy towards productivity shocks[32]. In a strictly inflation targeting regime the inflation risks generated by fiscal uncertainty are very costly. Building on the argument by Linnemann & Schabert (2003) discussed above, the monetary policy reacting to output mitigates inflation and reinforces the hedging property of real bonds to long-run consumption and leisure risks.

[32] Positive TFP shock pushes inflation down. Inflation drops more if $\phi_y > 0$ because the drop in marginal costs is accompanied by rise in interest rates from Taylor rule. Therefore, the correlation between consumption and inflation is stronger if $\phi_y > 0$. Bonds loose their real value more in bad times. Investors ask higher premium for holding bonds.

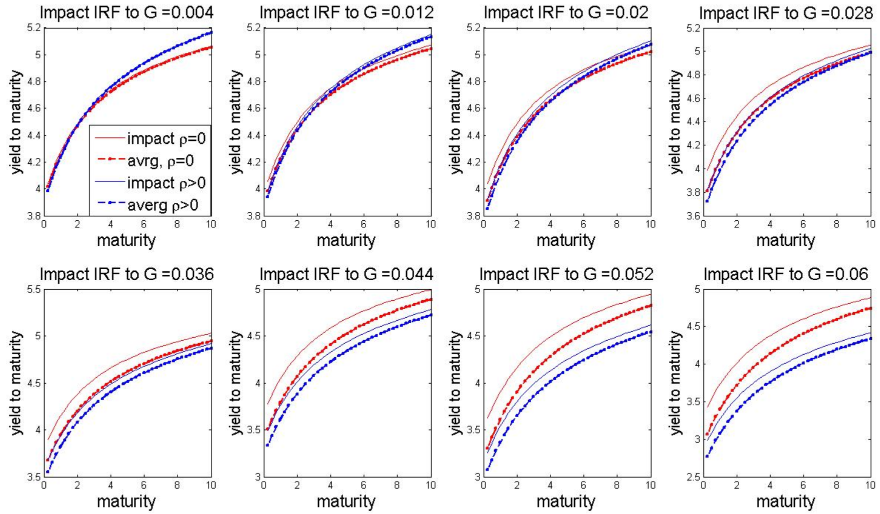


Figure 3.10: Wasteful Government Spending and The Term Structure of Nominal Interest Rates: The Role of Monetary Policy. The stochastic steady state of the term structure and the impact of increase in government spending on the yield curve. The red lines are the case of zero weight on output stabilization in the Taylor rule. The blue line correspond to the case of $\phi_y = 0.075$.

Spending Reversals

We introduce credible commitment of fiscal policy into the model such that government reduces its expenditure when government debt increases. The government spending is therefore an endogenous function of the government debt and fiscal policy decisions about government spending are history dependent. The benchmark model augmented by spending reversals predicts that there will be no crowding out of private investment by government. Households work more in a response to increases in aggregate demand. Higher government spending is financed through extra taxes and government debt. The price of debt rises to encourage additional savings. Expectations about future lower than steady state government spending imply lower future taxes and debt pushing the future expected interest rates down. Higher future disposable income makes households form expectations about future higher consumption. The intertemporaral smoothing assumption raises the current level of consumption and discourages savings.

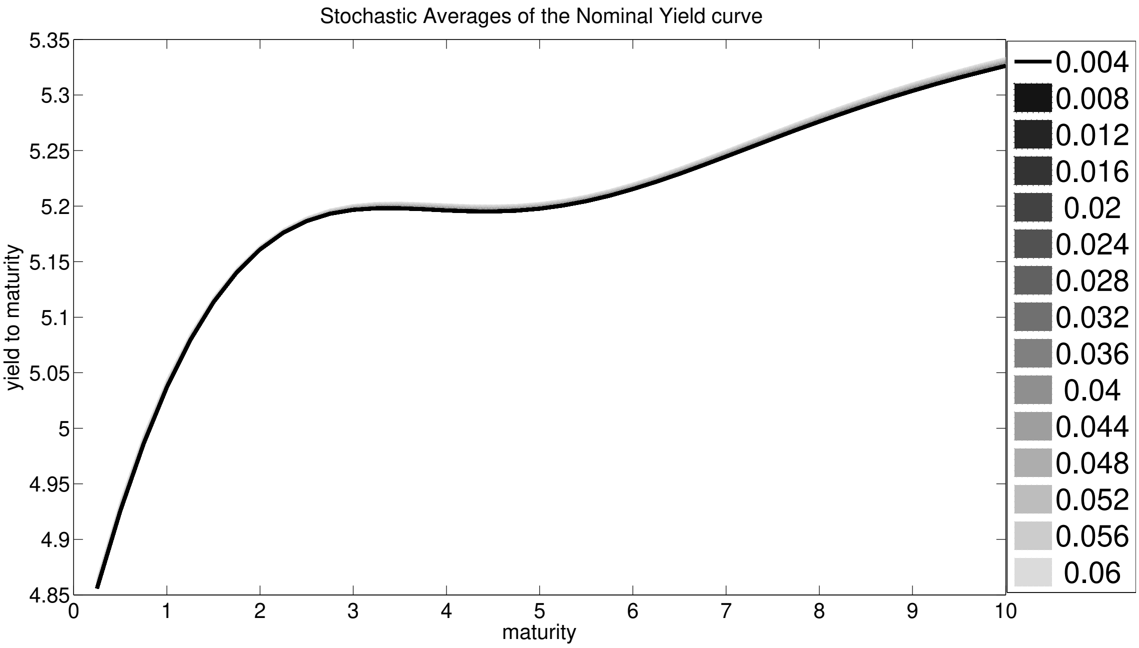
Figure 3.11 depicts the impact of growing uncertainty related to wasteful government spending shocks. In an economy with spending reversals *i*) the yield curve is immune to the degree of fiscal uncertainty, *ii*) the level of the yield curve lies higher than in the benchmark case so bonds are less demanded as an insurance. The nominal term premium is however lower than in the benchmark case.

The attribution analysis in the next section shows that fiscal policy commitment to finance temporarily higher spending by future austerity significantly decreases the price of risk related to uncertainty about government spending. Figure 3.11 demonstrate that the risk premiums are negligible. The history dependence in otherwise stochastic evolution of debt introduces into the model economy a new source of information. The increase in predictability of the evolution of debt and taxes mitigates the impact of uncertainty (higher order terms) on macroeconomic variables. This fact helps the investor to form a more accurate expectation. Larger time t conditional information set decreases the risk of bond miss-pricing, therefore the risk premia are lower.

3.6.2 Attribution Results

In this section we use the methodology developed in the section attribution analysis to quantitatively evaluate the conditional moments from the analytical deposition of the pricing kernel in equation (3.28).This decomposition allows

Figure 3.11: Term structure response to volatility of G's shock for $\phi_y = 0$



Note: The figure shows the mean of the term structure of interest rates as it varies under different size of volatility shocks for wasteful government expenditures when the fiscal policy is committed to reduce expenditures if the debt rises. The legend displays the volatility of the shock.

us to quantitatively evaluate the importance of the risk channels that form the term structure of interest rates.

Table 3.3: Attribution analysis.

Precautionary savings	Wasteful	Productive	G_S	G_N
Consumption risks	-0.5	-0.5	-11.1	-1.5
Inflation risks	-63.7	7.6	288	- 250
Long-run risk (C_t, N_t, b_t)	0.7	0.9	1.3	4.7
Total	-63.5	8.03	279	-248
Risk premiums				
Cov(consumption, inflation)	-0	1.4	14.7	-1.1
Cov(Consumption, Long-run risk)	-7.9	-9.0	52.2	-36.36
Cov(Inflation, Long-run risk)	-0.12	15.35	-37.5	-15.1
Total	-7.8	7.81	30.1	-52.28

Note: The table shows the decomposition of the bond risk premia (bond price relative to its steady state) based on the factors contribution. Numbers are in the basis points. Quantitatively precautionary savings are the main driver determining bond prices.

The table 3.3 shows the risk premia decomposition of the pricing equation 3.2.1 quantitatively. The conditional variance terms from the equation 3.28 are presented in the precautionary savings block and the covariance terms in the risk premium block. The numbers are expressed in basis points as the difference from the perfect foresight risk-less bond price. The table 3.3 demonstrates that bonds provide protection to uncertainties coming from wasteful government expenditures. The fact that households build up precautionary savings to uncertainties underlying the future development of purely wasteful government expenditure manifest in bonds by average 63.5 basis points drop in yields. In addition, bonds also provide hedge against wasteful government spending because the current drop/rise in consumption is accompanied by the rise/drop in future consumption and leisure.

In case of productive government spending bonds provide poor saving instruments. Investors (households) require actually risk compensation in terms of higher yield to save. This is because productive government expenditure rise inflation at the same time as long-run consumption drops and thus work as poor consumption smoothing instrument. Substitutable government spending generate sizable positive precautionary saving risk premia driven by inflation deteriorating the value of bonds. Further, bonds also provide hedge against substitutable and non-substitutable government spending because the drop in consumption implied by possible raise in these spending will be compensated by higher real return of bonds implied the drop in inflation.

Table 3.4: Bond premia shock specific decomposition.

Precautionary savings	B	$\phi_y = 0.93$	$\phi_y = 0$	TFP	Mark up	Pref	NTP
Cons risks	- 1.3	-0.9	- 1.3	-1.4	-0.2	-1.2	6
Infl risks	-85.4	-59.7	-78.5	-63.3	-100	-126.4	-10
Long-run risk (C_t, N_t, b_t)	+8.1	5.1	8.8	8.8	0	8.9	0
Δ pref	-2.1	0	0	0	0	-6.4	8
Total	-81	-55.5	-71	56	100	125	4
Risk premiums							
Cov(cons, infl)	0.2	-0.1	0.1	0.1	0.2	-1.43	6
Cov(Cons, Long-run risk)	- 29.4	-41.8	-64.6	-73.18	0	49.6	-9
Cov(Infl, Long-run risk)	43.5	-2.6	34.7	28.2	0.1	87.9	82
Cov(Long run, Δ in pref)	-36.4	0	0	0	0	-120	31
Cov(Cons, Δ in pref)	1.7	0	0	0	0	5.76	-12
Cov(Infl, Δ in pref)	1	0	0	0	0	3.44	-1
Total	-19	-44.5	-29	-44	0	25	96

Note: The table shows the decomposition of the bond risk premia (bond price relative to its steady state) based on the factors contribution and specific shock. We switch of all shocks but the one introducing the table column and calculate the decomposition. Numbers are in % of the overall shock contribution $\frac{f_i}{\sum_i f_i}$.

The table 3.4 decomposes the risk premia similarly as table 3.3 but this time it compares the risk premia imposed by other non-fiscal shocks. The table 3.4 demonstrates that the response of monetary policy to fluctuations in inflation and output is crucial in forming the risk premia in bond price. As the monetary policy affects how the inflation and thus real price of bonds response to the specific shock it determines if the bonds provide hedge or leverage to inflation risks.

3.7 Concluding Remarks

We develop a new method to decompose the pricing kernel into the precautionary savings and risk premia in terms of underlying macro variables. This allows us to provide a detailed economic story of how the risk (insurance) premium and nominal term premium are determined in the canonical macro-finance model. We apply this method to the variant of Rudebusch & Swanson (2012) augmented by the detailed structure of government expenditures. Further, as a sensitivity we redo our analysis in the extended model of **?**. We show that the success of the Rudebusch and Swanson (2012) model and its variants is driven by the ability of the model to generate large inflation risks for the long run consumption and leisure and that the price of inflation risks for the realized consumption is close to zero. We document the importance of the monetary policy fiscal mix for bond prices. Uncertainty about various types of government spending is priced in bonds through their ability to provide insurance against the fluctuations in investors wealth.

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3.A Note on Recursive preferences

The preferences are the crucial element driving large part of the results. Recursive preferences has been utilized increasingly in the asset pricing literature. Nevertheless, in macroeconomic literature Epstein Zin preferences belongs, yet, to group a of so called exotic preferences (see Backus (2014)). We provide detailed solution of the bond pricing equation and its second order approximation. The closed form second order solution to bond prices is useful because it helps us to better develop the intuition about the drivers of dynamics of the term structure of interest rates and relate them to macroeconomic fundamentals.

We lay out the recursive preferences as in Weil (1990). First, we use the utility transformation as in Rudebusch & Swanson (2012) that simplifies the work with utility kernels including labor. Next, we derive and log-linearize the stochastic discount factor (SDF). To substitute out the recursive element and to get SDF just as a function of macroeconomic fundamentals we log-linearize the value function and introduce the surprise operator as in Uhlig (2010). Consequently, using the method developed by Sutherland (2002) we derive the general form of second order approximation to the bond pricing equation. Finally, we merge the results to highlight the drivers of the yield curve dynamics.

3.A.1 Value Function Transformation

In the asset pricing literature, the recursive preferences are usually formulated in the following form (see Weil (1990), Epstein & Zin (1989), Bansal & Yaron (2004), Uhlig (2010), Guvenen (2009)),

$$\tilde{V} = \left\{ u(c_t, N_t)^{1-\gamma} + \beta [E_t \tilde{V}_{t+1}^{1-\psi}]^{\frac{1-\gamma}{1-\psi}} \right\}^{\frac{1}{1-\gamma}} \tag{A.1}$$

where ψ stands for the risk aversion and γ is the inverse of inter-temporal elasticity of substitution. In this paper we follow Rudebusch & Swanson (2012) and use slightly different form of value function

$$\tilde{V} = \left\{ u(C_t, N_t) + \beta [E_t \tilde{V}_{t+1}^{1-\psi}]^{\frac{1-\gamma}{1-\psi}} \right\}^{\frac{1}{1-\gamma}} \tag{A.2}$$

when using the additively separable period utility function it is useful to transform the value function as in Rudebusch & Swanson (2012) . We set $\frac{1-\psi}{1-\gamma} = 1-\alpha$ and define $V_t = \tilde{V}_t^{1-\gamma}$

$$V_t = u(C_t, N_t) + \beta(E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{A.3}$$

when $u(C_t, L_t) > 0$. If $u(C_t, L_t) < 0$, as in our benchmark calibration [33], the recursion takes the form:

$$V_t = u(C_t, L_t) - \beta(E_t[-V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{A.4}$$

To obtain the first order conditions, we solve for the constrain optimization problem.

3.A.2 Solving for the Bond Pricing Equation

There are several ways how to find optimal size of savings (bond purchases).

1. The social planner's problem formulation allows us to find the pricing kernel of the economy $m_{t+1} = \frac{\partial V_t / \partial C_{t+1}}{\partial V_t / \partial C_t}$ (see Caldara *et al.* (2012))
2. use Bellman equation - dynamic programming approach (see for exampleFerman (2011), Andreasen (2008), Tristani & Amisano (2010))
3. formulate the problem as lagrangian (see Rudebusch & Swanson (2012), Andreasen (2012)

3.A.3 Household Problem

$$v_t = u(c_t, N_t) + \beta(E_t[v_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}}$$

$$u(c_t, N_t, G_t^n) = e^{b_t}\left(\frac{(\tilde{c}_t)^{1-\gamma}}{1-\gamma} + \chi Z_t^{1-\gamma}\frac{(1-N_t)^{1+\eta}}{1+\eta} + \Omega(g_t^n)\right)$$

where $c_t$is the non-stationary aggregate consumption index, $Z_t$is the long run growth and $N_t$are hours worked, $G_t^n$are non-substitutable to consumption government spending. Notice that the additively separable utility function is not consistent with balanced growth path, multiplying hours worked by $Z_t^{1-\gamma}$makes both utility arguments grow at the same pace. The consumption index is defined as in case of government spending to be complementary to private consumption,

[33]the first order conditions will be correct however in either way

$$\tilde{c}_t = (c_t)^{\omega} (g_t^s)^{1-\omega}$$

or in case of substitutes

$$\tilde{c}_t = (c_t) + (g_t^s)$$

.

or the case of CES,

$$\tilde{c}_t = \left[\phi_c c_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)(g_t^s)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

$$\tilde{c}_t = Z_t \left[\phi_c Z^{\frac{\omega-1}{\omega}} C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c) Z^{\frac{\omega-1}{\omega}} (G_t^s)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$$

Defining the big cap letters as stationary variables and V_t must be homogenous of degree one,

$$V_t Z_t^{1-\gamma} = e^{b_t}\left(\frac{Z_t^{1-\gamma}(\tilde{C}_t)^{1-\gamma}}{1-\gamma} + \chi Z_t^{1-\gamma}\frac{(1-N_t)^{1+\eta}}{1+\eta} + \frac{Z_t^{1-\gamma}(G_t)^{1-\gamma}}{1-\gamma}\right) + \beta(E_t[(V_{t+1}Z_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}}$$

we can divide both sides of the equation by $Z_t^{1-\gamma}$ and define $\mu_{t+1} = \frac{Z_{t+1}}{Z_t}$.

$$V_t = e^{b_t}\left(\frac{(\tilde{C}_t)^{1-\gamma}}{1-\gamma} + \chi\frac{(1-N_t)^{1-\eta}}{1-\eta} + \frac{(G_t)^{1-\gamma}}{1-\gamma}\right) + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{A.5}$$

Stationarizing budget constraint, here I need to assume that households are payed for efficiency work and bonds are growing?

$$P_t C_t Z_t + E_t Q_{t,t+1} Z_{t+1} B_{t+1} \leq Z_t B_t + W_t N_t Z_t \tag{A.6}$$

Here we use the dynamic programming approach and define the constraint maximization problem of households as:

$$V_t = \max_{B_{t+1}, C_t, N_t}\{\frac{(\tilde{C}_t)^{1-\gamma}}{1-\gamma} + \chi\frac{(1-N_t)^{1-\eta}}{1-\eta} + \frac{(G_t)^{1-\gamma}}{1-\gamma} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} +$$

$$\lambda_t[P_t C_t + E_t Q_{t,t+1}\mu_{t+1} B_{t+1} \leq B_t + W_t N_t + T_t + \Pi_t]\}$$

For the constraint maximization problem it is useful to define value function in period $t+1$ in following way:

$$\begin{aligned} V_{t+1} &= \max_{B_{t+2},C_{t+1},N_{t+1}} \{ \frac{(\tilde{C_{t+1}})^{1-\gamma}}{1-\gamma} + \chi \frac{(1-N_{t+1})^{1-\eta}}{1-\eta} + \Omega(G^n_{t+1}) + \beta(E_{t+1}[(V_{t+2}\mu_{t+2}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} \\ &+ \lambda_{t+1}[P_{t+1}C_{t+1} + E_{t+1}Q_{t+1,t+2}\mu_{t+1,t+2}B_{t+2} - B_{t+1} - P_{t+1}W_{t+1}N_{t+1} - T_{t+1}]\} \end{aligned}$$

Households choose how much to safe by choosing the amount of bonds, $\frac{\partial V_t}{\partial B_{t+1}}$:

$$\lambda_t \mu_{t+1} Q_{t,t+1} = \beta \frac{1}{1-\alpha}[E_t(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}]^{\frac{\alpha}{1-\alpha}}(V_{t+1}\mu_{t+1}^{1-\gamma})^{-\alpha}(1-\alpha)\lambda_{t+1}\mu_{t+1}^{1-\gamma}$$

Agents decide how much to safe which delivers stochastic discount factor,

$$Q_{t,t+1} = \beta\zeta_t \frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}^{-\gamma}\left(\frac{(V_{t+1}\mu_{t+1}^{1-\gamma})}{R_t}\right)^{-\alpha} \tag{A.7}$$

where $\zeta_t = e^{b_{t+1}-b_t}$ is the preferences shock, $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation between period t and $t+1$, and certainty equivalent value of future consumption and leisure R_t is given by:

$$R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}} \tag{A.8}$$

Because of term $\left[\frac{V_{t+1}}{R_t}\right]^{-\alpha}$ [34], news at $t+1$ about consumption in $c_{t+2}, c_{t+3}\dots$ and leisure in $n_{t+2}, n_{t+3}\dots$ affects marginal utility of c_{t+1} and n_{t+1} relative to marginal utility of c_t and n_t. Good news at $t+1$ about future consumption and leisure is a positive shock to $R_{t+1}(V_{t+2})$, and therefore to $V_{t+1} = F(c_{t+1}, n_{t+1}; R_{t+1}(V_{t+2}))$. The more concave is the utility function and the more uncertain V_{t+1} is, the lower is the certainty equivalent R_t. Note that $R_t = V_{t+1}$ if there is no uncertainty on V_{t+1}.

There are two advantages to SDF of time-separable expected utility. First, it separates EIS from coefficient of relative risk aversion. Second, it is another source of risk premium, not just covariance with contemporaneous consumption growth, but also covariance with return to total wealth matters.

[34] next period's value relative to its certainty equivalent

By chaining the stochastic discount factor we can price bond of any maturity:

$$Q_{t,t+n} = \beta^n \left(\frac{C_{t+n}}{C_t} \right)^{-\gamma} \prod_{j=0}^{n} \frac{\zeta_{t+j}}{\pi_{t+j+1}} \left[\frac{R_{t+j}}{V_{t+j+1}} \right]^{\alpha} \tag{A.9}$$

3.A.4 Utility Kernel

Coub-Douglas Aggregator

The optimal level of consumption using $\tilde{c}_t = (c_t)^{\omega} (g_t^s)^{1-\omega}$,

$$\tilde{C_t}^{-\gamma} (c_t)^{\omega-1} (g_t^s)^{1-\omega} \omega = \lambda_t P_t$$

$$\frac{\tilde{C_t}^{-\gamma} \tilde{C_t}}{C_t} \omega = \lambda_t P_t$$

$$\frac{\tilde{C_t}^{1-\gamma} \omega}{C_t} = \lambda_t P_t$$

CES Aggregator

Shadow price for the CES aggregator, $\tilde{C}_t = \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c) (G_t^s)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}$

$$\tilde{C_t}^{-\gamma} \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c) (G_t^s)^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1}} \phi_c C_t^{\frac{\omega-1}{\omega}-1} = \lambda_t P_t$$

$$\tilde{C_t}^{-\gamma} \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c) (G_t^s)^{\frac{\omega-1}{\omega}} \right]^{\frac{1}{\omega-1} \frac{\omega}{\omega-1} \frac{\omega-1}{\omega}} \phi_c C_t^{\frac{\omega-1}{\omega}-1} = \lambda_t P_t$$

$$\tilde{C_t}^{-\gamma} \left[\tilde{C_t} \right]^{\frac{1}{\omega}} \phi_c C_t^{\frac{-1}{\omega}} = \lambda_t P_t$$

$$\tilde{C_t}^{-\gamma + \frac{1}{\omega}} \phi_c C_t^{\frac{-1}{\omega}} = \lambda_t P_t$$

Substituting into the equation A.7,
and for the CES ,

$$Q_{t,t+1} = \beta \left(\frac{\tilde{C_{t+1}}}{\tilde{C_t}} \right)^{-\gamma + \frac{1}{\omega}} \left(\frac{C_{t+1}}{C_t} \right)^{\frac{-1}{\omega}} \pi_t^{-1} \mu_{t+1}^{-\gamma} \left(\frac{(V_{t+1} \mu_{t+1}^{1-\gamma})}{[E_t (V_{t+1} \mu_{t+1}^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$

and for the substitutable,

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1} + G^s_{t+1}}{C_t + G^s_t} \right)^{-\gamma} \pi_t^{-1} \mu_{t+1}^{-\gamma} \left(\frac{(V_{t+1}\mu_{t+1}^{1-\gamma})}{[E_t(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$

Habits

External Habits

$$V_t = \max_{B_{t+1},C_t,N_t} \{ \frac{(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{1-\gamma}}{1-\gamma} + \chi \frac{(1-N_t)^{1-\eta}}{1-\eta} + \frac{(G_t)^{1-\gamma}}{1-\gamma} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} +$$

$$\lambda_t[P_tC_t + E_tQ_{t,t+1}\mu_{t+1}B_{t+1} \leq B_t + W_tN_t + T_t + \Pi_t]\}$$

Shadow price for the CES aggregator, $\tilde{C}_t = \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G^s_t\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$

$$(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G^s_t\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}} \phi_c C_t^{\frac{\omega-1}{\omega}-1} = \lambda_t P_t$$

$$(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G^s_t\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}\frac{\omega}{\omega-1}\frac{\omega-1}{\omega}} \phi_c C_t^{\frac{\omega-1}{\omega}-1} = \lambda_t P_t$$

$$(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\tilde{C}_t\right]^{\frac{1}{\omega}} \phi_c C_t^{\frac{-1}{\omega}} = \lambda_t P_t$$

$$Q_{t,t+1} = \beta \left(\frac{(\tilde{C_{t+1}} - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_t)^{-\gamma}\left[\tilde{C_{t+1}}\right]^{\frac{1}{\omega}}}{(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\tilde{C}_t\right]^{\frac{1}{\omega}}} \right) \left(\frac{C_{t+1}}{C_t}\right)^{\frac{-1}{\omega}} \pi_t^{-1}\mu_{t+1}^{-\gamma} \left(\frac{(V_{t+1}\mu_{t+1}^{1-\gamma})}{[E_t(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$

Internal Habits

$$\begin{aligned} V_t &= \max_{B_{t+1},C_t,N_t} \{ \frac{(\tilde{C}_t - b\left(\mu^*_{z,t}\right)^{-1}\tilde{C}_{t-1})^{1-\gamma}}{1-\gamma} + \chi \frac{(1-N_t)^{1-\eta}}{1-\eta} + \frac{(G_t)^{1-\gamma}}{1-\gamma} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} \\ &+ \lambda_t[P_tC_t + E_tQ_{t,t+1}\mu_{t+1}B_{t+1} \leq B_t + W_tN_t + T_t + \Pi_t]\} \end{aligned}$$

Shadow price for the CES aggregator, $\tilde{C}_t = \left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_t^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}}$

$$(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_t^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}}\phi_c C_t^{\frac{\omega-1}{\omega}-1} -$$

$$\beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\left(C_{t+1}^{\sim} - b\left(\mu_{z,t+1}^*\right)^{-1}\tilde{C}_t\right)^{-\gamma} b\left(\mu_{z,t+1}^*\right)^{-1}\left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_t^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{\omega-1}-}$$

$$(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\left[\phi_c C_t^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_t^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{1}{\omega-1}}\phi_c C_t^{\frac{-1}{\omega}} -$$

$$\beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\left(C_{t+1}^{\sim} - b\left(\mu_{z,t+1}^*\right)^{-1}\tilde{C}_t\right)^{-\gamma} b\left(\mu_{z,t+1}^*\right)^{-1}\left[\phi_c C_{t+1}^{\frac{\omega-1}{\omega}} + (1-\phi_c)\left(G_{t+1}^s\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega}{}}$$

$$(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\tilde{C}_t^{\frac{1}{\omega}}\phi_c C_t^{\frac{-1}{\omega}} -$$

$$\beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\left(C_{t+1}^{\sim} - b\left(\mu_{z,t+1}^*\right)^{-1}\tilde{C}_t\right)^{-\gamma} b\left(\mu_{z,t+1}^*\right)^{-\gamma}\tilde{C}_{t+1}^{\frac{1}{\omega}}\phi_c C_{t+1}^{\frac{-1}{\omega}} = \lambda_t P_t$$

Cob-Douglas Aggregator External Habits

The optimal level of consumption using $\tilde{c}_t = (c_t)^{\omega}\left(g_t^s\right)^{1-\omega}$,

$$V_t = \max_{B_{t+1},C_t,N_t}\left\{\frac{(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{1-\gamma}}{1-\gamma} + \chi\frac{(1-N_t)^{1-\eta}}{1-\eta} + \frac{(G_t)^{1-\gamma}}{1-\gamma} + \beta(E_t[(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}])^{\frac{1}{1-\alpha}} + \right.$$

$$\left.\lambda_t[P_tC_t + E_tQ_{t,t+1}\mu_{t+1}B_{t+1} \leq B_t + W_tN_t + T_t + \Pi_t]\right\}$$

$$(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}(C_t)^{\omega-1}\omega\left(G_t^s\right)^{1-\omega} = \lambda_t P_t$$

$$(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}\omega\frac{\tilde{C}_t}{C_t} = \lambda_t P_t$$

$$Q_{t,t+1} = \beta\left(\frac{(C_{t+1}^{\sim} - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_t)^{-\gamma}}{(\tilde{C}_t - b\left(\mu_{z,t}^*\right)^{-1}\tilde{C}_{t-1})^{-\gamma}}\right)\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right)\left(\frac{C_t}{C_{t+1}}\right)\pi_t^{-1}\mu_{t+1}^{-\gamma}\left(\frac{(V_{t+1}\mu_{t+1}^{1-\gamma})}{[E_t(V_{t+1}\mu_{t+1}^{1-\gamma})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}$$

3.A.5 Firm Problem

The production function of firm j is given by,

$$Y_t(j) = A_t K_t(j)^{\theta} \left(Z_t N_t(j)\right)^{1-\theta} \tag{A.10}$$

Further, we need to assume that the average (aggregate) output is given by, $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(j)dj\right]^{\frac{\epsilon}{\epsilon-1}}$, then the final good firms by solving the expenditure minimization problem derives the its demand for j-good,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

3.A.6 Log-linearizing SDF

LHS:

$$\bar{R}^{1-\alpha} e^{(1-\alpha)\hat{r}_t} \approx \bar{R}^{1-\alpha}(1 + (1-\alpha)\hat{r}_t) \tag{A.11}$$

RHS:

$$\bar{V}^{1-\alpha} E_t e^{(1-\alpha)\hat{v}_{t+1}} \approx \bar{V}^{1-\alpha}(1 + (1-\alpha)E_t\hat{v}_{t+1}) \tag{A.12}$$

Canceling out steady state delivers:

$$\hat{r}_t = E_t \hat{v}_{t+1} \tag{A.13}$$

Next, we log-linearize equation (A.9). RHS after taking Taylor expansion

$$st.st. + st.st E_t[\zeta_t - \gamma\Delta\hat{c}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{v}_{t+1} - \hat{r}_t) \tag{A.14}$$

Canceling out steady state and joining LHS with RHS we get log linearized price of one period bond:

$$q_{t,1} = \zeta_t - \gamma\Delta\hat{c}_{t+1} - \hat{\pi}_{t+1} - \alpha(\hat{v}_{t+1} - \hat{r}_t) \tag{A.15}$$

Next, we substitute equation A.13 into equation (A.9) to highlight that $\hat{v}_{t+1} - \hat{r}_t$ is the next periods value relative to its certainty equivalent [35]

$$q_{t,1} = E_t\left\{\zeta_t - \gamma\Delta\hat{c}_{t+1} - \hat{\pi}_{t+1}\right\} - \alpha(E_{t+1}\hat{v}_{t+1} - E_t v_{t+1}) \tag{A.16}$$

[35] note that the term $\hat{v}_{t+1} - \hat{r}_t$ in time t expectations equals to zero. This is given by the fact that the first order approximation eliminates uncertainty from the model and thus the $E_t v_{t+1} = R_t$. Agents expectations are up to the first order identical to certainty equivalent

By chaining the stochastic discount factor we derive the price of bond with any maturity n:

$$q_{t+n} = \sum_{j=1}^{n} E_t\zeta_{t+j} - \gamma\Delta^n E_t\hat{c}_{t+n} - E_t\sum_{j=1}^{n}\hat{\pi}_{t+j} - \alpha\left[\sum_{j=1}^{n}(\hat{v}_{t+j+1} - \hat{r}_{t+j})\right] \quad \text{(A.17)}$$

Note that risk aversion is denoted ψ and α is then:

$$\alpha = 1 - \frac{1-\psi}{1-\gamma} \quad \text{(A.18)}$$

so for the news to enter stochastic discount factor the risk aversion must be different of the inverse of intertemporal elasticity of substitution. γ and ψ are so called demands for smoothing parameters, intratemporally and intertemporally. Consider following hypothetical processes for consumption and leisure

1. coin flipped at $t = 0$ determines high or low consumption and leisure at all dates 1,2,3 ... T
2. T coins flipped at $t = 0$ determine high or low consumption and leisure at all dates 1,2,3 ... T
3. T coins flipped before each period to determine consumption and leisure that period

The first process implies intertemporally smooth path of consumption and leisure but is characterized by big time-zero volatility in V_t and thus dislike by risk averse agents. It will be preferred only if $1/\gamma$ is very small. In the second process all information is revealed at time $t = 0$ thus $E_t(V_{t+1})$ varies over time non-stochastically but the process features higher variation across time than the first one. The third process shares with the second one the volatility across time but differs in the timing of uncertainty resolution. When $\gamma < \psi$ agents prefer early resolution of uncertainty.

3.A.7 Log-linearizing the Value Function

The goal is to express bond prices as a function of macroeconomic fundamentals. Therefore, we need to eliminate the recursion. Lets assume that the period utility is additively separable CRRA.

$$V_t = \left[\frac{C^{1-\gamma}}{1-\gamma} - \chi\frac{N^{1+\eta}}{1+\eta}\right]\zeta_t + \beta(E_t[V_{t+1}^{1-\alpha}])^{\frac{1}{1-\alpha}} \tag{A.19}$$

Remember that $R_t = [E_t V_{t+1}^{1-\alpha}]^{\frac{1}{1-\alpha}}$ is the certainty equivalent of next period's utility. The log-linearized equation (3.A.7) around zero steady state is

$$\hat{v}_t = \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}\hat{\zeta}_t + \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}\hat{c}_t - \frac{\bar{\zeta}\bar{N}^{1-\eta}}{\bar{V}}\hat{n}_t + \beta\hat{r}_t \tag{A.20}$$

simplifying the notation

$$\hat{v}_t = a(\hat{\zeta}_t + \hat{c}_t) - b\hat{n}_t + \beta\hat{r}_t \tag{A.21}$$

where $a = \frac{\bar{\zeta}\bar{C}^{1-\gamma}}{\bar{V}}$ and $b = \frac{\bar{\zeta}\bar{N}^{1-\eta}\chi}{\bar{V}}$.

Solving the equation (A.20) forward we get:

$$\hat{v}_t = \sum_{j=0}^{\infty}\beta^j\left(a\hat{\zeta}_{t+j} + a\hat{c}_{t+j}\right) - \sum_{j=0}^{\infty}\beta^j b\hat{n}_{t+j} \tag{A.22}$$

Next, it is convenient to follow Uhlig (2010) and introduce the "surprise" operator $S_{t+k|t}$ for any random variable x, given by

$$S_{t+n|t} = E_{t+k}(x) - E_t(x) \tag{A.23}$$

thus for the period $t+1$, S_{t+1} is filtering out the surprise in conditional expectations and is defined

$$S_{t+1} = x_{t+1} - E_t[x_{t+1}] \tag{A.24}$$

Note that the surprise over n periods is simply

$$S_{t+n} = S_{t+n} + S_{t+n-1} + \ldots + S_{t+1} \tag{A.25}$$

Applying the filtering, using the equation (A.22) in the SDF, equation (A.15), we can show that the bond pricing equation is determined by the period consumption growth, inflation, exogenous preference shock and the surprise or news about the future consumption and labor.

$$q_{t,1} = \zeta_{t+1} - \gamma\Delta\hat{c}_{t+1} - \hat{\pi}_{t+1} - \alpha S_{t+1}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{c}_{t+j} - b\hat{n}_{t+j}\right]\right) \quad (A.26)$$

Notice that the labor enters the bond pricing equation which is usually the case only with non-separable preferences. Nevertheless, labor affect only higher order terms. Price of bond with maturity n is given by

$$q_{t+n} = \sum_{j=1}^{n}\zeta_{t+j} - \gamma\Delta^n\hat{c}_{t+n} - \sum_{j=1}^{n}\hat{\pi}_{t+j} - \alpha S_{t+n}\left(\sum_{j=0}^{\infty}\beta^j\left[a\hat{\zeta}_{t+j} + a\hat{c}_{t+j} - b\hat{n}_{t+j}\right]\right) \quad (A.27)$$

The revaluation in the expectations can be understood as well as the news or surprise. Investors require compensation for the uncertainty underlying the surprise component. Net effect of good news about $c_{t+2}, c_{t+3}\ldots$ and $n_{t+2}, n_{t+3},\ldots$ on marginal utility of c_{t+1} and n_{t+1} depends on α. If α is positive, news is a positive shock to SDF. Note also that news about c_{t+1} directly affect the consumption growth part of SDF but it also shows up in the second part of equation (A.27). If there is no news about c_{t+2}, c_{t+3} and n_{t+2}, n_{t+3} SDF reduces to $\beta\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\pi^{-1}$. Each period agents make expectations about future consumption and hours worked for the remaining life of the bond and compare it with the previous period execrations. The difference between this two is the update in expectations reflected in price of bonds. The update of expectations is sum of all news (surprises) over the remaining maturity of the bond about the life time stream of consumption and leisure.

3.A.8 Second Order Approximation to the Term Structure

The derivations rely on Sutherland (2002) who argues that first order approximate solutions are sufficient to derive second order approximate solutions to second moments. Second order accurate solutions for second moments can be obtained by considering first-order accurate solutions to realized values because terms of order two and above in the behaviour of realized values become terms of order three and above in the squares and cross products of realized values. The first part of the derivations, which is not explicitly working with EZ preferences, is in line with Hordahl *et al.* (2008). The price of bond with maturity

n is defined $P_t^{(n)} = E_t[Q_{t,t+n}]$; in the non-stochastic steady state $\bar{P} = \bar{Q}$. Lower case letters define logarithm of their upper case counterparts.

$$\begin{aligned}\bar{p}(1 + \hat{p}_{t,n} + \frac{1}{2}\hat{p}_{t,i}^2) &= E_t\left[\bar{q}(1 + \hat{q}_{t+n} + \frac{1}{2}\hat{q}_{t+n}^2)\right] \\ &= \bar{q}_t E_t\left[1 + \hat{q}_{t+n} + \frac{1}{2}\hat{q}_{t+n}^2\right]\end{aligned}$$

After canceling out steady state, we get:

$$\hat{p}_{t,n} = E_t[\hat{q}_{t,t+n} + \frac{1}{2}\hat{q}_{t,t+n}^2] - \frac{1}{2}\hat{p}_{t,n}^2$$

Up to the first order $\hat{p}_{t,n} = E_t\{\hat{q}_{t,t+n}\}$, thus we can substitute for the quadratic term $\hat{p}_{t,n}^2 = (E_t\{\hat{q}_{t,t+n}\})^2$. It follows that:

$$\hat{p}_{t,n} = \mathrm{E}_t\left[\hat{q}_{t,t+n} + \frac{1}{2}\hat{q}_{t,t+n}^2\right] - \frac{1}{2}(\mathrm{E}_t\hat{q}_{t,t+n})^2$$

From the last equation using the definition of variance [36] we can define price of one period bond.

$$\hat{p}_{t,n} = E_t[\hat{q}_{t,t+n}] + \frac{1}{2}\mathrm{Var}_t[\hat{q}_{t,t+n}] \tag{A.28}$$

using the definition of yield to maturity, $\widehat{ytm_t} = -(1/n)\hat{q}_{t,n}$ we can write equation (A.28)

$$\widehat{ytm}_t^n = -\frac{1}{n}E_t q_{t,t+n} - \frac{1}{2n}\mathrm{Var}_t(\hat{q}_{t,t+n}) \tag{A.29}$$

and use equation (A.27) and plug it into A.29 to get

$$\widehat{ytm}_t^n = -\frac{1}{n}\mathrm{E}_t\left\{\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right] - \gamma\Delta^n\hat{c}_{t+n} - \sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right] - \alpha S_{t+n}(\cdot)\right\} \tag{A.30}$$

$$-\frac{1}{2n}\mathrm{Var}_t\left(\sum_{j=1}^{n}\left[\hat{\zeta}_{t+n}\right] - \gamma\Delta^n\hat{c}_{t+n} - \sum_{j=1}^{n}\left[\hat{\pi}_{t,t+n}\right] - \alpha S_{t+n}(\cdot)\right) \tag{A.31}$$

Unconditional mean of the term structure is then

[36] $Var(x) = E[x^2] - (E[x])^2$

$$\begin{aligned}
E[\widehat{ytm}^n] &= -\frac{1}{2n}\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\zeta}_{t+j}\right)\right]-\frac{\gamma^2}{2n}\left[\mathrm{Var}_t\left(\Delta^n\hat{c}_{t+n}\right)\right]-\frac{1}{2n}\left[\mathrm{Var}_t\sum_{j=1}^{n}\left(\hat{\pi}_{t,t+j}\right)\right]\\
&+ -\frac{\alpha^2}{2n}E\left[\mathrm{Var}_tS_{t+n}\left(\cdot\right)\right]+\frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\Delta^n\hat{c}_{t+n}\right)\right]\\
&+ \frac{1}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right]-\frac{\gamma}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{c}_{t+n},\sum_{j=1}^{n}\hat{\pi}_{t+j}\right)\right]\\
&+ \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\zeta}_{t+j},S_{t+n}\left(\cdot\right)\right)\right]-\frac{\gamma\alpha}{n}E\left[\mathrm{Cov}_t\left(\Delta^n\hat{c}_{t+n},S_{t+n}\right)\right]\\
&- \frac{\alpha}{n}E\left[\mathrm{Cov}_t\left(\sum_{j=1}^{n}\hat{\pi}_{t+n},S_{t+n}\right)\right] \qquad \text{(A.32)}
\end{aligned}$$

S_{t+n} embodies the intensity of surprises from consumption, leisure and preference shocks over the maturity horizon. For a random variable ytm_t, the unconditional mean is simply the average of the realized yields. In contrast, the conditional mean of ytm_t is the expected value of ytm_t given a conditioning set of variables, Ω_t (shock realization). The term under the expectations in the equation (A.30) is on average zero. The term thus corresponds to the determinist steady state. The variance components represent the Jensen's inequality term and arise from the relative convexity of nominal bonds. Note also that even if we calculate mean of the stochastic steady state of the yield curve the variance and covariance is still conditional on the information in time t

Next, we rewrite the covariance terms using the definition for correlation. To make the equation more compact we rewrite the equation (A.32) using different notation. Note that $\sigma_{\Delta\hat{c}} = [Var_t(\Delta^n\hat{c}_{t+n})]^{\frac{1}{2}}$, $\rho_{\Delta\hat{c},S} = \mathrm{Corr}_t\left(\Delta^n\hat{c}_{t+n}, S_{t+n}\right)$ and other variables in equation (A.32) are rewritten analogically.

$$\begin{aligned}
E\widehat{ytm}_t^n &= -\frac{1}{2n}\left\{\sigma_{\hat{\zeta}}^2+\gamma^2\sigma_{\Delta\hat{c}}^2+\sigma_{\hat{\pi}}^2+\alpha^2\sigma_S^2\right\}+\frac{\gamma}{n}\sigma_{\hat{\zeta}}\sigma_{\Delta\hat{c}}\rho_{\hat{\zeta},\Delta\hat{c}}+\frac{1}{n}\sigma_{\hat{\zeta}}\sigma_{\hat{\pi}}\rho_{\hat{\zeta},\hat{\pi}}\\
&- \frac{\gamma}{n}\sigma_{\Delta\hat{c}}\sigma_{\hat{\pi}}\rho_{\Delta\hat{c},\hat{\pi}}+\frac{\alpha}{n}\sigma_{\hat{\zeta}}\sigma_S\rho_{\hat{\zeta},S}-\frac{\gamma\alpha}{n}\sigma_{\Delta\hat{c}}\sigma_S\rho_{\Delta\hat{c},S}-\frac{\alpha}{n}\sigma_{\hat{\pi}}\sigma_S\rho_{\hat{\pi},S} \qquad \text{(A.33)}
\end{aligned}$$

3.B Mid-Scale Model Based on Andreasen (2018)

3.B.1 Summary of the Andreasen Model Equilibrium

The Households

1. Value function

$$V_t = e^{b_t}\left(\frac{(C_t - b\mu_{z,t}^{-1}C_{t-1})^{1-\gamma}}{1-\gamma} + \chi\frac{(1-N_t)^{1-\eta}}{1-\eta}\right) + \beta(E_t[(V_{t+1})^{1-\alpha}])^{\frac{1}{1-\alpha}}$$

2. Marginal Utility of Consumption

$$(C_t - b\mu_{z,t}^{-1}C_{t-1})^{-\gamma} - \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} b\mu_{z,t+1}^{-1}\left(C_{t+1} - b\mu_{z,t+1}^{-1}C_t\right)^{-\gamma} = \lambda_t$$

3. Stochastic discount factor

$$Q_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\mu_{z,t+1}^{-1}\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}$$

4. Deposit rate

$$r_t^b = \frac{1}{Q_{t,t+1}}$$

5. Labor supply

$$\chi(1-N_t)^{-\eta} = W_t\lambda_t$$

6. Capital supply

$$0 = -q_t\mu_{z,t+1} + \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\frac{\lambda_{t+1}}{\lambda_t}$$
$$\left(r_{t+1}^k + q_{t+1}\left[(1-\delta) + \kappa_2\left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}}\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\kappa_2}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}}\right)^2\right]\right)$$

6b) Capital supply with end of the day stationary capital

$$0 = -q_t + \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\frac{\lambda_{t+1}}{\lambda_t}\mu_{z,t+1}^{-1}$$
$$\left(r_{t+1}^k + q_{t+1}\left[(1-\delta) + \mu_{z,t+1}^2\kappa_2\left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}}\mu_{z,t+1}^{-1}\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\kappa_2}{2}\left(\frac{I_{t+1}}{K_{t+1}}\mu_{z,t+1} - \frac{\bar{I}}{\bar{K}}\right)^2\right]\right)$$

7. Tobin q

$$q_t\left(1-\frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2-\kappa_1\left(\frac{I_t-\bar{I}}{\bar{I}}\right)\frac{I_t}{\bar{I}}-\kappa_2\left(\frac{I_t}{K_t}-\frac{\bar{I}}{\bar{K}}\right)\right)=1$$

8. Law of motion of capital

$$\mu_{z,t+1}K_{t+1}=(1-\delta)K_t+I_t-I_t\frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2-K_t\frac{\kappa_2}{2}\left(\frac{I_t}{K_t}-\frac{\bar{I}}{\bar{K}}\right)^2$$

8b) End of the day stationary

$$K_{t+1}=(1-\delta)K_t\mu_{z,t}^{-1}+I_t-I_t\frac{\kappa_1}{2}\left(\frac{I_t}{\bar{I}}-1\right)^2-K_t\frac{\kappa_2}{2}\left(\frac{I_t}{K_t}-\frac{\bar{I}}{\bar{K}}\mu_{z,t}^{-1}\right)^2$$

The Firms

9. Factor markets equilibrium

$$\frac{1-\theta}{\theta}=\frac{W_tN_t}{R_t^kK_t}$$

10. Optimal price

$$p_t^{opt}=(1+\lambda)\,\frac{K_t}{F_t}$$

11. Nominator

$$K_t=E_tY_tMC_t^r+\zeta\beta\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}^{-1}\left(\frac{(V_{t+1})}{\left[E_t(V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}e^{b_{t+1}-b_t}\left(\frac{\pi_{t+1}}{(\pi_{t-1}^{\iota}\pi^{1-\iota})}\right)^{\frac{1+\lambda}{\lambda}}K_{t+1}$$

12. Denominator

$$F_t=Y_t+\zeta\beta\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}^{-1}\left(\frac{(V_{t+1})}{\left[E_t(V_{t+1})^{1-\alpha}\right]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}e^{b_{t+1}-b_t}\left(\frac{\pi_{t+1}}{(\pi_{t-1}^{\iota}\pi^{1-\iota})}\right)^{\frac{1}{\lambda}}F_{t+1}$$

13. Price dispersion

$$S_t=(1-\zeta)\left(p_t^{opt}\right)^{-\frac{(1+\lambda)}{\lambda}}+\zeta\left(\frac{\pi_t}{\pi_{t-1}^{\iota_p}\pi^{1-\iota_p}}\right)^{\frac{(1+\lambda)}{\lambda}}S_{t-1}$$

14. Aggregate production function

$$S_t Y_t = A_t {K_t}^{\theta} N_t^{1-\theta}$$

14b) End of period capital stationary

$$S_t Y_t = \mu_t^{-\theta} A_t {K_t}^{\theta} N_t^{1-\theta}$$

15. Real Marginal Costs

$$MC_t^r = \frac{\left(R_t^k\right)^{\theta} W_t^{1-\theta}}{A_t \theta^{\theta} (1-\theta)^{1-\theta}}$$

Market clearing identities

16. Goods Market Clearing condition

$$Y_t = C_t + I_t + G_t$$

17. Monetary policy rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\,\bar{i} + (1-\rho_i)\left[\phi_\pi\left(\pi_t - \bar{\pi}\right) + \phi_y\left(\frac{y_t \mu_{zt}}{\bar{Y}\bar{\mu_z}} - 1\right) + \phi_{xhr}\left(eehpr_t^{40} - x_{t,40}\right)\right]$$

18. Recursive expression for unconditional expectations

$$x_{t,40} = (1-\gamma) eehpr_t^{40} + \gamma E_t[x_{t+1,40}]$$

19. Expected excess holding period return

$$eehpr_{t,40} = E_t \log\left(\frac{P_{t+1}^{39}}{P_t^{40}}\right) - i_t$$

20. Deposit rate paid by intermediary

$$r_t^b = i_t + \omega \times (eehpr_{t,40})$$

Exogenous processes

21. TFP
$$a_t = \rho_a a_{t-1} + \sigma_a e_t^a$$

22. Preferences Shock
$$b_t = \rho_b b_{t-1} + \sigma_b e_t^b$$

23. Trend growth
$$\mu_{z,t} = (1 - \rho_z)\bar{\mu} + \rho_z \mu_{z,t-1} + \sigma_\mu e_t^z$$

24. Government expenditure shock
$$g = \rho_g \bar{G} + \rho_g g_{t-1} + \sigma_g e_t^g$$

3.B.2 Trends

The economy has two sources of growth. Alongside the stochastic trend in productivity Z_t, which is assumed to be deterministic, $log\frac{Z_{t+1}}{Z_t} = \mu_{z,t}$, the economy also faces a deterministic trend in the relative price of investment Υ_t with $\mu_{t,\Upsilon} = (\frac{\Upsilon_t}{\Upsilon_{t-1}})$. As in Altig et. al. (2011) we define the overall measure of the productivity in the economy as,

$$Z_t^* = \Upsilon_t^{\frac{\alpha}{1-\alpha}} Z_t$$

which is in growth rates,

$$\frac{Z_t^*}{Z_{t-1}^*} = \frac{\Upsilon_t^{\frac{\alpha}{1-\alpha}} Z_t}{\Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}} Z_{t-1}}$$

from which follows μ
so the overall trend in the economy is characterized by

$$\mu_{t,z^+} = \frac{\alpha}{1-\alpha}\mu_{t,\Upsilon} + \mu_{t,z}$$

Labor augmenting growth comes from production function, $Y_t = A_t K_t^\theta (Z_t N_t)^{1-\theta}$. In addition, one unit of investment is transformed into Υ units of capital, where Υ is a stochastic process representing random and permanent investment-specific technological progress (index enhancing the quality of newly produced capital goods). Thus, $i_t \Upsilon_t$ is investment in efficiency units.

$$k_{t+1} = (1 - \delta)k_t + i_t \Upsilon_t$$

whereΥ is assumed to follow long run growth.

3.B.3 Balanced Growth Path

Production function with labor augmenting growth , $Y_t = A_t K_t^{\theta} \left(Z_t N_t\right)^{1-\theta}$ can be writen per unit of labor, $y_t = Y_t/N_t$and $k_t = K_t/N_t$,

$$y_t = k_t^{\theta} Z_t^{1-\theta}$$

writing it in the form of log differences (A_t is defined as stationary so it does not grow),

$$g_y = \theta g_k + (1-\theta) g_z$$

King, Plosser and Rebelo shows that without investment growth,

$$g_y = g_i = g_c$$

So if, Υ is constant, than, from $k_{t+1} = (1-\delta)k_t + i_t \Upsilon_t$

$$g_k = g_i$$

In case of investment specific technological change which operates through the capital deepening component of growth accounting equation. From the capital accumulation equation,

$$g_k = g_i + g_{\Upsilon}$$

Thus, Υ increases the effective capital stock. Since we know that $g_y = \theta g_k + (1-\theta) g_z$, then plugging in for g_k

$$g_y = \theta \left(g_i + g_{\Upsilon}\right) + (1-\theta) g_z$$

next I can use that $g_y = g_i$,

$$g_y = \theta \left(g_y + g_{\Upsilon}\right) + (1-\theta) g_z$$

simplifying,

$$g_y \left(1-\theta\right) = \theta g_{\Upsilon} + (1-\theta) g_z$$

$$g_y = g_z + \frac{\theta}{1-\theta} g_\Upsilon$$

HHs choose how much to save through deposits, $\frac{\partial V_t}{\partial B_{t+1}} = 0$,

$$\lambda_t E_t Q_{t,t+1} \frac{P_{t+1}}{P_t} \mu_{z,t+1} = \lambda_{t+1} \beta \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$

$$Q_{t,t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \mu_{z,t+1}^{-1} \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \tag{A.34}$$

HHs choose how much new capital to instal, $\frac{\partial V_t}{\partial K_{t+1}} = 0$,

$$0 = -q_t \lambda_t \mu_{z,t+1} + \beta \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$
$$\left(\lambda_{t+1} r_{t+1}^k + q_{t+1} \lambda_{t+1} \left[(1-\delta) - \left(K_{t+1} \kappa_2 \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right) (-1) K_{t+1}^{-2} I_{t+1} + \frac{\kappa_2}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right)^2 \right) \right] \right)$$

$$0 = -q_t \lambda_t \mu_{z,t+1} + \beta \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$
$$\left(\lambda_{t+1} r_{t+1}^k + q_{t+1} \lambda_{t+1} \left[(1-\delta) - \left(\kappa_2 \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right) (-1) \frac{I_{t+1}}{K_{t+1}} + \frac{\kappa_2}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right)^2 \right) \right] \right)$$

$$0 = -q_t \lambda_t \mu_{z,t+1} + \beta \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha}$$
$$\left(\lambda_{t+1} r_{t+1}^k + q_{t+1} \lambda_{t+1} \left[(1-\delta) + \kappa_2 \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\kappa_2}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right)^2 \right] \right)$$

$$-q_t \mu_{z,t+1} + \beta \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} \frac{\lambda_{t+1}}{\lambda_t}$$
$$\left(r_{t+1}^k + q_{t+1} \left[(1-\delta) + \kappa_2 \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right) \frac{I_{t+1}}{K_{t+1}} - \frac{\kappa_2}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}} \right)^2 \right] \right) = 0$$

HHs choose how much to invest, $\frac{\partial V_t}{\partial I_t} = 0$,

$$-\lambda_t + q_t\lambda_t \left(1 - \frac{\kappa_1}{2}\left(\frac{I_t - \bar{I}}{\bar{I}}\right)^2 - I_t\kappa_1 \left(\frac{I_t - \bar{I}}{\bar{I}}\right)\frac{1}{\bar{I}} - K_t\kappa_2\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)\frac{1}{K_t}\right) = 0$$

simplify,

$$-\lambda_t + q_t\lambda_t \left(1 - \frac{\kappa_1}{2}\left(\frac{I_t - \bar{I}}{\bar{I}}\right)^2 - \kappa_1 \left(\frac{I_t - \bar{I}}{\bar{I}}\right)\frac{I_t}{\bar{I}} - \kappa_2\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)\right) = 0$$

$$q_t \left(1 - \frac{\kappa_1}{2}\left(\frac{I_t - \bar{I}}{\bar{I}}\right)^2 - \kappa_1 \left(\frac{I_t - \bar{I}}{\bar{I}}\right)\frac{I_t}{\bar{I}} - \kappa_2\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)\right) = 1 \qquad (A.35)$$

3.B.4 Firm Problem

Lets assume model without capital for now, the production function is thus given by,

$$Y_t(j) = A_t K_t(j)^{\theta} \left(Z_t N_t(j)\right)^{1-\theta} \qquad (A.36)$$

Further, we need to assume that the average (aggregate) output is given by, $Y_t = \left[\int_0^1 Y_t^{\frac{\epsilon-1}{\epsilon}}(j)dj\right]^{\frac{\epsilon}{\epsilon-1}}$, then the final good firms by solving the expenditure minimization problem derives the its demand for j-good,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

Intermediate Good Domestic Producers

We break the problem into two subproblems. As a cost minimizer and as a price setting firm.

Cost Minimizer The cost minimization problem,

$$Y_t(j) = A_t K_t(j)^{\theta} \left(Z_t N_t(j)\right)^{1-\theta} \qquad (A.37)$$

$$\min_{N_t(j)} W_t N_t(j) + R_t^k K_t + MC_t^r(j)\left[Y_t(j) - A_t K_t(j)^{\theta} \left(Z_t N_t(j)\right)^{1-\theta}\right]$$

The $MC_t(j)$ is the multiplier associated with the constraint. The firms chooses its demand for labor,

$$W_t = MC_t^r(j)(1-\theta)Z_t A_t K_t(j)^{\theta}\left(Z_t N_t(j)\right)^{-\theta} \tag{A.38}$$

The firm demand for capital,

$$R_t^k = MC_t^r(j)A_t\theta K_t(j)^{\theta-1}\left(Z_t N_t(j)\right)^{1-\theta} \tag{A.39}$$

Rewrting labor demand,

$$W_t N_t(j) = MC_t^r(j)(1-\theta)Y_t(j)$$

Rewriting capital demand,

$$R_t^k K_t(j) = MC_t^r(j)\theta Y_t(j)$$

Total costs are thus,

$$TC_t(j) = \left[MC_t^r(j)(1-\theta) + MC_t^r(j)\theta\right]Y_t(j)$$

$$TC_t(j) = \left[MC_t^r(j)\right]Y_t(j) \tag{A.40}$$

Marginal costs are defined as a change in total cost when output changes, $\frac{dTC_t(j)}{dY_t(j)} = MC_t^r(j)$, which shows that the lagrange multiplier equals to real marginal costs.

From equation A.38 and equation A.39 we get the factor market equilibrium condition,

$$\boxed{\frac{1-\theta}{\theta} = \frac{W_t N_t(j)}{R_t^k K_t(j)}}$$

Marginal Costs Since the factor prices are common for all the firms, the ratio of $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(j)}{K_t(j)}$ is the same for all firms.

Plugging the factor demands into production function,

$$Y_t(j) = A_t\left(\frac{MC_t^r(j)\theta Y_t(j)}{R_t^k}\right)^{\theta}\left(\frac{MC_t^r(j)(1-\theta)Y_t(j)}{W_t}\right)^{1-\theta}$$

we can cancel out $Y_t(j)$ and put the ratios after common power,

$$\left(\frac{MC_t^r(j)(1-\theta)}{W_t}\right)^{-1} = A_t\left(\frac{\frac{MC_t^r(j)\theta}{R_t^k}}{\frac{MC_t^r(j)(1-\theta)}{W_t}}\right)^{\theta}$$

$$\left(\frac{MC_t^r(j)(1-\theta)}{W_t}\right)^{-1} = A_t\left(\frac{\theta W_t}{(1-\theta)R_t^k}\right)^{\theta}$$

$$(MC_t^r(j))^{-1} = A_t\left(\frac{\theta W_t}{(1-\theta)R_t^k}\right)^{\theta}\frac{(1-\theta)}{W_t}$$

$$(MC_t^r(j))^{-1} = A_t\frac{W_t^{\theta-1}\theta^{\theta}}{\left(R_t^k\right)^{\theta}(1-\theta)^{\theta-1}}$$

$$MC_t^r = \int_0^1 MC_t^r(j)dj = \frac{\left(R_t^k\right)^{\theta}W_t^{1-\theta}}{A_t\theta^{\theta}(1-\theta)^{1-\theta}} \tag{A.41}$$

Price Setter We follow Calvo (1983) when laying out the firms price setting problem. We denote $\tilde{P}_t(j)$ the price set a by a firm j adjusting its price in period t. Under the Calvo price-setting structure, $P_{t+k}(j) = \tilde{P}_t(j)$ with probability θ^k for $k = 0, 1, 2, \ldots$. Since all the firms resetting price in the specific period will choose in equilibrium the same price so by symetry we can drop j subscript after optimization. The non-optimizing firms let

$$P_t(j) = P_{t-1}(j)\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\iota}\bar{\pi}^{1-\iota}$$

Firms thus firms index their price partialy the average of steady state inflation and last period inflation. Shifting prices one period forward,

$$P_{t+1}(j) = P_t(j)\left(\frac{P_t}{P_{t-1}}\right)^{\iota}\bar{\pi}^{1-\iota}$$

Writing the indexation sheme in terms of inflation.

$$P_t(j) = P_{t-1}(j)\pi_{t-1}^{\iota}\bar{\pi}^{1-\iota}$$

The price optimizing firm chooses its optimal price to maximize profits but knowing that the fixed price will be changing based on the last inflation, thus

$$\tilde{P}_{t|k=0}\left(\frac{P_t}{P_{t-1}}\right)^{\iota}_{|k=1}\left(\frac{P_{t+1}}{P_t}\right)^{\iota}_{|k=2}\left(\frac{P_{t+2}}{P_{t+1}}\right)^{\iota}_{|k=3}\dots$$

which can be writen more compactly in several ways, for instance Cogley Sbordone in AER,

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\tilde{P}_t\prod_{l=1}^{k}\left(\frac{P_{t+l-1}}{P_{t+l-2}}\right)^{\iota}\right\}$$

in terms of inflation,

$$\tilde{P}_{t|k=0}\left(\pi_t\right)^{\iota}_{|k=1}\left(\pi_{t+1}\right)^{\iota}_{|k=2}\left(\pi_{t+2}\right)^{\iota}_{|k=3}\dots$$

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\tilde{P}_t\prod_{l=1}^{k}\left(\pi_{t+l-1}\right)^{\iota}\right\}=\tilde{P}_{t|k=0}+\theta^1\tilde{P}_{t|k=1}\pi_t^{\iota}+\theta^2\tilde{P}_{t|k=2}\pi_t^{\iota}\pi_{t+1}^{\iota}\dots$$

We could also use different notation and write the problem in the following way,

$$\tilde{P}_{t|k=0}\left(\frac{P_t}{P_{t-1}}\right)^{\iota}_{|k=1}\left(\frac{P_{t+1}}{P_t}\right)^{\iota}_{|k=2}\left(\frac{P_{t+2}}{P_{t+1}}\right)^{\iota}_{|k=3}\dots$$

$$\tilde{P}_{t|k=0}\left(\frac{P_{t+2}}{P_{t-1}}\right)^{\iota}_{|k=3}=\tilde{P}_{t|k=0}\pi^{\iota}_{t-1,t+2}$$

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\tilde{P}_t\left(\frac{P_{t+k-1}}{P_{t-1}}\right)^{\iota}\right\}$$

which is in terms of inflation

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{\tilde{P}_t\pi^{\iota}_{t-1,t+k-1}\pi^{1-\iota}\right\}$$

When choosing new price the firm wants to maximize the present value of future profits, condition on that price being effective plus the profits over all other state when the firm can reset its price. Profits in the state of the world when $P_t \neq \tilde{P}_t$ are irrelevant as $\frac{dX_t}{dP_t}=0$.

$$\max_{\tilde{P}_t(j)}\sum_{k=0}^{\infty}\theta^k E_t\left\{Q_{t,t+k}\left[Y_{t+k}(j)\tilde{P}_t(j)\left(\pi^{\iota}_{t-1,t+k-1}\pi^{1-\iota}\right)-TC^n_{t+k}(j)\right]\right\}+X_t$$

subject to (firm specific production function and j-good demand function)

$$Y_t(j) = A_t K_t(j)^{\theta} N_t^{1-\theta}(j) \tag{A.42}$$

$$Y_{t+k}(j) = \left(\frac{\tilde{P}_t(j)}{P_{t+k}} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) \right)^{-\epsilon} Y_{t+k}$$

Taking the derivative wrt. $\tilde{P}_t$,

$$\max_{\tilde{P}_{H,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[\frac{\partial Y_{t+k}(j)}{\partial \tilde{P}_t(j)} \frac{\partial \tilde{P}_t(j)}{\partial \tilde{P}_t(j)} - \underbrace{\frac{\partial TC_{t+k}^n(j)}{\partial Y_{t+k}(j)}}_{MC_{t+k}(j)} \frac{\partial Y_{t+k}(j)}{\partial \tilde{P}_t(j)} \right] \right\}$$

$$\max_{\tilde{P}_t(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left[Y_{t+k}(j) \tilde{P}_t(j) \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) - TC_{t+k}^n(j) \right] \right\}$$

Plugging the constraints in the objective function to simplify the chain rules, delivers,

$$\max_{P_t(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \begin{bmatrix} \tilde{P}_t(j)^{-\epsilon+1} \left(\left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) \right)^{-\epsilon+1} Y_{t+k} P_{t+k}^{\epsilon} - \\ MC_{t+k}(j) \left(\frac{\tilde{P}_t(j)}{P_{t+k}} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) \right)^{-\epsilon} Y_{t+k} \end{bmatrix} \right\}$$

where taking derivative of the demand function,

$$\begin{aligned} \frac{\partial Y_{t+k}(j)}{\partial \tilde{P}_t(j)} &= -\epsilon \left(\frac{\tilde{P}_t(j)}{P_{t+k}} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) \right)^{-\epsilon-1} \frac{\left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right)}{P_{t+k}} Y_{t+k} \\ &= -\epsilon (\tilde{P}_t(j))^{-\epsilon-1} \left(\frac{\left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right)}{P_{t+k}} \right)^{-\epsilon} Y_{t+k} \end{aligned}$$

which we can then use to plug in the

$$\max_{\tilde{P}_{H,t}(j)} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \begin{bmatrix} (1-\epsilon)(\tilde{P}_t(j))^{-\epsilon} \left(\left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right) \right)^{-\epsilon+1} Y_{t+k} P_{t+k}^{\epsilon} + \\ MC_{t+k}(j) \epsilon (\tilde{P}_t(j))^{-\epsilon-1} \left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota} \right)} \right)^{\epsilon} Y_{t+k} \end{bmatrix} \right\}$$

we cannot devide by the variables which vary over k (Y_{t+k},P_{t+k}, $MC_{t+k}(j)$), thus deviding by $1-\epsilon$ and $(\tilde{P}_t(j))^{-\epsilon}$

$$\sum_{k=0}^{\infty}\theta^k E_t\left\{Q_{t,t+k}\left(\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)\right)^{-\epsilon+1}Y_{t+k}P_{t+k}^{\epsilon}\left[\tilde{P}_t(j)+\frac{\epsilon}{1-\epsilon}MC_{t+k}(j)\right]\right\}=0$$

Expressing for $\tilde{P}_t(j)$ and using the fact that in equilibrium all the firms that reset the price choose the same price we get the optimal reset price in terms of aggregate output.

$$\tilde{P}_t=\frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon}Y_{t+k}MC_{t+k}(j)}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)\right)^{-\epsilon+1}Y_{t+k}P_{t+k}^{\epsilon}} \tag{A.43}$$

We can further use the fact that firms which reset prices face the same demand for their j-goods, multiplying by both denominator and nominator with $(\tilde{P}_t)^{-\epsilon}$

$$\tilde{P}_t=\frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{\tilde{P}_t}{P_{t+k}}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)\right)^{-\epsilon}Y_{t+k}MC_{t+k}^{n}(j)}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{\tilde{P}_t}{P_{t+k}}\right)^{-\epsilon}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)^{-\epsilon+1}Y_{t+k}} \tag{A.44}$$

and using the fact that $\tilde{Y}_{t+k}(j)=\left(\frac{\tilde{P}_t}{P_{t+k}}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)\right)^{-\epsilon}Y_{t+k}$

$$\tilde{P}_t=\frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\tilde{Y}_{t+k}(j)MC_{t+k}^{n}(j)}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\tilde{Y}_{t+k}(j)}=\frac{\epsilon}{1-\epsilon}\sum_{k=0}^{\infty}\phi_{t+k}(j)MC_{t+k}^{n}(j) \tag{A.45}$$

where $\frac{\epsilon}{1-\epsilon}$is the mark-up implied by monopolistic competition and ϕ_{t+k}says that the optimal price is a weighted avearge of current and expecreted future nominal marginal costs. Weights depend on expected demand in the future and how much firm discounts profits.

Price Adjustment Gap We want to have expression with well defined steady state and thus get rid of prices. Lets use the equation with aggregate output.

Next, I define real marginal costs as $MC_t^n = MC_t^r P_t$ Further, I write the price choosen by reseters relative to aggregate price index (devide both sides by P_t)[37].

$$\frac{\tilde{P}_t}{P_t} = \frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon} Y_{t+k}\overbrace{\frac{MC_{t+k}(j)}{P_{t+k}}}^{MC_{t+k}^r(j)} P_{t+k}}{P_t\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right) Y_{t+k}} \quad (A.46)$$

just rearanging and collecting terms,

$$\frac{\tilde{P}_t}{P_t} = \frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon} Y_{t+k}}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right) Y_{t+k}}\frac{P_{t+k}}{P_t} MC_{t+k}^r(j) \quad (A.47)$$

deviding and multiplying by P_t^{ϵ}

$$p_t = \frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{P_t}\frac{1}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon} Y_{t+k}}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\left(\frac{P_{t+k}}{P_t}\frac{1}{\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right) Y_{t+k}}\frac{P_{t+k}}{P_t} MC_{t+k}^r(j) \quad (A.48)$$

using the definition of inflation $\frac{P_{t+k}}{P_t} = \pi_{t,t+k}$

$$p_t = \frac{\epsilon}{1-\epsilon}\frac{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\pi_{t+k}^{\epsilon+1}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)^{-\epsilon} Y_{t+k}}{\sum_{k=0}^{\infty}\theta^k E_t Q_{t,t+k}\pi_{t+k}^{\epsilon}\left(\pi_{t-1,t+k-1}^{\iota}\pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k}} MC_{t+k}^r(j)$$

and where the nominal stochastic discount factor is

$$Q_{t,t+1} = \beta e^{b_{t+1}-b_t}\left(\frac{C_{t+1}}{C_t}\right)^{\gamma}\frac{1}{\pi_{t+1}}\mu_{t+1}^{-1}\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}$$

Next, the real marginal costs need to be aggregated, and because of the constant return to scale assumption marginal costs are independent from the quantity produced and $MC_t^r = \int_0^1 MC_t^r(j)dj$

[37]Some papers define the relative price with respect to P_{t-1}. Ascari calls this ratio PRICE ADJUSTMENT GAP

$$\tilde{p}_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon+1} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} Y_{t+k}}{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k}} MC_{t+k}^r$$

Recursive Philips Curve

$$\tilde{p}_t = \frac{\epsilon}{\epsilon - 1} \frac{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon+1} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} Y_{t+k}}{\sum_{k=0}^{\infty} \theta^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k}} MC_{t+k}^r = \frac{\epsilon}{\epsilon - 1} \frac{K_t}{F_t} \tag{A.49}$$

The nominator can be writen as,

$$K_t = \sum_{k=0}^{\infty} (\zeta\beta)^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon+1} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k} MC_{t+k}^r \tag{A.50}$$

We can write this equation recursively using real discount bonds,

$$\begin{aligned} K_t = (\zeta\beta)^0 E_t \underbrace{Q_{t,t+0}}_{=1} \underbrace{\pi_{t,t+0}^{\epsilon}}_{=1} Y_{t+0} MC_{t+0}^r + \\ + \zeta\beta \sum_{k=1}^{\infty} (\zeta\beta)^{k-1} E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon+1} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} Y_{t+k} MC_{t+k}^r \end{aligned}$$

Starting the sum one period back and using real discount bond,

$$K_t = E_t Y_t MC_t^r + \zeta\beta \sum_{k=0}^{\infty} (\zeta\beta)^k E_t Q_{t,t+k+1}^r \pi_{t,t+k+1}^{\epsilon} \left(\pi_{t-1,t+k}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} Y_{t+k+1} MC_{t+k+1}^r \tag{A.51}$$

Recognise that

$$K_{t+1} = \sum_{k=0}^{\infty} (\zeta\beta)^k E_{t+1} Q_{t+1,t+1+k}^r \pi_{t+1,t+1+k}^{\epsilon} \left(\pi_{t,t+k}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} Y_{t+k+1} MC_{t+k+1}^r \tag{A.52}$$

Note that, $\pi_{t+1,t+k+1} = \frac{P_{t+k+1} P_t}{P_{t+1} P_t} = \frac{\pi_{t,t+k+1}}{\pi_{t+1}}$, and

$$Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+2} \dots$$

$$Q_{t+1,t+k+1} = Q_{t+1,t+2}Q_{t+2,t+3}\dots$$

$\sum_{k=0}^{\infty} Q_{t,t+k+1} = Q_{t,t+1} \sum_{k=0}^{\infty} Q_{t+1,t+k+1}$[38] .

Thus the second part of K_t can be writen as $(\pi_{t+1,t+k+1}\pi_{t+1} = \pi_{t,t+k+1})$ and, $(\pi^{\iota}_{t-1,t}\pi^{\iota}_{t,t+1}\pi^{\iota}_{t+1,t+2})\pi^{1-\iota}$

$$K_t = E_t Y_t MC^r_t +$$
$$\zeta\beta Q^r_{t,t+1}\pi^{\epsilon}_{t+1}\left(\pi^{\iota}_{t-1,t}\pi^{1-\iota}\right)^{-\epsilon}\sum_{k=0}^{\infty}(\zeta\beta)^k E_t Q^r_{t+1,t+k+1}\left(\pi_{t+1,t+k+1}\right)^{\epsilon}\left(\pi^{\iota}_{t,t+k}\pi^{1-\iota}\right)^{-\epsilon} Y_{t+k+1}MC^r_{t+k+1}$$

Tranforming the second part of equation (A.51) to fit the equation (A.52),

$$K_t = Y_t MC^r_t +$$
$$\zeta\beta Q^r_{t,t+1}\pi^{\epsilon}_{t+1}\left(\pi^{\iota}_{t-1,t}\pi^{1-\iota}\right)^{-\epsilon}\underbrace{\sum_{k=0}^{\infty}(\theta\beta)^k E_t Q^r_{t+1,t+k+1}\pi^{\epsilon}_{t+1,t+k+1}\left(\pi^{\iota}_{t,t+k}\pi^{1-\iota}\right)^{-\epsilon} Y_{t+k+1}MC^r_{t+k+1}}_{K_{t+1}}$$

$$K_t = Y_t MC^r_t + \tag{A.53}$$
$$+\zeta\beta Q^r_{t,t+1}\pi^{\epsilon}_{t+1}\left(\pi^{\iota}_{t-1,t}\pi^{1-\iota}\right)^{-\epsilon}\underbrace{\sum_{k=0}^{\infty}(\theta\beta)^k E_t Q^r_{t+1,t+k+1}\pi^{-\epsilon}_{t+1,t+k+1} Y_{t+k+1}MC^r_{t+k+1}}_{K_{t+1}} \tag{A.54}$$

Now we can use (A.52) in (A.53) to write the equation (3.B.4) recursively (β is in the SDF),

$$\boxed{K_t = E_t Y_t MC^r_t + \zeta Q^r_{t,t+1}\pi^{\epsilon}_{t+1}\left(\pi^{\iota}_{t-1,t}\pi^{1-\iota}\right)^{-\epsilon} K_{t+1}} \tag{A.55}$$

[38]

$$Q_{t,t+k} = Q_{t,t+1}Q_{t+1,t+k} =$$
$$\beta\left(\frac{C_{t+k}}{C_{t+1}}\right)^{-\gamma}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+k}}\mu^{-1}_{t+1}\mu^{-1}_{t+k}\left(\frac{(V_{t+k})}{[E_t(V_{t+k})^{1-\nu}]^{\frac{1}{1-\nu}}}\right)^{-\nu}\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\nu}]^{\frac{1}{1-\nu}}}\right)^{-\nu}$$

We need to do the same excercise for the denominator F_t,

$$F_t = \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k} \pi_{t,t+k}^{\epsilon} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k} \tag{A.56}$$

Next we notice that,

$$F_{t+1} = \sum_{k=0}^{\infty} \zeta^k E_t Q_{t+1,t+1+k} \pi_{t+1+k}^{\epsilon} \left(\pi_{t,t+k}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+1+k} \tag{A.57}$$

To rewrite equation (A.56) recursively, we expand the sum in the equation (A.56) to get,

$$F_t = Y_t + \zeta \sum_{k=1}^{\infty} \zeta^{k-1} E_t Q_{t,t+k} \pi_{t+k}^{\epsilon} \left(\pi_{t-1,t+k-1}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k} \tag{A.58}$$

Shifting the sum one period back (starting from zero),

$$F_t = Y_t + \zeta \sum_{k=0}^{\infty} \zeta^k E_t Q_{t,t+k+1} \pi_{t+k+1}^{\epsilon} \left(\pi_{t-1,t+k}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k+1} \tag{A.59}$$

Next, we use the fact that $Q_{t,t+k} = Q_{t,t+1} Q_{t+1,t+k}$ and $\pi_{t+1,t+k+1}\pi_{t+1} = \pi_{t,t+k}$to get,

$$F_t = Y_t + \zeta Q_{t,t+1} \pi_{t+1}^{\epsilon} \left(\pi_{t-1,t}^{\iota} \pi^{1-\iota}\right)^{-\epsilon} \sum_{k=0}^{\infty} \theta^k E_t Q_{t+1,t+k+1} \pi_{t+1,t+k+1}^{\epsilon} \left(\pi_{t,t+k}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} Y_{t+k+1} \tag{A.60}$$

Now we can plug equation (A.57) into (A.60) to get the recursive formulation of F_t,

$$F_t = Y_t + \zeta Q_{t,t+1} \pi_{t+1}^{\epsilon} \left(\pi_{t-1,t}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} F_{t+1}$$

or using the definition of SDF,

$$F_t = Y_t + \zeta \beta \frac{\lambda_{t+1}}{\lambda_t} \mu_{t+1}^{-1} \left(\frac{(V_{t+1})}{[E_t (V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}} \right)^{-\alpha} e^{b_{t+1}-b_t} \frac{\pi_{t+1}^{\epsilon}}{\pi_{t+1}} \left(\pi_{t-1,t}^{\iota} \pi^{1-\iota}\right)^{-\epsilon+1} F_{t+1} \tag{A.61}$$

$$F_t = Y_t + \zeta\beta\frac{\lambda_{t+1}}{\lambda_t}\mu_{t+1}^{-1}\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} e^{b_{t+1}-b_t}\left(\frac{\pi_{t+1}}{\left(\pi_{t-1,t}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon-1} F_{t+1} \tag{A.62}$$

Elastisticy of substitution notation For our RS replication code

$$\frac{1+\lambda}{\lambda} = \epsilon$$

thus

$$\frac{1+\lambda}{\lambda} - 1 = \frac{1}{\lambda} = \epsilon - 1$$

$$\frac{1+\lambda}{\lambda} + 1 = \epsilon + 1$$

$$\frac{\epsilon}{\epsilon-1} = \frac{\frac{1+\lambda}{\lambda}}{\frac{1+\lambda}{\lambda}-1} = \frac{\frac{1+\lambda}{\lambda}}{\frac{1}{\lambda}} = (1+\lambda)$$

3.B.5 Aggregation

$$Y_t(j) = A_t K_t(j)^{\theta} N_t^{1-\theta}(j) \tag{A.63}$$

Using the demand function for good j,

$$\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t = A_t\left(\frac{K_t(j)}{N_t(j)}\right)^{\theta} N_t(j) \tag{A.64}$$

Using the fact that $\frac{1-\theta}{\theta}\frac{R_t}{W_t} = \frac{N_t(j)}{K_t(j)}$ and thus the ratio is common for all firms,

$$\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t = A_t\left(\frac{K_t(j)}{N_t(j)}\right)^{\theta} N_t(j) \tag{A.65}$$

$$\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj Y_t = A_t\left(\frac{K_t(j)}{N_t(j)}\right)^{\theta} \int_0^1 N_t(j)dj \tag{A.66}$$

$$S_t Y_t = A_t {K_t}^{\theta} N_t^{1-\theta} \tag{A.67}$$

Stationarizing production function,

$$S_t Y_t Z_t = A_t\left(Z_t K_t\right)^{\theta}\left(Z_t N_t\right)^{1-\theta}$$

dividing by Z_t delivers,

$$S_t Y_t = A_t \left(K_t\right)^{\theta} \left(N_t\right)^{1-\theta}$$

Price dispersion

Lets define price dispersion, S_t:

$$S_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$$

where ϵ is the elasticity of substitution between differenciated good j. Next, using the 'Calvo result' (proportion of firms changing its price), we can write price dispersion recursively as:

$$S_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} dj$$

$$S_t = \zeta(1-\zeta)\left(\frac{\tilde{P}_{t-1}(j)}{P_t}\right)^{-\epsilon} + \zeta^2(1-\zeta)\left(\frac{\tilde{P}_{t-2}(j)}{P_t}\right)^{eqnarray*}$$

where $(1-\zeta)$ is the probability that the firm will be able to change price and . Price dispersion can be written recursively as

$$S_t = (1-\zeta)\left(\frac{\tilde{P}_t(j)}{P_t}\right)^{-\epsilon} + \zeta\left(\frac{P_{t-1}\left(\pi_{t-1}^{\iota}\pi^{1-\iota}\right)}{P_t}\right)^{-\epsilon} S_{t-1}$$

$$S_t = (1-\zeta)\left(\frac{\tilde{P}_t(j)}{P_t}\right)^{-\epsilon} + \zeta\left(\frac{\pi_t}{\left(\pi_{t-1}^{\iota}\pi^{1-\iota}\right)}\right)^{\epsilon} S_{t-1} \tag{A.68}$$

Now we can use the domestic aggregate price index to substitute out the ratio of prices and write everything in terms of inflation. Using the definition of aggregate price index and $\tilde{p}_t = \frac{\tilde{P}_t(j)}{P_t}$

$$P_t = \int_0^1 \left[P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\epsilon}}$$

$$P_t^{1-\varepsilon} = (1-\zeta)\left(\tilde{P}_t(j)\right)^{1-\varepsilon} + \zeta\left(P_{t-1}\left(\pi_{t-1}^{\iota}\pi^{1-\iota}\right)\right)^{1-\epsilon}$$

$$1 = (1-\zeta)\left(\frac{\tilde{P}_t(j)}{P_t}\right)^{1-\epsilon} + \zeta\left(\frac{P_{t-1}}{P_t}\left(\pi_{t-1}^{\iota}\pi^{1-\iota}\right)\right)^{1-\epsilon}$$

$$\tilde{p_t} = \left[\frac{1-\theta \left(\frac{\pi_t}{\left(\pi_{t-1}^{\iota}\pi^{1-\iota}\right)} \right)^{\epsilon-1}}{1-\theta} \right]^{\frac{1}{1-\epsilon}}$$

Market Clearing Condition

$$Y_t = C_t + I_t + G_t$$

3.B.6 Excess Holding Period Return

Let i_t be the nominal interest rate. Real interest rate is given by $r_t = i_t - \pi_{t+1}$or in gross terms, $R_t = \frac{I_t}{\pi_{t+1}}$. The price of nominal ten year bond can be written recursively, $P_t^n = e^{-i_t} E_t P_{t+1}^{n-1}$ which is in logs $p_t^n = -i_t + logE_t p_{t+1}^{n-1}$. In general, the price of nominal ten year bond,

$$P_t^n = Q_{t,t+n} = \frac{1}{(1+ytm_t)^n} = \prod_{k=0}^{n-1} \frac{1}{(1+i_{t+k})}$$

$$P_t^n = Q_{t,t+n} = \frac{1}{(1+ytm_t)^n} = \frac{1}{(1+i_t)} \frac{1}{(1+i_{t+1})} \frac{1}{(1+i_{t+2})} + \dots$$

where ytm_t is the yield to maturity on ten year bond.

$$\log P_t^n = \log Q_{t,t+n} = \log \frac{1}{(1+ytm_t)^n}$$

$$\log P_t^n = \log(1+ytm_t)^{-n}$$

$$\log P_t^n = -n \log(1+ytm_t)$$

thus

$$ytm_t^n = -\frac{1}{n} p_t^n = -\frac{1}{n} q_{t,t+n}$$

In the dynare code the SDF defines $\exp(p_t^n)$ which is then $\exp(p_t^n) = \exp\left((-n) ytm_t^n\right)$.

The excess holding period return is holding return of bond with maturity n for one period above the risk free one period interest rate. In other words for how much I can sell the bond compared to for how much I bought the bond versus buying one year risk free bond.

defined as

$$ehpr_t = \frac{P_t^{39}}{P_{t-1}^{40}} - (1 + i_{t-1})$$

$$ehpr_t = \left(\frac{P_t^{39}}{P_{t-1}^{40}} - 1\right) - i_{t-1}$$

$$ehpr_t = \left(\frac{\frac{1}{(1+ytm_t)^{39}}}{\frac{1}{(1+ytm_t)^{40}}} - 1\right) - i_{t-1}$$

$$ehpr_t = \left(\frac{(1 + ytm_t)^{40}}{(1 + ytm_t)^{39}} - 1\right) - i_{t-1}$$

$$ehpr_t = ytm_t^1 - i_{t-1} = 0 + RP$$

In the dynare code

$$ehpr_t = \left(\frac{\exp\left((39)ytm_t^{39}\right)}{\exp\left((40)ytm_t^{40}\right)} - 1\right) - i_{t-1}$$

Expected Excess Period Return

$$eehpr_t = \left(\frac{E_t P_{t+1}^{39}}{P_t^{40}} - 1\right) - i_t \tag{A.69}$$

Steady State

$$ehpr = \left(\frac{\exp\Big((-39)\log(\frac{1}{\beta})\Big)}{\exp\Big((-40)\log(\frac{1}{\beta})\Big)} - 1\right) - i_{t-1}$$

$$ehpr = \left(\left(\frac{1}{\beta}\right)^{-39+40} - 1\right) - \log(\frac{1}{\beta})$$

$$ehpr = \left(\frac{1}{\beta} - 1\right) - \log(\frac{1}{\beta}) \approx 0$$

3.B.7 Financial Intermediary

The financial intermediary invests one period deposits from households (bonds/deposites in HHs optimization problem) into the short and long term government bonds. This intermediary pays the deposite rate to households r_t^b which comes out from the stochastic discount factor (deposite supply equation). The intermediary invests into short (on quarter) and long bonds with 10 year maturity.

As deposits are for one period he needs to liquidate his portfolio every period. Thus he receives holding period return from the bonds in his portfolio. Because financial intermediary is risk neutral and owned by households his return is equal to

$$r_t^b = (1-\omega) \times hr_{t,1} + \omega \times hr_{t,40} \tag{A.70}$$

where the expected holding period return is,

$$hr_{t,40} = E_t \log\left(\frac{P_{t+1}^{39}}{P_t^{40}}\right) \tag{A.71}$$

From the equation A.70 we can derive the link between the nominal interest rate (monetary policy instrument) and the return on the HHs deposite (return on the portfolio). Using the definition of expected excess holding period return, equation A.69,

$$eehpr_{t,40} = E_t \log\left(\frac{P_{t+1}^{39}}{P_t^{40}}\right) - i_t$$

Substituting in the equation A.71

$$eehpr_{t,40} = hr_{t,40} - i_t$$

Next, notice that the expected excess holding period return for one period deposite/bond over the risk free rate is,

$$eehpr_{t,1} = E_t \log\left(\frac{P_{t+1}^{0}}{P_t^{1}}\right) - i_t = 0$$

$$hr_{t,1} = i_t$$

thus we can simplify the equation A.70 for return on bond portfolio by plugging in for $hr_{t,1}$,

$$r_t^b = (1-\omega) \times i_t + \omega \times (eehpr_{t,40} + i_t)$$

$$r_t^b = i_t + \omega \times (eehpr_{t,40}) \tag{A.72}$$

Steady state of i_t

$$r^b = i + \omega \times (eehpr_{t,40})$$

$$e e \bar{h} p r = 0$$

$$r^b = \log\left(\frac{\bar{\pi}}{\beta\mu^{-\gamma}}\right) = i$$

Steady state value of monetary policy instrument,

3.B.8 Central Bank

Central bank pins down i_t but HHs and firms use for discounting and pricing assets r_t^b (which is potentialy higher because it reflects also the excess period return).

$$i_t - \bar{i} = \rho_i \left(i_{t-1} - \bar{i}\right) + (1-\rho_i)\left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(\frac{y_t \mu_t}{\bar{Y}\bar{\mu}} - 1\right) + \phi_{xhr}\left(eehpr_t^{40} - E\left[eehpr_t^{40}\right]\right)\right]$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\,\bar{i} + (1-\rho_i)\left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(\frac{y_t \mu_t}{\bar{Y}\bar{\mu}} - 1\right) + \phi_{xhr}\left(eehpr_t^{40} - E\left[eehpr_t^{40}\right]\right)\right]$$

where $E\left[ehpr_t^L\right]$ should be the deterministic steady state.

Andreasen shows that the unconditional expectation of expected excess holding period return can be desribed by the recursive relationship

$$x_{t,40} = (1-\gamma)eehpr_t^{40} + \gamma E_t[x_{t+1,40}]$$

$$i_t = \rho_i i_{t-1} + (1-\rho_i)\,\bar{i} + (1-\rho_i)\left[\phi_\pi \left(\pi_t - \bar{\pi}\right) + \phi_y \left(\frac{y_t \mu_t}{\bar{Y}\bar{\mu}} - 1\right) + \phi_{xhr}\left(eehpr_t^{40} - x_{t,40}\right)\right]$$

In steady state the recurisve relationship implies,

$$x_{40} - \gamma E_t[x_{40}] = (1-\gamma)eehpr^{40}$$

$$x_{40} = eehpr^{40}$$

3.B.9 Frish Elasticity

The Frisch elasticity of labor supply measures the percentage change in hours worked due to the percentage change in wages, holding constant the marginal utility of wealth (i.e., the multiplier on the budget constraint λ). It can be derived from the HHs problem by differentiating labor supply equation and treating lagrange multiplier as constant. In general the formula is:

$$\epsilon = \frac{u_n}{\bar{N}(u_{nn} - \frac{u_{nc}^2}{u_{cc}})}$$

because of the separability in preferences in our case the cross-derivative will fall out.

Separable Preferences

$$u(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi_0 Z_t^{1-\gamma} \frac{(L_{max} - N_t)^{1-\chi}}{1-\chi}$$

$$\epsilon = \frac{\chi_0 Z^{1-\gamma}(L_{max} - \bar{N})^{-\chi}}{\chi_0 Z^{1-\gamma} \bar{N}[\chi(L_{max} - \bar{N})^{-\chi-1} - \frac{0}{u_c c}]} = \frac{L_{max} - \bar{N}}{\chi \bar{N}}$$

Keep in mind that if you define the utility kernel, as \begin{equation}

$$u(C_t, N_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \chi_0 Z_t^{1-\gamma} \frac{(N_t)^{1-\chi}}{1-\chi}$$

then Frisch elasticity is independent of hours worked. \begin{equation}

$$\epsilon = \frac{\chi_0 Z^{1-\gamma}(\bar{N})^{-\chi}}{\chi_0 Z^{1-\gamma} \bar{N}[\chi(\bar{N})^{-\chi-1} - \frac{0}{u_c c}]} = \frac{1}{\chi}$$

3.B.10 End of the Day Stationarizing

Here we stationarize capital not as in RBC literature (King and Rebelo (1999)) but as in Altig et. al. (2005) and $K_t = \frac{k_t}{Z_{t-1}}$

3.B.11 Household Problem

When maximizing life time utility they face following two constraints writen up as stationarized tranding variables. All the quantities are devided by Z_t

$$C_t + E_t Q^r_{t,t+1} \frac{P_{t+1}}{P_t} \mu_{z,t+1} B_{t+1} + I_t \leq B_t + W_t N_t + r^k_t K_t \tag{A.73}$$

and law of motion of capital which reads in its non-stationary form as,

$$k_{t+1} = (1-\delta)k_t + i_t - i_t \frac{\kappa_1}{2}\left(\frac{i_t - \bar{i}}{\bar{i}}\right)^2 - i_t \frac{\kappa_2}{2}\left(\frac{i_t}{k_t} - \frac{\bar{i}}{\bar{k}}\right)^2 \tag{A.74}$$

now we replace the growing variables by their stationary counterparts, $K_t Z_{t-1} = k_t$

$$\frac{k_{t+1}}{Z_t} = (1-\delta)\frac{k_t}{Z_t}\frac{Z_{t-1}}{Z_{t-1}} + \frac{i_t}{Z_t} - \frac{i_t}{Z_t}\frac{\kappa_1}{2}\left(\frac{i_t}{\bar{i}}\frac{\bar{Z}}{Z_t} - 1\right)^2 - \frac{k_t}{Z_t}\frac{Z_{t-1}}{Z_{t-1}}\frac{\kappa_2}{2}\left(\frac{\frac{i_t}{Z_t}}{\frac{k_t}{Z_{t-1}}}\frac{Z_t}{Z_{t-1}} - \frac{\bar{I}}{\bar{K}}\right)^2 \tag{A.75}$$

$$K_{t+1} = (1-\delta)K_t \mu_{z,t}^{-1} + I_t - I_t \frac{\kappa_1}{2}\left(\frac{I_t}{\bar{I}} - 1\right)^2 - \mu_{z,t}^{-1} K_t \frac{\kappa_2}{2}\left(\frac{I_t}{K_t}\mu_{z,t} - \frac{\bar{I}}{\bar{K}}\right)^2 \tag{A.76}$$

where I_t represents investment and K_t is capital stock, prices are defined as real. Andreasen considers both type of adjustment costs. It is costly to change investment from its steady state value. The capital adjustment cost function depned on the quantity of investment relative to the installed capital stock, that is, on the ratio between the new capital to be installled and the captial stock already installed. Thus, if firm instals more new capital than depretiation it has to pay instalment costs.

Here we use the dynamic programming approach and define the constraint maximization problem of households as:

$$V_t = \max_{B_{t+1}, C_t, N_t} \left\{ \frac{(C_t - b\mu_{z,t}^{-1} C_{t-1})^{1-\gamma}}{1-\gamma} + \chi \frac{(1-N_t)^{1-\eta}}{1-\eta} + \beta (E_t[(V_{t+1})^{1-\alpha}])^{\frac{1}{1-\alpha}} - \right.$$

$$-\lambda_t[C_t + E_t Q_{t,t+1}\frac{P_{t+1}}{P_t}\mu_{z,t+1}B_{t+1} + I_t - B_t - W_t N_t - r_t^k K_t \mu_{z,t}^{-1}]+$$

$$+q_t\lambda_t\left((1-\delta)K_t\mu_{z,t}^{-1} + I_t - I_t\frac{\kappa_1}{2}\left(\frac{I_t}{\bar{I}}-1\right)^2 - \mu_{z,t}^{-1}K_t\frac{\kappa_2}{2}\left(\frac{I_t}{K_t}\mu_{z,t} - \frac{\bar{I}}{\bar{K}}\right)^2 - K_{t+1}\right)$$

where we define Tobin's Q marginal ratio as the ratio of the two multipliers associated with our constraints, thus $Q_t = q_t\lambda_t$. We need to devide by λ_t to put the shedow value of having extra unit of capital into the consumption units.

Again, it helps to write down,

$$V_{t+1} = \max_{B_{t+2},C_{t+1},N_{t+1}} \frac{\left(C_{t+1} - b\mu_{z,t+1}^{-1}C_t\right)^{1-\gamma}}{1-\gamma} + \chi\frac{(1-N_{t+1})^{1-\eta}}{1-\eta} + \beta(E_{t+1}[(V_{t+2})^{1-\alpha}])^{\frac{1}{1-\alpha}}$$
$$-\lambda_{t+1}[C_{t+1} + E_{t+1}Q_{t+1,t+2}\frac{P_{t+2}}{P_{t+1}}\mu_{z,t+2}B_{t+2} + I_{t+1} - B_{t+1} - W_{t+1}N_{t+1} - r_{t+1}^k K_{t+1}\mu_{z,t+1}^{-1}]$$
$$+q_{t+1}\lambda_{t+1}\left((1-\delta)K_{t+1}\mu_{z,t+1}^{-1} + I_{t+1} - I_{t+1}\frac{\kappa_1}{2}\left(\frac{I_{t+1}-\bar{I}}{\bar{I}}\right)^2\right)$$
$$-q_{t+1}\lambda_{t+1}\mu_{z,t+1}^{-1}K_{t+1}\frac{\kappa_2}{2}\left(\frac{I_{t+1}}{K_{t+1}}\mu_{z,t+1} - \frac{\bar{I}}{\bar{K}}\right)^2 - q_{t+1}\lambda_{t+1}K_{t+2}$$

HHs choose how much to consume, $\frac{\partial V_t}{\partial C_t} = 0$, in case of internal habit

$$(C_t - b\mu_{z,t}^{-1}C_{t-1})^{-\gamma} + \frac{1}{1-\alpha}\beta(E_t[(V_{t+1})^{1-\alpha}])^{\frac{1}{1-\alpha}-1}(1-\alpha)V_{t+1}^{-\alpha}b\mu_{z,t+1}^{-1}\left(C_{t+1} - b\mu_{z,t+1}^{-1}C_t\right)^{-\gamma} = \lambda_t$$

Simplifying, we get the **marginal utility of consumption for internal habits**

$$(C_t - b\mu_{z,t}^{-1}C_{t-1})^{-\gamma} - \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} b\mu_{z,t+1}^{-1}\left(C_{t+1} - b\mu_{z,t+1}^{-1}C_t\right)^{-\gamma} = \lambda_t \tag{A.77}$$

For the external habit,

$$(C_t - b\mu_{z,t}^{-1}C_{t-1})^{-\gamma} = \lambda_t \tag{A.78}$$

HHs choose how much to work, $\frac{\partial V_t}{\partial N_t} = 0$

$$\chi(1-N_t)^{-\eta} = W_t\lambda_t$$

HHs choose how much to save through deposits, $\frac{\partial V_t}{\partial B_{t+1}} = 0$,

$$\lambda_t E_t Q_{t,t+1}\frac{P_{t+1}}{P_t}\mu_{z,t+1} = \lambda_{t+1}\beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}$$

$$Q_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t}{P_{t+1}}\mu_{z,t+1}^{-1}\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} \qquad \text{(A.79)}$$

HHs choose how much new capital to instal, $\frac{\partial V_t}{\partial K_{t+1}} = 0$,
Putting $\mu_{z,t+1}^{-1}$ in front of the big bracket and the other μ's inside the brackets

$$-q_t + \beta\left(\frac{(V_{t+1})}{[E_t(V_{t+1})^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha}\frac{\lambda_{t+1}}{\lambda_t}\mu_{z,t+1}^{-1}$$
$$\left(r_{t+1}^k + q_{t+1}\left[(1-\delta) + \mu_{z,t+1}^2\kappa_2\left(\frac{I_{t+1}}{K_{t+1}} - \frac{\bar{I}}{\bar{K}}\mu_{z,t+1}^{-1}\right)\frac{I_{t+1}}{K_{t+1}} - \frac{\kappa_2}{2}\left(\frac{I_{t+1}}{K_{t+1}}\mu_{z,t+1} - \frac{\bar{I}}{\bar{K}}\right)^2\right]\right) = 0$$

HHs choose how much to invest, $\frac{\partial V_t}{\partial I_t} = 0$,

$$-\lambda_t + q_t\lambda_t\left(1 - \frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2 - I_t\kappa_1\left(\frac{I_t-\bar{I}}{\bar{I}}\right)\frac{1}{\bar{I}} - K_t\mu_{z,t}^{-1}\kappa_2\left(\frac{I_t}{K_t}\mu_{z,t} - \frac{\bar{I}}{\bar{K}}\right)\frac{\mu_{z,t}}{K_t}\right) = 0$$

simplify,

$$-\lambda_t + q_t\lambda_t\left(1 - \frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2 - \kappa_1\left(\frac{I_t-\bar{I}}{\bar{I}}\right)\frac{I_t}{\bar{I}} - \kappa_2\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)\right) = 0$$

$$q_t\left(1 - \frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2 - \kappa_1\left(\frac{I_t-\bar{I}}{\bar{I}}\right)\frac{I_t}{\bar{I}} - \kappa_2\left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}}\right)\right) = 1 \qquad \text{(A.80)}$$

Production Function

$$S_tY_tZ_t = A_t\left(Z_{t-1}K_t\right)^{\theta}\left(Z_tN_t\right)^{1-\theta}$$

$$S_tY_tZ_t = Z_t^{1-\theta}Z_{t-1}^{\theta}A_t\left(K_t\right)^{\theta}\left(N_t\right)^{1-\theta}$$

dividing by Z_t

$$S_tY_t = Z_t^{-\theta}Z_{t-1}^{\theta}A_t\left(K_t\right)^{\theta}\left(N_t\right)^{1-\theta}$$

and writing in terms of growth of Z_t,

$$S_tY_t = \mu_t^{-\theta}A_t\left(K_t\right)^{\theta}\left(N_t\right)^{1-\theta}$$

3.B.12 Trends

The economy has two sources of growth. Alongside the stochastic trend in productivity Z_t, the economy also faces a deterministic trend in the relative price of investment Υ_t with $\mu_{t,\Upsilon} = \log(\frac{\Upsilon_t}{\Upsilon_{t-1}})$. As in Altig et. al. (2011) we define the overall measure of the productivity in the economy as *(why this functional form I do not understand, also proof that this implies balanced growth path would help)*

$$Z_t^{+} = \Upsilon_t^{\frac{\alpha}{1-\alpha}}Z_t$$

which is in growth rates,

$$\frac{Z_t^{+}}{Z_{t-1}^{+}} = \frac{\Upsilon_t^{\frac{\alpha}{1-\alpha}}Z_t}{\Upsilon_{t-1}^{\frac{\alpha}{1-\alpha}}Z_{t-1}}$$

so the overall trend in the economy is characterized by

$$\mu_{t,z^{+}} = \frac{\alpha}{1-\alpha}\mu_{t,\Upsilon} + \mu_{t,z}$$

3.B.13 Steady state

$$S = (1-\zeta)\left(\frac{P^{*}}{P}\right)^{-\epsilon} + \zeta(\frac{\pi}{(\pi^{\iota}\pi^{1-\iota})})^{\epsilon}S$$

$$S - \zeta S = (1-\zeta)\left(\frac{P^{*}}{P}\right)^{-\epsilon}$$

$$S = \frac{(1-\zeta)\left(\frac{P^{*}}{P}\right)^{-\epsilon}}{1-\zeta} \tag{A.81}$$

$$K = YMC^r + \zeta\beta\left(\frac{\pi}{\pi}\right)^{\epsilon+1} K$$

$$K = \frac{YMC^r}{1-\zeta\beta} \tag{A.82}$$

$$MC = \frac{\left(R_t^k\right)^{\theta} W_t^{1-\theta}}{A_t \theta^{\theta}(1-\theta)^{1-\theta}}$$

$$W = \left[\frac{\theta^{\theta}(1-\theta)^{1-\theta}}{(R^k)^{\theta}} MC\right]^{\frac{1}{1-\theta}} \tag{A.83}$$

$$R_t^k K_t(j) = \frac{W_t N_t(j)}{\frac{1-\theta}{\theta}}$$

$$\frac{\theta WN}{(1-\theta)R^K} = K \tag{A.84}$$

$$\frac{1}{[R^k + (1-\delta)]} = \beta\mu^{-1}\frac{C^{-\gamma}}{C^{-\gamma}}\left(\frac{(V}{[E_t(V)^{1-\alpha}]^{\frac{1}{1-\alpha}}}\right)^{-\alpha} \tag{A.85}$$

$$\boxed{\frac{1}{\mu^{-1}\beta} - (1-\delta) = \left[R^k\right]} \tag{A.86}$$

$$Y = C + \mu_{t+1}K - (1-\delta)K + G$$

$$Y = C + (\mu_{t+1} - 1 - \delta)K + G$$

$$\boxed{Y\left[1 - (\mu_{t+1} - 1 - \delta)\frac{K}{Y} - \frac{G}{Y}\right] = C} \tag{A.87}$$

$$(\tilde{p}_t) = \frac{\epsilon}{\epsilon - 1}\frac{K_t}{F_t} \tag{A.88}$$

$$K = YMC^r + \zeta\beta\mu_z^{-1}\left(\frac{\pi}{\pi^{\iota}\pi^{1-\iota}}\right)^{\epsilon+1} K$$

$$K = \frac{YMC^r}{(1-\zeta\beta\mu_z^{-1})}$$

$$F = \frac{Y}{(1-\zeta\beta\frac{\pi}{\pi})}$$

$$(\tilde{p})\,\frac{\epsilon-1}{\epsilon}=\frac{\frac{YMC^r}{(1-\zeta\beta)}}{\frac{Y}{(1-\zeta\beta)}}$$

$$(\tilde{p})\,\frac{\epsilon-1}{\epsilon}=\frac{\frac{MC^r}{(1-\zeta\beta)}}{\frac{1}{(1-\zeta\beta)}}$$

$$(\tilde{p})\,\frac{\epsilon-1}{\epsilon}=\frac{(1-\zeta\beta)MC^r}{(1-\zeta\beta)}$$

$$(\tilde{p})\,\frac{\epsilon-1}{\epsilon}\frac{(1-\zeta\beta)}{(1-\zeta\beta)}=MC^r$$

using λ's,

$$(\tilde{p})\,\frac{1}{1+\lambda}=MC^r$$

$$\mu K_{t+1}=(1-\delta)K_t+I_t-I_t\frac{\kappa_1}{2}\left(\frac{I_t-\bar{I}}{\bar{I}}\right)^2-K_t\frac{\kappa_2}{2}\left(\frac{I_t}{K_t}-\frac{\bar{I}}{\bar{K}}\right)^2$$

$$\mu K=(1-\delta)K+I-0-0$$

$$\mu K-(1-\delta)K=I$$

$$\boxed{I=(\mu-1+\delta)K}$$

$$-q\lambda+\beta\left(\lambda r^k+q\lambda\left[(1-\delta)\right]\right)=0$$

dividing by λ,

$$-q+\beta\left(r^k+q\left[(1-\delta)\right]\right)=0$$

$$q-\beta q\left[(1-\delta)\right]=\beta r^k$$

$$q\left(1-\beta\left[(1-\delta)\right]\right)=\beta r^k$$

$$q=\frac{\beta r^k}{1-\beta\left[(1-\delta)\right]}$$

$$q_t \left(\frac{\kappa_1}{2} \left(\frac{I_t - \bar{I}}{\bar{I}} \right)^2 + \kappa_1 \left(\frac{I_t - \bar{I}}{\bar{I}} \right) \frac{I_t}{\bar{I}} + \kappa_2 \left(\frac{I_t}{K_t} - \frac{\bar{I}}{\bar{K}} \right) - 1 \right) = 1 \tag{A.89}$$

$$q\,(1) = 1 \tag{A.90}$$

$$\boxed{q = 1}$$

$$-1 = \frac{\beta r^k}{\beta(1-\delta) - 1}$$

$$1 - \beta(1-\delta) = \beta r^k$$

$$\boxed{r^k = \frac{1}{\beta} - (1-\delta)}$$

Chapter 4

Determinants of Fiscal Multipliers Revisited

4.1 Introduction

After the introduction of the $750 billion US fiscal stimulus package in 2009 there has been a renewed interest in the effectiveness of fiscal policy in the environment of ultra-low interest rates. Several authors show that the size of fiscal multipliers is significantly higher when the economy is at a zero lower bound (ZLB) of the nominal interest rate (see Eggertsson (2011), Erceg & Linde (2014), Christiano *et al.* (2011) or Woodford (2011)), making a case for the ability of fiscal policy to curb the adverse effects of financial crisis. The economic consensus on fiscal multipliers in normal times is, that they tend to be small. This is for two reasons: one, increases in government expenditure need to be financed, and thus come with a negative wealth effect, which crowds out consumption and decreases demand; two, a fiscal expansion, increasing inflation and output, triggers an endogenous response of the monetary authority, which raises interest rates, offsetting some of the expansionary effect of fiscal policy. In times when the economy is at the zero lower bound, such endogenous dampening response of monetary policy is absent, as the nominal interest rate stuck at the lower bound and thus constant; in such case, an increase in

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(expected) inflation, resulting from a fiscal expansion, leads to a drop in the real interest rate, which further stimulates demand and thus increases fiscal multipliers.

This paper extends the New Keynesian model of Eggertsson (2011) and studies the size of various types of fiscal multipliers, in normal times, when the nominal interest rate is positive, and when the economy is at the zero lower bound. We calibrate our model to the US economy and study four different types of fiscal multipliers: a government spending, a payroll tax, a sales tax, and a financial asset tax multiplier. We document that the size of fiscal multipliers at the ZLB crucially depends on the slope of the Phillips curve, with a flatter Phillips curve being associated with smaller multipliers. This is because in the context of the New Keynesian model an, e.g., increase in government spending can raise output owing to a rise in expected inflation which, at the zero lower bound, decreases the real interest rate, stimulating consumption and output. A flatter Phillips curve attenuates the inflation channel and, thus, decreases the value of the multiplier. A sufficiently flat Phillips curve, consistent with recent empirical estimates, delivers a spending multiplier at or below one and a consumption tax cut multiplier that is strictly below one.

The reasons behind the flattening of the Phillips curve that we consider in our model are consistent with both the macroeconomic and microeconomic empirical evidence. In particular, we do not obtain a flatter Phillips curve from employing a higher degree of nominal rigidity; instead, it results from an increase in the degree of strategic complementarity in price-setting, invoked in the model through assumptions of (i) a specific labour market[1] and (ii) decreasing returns-to-scale in production. There is a growing macroeconomic literature suggesting a flattening of the Phillips curve (see, e.g., Blanchard *et al.* (2015), among others), i.e. a weaker link between economic activity and inflation. The reasons and implications of the flattening of the Phillips curve have been primarily examined for the (lack of) inflation after the crisis or more generally, for monetary policy strategy (Blanchard *et al.* (2015)). We document that this consideration is equally consequential for fiscal multipliers. This macroeconomic literature on the flattening of the Phillips curve

[1]In general, the labour market can be modeled either as an economy-wide or specific labour market. An economy-wide labour market (one type of labour for all firms) implies strategic substitutability in price-setting i.e. an individual firm which observes a rise in the prices of goods of the other firms will lower the price of its own good. In contrast, a specific factor market leads to the synchronisation of prices across firms which implies a case of strategic complementarity.

is supported by a growing microeconomic literature suggesting that strategic complementarity is an important factor in how firms set prices, and that a high degree of strategic complementarity results in a flat Phillips curve (Coricelli & Horvath (2010), Woodford (2003)). Using micro-level Belgian consumer prices data, Amiti *et al.* (2019) develop a general theoretical framework and empirical identification strategy to directly estimate firm price responses to changes in prices of their competitors. Their results suggest an elasticity of more than one-third in response to the price changes of its competitors (i.e. strategic complementarity) and an elasticity of nearly two-thirds in response to its own cost shocks. Interestingly, this 'strategic complementarity' elasticity increases to one-half for large firms.[2]

Our results suggest that the empirically relevant reasons for a flattening of the Phillips curve, that we incorporate in our model, lead to smaller fiscal multipliers at the ZLB. More generally, we present detailed results for multipliers for our four types of fiscal instruments, in both normal and ZLB times, and show how they are influenced by the different settings of specific versus economy-wise labour market and constant versus decreasing returns to scale.[3] We also present evidence that shows that the level of steady-state government spending-to-GDP ratio affects the size of the resulting multiplier.[4] Finally, we

[2]In addition, based on a survey conducted for nearly 11 000 firms in the Euro Area, Fabiani *et al.* (2006) find that the prices of around 30 percent of Euro Area firms are shaped by competitors prices, while the remaining 70 percent of the firms set prices according to markup (see Alvarez *et al.* (2006), where this result is discussed, too). Overall, this empirical evidence suggests that strategic complementarity plays an important role for firms' price setting behaviour. Strategic complementarity in price-setting also helps to jointly match the micro-evidence on the frequency of firms' price adjustment and the low estimates on the slope of the New Keynesian Phillips curve (NKPC) (see Linde & Trabandt (2018)). See Nakamura & Steinnsson (2008) who estimated a duration of price rigidity is about 2-3 quarters using US micro data. Estimates on the slope of the NKPC vary between 0.009-0.04 (see, e.g., Adolfson & Laséen (2005), Altig *et al.* (2011), Gali & Gertler (1999), Woodford (2003)).

[3]Our model version with decreasing returns in labour is equivalent to a model with firm-specific fixed capital (and variable input labour), which Altig *et al.* (2011) consider important in reconciling the micro-evidence on the frequency of price changes with the macro evidence on the slope of the Phillips curve. The decreasing returns to scale of technology implies a flatter Phillips curve, again giving rise to smaller multipliers compared to the constant-returns-to-scale assumption of Eggertsson (2011).

[4]Many influential papers, such as Eggertsson (2011) and Woodford (2011), assume a zero government spending-to-GDP ratio when calculating fiscal multipliers. However, US post-war data show that the government spending-to-GDP ratio ranges between 17-20 per cent. Not accounting for a positive government spending-to-GDP ratio distorts the correct size of the private consumption-to-GDP ratio based on the aggregate resource constraint and has an impact on the effective value of the elasticity of intertemporal substitution (IES). Using our model, we show that allowing for positive government spending-to-GDP ratio has non-negligible effects on the size of the government spending multiplier. Interestingly, this issue is largely overlooked in the empirical literature. For example, the existing meta-analyses on

present results from robustness checks in terms of the solution method used to compute fiscal multipliers, considering multipliers that are computed not only from a linear solution method but also from more accurate global solution methods.

Our work is closely related to Boneva *et al.* (2016) and Ngo (2019), who also study the consequences of a flattening of the Phillips curve for fiscal multipliers, which, however, in their setting is due to an increase in price rigidity parameters. Two further, recently published papers also emphasize the importance of the slope of the Phillips curve for the conduct of monetary policy at the zero lower bound, or for the value of the fiscal multiplier. Belgibayeva & Horvath (2019) explore how the degree of strategic complementarity in price-setting affects optimal monetary policy in a New Keynesian model with wage and price setting frictions. Linde & Trabandt (2018) find that strategic complementarity, introduced via a Kimball consumption basket instead of the constant-elasticity-of-substitution (CES) aggregator, accounts for the difference between the value of the multiplier calculated from the linear and non-linear solution of the model.

Other related contributions include Miao & Ngo (2019), who find that the multipliers behave differently in the non-linear Calvo and Rotemberg models. Surprisingly, they find that the mutliplier is increasing (decreasing) with the duration of the ZLB in the Calvo (Rotemberg) model. They also find that the spending multiplier is a non-linear function of the persistence of the government spending shock. Eggertsson & Singh (2011) argue that the multipliers do not differ a lot across the linear and non-linear New Keynesian models (with either Calvo or Rotemberg pricing) as long as we consider empirically realistic calibration of the models. Boneva *et al.* (2016) also show the sign and size of the multipliers with respect to the slope of the NKPC and the duration of the zero lower bound using the linear and non-linear New Keynesian model with Rotemberg pricing. Importantly, they show that the labour tax cut multiplier is negative for empirically realistic durations of the zero lower bound in the linear as well as the non-linear New Keynesian model. Ngo (2019) uses US data to calculate the unconditional probability of hitting the zero lower bound and calibrates a model with occasionally binding zero lower bound constraint. He finds a government spending multiplier of around 1.25, which is larger than the one in the model without occasionally binding constraint or transient government spending shocks. He also confirms the finding of Miao & Ngo (2019)

the fiscal multipliers do not mention the possible effect of government spending-to-GDP ratio on the size of multiplier (Gechert (2015) and Gechert & Rannenberg (2018)).

regarding the nonlinearity of the multiplier with respect to the persistence of the government spending shock. The focus of our paper differentiates us from the previous papers. In particular, we explore how the recent flattening of the Phillips curve as resulting from a higher degree of strategic complementarity, and show that this affects the size of fiscal multipliers significantly.

Hills & Nakata (2018) show that the government spending multiplier is very sensitive to the inclusion of interest rate smoothing in the Taylor rule. Once one allows for inertia in the interest rate rule, the multiplier decreases from 1.9 to 0.5. Leeper *et al.* (2017) estimate fiscal multipliers using Bayesian methods on US data. With several combinations of model specifications and different priors they find impact multipliers of about 1.4. Further, they find that multipliers are much higher in a regime with passive monetary and active fiscal policy relative to a regime with active monetary and passive fiscal policy.

The paper proceeds as follows. Section 2 lays out our modelling framework, while section 3 describes the equilibrium of the model. Section 4 discusses intuition and economic channels at play to help interpret fiscal multipliers. Section 5 focuses on the calibration of the model. Section 6 contains the numerical results as well as an explanation of the sign and magnitude of fiscal multipliers. Section 7 presents results from a non-linear solution method to verify robustness of our results. Section 8 provides concluding remarks. An Appendix with the model derivations can be found at the end of the paper.

4.2 The Log-Linear Model

We log-linearise a basic New Keynesian model as in Eggertsson (2011) around its non-stochastic zero inflation steady state. The New-Keynesian IS curve along with the log-linear aggregate resource constraint, $\hat{Y}_t = (1-g)\hat{C}_t + \hat{G}_t$, yields the aggregate demand curve:

$$\hat{Y}_t - E_t\hat{Y}_{t+1} = \hat{G}_t - E_t\hat{G}_{t+1} - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \check{\sigma}\chi^S\left[E_t\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \check{\sigma}\chi^A\hat{\tau}_t^A. \quad (4.1)$$

In the expression above, Y_t stands for output, C_t for consumption, i_t for nominal interest rate, π_t for inflation, G_t for government spending and $g \equiv 1 - \bar{C}/\bar{Y} = \bar{G}/\bar{Y} > 0$ is the steady state government spending-to-GDP ratio. Parameter $\sigma \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}\bar{C}}$ is the IES of consumption. $\check{\sigma} \equiv \sigma(1-g)$ is the IES re-scaled by the government spending-to-GDP ratio.

Variables with a hat are defined as: $\hat{Y}_t \equiv \log(Y_t/\bar{Y})$, $\hat{C}_t \equiv \log(C_t/\bar{C})$,

$\hat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$, $\hat{\tau}_t^i \equiv \tau_t^i - \bar{\tau}^i$, $i \in \{A, S, W\}$ and $r_t^e \equiv \log \beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$ where $\hat{\xi}_t \equiv \log(\xi_t/\bar{\xi})$.[5] The $\chi^S \equiv \frac{1}{1+\bar{\tau}^S}$, $\chi^A \equiv \frac{1-\beta}{1-\bar{\tau}^A}$ are constants scaling the sales and capital taxes.

The NKPC (or aggregate supply—AS curve) is given by:

$$\pi_t = \kappa \hat{Y}_t + \kappa\psi(\chi^W \hat{\tau}_t^W + \chi^S \hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t \pi_{t+1}, \tag{4.2}$$

with

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)\vartheta}{\alpha}; \vartheta \equiv \frac{\check{\sigma}^{-1} + \phi(1+\omega) - 1}{1 + \omega_y \theta}; \qquad \psi \equiv \frac{1}{\check{\sigma}^{-1} + \phi(1+\omega) - 1};$$

$$\omega_y \equiv \phi(1 + \mathcal{I}\omega) - 1; \qquad \omega \equiv \frac{\bar{v}_{ll}\bar{l}}{\bar{v}_l}; \qquad \chi^W \equiv \frac{1}{1 - \bar{\tau}^W}.$$

The production function is given by $y_t = l_t^{1/\phi}$.[6] ϕ governs the degree of the returns-to-scale in technology production ($\phi = 1$ is CRS, constant returns-to-scale; $\phi > 1$ is DRS, decreasing returns-to-scale). ω is the elasticity of the marginal disutility of work. ω_y is defined similar to ω but also allows for DRS (for CRS $\omega_y = \omega$). χ^W scales labour taxes. β is the discount factor which is used to discount future utilities and profit streams to the present and θ is the elasticity of substitution among intermediary goods. κ is called the slope of the NKPC.

The slope of the Phillips curve is governed by the assumption of the factor market.[7] It can be shown (see, e.g. Woodford (2003) and below) that the slope of the NKPC is smaller with a higher degree of strategic complementarity—firms adjust quantities more than prices in response to shocks. Consequently,

[5] $\hat{\tau}_t^A$ is defined such that a one percent increase in capital income per year is comparable with the tax on labour income.

[6] More generally the production function of firm i can be written as $y_t(i) = k_t(i) f(l_t(i)/k_t(i))$ where f is an increasing and concave function. We abstract from total factor productivity, as it is not in the focus of the present paper. Index i reflects the fact that either capital or labour can be firm-specific in our setup. In line with Woodford (2003, 2005, 2011) we make two assumptions. First, in the case of a specific labour market there exists a rental market for capital while the rental market does not exist in the case of an economy-wide labour market with firm-specific capital. Second, capital is normalised to one in the case of a specific labour market.

[7] Factor market means labour market in this paper. However, instead of assuming a firm-specific labour market we can arrive at similar results under the alternative assumption of a homogeneous (or economy-wide) labour market with firm-specific (fixed) capital and decreasing returns in production.

the impact of fiscal measures, which alter the marginal cost in the NKPC, on inflation and expected inflation is also smaller.

An economy-wide factor market (one type of factor for all firms) implies strategic substitutability in price-setting (or, equivalently, a steeper Phillips curve) i.e. an individual firm which experiences a rise in the prices of goods of the other firms will decrease the price of its own good. On the other hand, a specific factor market leads to the synchronization of prices across firms which implies a case of strategic complementarity. Strategic complementarity represents an important factor in how firms set prices (see empirical evidence for the US by Amiti *et al.* (2019) and for Europe by Fabiani *et al.* (2006)). An economy-wide factor market implies a steeper Phillips curve than a firm-specific one.

Let $\mathcal{I}$ be an indicator variable which takes the value of one when we assume strategic complementarity, owing to a specific labour market. The case of $\mathcal{I} = 0$ corresponds to the setup with an economy-wide labour market. $\vartheta < 1$ means that there is some degree of strategic complementarity which is supported by empirical evidence (see, Woodford (2003)). The case of strategic substitutability, $\vartheta > 1$, is not covered here because it is not supported by data.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 1$ the Eggertsson (2011) setup is derived. Note that only the content of parameters $\check{\sigma}$, κ, ϑ and ψ changes when we generalise Eggertsson (2011) for positive long-run government spending and DRS. Table (4.1) provides an overview how the slope of NKPC (κ) changes due to the various assumptions (economy-wide versus specific labour market and CRS versus DRS): estimates for the slope of New Keynesian Phillips curve vary between

Table 4.1: The effect of various labour market assumptions (economy-wide/specific or, equivalently, steeper/flatter Phillips curve) and production technology (constant or decreasing returns-to-scale) assumptions on the value of the slope of the New Keynesian Phillips curve.

	Economy-wide	Specific
	$\mathcal{I} = 0$	$\mathcal{I} = 1$
CRS ($\phi = 1$)	0.1999	0.0095
DRS ($\phi = 1.5$)	0.0386	0.0076

0.0076-0.1999 (see e.g. Linde & Trabandt (2018) for a collection of estimates for the US). We make the following observations. First, we do not consider the economy-wide labour market with CRS to calculate fiscal multipliers because the slope of the NKPC in that case is out of range of the empirical estimates. Second, DRS is a substantial source of strategic complementarity even in the case of an economy-wide labour market. Third, a specific labour market implies a substantial degree of strategic complementarity with either CRS or DRS. It is important to note that the flattening of the Phillips curve could, alternatively, occur due to a rise in price rigidity parameter as analyzed in Boneva *et al.* (2016) and Ngo (2019).

Monetary policy follows Taylor rule, generalized to allow for the case of a zero lower bound:

$$i_t = \max\{0, r_t^e + \phi_\pi \pi_t + \phi_Y \hat{Y}_t\}, \tag{4.3}$$

where $\phi_\pi > 1$ and $\phi_Y > 0$ and the max operator refers to the zero lower bound on the nominal interest rate.

4.3 Description of the Equilibrium

We analyse a short-run and a long-run equilibrium. Initially, we are in steady state ($t = 0$). Then, from time $t = 1$, for some interval, $0 < t < T$, which we can call the *short-run* (see subscript S), a shock hits the economy. That is, when $t < T$ the shock is described by an exogenous decrease in $r_t^e = r_S^e < 0$ with T denoting the stochastic date at which the shock vanishes.

In period t, the shock persists with probability μ or dies out with $1 - \mu$ for all $t < T$. In the short-run, the zero lower bound on nominal interest can be either binding ($i_t = i_S = 0$) or not binding ($i_t = i_S > 0$). In the non-binding case, the nominal interest is governed by the Taylor rule. For time, $t \geq T$, variables take on their *long-run* steady-state values. We proceed to describe the equilibria under positive and zero nominal interest rates.

Positive Interest rate. We assume that inflation and output are linear functions of the fiscal variables, $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$:

$$\pi_S = A_\pi \hat{F}_S, \tag{4.4}$$

$$\hat{Y}_S = A_Y \hat{F}_S, \tag{4.5}$$

where A_π and A_Y are coefficients to be determined.

The fiscal instrument F follows an AR(1) process:

$$F_{t+1} = F_t^{\rho} \exp(\varepsilon_{t+1}) \tag{4.6}$$

where ρ measures persistence and ε is an *i.i.d.* shock with zero mean and constant variance.

The fiscal multipliers are computed separately, e.g., a sales tax cut is computed under the assumption of no change in other fiscal instruments. Also, we assume that changes in spending (or taxes) are offset by present or future lump-sum taxes/transfers, i.e. the Ricardian evidence holds.

Zero nominal interest rate. In period t and $t+1$ variable $\hat{X}_i = \{\hat{F}_i, \hat{Y}_i, \pi_i\}$ with $\hat{F}_i = \{\hat{G}_i, \hat{\tau}_i^W, \hat{\tau}_i^S, \hat{\tau}_i^A\}$ for $i \in \{t, t+1\}$ are taking, respectively, the following values:

$$\hat{X}_t = \begin{cases} \hat{X}_t = \hat{X}_S, & 0 < t < T, \text{ zero bound binding,} \\ \hat{X}_t = 0, & t \geq T, \text{ zero bound not binding,} \end{cases}$$

and

$$\hat{X}_{t+1} = \begin{cases} (1-\mu)\hat{X}_S = 0, & \text{with probability } 1-\mu, \ \hat{X}_{t+1} \text{ reverts to steady state,} \\ \mu\hat{X}_S, & \text{with probability } \mu \text{ zero bound continues to bind.} \end{cases}$$

It is necessary to formulate conditions under which the zero bound binds. Condition *C1* ensures that the shock in r_S is large enough to make the zero bound binding even with an expansionary fiscal policy:[8]

$$\begin{aligned} r_t^e < & -\frac{\kappa\check{\sigma}^{-1}(1-\mu)(\check{\sigma}-\psi)\phi_\pi + [(1-\mu)(1-\beta\mu) - \kappa\psi\mu]\phi_Y}{\kappa\mu(\phi_\pi - \mu) + [1 + \check{\sigma}\phi_Y - \mu](1-\beta\mu)}[\hat{G}_t - \check{\sigma}\chi^S\hat{\tau}_t^S] \\ & - \frac{(1-\mu)\kappa\psi\phi_\pi + \check{\sigma}\mu\kappa\psi\phi_Y}{\kappa\check{\sigma}(\phi_\pi - \mu) + [1 + \check{\sigma}\phi_Y - \mu](1-\beta\mu)}\chi^W\hat{\tau}_t^W \\ & - \frac{\phi_\pi\kappa + (1-\beta\mu)\phi_Y}{\kappa\check{\sigma}(\phi_\pi - \mu) + [1 + \check{\sigma}\phi_Y - \mu](1-\beta\mu)}\check{\sigma}\chi^A\hat{\tau}_t^A \end{aligned}$$

while condition *C2* makes sure that the crises do not last for too long[9]:

$$L(\mu) \equiv (1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa > 0. \tag{4.7}$$

[8] This condition can be derived by substituting equations (4.8) and (4.9) into the Taylor rule, equation (A.6).

[9] Condition *C2* also facilitates i) the avoidance of the deflationary black hole which would arise at $\bar{\mu}$ that satisfies $L(\bar{\mu}) = 0$ and ii) ensures that the coefficient on r_t^e in equation (4.11) is positive so that $r_t^e < 0$ is satisfied.

Proposition 4.1. *In the short-run when $i_S > 0$ and C1 does not hold, the equilibrium π_S, $\hat{Y}_S$ and i_S are described, respectively, by:*[10]

$$\pi_S = \mathcal{A}\hat{G}_S + \mathcal{B}\hat{\tau}_S^S + \mathcal{C}\hat{\tau}_S^W + \mathcal{D}\hat{\tau}_S^A, \qquad \mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D} > 0 \text{ (constants)}, \tag{4.8}$$

$$\begin{aligned}\hat{Y}_S = & \frac{\kappa\psi[\phi_\pi - \rho] + (1-\rho)(1-\beta\rho)}{[1+\check{\sigma}\phi_Y - \rho](1-\beta\rho) + \kappa\check{\sigma}[\phi_\pi - \rho]}\hat{G}_S \\ & - \frac{\kappa\psi(\phi_\pi - \rho) + (1-\rho)(1-\beta\rho)}{[1+\check{\sigma}\phi_Y - \rho](1-\beta\rho) + \kappa\check{\sigma}(\phi_\pi - \rho)}\check{\sigma}\chi^S\hat{\tau}_S^S \\ & - \frac{\kappa\psi\chi^W\check{\sigma}(\phi_\pi - \rho)}{[1+\check{\sigma}\phi_Y - \rho](1-\beta\rho) + \kappa\check{\sigma}(\phi_\pi - \rho)}\hat{\tau}_S^W \\ & + \frac{\check{\sigma}\chi^A(1-\beta\rho)}{[1+\check{\sigma}\phi_Y - \rho](1-\beta\rho) + \check{\sigma}\kappa(\phi_\pi - \rho)}\hat{\tau}_S^A\end{aligned} \tag{4.9}$$

and

$$i_S = i_S^e + \phi_\pi\pi_S + \phi_Y\hat{Y}_S. \tag{4.10}$$

Similarly, in the short-run when $i = 0$, C1 and C2 hold, the equilibrium is as follows:

$$\pi_S = \check{\mathcal{A}}\hat{G}_S + \check{\mathcal{B}}\hat{\tau}_S^S + \check{\mathcal{C}}\hat{\tau}_S^W + \check{\mathcal{D}}\hat{\tau}_S^A + \check{\mathcal{E}}r_S^e, \quad \check{\mathcal{A}},\check{\mathcal{B}},\check{\mathcal{C}},\check{\mathcal{D}},\check{\mathcal{E}} > 0 \text{ (constants)},$$

$$\begin{aligned}\hat{Y}_S = & \frac{(1-\mu)(1-\beta\mu) - \mu\kappa\psi}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{G}_S + \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^W \\ & - \frac{[(1-\mu)(1-\beta\mu) - \mu\kappa\psi]\chi^S}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\check{\sigma}\hat{\tau}_S^S + \frac{(1-\beta\mu)\chi^A}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^A \\ & + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e\end{aligned} \tag{4.11}$$

and

$$i_S = 0.$$

For the proof, we use the method of undetermined coefficients. In particular, we derive equation 4.9 through the combination of equations 4.1, 4.2 and 4.10. Equation 4.11 can be obtained using equations 4.1, 4.2 and $i_S = 0$. A similar procedure can be used to generate the expressions for inflation for both $i > 0$

[10] In the interest of space we do not report coefficients $\mathcal{A},\mathcal{B},\mathcal{C},\mathcal{D}$ ($\check{\mathcal{A}},\check{\mathcal{B}},\check{\mathcal{C}},\check{\mathcal{D}},\check{\mathcal{E}}$). To derive the fiscal multipliers it is sufficient to have the expressions for output.

and $i = 0$.

Note that the fiscal multiplier can be derived as $d\hat{Y}_S/d\hat{F}_S$ with $\hat{F}_S = \{\hat{G}_S, \hat{\tau}^W_S, \hat{\tau}^S_S, \hat{\tau}^A_S\})$ using equations (4.9) and (4.11) for $i > 0$ and $i = 0$ cases, respectively. We follow Eggertsson (2011) in assuming that the persistence parameters for the exogenous processes of fiscal instruments equal the parameter of the probability of remaining in a ZLB scenario, $\rho = \mu$. An approximate equilibrium that is correct up to the first order is a collection of stochastic processes for $\{\hat{Y}_t, \pi_t, i_t, r^e_t\}$ that solves equations (4.1)-(A.6) given paths for fiscal policy, $\{\hat{G}_t, \hat{\tau}^W_t, \hat{\tau}^S_t, \hat{\tau}^A_t\}$.

4.4 Intuition for the Multipliers

This section provides an illustration of the main mechanisms in our model to develop intuition for the section 6, where we present results on the values of multipliers for our four fiscal instruments, based on the calibration of our model reported in section 5.

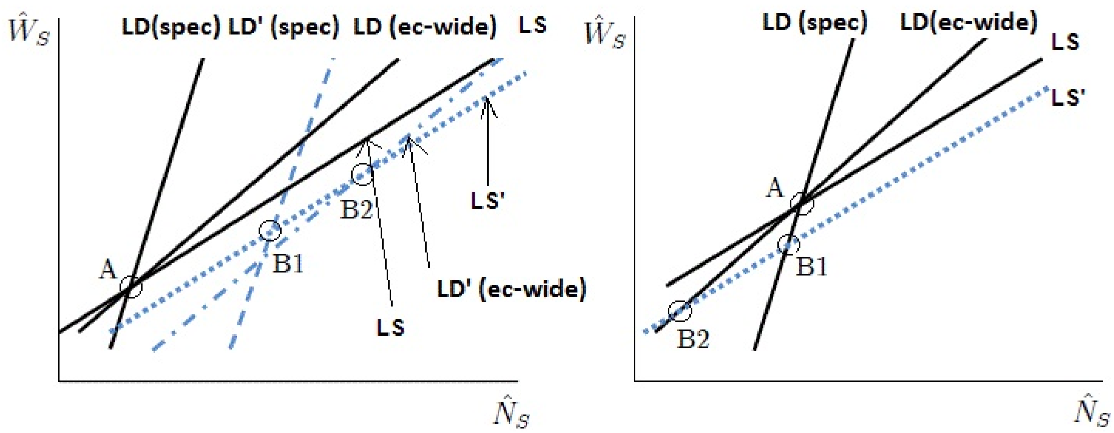
We start by discussing why the labour demand is upward-sloping at the peculiar environment of the zero lower bound. We then elaborate on the effects of the degree of strategic complementarity on the slope of the labour demand.

4.4.1 Upward-Sloping Labour Demand at the Zero Lower Bound

We build upon the intuition from Eggertsson (2011). To better understand the argument in case of the zero lower bound, it is useful to start with describing normal times, i.e. when the nominal interest rate is positive and is determined through an interest rate rule. In this case, labour demand is downward-sloping relationship in the real wage-labour system. The story could, alternatively, also be told in terms of aggregate demand (AD), which is a downward-sloping relationship in an inflation-output system. In such setting, a decrease in inflation implies that the nominal interest is cut more than the fall in inflation, in line with the logic of the Taylor rule (the coefficient on inflation is higher than one, $\phi_\pi > 1$, see the equation A.6). A lower nominal interest rate thus results in a lower real interest rate, stimulating aggregate demand. Thus, the labour demand or AD has a negative slope.

Woodford (2011) provides an alternative explanation for why, at positive interest rates, the government spending multiplier is equal to at most exactly one or below one at positive interest rates in the sticky-price model. The intuition for this proceeds as follows. The multiplier is exactly one as long as the real interest rate is fixed because consumption will not change through the Euler equation (the negative wealth effect of higher government spending on private consumption is eliminated). Then the spending multiplier can be simply derived from the aggregate resource constraint and takes on the value of one. When the real interest rate is allowed to change then higher spending will trigger a higher nominal and, thus, through the Taylor rule, real interest rate, crowding out private consumption. In this case, the multiplier is typically lower than one, as long as consumption and hours worked are separable in the utility function implying that they are substitutes[11].

Figure 4.1: Labour demand and supply at the zero lower bound



Notes: Left panel: an increase in government spending. Right panel: a decrease in labour tax. In both panels LD refers to labour demand while LS is labour supply. Spec. refers to specific while ec-wide refers to economy-wide labour market. An increase in government spending shifts both LS and LD to the right while the labour tax-cut shifts only the labour supply. The higher is strategic complementarity in price-setting (the case of specific labour market relative to economy-wide labour market) the steeper is the labour demand and the flatter is the Phillips curve.

The previous intuition changes at the zero lower bound: a reduction in in-

[11]Complementarity between consumption and hours worked can imply a multiplier of one or slightly higher than one with positive interest rates, see the discussion of Christiano *et al.* (2011).

flation is no longer counteracted by the Taylor rule. When the nominal interest rate is fixed, a deflationary policy implies higher real interest rates, depressing labour demand and aggregate demand. Figure 4.1 provides a graphical illustration of the effects of higher government purchases and lower taxes on labour demand and supply at the zero lower bound. The left (right) panel of Figure 4.1 shows the effects of higher government purchases (lower labour taxes) on the labour demand and supply. The initial situation is denoted by solid lines. The labour tax-cut does not have an effect on the labour demand (or AD) equation while government purchases affects both LD and LS.

A labour tax-cut which reduces marginal costs[12], shifts labour supply to the right, and is thus deflationary. Contrary to the conventional wisdom of New Keynesian models in normal times, the model predicts that cuts in the payroll tax are contractionary at the zero lower bound.

Next, we proceed to study the effects of higher government expenditure which affects both LD and LS. Higher government spending has a strong negative wealth effect, making the representative household reduce consumption and leisure, as both of them are normal goods. The decrease in leisure automatically leads to a rise hours worked, as the time endowment is fixed. In other words, the household wants to insure against the negative wealth effect by working more (LS shifts to the right). Despite crowding out consumption, the higher government spending raises aggregate demand overall, which would induce firms to raise their prices in a flexible price environment. However, because firms face nominal rigidities in their price setting, output is demand determined, and firms respond to higher aggregate demand by producing more: they demand more labour, so that LD shifts to the right.

4.4.2 The Degree of Strategic Complementarity and the Size of Multipliers

To highlight the importance of the degree of strategic complementarity for the size of fiscal multipliers we study the labour market equilibrium analytically and graphically. Combining the log-linear Euler equation, the NKPC and market clearing equations, we obtain the inverse labour demand curve:

$$\hat{W}_S = \Lambda\mu\phi^{-1}\hat{N}_S - \Lambda(1-\mu)^{-1}r^e_S - \Lambda\breve{\sigma}^{-1}\hat{G}_S + \Lambda\chi^S\hat{\tau}^S_S - \Lambda\chi^A(1-\mu)^{-1}\hat{\tau}^A_S \quad (4.12)$$

[12]In our setup there is no technology shock, and production is a function of labour input only, so the real wage equals real marginal costs.

where $\Lambda \equiv \frac{1-\beta(1-\mu)}{\kappa\psi}$. Equation 4.12 shows that the slope of the labour demand is influenced by the degree of strategic complementarity in price setting. In particular, higher strategic complementarity lowers κ, i.e., flattens the Phillips curve, which raises the slope of the labour demand, Λ. labour demand is affected by the discount factor shock (see the r_S^e term in equation 4.12) while labour supply (see equation 4.13) is not. Government spending, $\hat{G}_S$, labour taxes, $\hat{\tau}_S^W$, and consumption taxes, $\hat{\tau}_S^S$ appear in both labour demand and supply equations while the tax rate on bonds, $\hat{\tau}_S^A$ shows up only in the labour demand equation.

Similarly, let us substitute the log-linear market clearing for consumption into the log-linear intratemporal condition to arrive at the inverse labour supply:

$$\hat{W}_S = \left[\frac{\omega\phi + \check{\sigma}^{-1}}{\phi}\right]\hat{N}_S + \chi^W\hat{\tau}_S^W + \chi^S\hat{\tau}_S^S - \check{\sigma}^{-1}\hat{G}_S. \tag{4.13}$$

Equation 4.13 shows that the value of κ does not influence labour supply. However, it enters labour demand through Λ. For the rest of this sub-section we assume that there is DRS in both types of labour market. It remains true that strategic complementarity is higher with firm-specific labour market. Formally, this means that the value of κ in case of an economy-wide labour market –denoted as κ^{ew}– is higher than the κ under firm-specific labour market – denoted κ^{sp}):

$$\kappa^{\text{sp}} < \kappa^{ew}. \tag{4.14}$$

To see why inequality (4.14) is true one can recall the definitions of κ^{sp} and κ^{ew}:

$$\kappa^{\text{sp}} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi(1+\omega)-1+\check{\sigma}^{-1}}{1+\omega_y\theta}; \quad \kappa^{ew} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\frac{\phi(1+\omega)-1+\check{\sigma}^{-1}}{1+(\phi-1)\theta}, \tag{4.15}$$

where the difference between κ^{sp} and κ^{ew} lies in their denominator:

$$\omega_y \equiv \phi(1+\omega)-1 > (\phi-1). \tag{4.16}$$

The latter is always satisfied because $\omega > 0$. It follows that $\Lambda^{ew} < \Lambda^{\text{sp}}$ holds, and the slope of the labour demand under a firm-specific factor market is higher than the one with an economy-wide labour market:

$$\left(\frac{\partial\hat{W}_S}{\partial\hat{N}_S}\right)_{ew} < \left(\frac{\partial\hat{W}_S}{\partial\hat{N}_S}\right)_{\text{sp}}.$$

Looking at the labour demand (LD) and supply (LS) equations (4.12) and (4.13) we find:

$$(\breve{\sigma}^{-1})^{LS} < (\Lambda\breve{\sigma}^{-1})^{LD-ew} < (\Lambda\breve{\sigma}^{-1})^{LD-\text{sp}}. \tag{4.17}$$

Let us return to the case of an increase in government spending, which is depicted on the left panel of Figure (4.1). The relation in equation (4.17) tells us that a rise in government purchases will change wages more, ceteris paribus, through the labour demand (see the constant terms multiplying $\hat{G}_S$ in equation (4.12)) in the specific labor market case relative to the economy wide case. Hence, the labour demand curve in the firm-specific labour market setting ($LD-$sp) shifts to the right by more than the labour demand under the economy-wide factor market ($LD-ew$ see dashed-dotted line). The relation in equation (4.17) also indicates that labor supply shifts less than labor demands (with either economy-wide or firm-specific labor markets).

Figure (4.1) also shows that labour expands more under the economy-wide factor market as the labour demand curve in the economy wide case is flatter than the firm-specific one (see equilibrium points B2 and B1, respectively). Overall, we conclude that the rise in labour demand and supply due to higher government purchases leads to higher output produced under economy-wide labour market relative to the firm-specific labour market. Intuitively, the higher is the slope of the NKPC, the higher is the rise in inflation, resulting from increases in the marginal cost (through the NKPC) and, thus, the lower is the real interest rate stimulating private spending at the ZLB.

The right panel of Figure (4.1) displays the effects of a cut in labour taxes. The labour tax rate appears only in the labour supply equation, so that labour demand is not affected. Due to the fact that labour demand in the economy-wide case is flatter, the rightward shift of the labour leads to larger recession (see equilibrium point B2) relative to the specific factor market outcome (B1).[13] Alternatively, this can be explained as follows. The labour tax cut decreases

[13]Note that, at the zero lower bound, the response of labour to a payroll tax decrease is undoubtedly negative for the case of the linear solution described here (due to the omission of labour contracts from the model, i.e. lack of a downward nominal wage rigidity). This is also the case in the linear solution of Eggertsson (2011). More generally, however, this may not be the case in the exact nonlinear environment. Boneva *et al.* (2016) show that, when using a fully nonlinear solution, a payroll tax cut leads to an *increase* in employment. We confirm the results of Boneva *et al.* (2016) using our global solution: based on the scenario computed in Table 4.5, we find that employment indeed slightly increases in response to the payroll tax decrease at the zero lower bound. Section 7 discusses our robustness checks and implied results on fiscal multipliers from the global method in detail.

marginal costs, and thus leads to a drop in inflation. This drop is larger in case of a steeper Phillips curve, so that it causes a deeper recession in the case of the economy-wide labour market. Note that the sales tax cut works similar to the increase in government spending, but has smaller positive effects. Capital tax cuts are deflationary, similar to labour tax cuts, but lead to multipliers close to zero.

4.4.3 The Effects of the Returns-to-Scale on the Value of the Multiplier

The assumption of either CRS or DRS technology is equivalent to assuming a lower or higher degree of strategic complementarity, respectively, and the previous arguments apply. Returns to scale are governed by parameter ϕ, where $1/\phi$ is the coefficient on labour in the production function, $y_t = l_t^{1/\phi}$. Having previously defined parameter $\omega_y \equiv \phi(1 + \mathcal{I}\omega) - 1$, one can shown that under CRS, with $\phi = 1$, $\omega_y \equiv 0$ for the case of an economy-wide labour market ($\mathcal{I} = 0$), and, $\omega_y \equiv \omega$, for the case of a specific labour market ($\mathcal{I} = 1$). Instead, under DRS, with $\phi > 1$, $\omega_y \equiv \phi - 1$ for the case of an economy-wide labour market ($\mathcal{I} = 0$), and, $\omega_y \equiv \phi(1+\omega)-1$, for the case of a specific labour market ($\mathcal{I} = 1$). It can thus be seen that, ω_y is, for each labour market assumption, larger in the case of DRS compared to CRS, so that according to equation (4.15) the Phillips curve slope is smaller. This is also summarised in Table 4.2. It is important to note that the economy wide labour market with DRS delivers a lower degree of strategic complementarity than specific labour market with CRS.

4.4.4 Introducing Positive Government Purchases-to-GDP Ratio

Instead of assuming zero government spending-to-GDP ratio as in previous papers (see e.g. Eggertsson (2011) and Woodford (2011)) we introduce a positive 20 per cent g ratio, which is in line with post-war US data. This also helps us to have a more reasonable calibration for the steady-state consumption-to-GDP ratio. The introduction of a positive government purchases-to-GDP ratio ($g > 0$) modifies the slopes of the demand and the supply of labour as well

as re-scales the size of the government spending. In the numerical exercises below (cf. Table 4.4), we find that the introduction of $g > 0$ has an only minor quantitative effects on the multipliers in case of positive nominal interest rate. However, in the case of constant nominal interest rate the multipliers are smaller in (absolute) value when $g > 0$, because positive g reduces the intertemporal elasticity of substitution (IES) and the representative agent responds less to changes in the real interest rate by changing its consumption. One can notice that higher g would raise the slope of the NKPC as well as the multiplier. So there are two opposing effects. The total of effect of higher g on the multiplier is negative, however. $\check{\sigma}$ governs the strength of the wealth effect of the government spending shock on consumption.
To see this more clearly recall the log-linear aggregate resource constraint, $\hat{Y}_t = (1-g)\hat{C}_t + \hat{G}_t$ and differentiate it with respect to $\hat{G}_t$. We obtain the government spending multiplier and it is apparent that it depends negatively on g:

$$\frac{d\hat{Y}_t}{d\hat{G}_t} = 1 + (1-g)\frac{d\hat{C}_t}{d\hat{G}_t}$$

The previous formula shows that the consumption multiplier, $\frac{d\hat{C}_t}{d\hat{G}_t}$, is scaled by g. Christiano *et al.* (2011) explain that lower values of $\check{\sigma}$ lead to lower government spending multipliers. In total, it seems that the second effect (wealth effect) dominates in the case of introducing $g > 0$.

4.5 Calibration

We follow Eggertsson (2011) who estimated the linearised model to match a 30 percent drop in output and a 10 percent drop in inflation, as experienced during the Great Depression. The values are summarised in Table 4.2:

Table 4.2: Parameterisation of the model

β	σ	ω	ρ	ϕ_π	ϕ_Y	$1/\phi$
0.9970	0.86	1.5692	0.9030	1.5	0.5/4	2/3
α	μ	g	$\bar{\tau}^S$	$\bar{\tau}^A$	$\bar{\tau}^W$	θ
0.7747	0.9030	0.2	0.05	0	0.2	12.7721

Notes: g is from Christiano et al. (2011). ϕ is from Woodford (2003).

In addition to the 'Great Depression'-scenario, Eggertsson and Singh (2016) also consider an additional empirically relevant calibration scenario, which is the 'Great Recession'-scenario, whereby US output and inflation dropped about -10 percent and -2 percent, respectively. In Table 4.5 we provide results based on a fully non-linear solution, for such a 'Great Recession'-sized output drop.[14] In the non-linear solution of the model one needs to assign values to the size of the fiscal shocks, which we set in the range of $[0.001(1-\beta), 0.01]$, which is consistent with the Bayesian estimates of Zubairy (2014) on post-war US data.

4.6 Results

Based on the calibration just outlined, we compute fiscal multipliers for a number of comparison scenarios, summarised in Tables 4.3 and 4.4. Four main results emerge.

Result 1. Table 4.3 documents, that under positive nominal interest ($i_t > 0$), the government spending and sales tax multipliers are higher the flatter the Phillips curve in the underlying model, or, respectively, the higher the degree of strategic complementarity. In particular, the government spending multiplier and the sales tax multiplier in Table 4.4 are given by 0.6772 and 0.4448 respectively, for the case of a high degree of strategic complementarity and a flat Phillips curve, coming from the assumption of a specific labour market ($\mathcal{I} = 1$). In contrast, for the low degree of strategic complementarity and steeper Phillips curve, coming from the assumption of an economy-wide labour market, the resulting multipliers are lower, 0.6108 and 0.4012, respectively. This is in line with the basic intuition on how the monetary authority reacts to the state of the economy, as described by the Taylor rule. Under a steep Phillips curve, when an expansionary fiscal policy shifts out the AD curve, the resulting inflation increase is relatively large. The central bank reacts to this increase in inflation with a relatively strong increase in the nominal interest rate, which (because this translates into an increase in the real interest rate in a world of sticky prices) contracts output and offsets part of the fiscally-driven expansion – because of the strong response of the monetary authority, the implied multipliers are relatively small. In contrast, when the Phillips curve is flat, inflation rises only little in response to the fiscal expansion, and the offsetting effect from monetary policy are mild – the implied multipliers are

[14] The output drop of 10 percent is achieved by choosing the size of the shock that puts the economy into a ZLB scenario, accordingly. Since we keep all parameters constant to the ones of Eggertsson (2011), reported in Table 4.2, and only vary one parameter (the size of the ZLB-shock), the inflation drop is not fully matched.

larger. It should be noted, however, that, while intuitive, there is no guarantee that the government spending or the sales tax multiplier are always larger under a flatter Phillips curve. E.g., Linnemann & Schabert (2003) show that for very persistent government spending increases, labour supply shifts out strongly, due to the negative wealth effect of the government spending shock (leisure decreases, so one has to work more). Recall from Figure 4.1 that the economy-wide labour market (the steep PC scenario) implied a flat LD curve. If the outward shift in labour supply is large because of a large negative wealth effect, it may actually be the case that the real wage, and, in consequence, marginal cost and inflation, all decrease. In this case, the endogenous response of monetary policy implies that the multiplier is larger for a steeper Phillips curve. Miao and Ngo (2019) and Ngo (2019) similarly document the described nonlinearities of the multiplier with respect to the persistence of the government spending shock. Even if we have now discussed various reasons for the directions in which fiscal multipliers differ across steep versus flat Phillips curve slopes, we want to emphasize that, overall, our results from Tables 4.3 and 4.4 indicate, that, in normal times, at positive interest rates, fiscal multipliers are similar across scenarios; the quantitative differences in the various multipliers in normal times are minor.

Result 2. When the zero lower bound on nominal interest becomes binding, the government spending, and the sales tax cut multipliers are higher in the case of a steeper slope of the Phillips curve, or, equivalently with a lower degree of strategic complementarity. Table 4.3 shows this to be the case for the economy-wide labour markets ($\mathcal{I} = 0$, steep PC, low degree of strategic complementarity): the spending multiplier equals 1.7350, the labour tax cut multiplier -0.3219, and the sales tax cut multiplier 1.1396. For the case of the firm-specific ($\mathcal{I} = 1$, flat PC, high degree of strategic complementarity) the resulting multipliers are 1.0767, -0.0336 and 0.7073, respectively.[15] This exercise implies that, in both cases, a unit of government purchases brings more than one unit of GDP, but more so when strategic complementarity is low. Whereas, the case of high degree of strategic complementarity leads to an

[15]Multipliers with either low or high degrees of strategic complementarity in the case of DRS are not directly comparable with $\mu = 0.903$ (the estimated value of Eggertsson (2011) and our baseline parametrisation) when $i_t = 0$ because C2 is not satisfied. Table 3, instead, uses a value of $\mu = 0.80$, under which C2 is satisfied again.

In the absence of a specific factor market ($\mathcal{I} = 0$), $g > 0$ and DRS ($\phi = 3/2$) the maximum value of μ that satisfies condition C2 is 0.85. For $\mu = 0.85$ the multiplier is implausibly large. Hence, we use the somewhat lower but empirically still plausible value of $\mu = 0.8$ of Christiano *et al.* (2011) for comparison.

Table 4.3: Fiscal multipliers with high ($\mathcal{I} = 1$: specific labour market; flat Phillips curve) and low ($\mathcal{I} = 0$: economy-wide labour market; steep Phillips curve) degree of strategic complementarity

	outside ZLB, DRS		ZLB, DRS	
	Strategic complementarity:			
Multipliers	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.6772	0.6108	1.0767	1.7350
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0173	0.0706	-0.0336	-0.3219
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.4448	0.4012	0.7073	1.1396
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0068	-0.0055	-0.0115	-0.0218

Notes: For the estimated value of μ in Eggertsson (2011) (our baseline calibration), condition C2 is not satisfied in case of a lower degree of strategic complementarity. Hence, the comparison is accomplished using a lower value of $\mu = .8$ from Christiano et al. (2011). The comparison is made for the case of DRS because C2 in the case of CRS and a lower degree of strategic complementarity is satisfied for the maximum of $\mu = .69$ which may be empirically implausible.

Table 4.4: The effect of constant-returns-to-scale (CRS, $\phi = 1$: steep Phillips curve) versus decreasing-returns-to-scale (DRS, $\phi = 1.5$: flat Phillips curve), and the effect of positive government spending-to-GDP ratio on the multipliers

	Constant Returns (steep PC)		Decreasing Returns (flat PC)	
Multipliers	no ZLB	ZLB	no ZLB	ZLB
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g = 0$	0.4650	2.2858	0.4447	1.9464
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5208	1.8182	0.5013	1.6366
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0815	-1.0242	0.0472	-0.4145
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3818	1.8768	0.3659	1.5982
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0104	-0.0863	-0.0107	-0.0622

Notes: Grey cells contain the values computed from the fiscal multiplier formulas of Eggertsson (2011). Each multiplier is calculated under the assumption of a specific labour market.

only mild multiplier effects (the multiplier is slightly higher than one).

Further, the payroll tax-cut multiplier is less negative in the case of a lower degree of strategic complementarity (see -0.03 in the same Table). The latter is consistent with Christiano (2011), who finds in a model similar to ours but containing wage rigidities, that the payroll tax-cut multiplier may be slightly negative or close to zero.

The empirical SVAR literature finds, however, labour tax cuts to have positive effects on the economy. Using the SVAR models with different identifying assumptions regarding tax shocks based on US data, Mertens & Ravn (2012) and Romer & Romer (2010) find that tax-cuts are stimulative. The model in our paper does not address the problem of the negative payroll tax-cut multiplier. Kaszab (2016) modifies the basic New Keynesian model by adding non-Ricardian households and wage rigidity and finds that this model extension changes the sign of the payroll tax-cut multiplier from negative to positive. Wieland (2019) provides empirical evidence on the contractionary effects of negative supply shocks, such as rises in oil prices and the Great East Japan earthquake at the zero lower bound. The standard New Keynesian model predicts the opposite: negative supply shocks are expansionary. Wieland (2019) argues that the inclusion of financial frictions in the New Keynesian model leads to the results in line with the empirical evidence.

Result 3. When the government spending-to-output ratio is positive ($g > 0$), multipliers are higher than with $g = 0$, in the case of positive interest rates for both CRS and DRS. At zero nominal interest rate the government spending multiplier is higher with CRS relative to DRS (irrespective of a positive or zero choice for g). In the case of zero nominal interest rate, the difference is larger between the size of government spending multipliers across CRS and DRS with $g = 0$ than with $g > 0$.

The comparison of the multipliers with positive or zero government spending-to-output ratio can be found in Table 4.4. This Table makes use of the baseline calibration of μ so that our results are comparable to the ones in Eggertsson (2011). The models of Eggertsson (2011) and Woodford (2011) calculate fiscal multipliers under the assumption of a zero steady-state government spending-to-GDP ratio ($g = 0$). Instead, in this paper we also consider the empirically more realistic case of positive steady-state government purchases-to-GDP ratio and show that $g > 0$ has non-negligible impact on the size of the government spending multiplier. When $g > 0$ the value of IES, $\check{\sigma} = \sigma(1 - g)$, declines and consumers are less willing to substitute present consumption for future consumption after the positive government spending shock, even if the negative wealth effect forces consumers to do so. Thus, a lower $\check{\sigma}$ results in a smaller consumption loss and a higher multiplier when $i > 0$.

In contrast, multipliers in the case of $i = 0$ become smaller with $g > 0$. When $i = 0$, expansionary fiscal policy leads to a rise in inflation, which –in the absence of a Taylor rule– implies a decline in the real rate. A smaller real rate serves as an incentive for households to consume more in the present and, thereby, increases the multiplier. However, as our results presented in Table 4.4 show, this incentive is less strong with smaller a IES ($\check{\sigma} < \sigma$ due to $g > 0$).

Result 4. Multipliers (in absolute value) in the case of DRS are lower than those for CRS irrespective of whether $i > 0$ or $i = 0$. The presence of DRS in production can itself imply strategic complementarity even in the absence of a specific labour market because DRS reduces κ (see the term, $(\phi - 1)\theta$, in the denominator of κ in Equation (4.2)). Multipliers in case of $i > 0$ do not differ a lot across CRS and DRS. However, for $i = 0$ we observe that the government spending multiplier in case of $g = 0$ with DRS (1.94) is lower than with CRS (2.28) and the largest is the difference for payroll tax cut (-1.02 and -0.41 for CRS and DRS, respectively).

4.7 Robustness Checks – Results on Fiscal Multipliers Obtained from Non-Linear Solution Method

This section presents results from a robustness exercise with respect to the solution method. So far, the results presented stem from a log-linear approximation, for which a closed-form solution can be derived. A number of authors have computed fiscal multipliers also in a fully non-linear setting[16], with somewhat differing findings. While Eggertsson & Singh (2011) find that multipliers from a linear model are similar to their non-linear counterparts, other contributions have found significant differences, namely, that multipliers tend to be smaller when computed from a non-linear method (see, e.g. Boneva *et al.* (2016) and Linde & Trabandt (2018)). As a consequence, we also derive numerical results, equivalent to the ones presented in Table 4.3 and 4.4, but computed from a global approximation method. The method used is time iteration (cf. Coleman (1990, 1991), which amounts to computing, given some

[16] A non-exhaustive list of references includes, Miao & Ngo (2019), Fernández-Villaverde *et al.* (2015), Boneva *et al.* (2016), Eggertsson and Singh (2016), Nakata (2017), Throckmorton & Richter (2016), Linde & Trabandt (2018), and Belgibayeva & Horvath (2019).

initial guesses, the solution to the exact non-linear system of first order and equilibrium conditions over a grid of fixed points, and then iterating on the guesses until convergence. We choose 31 gridpoints for the endogenous state variable (price dispersion) and 5 gridpoints for each of the four exogenous state variables, G_t, τ_t^W, τ_t^S, τ_t^A. The exogenous continuous AR(1) processes are discretized using the method of Rouwenhorst (1995). We use linear interpolation for computing the solution in between gridpoints. We iterate on guesses of the conditional expectation appearing in the Euler equation, and in the two auxiliary equations of the Calvo price setting problem. The algorithm is layed out in detail in Rabitsch (2012) and Rabitsch (2016). Tables 4.5 and 4.6 repeat the exercises of section 6 and present fiscal multipliers for various scenarios from the global solution. As is well known, in a non-linear setting, the size of shocks affects the solution and thus the size of fiscal multipliers. Unless noted otherwise, in the computations below we set $\sigma_{G,t} = 0.01$, $\sigma_{\tau^W,t} = 0.009$, $\sigma_{\tau^S,t} = 0.009$, and $\sigma_{\tau^A,t} = 0.001(1-\beta)$. Else, parameters take on the values summarised in the calibration section, Table 4.2. Table4.5 presents the results for the different degrees of strategic complementarity, from the assumptions of either a firm-specific ($\mathcal{I} = 1$) or an economy-wide ($\mathcal{I} = 0$) labour market. The upper part presents multipliers at the zero lower bound for a 'Great Depression' scenario, where the size of the shock that puts the economy into a ZLB is such that output drops by about 30 percent – the table also reports the size of the ZLB-shock, and the implied drops in output and (annualized) inflation in percent. Unfortunately, for the 'Great Depression' scenario, the solution for the economy-wide labour market ($\mathcal{I} = 0$) cannot be obtained at the given set of parameters. We do not find this surprising, as, in fact, because of the steepness of the Phillips curve in the economy-wide labor market setting under the given set of parameters, the implied changes in inflation that accompany a 30 percent drop in output, would be enormous.[17] Table 4.5 thus proceeds in two steps.

[17] To make this point more precisely: we also computed the sets of multipliers from a quasi-nonlinear solution, 'Occbin', of Guerrieri & Iacoviello (2015). In this case, a solution can be obtained, and it provides an indication of what may be the source of the difficulties of solving this model-scenario fully non-linearly: in the Occbin-solution of this scenario, an output drop of 30 would be accompanied by a 21 percent drop in inflation. This indicates a clear counterfactual behavior of the economy-wide model version under this set of parameters. One would, in fact, need to re-calibrate this model version, to obtain realistic scenarios of a -30 percent output and -10 percent inflation response. Eggertsson & Singh (2011) follow this strategy, estimating the set of parameters needed to achieve such Great Depression scenario (even though not for a model version of economy-wide labor markets). This is, however, not our main exercise. We are interested in portraying how fiscal multipliers are affected as the slope of the Phillips curve steepens. A re-calibration of the economy-wide model version,

The upper part, presenting the 'Great Depression'-scenario results for the case of $\mathcal{I} = 1$, allows contrasting the global results for this case to the multipliers obtained from the linear method (summarised in Table 4.3). The lower part compares multipliers from scenarios $\mathcal{I} = 1$ and $\mathcal{I} = 0$, for a setting where they can be computed in both cases (a 'Great Recession' scenario, of a ZLB-shock sized such that a drop of output of 10 percent results; in addition, the shock sizes are scaled by one-half their regular size). The latter scenario allows a direct comparison between the cases of flat versus steep Phillips curves (respectively, high versus low degrees of strategic complementarity) in the global solution. Finally, Table 4.6 presents the parallel set of results for the cases of CRS versus DRS – always under a 'Great Depression' scenario.

The following set of results emerges: multipliers in normal times, when the nominal interest rate is positive, are roughly similar in size compared to the multipliers obtained from the linear solution; when the interest rate is at a ZLB, the multipliers are typically substantially smaller than under the linear method throughout. We thus confirm the insights from Boneva *et al.* (2016) or Linde & Trabandt (2018). Nonetheless, almost all main results established in section 6 for the linear method, as well as the ordering of multipliers across the different scenarios, continue to hold. Table 4.5 documents that the government spending and sales tax multiplier in normal times is higher under a flat Phillips curve or high degree of strategic complementarity (Result 1). Table 4.5 and 4.6 document that, at the zero lower bound, multipliers are larger in absolute magnitude (compared to normal times), because the monetary authority no longer counteract the effects of a fiscal stimulus at fixed nominal interest rates; now, a steeper Phillips curve implies a larger inflation increase in response to a fiscal expansion, so that multipliers are larger in this case (Result 2). We continue to find that government spending multipliers computed for the case of a positive government-spending-to-GDP ratio ($g > 0$) exceed their counterparts when $g = 0$ in normal times, at positive interest rates (Result 3). Unlike in the results based on the linear method, this situation does not change when turning to times of a binding ZLB: they continute to be larger for the case of $g > 0$ compared to $g = 0$. Finally, multipliers continue to be lower under DRS than under CRS, irrespective of whether $i > 0$ or $i = 0$ (see Result 4).

so that the inflation response is more in line with the experience in the Great Depression would then require a parameter combination that implies a somewhat less steep Phillips curve again.

Table 4.5: Results from the global solution: fiscal multipliers with high ($\mathcal{I} = 1$: specific labour market; flat Phillips curve) and low ($\mathcal{I} = 0$: economy-wide labour market; steep Phillips curve) degree of strategic complementarity

Multipliers	Strategic complementarity: High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)	High degree ($\mathcal{I} = 1$) (flat PC)	Low degree ($\mathcal{I} = 0$) (steep PC)
			Great Depression scenario	
	outside ZLB, DRS		ZLB, DRS	
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5764	–	1.3266	–
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0240	–	-0.0706	–
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3081	–	0.4220	–
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0098	–	-0.0155	–
Size of ZLB-shock,	–	–	0.1137	–
implied change in Y,	–	–	-30.0154	–
implied change in π	–	–	-5.0099	–
			Great Recession scenario	
	outside ZLB, DRS		ZLB, DRS	
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5555	0.4928	0.8925	1.0151
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0381	0.0990	-0.0245	-0.1437
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.2925	0.2626	0.3962	0.4640
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0085	-0.0057	-0.0135	-0.0184
Size of ZLB-shock,	–	–	0.0436	0.0309
implied change in Y,	–	–	-10.0017	-10.0000
implied change in π	–	–	-1.5657	-8.0830

Table 4.6: Results from the global solution: The effect of constant-returns-to-scale (CRS, $\phi = 1$: steep Phillips curve) versus decreasing-returns-to-scale (DRS, $\phi = 1.5$: flat Phillips curve), and the effect of positive government spending-to-GDP ratio on the multipliers

	Constant Returns (steep PC)		Decreasing Returns (flat PC)	
Multipliers	no ZLB	ZLB	no ZLB	ZLB
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g = 0$	0.4679	1.4380	0.4490	1.3929
Gov. spending, $\frac{d\widehat{Y}_t}{d\widehat{G}_t}, g > 0$	0.5922	1.5150	0.5761	1.4268
Payroll tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^W}$	0.0665	-0.4269	0.0387	-0.1563
Sales tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^S}$	0.3850	0.7988	0.3700	0.7852
Capital tax cut, $\frac{d\widehat{Y}_t}{-d\widehat{\tau}_t^A}$	-0.0109	-0.0366	-0.0111	-0.0294
Size of ZLB-shock,	–	0.0184	–	0.0265
implied change in Y,	–	-30.0046	–	-30.0006
implied change in π	–	-8.3078	–	-5.3877

4.8 Concluding Remarks

We generalize the New Keynesian model of Eggertsson (2011), calibrate it to US data and show how the size of fiscal multipliers depends on the slope of the Phillips curve. The variations in the slope of the Phillips curve we consider result from differing degrees of strategic complementarity in price setting, from assuming either a firm-specific or an economy-wide labour market, or from considering a constant-returns-to-scale versus a decreasing-returns-to scale production function. Using our extended model, we calibrate two scenarios: a scenario of normal times, with positive interest rates, and a scenario of crisis times, in which a shock moves the economy temporarily into a state of a deep recession, at which the zero lower bound is binding.

The previous literature finds very high fiscal multipliers when the economy is at the zero lower bound. We show that the introduction of strategic complementarity reduces multipliers at the zero lower bound due to the fact that higher strategic complementarity decreases the slope of the Phillips curve and the fiscal stimulus induces less inflation and a smaller reduction in the real interest rate, which is the driver of private spending.

Outside the zero lower bound (in normal times) multipliers are not much different either with high or low degree of strategic complementarity and remain below one. The payroll tax-cut multiplier is also less negative (smaller in absolute value) in case of a higher degree of strategic complementarity at the zero lower bound. Overall, our findings suggest that the size of fiscal multipliers are quite sensitive to degree of strategic complementarity in price setting at the zero lower bound.

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4.A Technical Appendix

4.A.1 Abbreviations

Some notations are here to explain shorthands in this appendix:

$$\begin{aligned}\text{AD} &= \text{Aggregate Demand}\\ \text{AS} &= \text{Aggregate Supply}\\ g &\equiv \frac{G}{Y} \text{ steady-state government spending-to-GDP ratio}\\ \text{CRS} &= \text{constant returns to scale technology}\\ \text{DRS} &= \text{decreasing returns to scale technology}\end{aligned}$$

4.A.2 Derivation of the AD Curve when $g > 0$

Note that economy-wide or specific labour market will influence the AS curve (derived in detail below) and AD curve is only affected by the choice of g (positive or zero).

The AD curve is the loglinear version of Euler equation based on separable preferences. The consumption Euler equation can be written as:

$$E_t\left\{\frac{u'(Y_{t+1}-G^N_{t+1})}{u'(Y_t-G^N_t)}(1-\tau^A_{t+1})R^{-1}_{t+1}\right\} = \beta^{-1}E_t\left\{\frac{\xi_t}{\xi_{t+1}}\frac{(1+\tau^S_{t+1})}{(1+\tau^S_t)}\frac{P_{t+1}}{P_t}\right\}.$$

In the previous equation we substituted in the aggregate resource constraint ($Y_t = C_t + G_t^N$) for consumption (C_t).

The previous equation can be log-linearised around the zero inflation non-stochastic steady-state as:

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \check{\sigma}(i_t - E_t\pi_{t+1} - r_t^e) + (\widehat{G}_t - E_t\widehat{G}_{t+1}) + \check{\sigma}\chi^S(\widehat{\tau}_{t+1}^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A\widehat{\tau}_t^A. \quad \text{(A.1)}$$

In the previous equation the following definitions are applied:

$$\check{\sigma} \equiv -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}}\frac{\overline{C}}{\overline{Y}} = -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}}s_C = -\frac{\overline{u}_c}{\overline{u}_{cc}\overline{C}}(1-g) = \sigma(1-g),$$
$$\chi^S \equiv \frac{1}{1+\bar{\tau}^S}, \qquad \chi^W \equiv \frac{1}{1-\bar{\tau}^W}, \qquad \chi^A \equiv \frac{1-\beta}{1-\bar{\tau}^A}.$$

Variables with a hat denote percentage deviation from steady-state: e.g. $\widehat{Y}_t \equiv \log(Y_t/\bar{Y}) \approx (Y_t - \bar{Y})/\bar{Y}$ where the upper bar denotes steady-state. Note that government spending is denifed relative to steady-state GDP as in Eggertsson: $\widehat{G}_t \equiv (G_t - \bar{G})/\bar{Y}$. The tax rates are already in per cent so they are defined as deviation from their steady-states: $\hat{\tau}_t^i \equiv \tau_t^i - \bar{\tau}^i$, $i \in \{A, S, W\}$. The discount factor shock which makes the zero lower bound binding is defined as: $r_t^e \equiv \log\beta^{-1} + E_t(\hat{\xi}_t - \hat{\xi}_{t+1})$ where $\hat{\xi}_t \equiv \log(\xi_t/\bar{\xi})$. Inflation is defined as: $\pi_t = \log(P_t/P_{t-1})$.

Government spending is wasteful spending in our paper (denoted with superscript N in Eggertsson (2011), we simply dropped the superscript N from $\widehat{G}_t$). We can see that the introduction of positive g results in a redefinition of the intertemporal elasticity of substitution (the original IES is σ and the redefined one is $\check{\sigma} \equiv \sigma(1-g)$):

4.A.3 Derivation of AS Curves

Economy-Wide Labour Market ($g = 0$ and CRS)

It is the same as in Woodford (2011) who sketches the derivation. It can also be found more detailed in Woodford (2003). The AS curve for economy-wide factor market in Eggertsson (2011) is achieved by setting $\omega\theta = 0$ in the definition of κ which can be found in his footnote 13.

Firm-Specific Labour Market ($g = 0$ and CRS)

This is the same as the one in Eggertsson (2011).

Firm Specific Labour Market ($g > 0$ and DRS)

This is the most general case and it is derived here (note that the $g = 0$, CRS and economy-wide labour market are simply parameter restrictions of this more general setup). Let us start from the FOC of intermediary firm i (this is the optimality condition of the Calvo firm which chooses the price p_t^* optimally at time t taking into account with probability α it will stuck with this optimal price for T periods $T > t$:

$$\sum_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \left[\frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} mc_{t,T}(i)\right] = 0,$$

and let us focus on the terms in the square bracket, []:

$$\begin{aligned}
0 &= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{(W_T/P_T)}{(1/\phi)[l_T(j)]^{1/\phi-1}} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{\frac{1+\tau_T^S}{1-\tau_T^W} \frac{v_l(l_T(j))}{u_c(Y_T-G_T)}}{(1/\phi)[l_T(j)]^{(1-\phi)/\phi}} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{1+\tau_T^S}{1-\tau_T^W} \frac{v_l\left(l_T(j)\right)}{u_c(Y_T-G_T)} \phi [l_T(j)]^{(\phi-1)/\phi} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{1+\tau_T^S}{1-\tau_T^W} \frac{v_l\left(l_T(j)\right)}{u_c(Y_T-G_T)} \phi [Y_T(j)]^{(\phi-1)} \\
&= \frac{p_t^*}{P_T} - \frac{\theta}{\theta-1} \frac{1+\tau_T^S}{1-\tau_T^W} \frac{v_l\left(\left[\left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T\right]^{\phi}\right)}{u_c(Y_T-G_T)} \phi \left[\left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T\right]^{(\phi-1)}
\end{aligned}$$

where in the first line we made us of the definition of the marginal cost: $mc_t = (W_t/P_t)/MPL_t$ with W_t/P_t meaning the real wage and MPL_t denoting the marginal product of labour derived from the DRS production function in the main text. Note that we substituted the intratemporal condition for the real wage in the second row and used the production function in the fourth row. The last row uses the demand curve of variety i.

Next we log-linearise the FOC[18] as follows:

$$\hat{p}_t^* - \log\left(\prod_{i=1}^{T}\Pi_{t+i}\right) = \frac{v_{ll}}{v_l}\bar{l}\hat{l}_T - \frac{u_{cc}C}{u_c}\frac{Y}{C}\hat{Y}_T + (\phi-1)\hat{Y}_T + \frac{u_{cc}}{u_c}C\frac{Y}{C}\hat{G}_T$$
$$+ \left[-\theta\phi\frac{v_{ll}\bar{l}}{v_l} - \theta(\phi-1)\right]\left[\hat{p}_t^* - \log\left(\prod_{i=1}^{T}\Pi_{t+i}\right)\right]$$
$$+ \frac{1}{1+\tau^S}\hat{\tau}_T^S + \frac{1}{1-\tau^W}\hat{\tau}_T^W.$$

where $\hat{p}_t^* \equiv \log(p_t^*/P_t)$, $\hat{l}_T \equiv \frac{l_T-\bar{l}}{\bar{l}}$, $\hat{Y}_T \equiv \frac{Y_T-\bar{Y}}{\bar{Y}}$, $\hat{G}_T \equiv \frac{G_T-\bar{G}}{\bar{Y}}$, $\hat{\tau}_T^i \equiv \tau_T - \bar{\tau}$, $i = \{S, W\}$. Let us re-arrange some terms:

$$\left[1+\theta\phi\frac{v_{ll}\bar{l}}{v_l} + \theta(\phi-1)\right]\hat{p}_t^* = \frac{v_{ll}}{v_l}\bar{l}\hat{l}_T - \frac{u_{cc}C}{u_c}\frac{Y}{C}\hat{Y}_T + (\phi-1)\hat{Y}_T + \frac{u_{cc}}{u_c}C\frac{Y}{C}\hat{G}_T$$
$$+ \left[1+\theta\phi\frac{v_{ll}\bar{l}}{v_l} + \theta(\phi-1)\right]\sum_{\tau=t+1}^{T}\pi_\tau + \frac{1}{1+\tau^S}\hat{\tau}_T^S + \frac{1}{1-\tau^W}\hat{\tau}_T^W$$

where $\log\left(\prod_{i=1}^{T}\Pi_{t+i}\right) \equiv \sum_{\tau=t+1}^{T}\pi_\tau$. In the next, we introduce notations for the elasticities:

$$\hat{p}_t^*[1+\theta\phi\omega+\theta(\phi-1)] = \left[\omega\phi + \breve{\sigma}^{-1} + (\phi-1)\right]\hat{Y}_T - \breve{\sigma}^{-1}\hat{G}_T$$
$$+ [1+\theta\phi\omega+\theta(\phi-1)]\sum_{\tau=t+1}^{T}\pi_\tau + \chi^S\hat{\tau}_T^S + \chi^W\hat{\tau}_T^W$$

where $\breve{\sigma} \equiv -\frac{\bar{u}_c}{\bar{u}_{cc}C}\frac{C}{Y} = \sigma(1-g)$, $\omega \equiv \frac{v_{ll}\bar{l}}{v_l}$, $\chi^S \equiv \frac{1}{1+\bar{\tau}^S}$, $\chi^W \equiv \frac{1}{1-\bar{\tau}^W}$.

Further, let us work again with the full expression:

$$\hat{p}_t^* = (1-\alpha\beta)\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}\left[[1+\omega_y\theta]^{-1}\widehat{mc}_T + \sum_{\tau=t+1}^{T}\pi_\tau\right]$$
$$= \left(\frac{1-\alpha\beta}{1+\omega_y\theta}\right)\sum_{T=t}^{\infty}(\alpha\beta)^{T-t}E_t\widehat{mc}_T + \sum_{T=t+1}^{\infty}(\alpha\beta)^{T-t}\pi_T \qquad \text{(A.2)}$$

where $\omega_y \equiv \phi\omega + (\phi-1) = \phi(1+\omega) - 1$ and

$$\widehat{mc}_T = \left[\omega\phi + \breve{\sigma}^{-1} + (\phi-1)\right]\hat{Y}_T - \breve{\sigma}^{-1}\hat{G}_T + \chi^S\hat{\tau}_T^S + \chi^W\hat{\tau}_T^W.$$

[18]Note that it is enough to log-linearise the expression in the square bracket due to the fact that the steady-state in the squared bracket is zero and therefore all loglinear terms outside the bracket would be multiplied by zero.

Let us then quasi-difference the equation (A.2) to obtain:

$$\hat{p}_t^* = \left(\frac{1-\alpha\beta}{1+\omega_y\theta}\right)\widehat{mc}_t + \alpha\beta E_t\pi_{t+1} + \alpha\beta E_t\hat{p}_{t+1}^*$$

which together with the log-linear version of the price index,

$$\pi_t = \frac{1-\alpha}{\alpha}\hat{p}_t^*$$

results in what we call NKPC:

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}_t^W + \chi^S\hat{\tau}_t^S - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1} \tag{A.3}$$

where the parameters for separable preferences are

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)\vartheta}{\alpha}; \vartheta \equiv \frac{\phi(1+\omega)-1+\check{\sigma}^{-1}}{1+\omega_y\theta}; \qquad \psi \equiv \frac{1}{\phi(1+\omega)-1+\check{\sigma}^{-1}};$$

$$\omega_y \equiv \phi(1+\mathcal{I}\omega)-1; \qquad \omega \equiv \frac{\bar{v}_{ll}\bar{l}}{\bar{v}_l}; \qquad \chi^W \equiv \frac{1}{1-\bar{\tau}^W},$$

where $\mathcal{I}$ is an indicator variable which takes on the value of one when we assume specific labour market. DRS in production, $\phi > 1$, can also induce strategic complementarity even under economy-wide labour markets.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 1$ the setup of Eggertsson (2011) is obtained.

For $\phi = 1$, $g = 0$, $\mathcal{I} = 0$ the setup of Woodford (2011) is obtained.

4.B Derivation of Fiscal Multipliers in Tables 3 and 4

4.B.1 Short run, positive nominal interest, $i > 0$

To derive multipliers under positive nominal interest rate we re-write the AD curve using the method of undetermined coefficients:

$$\pi_S = A_\pi\hat{F}_S, \tag{A.4}$$

$$\hat{Y}_t = A_Y\hat{F}_S, \tag{A.5}$$

for $\hat{F}_S = \{\hat{G}_S, \hat{\tau}_S^W, \hat{\tau}_S^S, \hat{\tau}_S^A\}$ and the Taylor rule

$$i_t = r_t^e + \phi_\pi \pi_t + \phi_Y \hat{Y}_t, \tag{A.6}$$

to express output as a function of the fiscal variable $\hat{F}_S$. The fiscal multiplier is given by A_Y.

Government Spending

Let us substitute for π_t and $E_t\hat{Y}_{t+1}$ equations (A.4) and (A.5) and for i_{t+1} the Taylor rule (equation (A.6)) in the AD formula: (see equation (A.1)):

$$\begin{aligned}\widehat{Y}_t &= \rho A_Y \widehat{G}_t + (1-\rho)\widehat{G}_t - \check{\sigma}\left(r_t^e + \phi_\pi A_\pi \widehat{G}_t + \phi_Y \widehat{Y}_t - r_t^e\right)\\ &\quad + \check{\sigma} A_\pi \rho \widehat{G}_t + \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A \widehat{\tau}_t^A\end{aligned}$$

where we used the method of undetermined coefficients—described in the main text—when substituting $A_\pi \widehat{G}_t$ for π_t and $\rho A_Y \widehat{G}_t$ for $E_t\widehat{Y}_{t+1}$. The latter also made use of the fact that government spending—similarly to other fiscal instruments—follows an AR(1) process with a persistence parameter ρ. Under positive nominal interest rates the discount factor is not time-varying and does not deviate from its steady-state, $r_t^e = 0$.

In the next we plug in the guess for time $t+1$ variables:

$$\begin{aligned}[1+\check{\sigma}\phi_2]\hat{Y}_t &= A_Y \rho \widehat{G}_t - \check{\sigma} A_\pi \phi_\pi \widehat{G}_t + \check{\sigma} A_\pi \rho \widehat{G}_t + (\widehat{G}_t - \rho \widehat{G}_t)\\ &\quad + \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A \widehat{\tau}_t^A\end{aligned}$$

where we set fiscal instruments other than government spending equal to zero ($\widehat{\tau}_t^W = \widehat{\tau}_t^S = \widehat{\tau}_t^A = 0$) and obtain:

$$[1+\check{\sigma}\phi_2]\hat{Y}_t = A_Y \rho \widehat{G}_t - A_\pi \check{\sigma}[\phi_\pi - \rho]\widehat{G}_t + (1-\rho)\widehat{G}_t \tag{A.7}$$

To proceed we need a formula that replaces A_π as a linear function of A_Y. To do so, we need to re-write the NKPC using undetermined coefficients. First, recall NKPC and use equation (A.4) and (A.5) to substitute for $\hat{Y}_t$, π_t and π_{t+1} together with the AR(1) process for the fiscal shock.

$$(1-\beta\rho)A_\pi \widehat{G}_t = [\kappa A_Y - \kappa\psi\check{\sigma}^{-1}]\widehat{G}_t.$$

Then it follows that $A_\pi = \frac{\kappa A_Y - \kappa\psi\check{\sigma}^{-1}}{1-\beta\rho}$ that can be inserted into equation

(A.7):

$$\begin{aligned}[1+\check{\sigma}\phi_2]\hat{Y}_t &= A_Y\rho\widehat{G}_t - \frac{(\kappa A_Y - \kappa\psi\check{\sigma}^{-1})}{1-\beta\rho}\check{\sigma}[\phi_\pi-\rho]\widehat{G}_t + (1-\rho)\widehat{G}_t \\ &= \left[A_Y\rho - \frac{(\kappa A_Y - \kappa\psi\check{\sigma}^{-1})}{1-\beta\rho}\check{\sigma}[\phi_\pi-\rho] + (1-\rho)\right]\widehat{G}_t\end{aligned}$$

And

$$A_Y = \frac{A_Y\rho - \frac{(\kappa A_Y - \kappa\psi\check{\sigma}^{-1})}{1-\beta\rho}\check{\sigma}[\phi_\pi-\rho] + (1-\rho)}{1+\check{\sigma}\phi_2}$$

And

$$A_Y\left[1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\kappa\check{\sigma}[\phi_\pi-\rho]}{(1-\beta\rho)(1+\check{\sigma}\phi_2)}\right] = \frac{\kappa\psi[\phi_\pi-\rho]}{(1-\beta\rho)(1+\check{\sigma}\phi_2)} + \frac{(1-\rho)}{(1+\check{\sigma}\phi_2)}$$

Finally

$$\begin{aligned}A_Y &= \frac{\frac{\kappa\psi[\phi_\pi-\rho]}{(1-\beta\rho)(1+\check{\sigma}\phi_2)} + \frac{(1-\rho)}{(1+\check{\sigma}\phi_2)}}{1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\kappa\check{\sigma}[\phi_\pi-\rho]}{(1-\beta\rho)(1+\check{\sigma}\phi_2)}} \\ &= \frac{\kappa\psi[\phi_\pi-\rho] + (1-\rho)(1-\beta\rho)}{(1-\beta\rho)(1+\check{\sigma}\phi_2) - \rho(1-\beta\rho) + \kappa\check{\sigma}[\phi_\pi-\rho]}\end{aligned}$$

which is the same as the one reported by Eggertsson (2011). Note that extensions in our paper modify the content of σ and κ. In particular, when allowing for positive g, the σ changes to $\check{\sigma} \equiv \sigma(1-g)$. Further, the introduction of either DRS or specific labour market leads to lower κ implying higher degree of strategic complementarity in price-setting.

Labour Tax Cut

Recall the AD curve:

$$\begin{aligned}[1+\check{\sigma}\phi_2]\hat{Y} &= A_Y\rho\widehat{G}_t + \check{\sigma}r_t^e - \check{\sigma}A_\pi\phi_\pi\widehat{G}_t + \check{\sigma}A_\pi\rho\widehat{G}_t - \check{\sigma}r_t^e + (\widehat{G}_t - \rho\widehat{G}_t) \\ &+ \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) + \check{\sigma}\chi^A\widehat{\tau}_t^A.\end{aligned}$$

As we focus only on $\widehat{\tau}_t^W$only we can set $\widehat{\tau}_t^S = \widehat{\tau}_t^A = \widehat{G}_t = 0$:

$$[1+\check{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \check{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^W + \check{\sigma}A_\pi\rho\widehat{\tau}_t^W$$

and use NKPC to obtain $A_\pi = \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]$ which can be substituted back to the previous equation to arrive at:

$$[1+\check{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \check{\sigma}r_t^e - \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\check{\sigma}(\phi_\pi - \rho)\widehat{\tau}_t^W + \check{\sigma}r_t^e$$

or

$$[1+\check{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\widehat{\tau}_t^W - \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\check{\sigma}(\phi_\pi - \rho)\widehat{\tau}_t^W$$

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\check{\sigma}\phi_2} - \frac{\frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\check{\sigma}(\phi_\pi - \rho)}{1+\check{\sigma}\phi_2}\right]\widehat{\tau}_t^W$$

$$A_Y = \frac{A_Y\rho}{1+\check{\sigma}\phi_2} - \frac{\frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^W\right]\check{\sigma}(\phi_\pi - \rho)}{1+\check{\sigma}\phi_2}$$

$$\left[1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\frac{\kappa}{1-\beta\rho}\check{\sigma}(\phi_\pi - \rho)}{1+\check{\sigma}\phi_2}\right]A_Y = -\frac{\frac{\kappa}{1-\beta\rho}\psi\chi^W\check{\sigma}(\phi_\pi - \rho)}{1+\check{\sigma}\phi_2}$$

$$\begin{aligned}
A_Y &= \frac{-\frac{\kappa\psi\chi^W\check{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1+\check{\sigma}\phi_2)}}{1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\kappa\check{\sigma}(\phi_\pi-\rho)}{(1-\beta\rho)(1+\check{\sigma}\phi_2)}} \\
&= -\frac{\kappa\psi\chi^W\check{\sigma}(\phi_\pi - \rho)}{(1-\beta\rho)(1+\check{\sigma}\phi_2) - \rho(1-\beta\rho) + \kappa\check{\sigma}(\phi_\pi - \rho)} \\
&= -\frac{\kappa\psi\chi^W\check{\sigma}(\phi_\pi - \rho)}{(1-\beta\rho)(1-\rho+\check{\sigma}\phi_2) + \kappa\check{\sigma}(\phi_\pi - \rho)}
\end{aligned}$$

which is the same as the one reported in the main text.

Sales Tax Cut

Recall the AD curve:

$$\begin{aligned}
[1+\check{\sigma}\phi_2]\hat{Y} &= A_Y\rho\widehat{\tau}_t^S - \check{\sigma}r_t^e - \check{\sigma}A_\pi\phi_\pi\widehat{\tau}_t^S + \check{\sigma}A_\pi\rho\widehat{\tau}_t^S + \check{\sigma}r_t^e + \check{\sigma}\chi^S(\rho\widehat{\tau}_t^S - \widehat{\tau}_t^S) \\
&= A_Y\rho\widehat{\tau}_t^S - \check{\sigma}A_\pi(\phi_\pi - \rho)\widehat{\tau}_t^S + \check{\sigma}\chi^S(\rho - 1)\widehat{\tau}_t^S \\
&= \left[A_Y\rho - \check{\sigma}A_\pi(\phi_\pi - \rho) + \check{\sigma}\chi^S(\rho - 1)\right]\widehat{\tau}_t^S
\end{aligned}$$

Using the NKPC $A_\pi = \frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^S\right]$:

$$[1+\check{\sigma}\phi_2]\hat{Y} = \left[A_Y\rho - \check{\sigma}\frac{\kappa}{1-\beta\rho}\left[A_Y + \psi\chi^S\right](\phi_\pi - \rho) + \check{\sigma}\chi^S(\rho - 1)\right]\widehat{\tau}_t^S$$

Now we can express for A_Y by collecting terms on both RHS and LHS:

$$A_Y = \frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \breve{\sigma}\frac{\kappa}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}\left[A_Y + \psi\chi^S\right](\phi_\pi - \rho) + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}$$

$$A_Y\left[1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}\right] = -\frac{\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)} + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}$$

$$\begin{aligned} A_Y &= -\frac{\frac{\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)} + \frac{\breve{\sigma}\chi^S(\rho-1)}{1+\breve{\sigma}\phi_2}}{1 - \frac{\rho}{1+\breve{\sigma}\phi_2} + \frac{\kappa\breve{\sigma}(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)}} \\ &= \frac{-\kappa\breve{\sigma}\psi\chi^S(\phi_\pi-\rho) - \breve{\sigma}\chi^S(1-\rho)(1-\beta\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho) - \rho(1-\beta\rho) + \kappa\breve{\sigma}(\phi_\pi-\rho)} \\ &= \frac{-[\kappa\psi(\phi_\pi-\rho) + (1-\rho)(1-\beta\rho)]\breve{\sigma}\chi^S}{(1+\breve{\sigma}\phi_2-\rho)(1-\beta\rho) + \kappa\breve{\sigma}(\phi_\pi-\rho)} \end{aligned}$$

which is the same as the one reported in the main text.

Capital Tax Cut

Recall AD curve:

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\hat{\tau}_t^A - \breve{\sigma}r_t^e - \breve{\sigma}A_\pi\phi_\pi\hat{\tau}_t^A + \breve{\sigma}A_\pi\rho\hat{\tau}_t^A + \breve{\sigma}r_t^e + \breve{\sigma}\chi^A\hat{\tau}_t^A$$

or

$$[1+\breve{\sigma}\phi_2]\hat{Y}_t = A_Y\rho\hat{\tau}_t^A - \breve{\sigma}A_\pi\phi_\pi\hat{\tau}_t^A + \breve{\sigma}A_\pi\rho\hat{\tau}_t^A + \breve{\sigma}\chi^A\hat{\tau}_t^A$$

or

$$\begin{aligned} [1+\breve{\sigma}\phi_2]\hat{Y}_t &= \left[A_Y\rho - \breve{\sigma}A_\pi(\phi_\pi-\rho) + \breve{\sigma}\chi^A\right]\hat{\tau}_t^A \\ \hat{Y}_t &= \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\breve{\sigma}A_\pi(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2} + \frac{\breve{\sigma}\chi^A}{1+\breve{\sigma}\phi_2}\right]\hat{\tau}_t^A \end{aligned}$$

and using NKPC, $A_\pi = \frac{\kappa A_Y}{1-\beta\rho}$:

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\breve{\sigma}\frac{\kappa A_Y}{1-\beta\rho}(\phi_\pi-\rho)}{1+\breve{\sigma}\phi_2} + \frac{\breve{\sigma}\chi^A}{1+\breve{\sigma}\phi_2}\right]\hat{\tau}_t^A$$

or

$$\hat{Y}_t = \left[\frac{A_Y\rho}{1+\breve{\sigma}\phi_2} - \frac{\breve{\sigma}\kappa A_Y(\phi_\pi-\rho)}{(1+\breve{\sigma}\phi_2)(1-\beta\rho)} + \frac{\breve{\sigma}\chi^A}{1+\breve{\sigma}\phi_2}\right]\hat{\tau}_t^A$$

And we can express for A_Y:

$$A_Y \left[1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\check{\sigma}\kappa(\phi_\pi - \rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho)}\right] = \frac{\check{\sigma}\chi^A}{1+\check{\sigma}\phi_2}$$

Finally,

$$\begin{aligned} A_Y &= \frac{\frac{\check{\sigma}\chi^A}{1+\check{\sigma}\phi_2}}{1 - \frac{\rho}{1+\check{\sigma}\phi_2} + \frac{\check{\sigma}\kappa(\phi_\pi-\rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho)}} \\ &= \frac{\check{\sigma}\chi^A(1-\beta\rho)}{(1+\check{\sigma}\phi_2)(1-\beta\rho) - \rho(1-\beta\rho) + \check{\sigma}\kappa(\phi_\pi - \rho)} \\ &= \frac{\check{\sigma}\chi^A(1-\beta\rho)}{(1-\rho+\check{\sigma}\phi_2)(1-\beta\rho) + \check{\sigma}\kappa(\phi_\pi-\rho)} \end{aligned}$$

which is the same as the one reported in the main text.

4.B.2 Short Run, Zero Nominal Interest, $i = 0$

Fiscal policy is activated (e.g. government spending is higher than its steady-state $\widehat{G}_S > 0$) as long as the zero lower bound on the nominal interest rate is binding:

$$\widehat{G}_t = \hat{F}_S > 0 \text{ for } 0 < t < T^e,$$

$$\widehat{G}_t = 0 \text{ for } t \geq T^e,$$

where $\hat{F}_S = \{\hat{G}_S, \hat{\tau}^W_S, \hat{\tau}^S_S, \hat{\tau}^A_S\}$.

Different from the case of positive interest rate the discount factor in this section is the source of the deflationary shock that makes the zero lower bound on the nominal interest rate binding and is negative, $r^e_t < 0$.

Recall that the NKPC is given by

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}^W_t + \chi^S\hat{\tau}^S_t - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1},$$

and the AD is written as:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r^e_t\right) + \chi^S\check{\sigma}E_t\left[\hat{\tau}^S_{t+1} - \hat{\tau}^S_t\right] + \chi^A\check{\sigma}\hat{\tau}^A_t.$$

Government Spending

The short-run AD and AS equations when the zero bound binds can be written as (ignoring taxes):

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r^e_S + (1-\mu)\widehat{G}_S$$

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa\widehat{Y}_S - \kappa\psi\check{\sigma}^{-1}\widehat{G}_S}{1-\beta\mu}\right] + \check{\sigma}r^e_S + (1-\mu)\widehat{G}_S$$

or

$$(1-\mu)\widehat{Y}_S - \frac{\check{\sigma}\mu\kappa\widehat{Y}_S}{1-\beta\mu} = -\frac{\mu\kappa\psi\widehat{G}_S}{1-\beta\mu} + \check{\sigma}r^e_S + (1-\mu)\widehat{G}_S$$

or

$$\left[(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa\right]\widehat{Y}_S = \left[(1-\mu)(1-\beta\mu) - \mu\kappa\psi\right]\widehat{G}_S + (1-\beta\mu)\check{\sigma}r^e_S$$

Then, the government spending multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\widehat{G}_S} = \frac{(1-\mu)(1-\beta\mu) - \mu\kappa\psi}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

Labour Tax Cut

Recall AS

$$\pi_t = \kappa\hat{Y}_t + \kappa\psi(\chi^W\hat{\tau}^W_t + \chi^S\hat{\tau}^S_t - \check{\sigma}^{-1}\hat{G}_t) + \beta E_t\pi_{t+1}$$

and the AD is

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r^e_t\right) + \chi^S\check{\sigma}\left[\hat{\tau}^S_{t+1} - \hat{\tau}^S_t\right] + \chi^A\check{\sigma}\hat{\tau}^A_t$$

The AD and AS equations when the zero bound binds can be written as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r^e_S$$

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S + \kappa\psi\chi^W\hat{\tau}^W_S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa\widehat{Y}_S + \kappa\psi\chi^W\hat{\tau}^W_S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa\widehat{Y}_S + \kappa\psi\chi^W\hat{\tau}^W_S}{1-\beta\mu}\right] + \check{\sigma}r^e_S$$

After collecting terms we obtain:

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \frac{\check{\sigma}\mu\kappa\psi\chi^W}{1-\beta\mu}\hat{\tau}^W_S + \check{\sigma}r^e_S$$

or

$$\begin{aligned}\widehat{Y}_S &= \frac{\frac{\check{\sigma}\mu\kappa\psi\chi^W}{1-\beta\mu}}{(1-\mu)-\frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}^W_S + \frac{\check{\sigma}}{(1-\mu)-\frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r^e_S \\ &= \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa}\hat{\tau}^W_S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa}r^e_S\end{aligned}$$

Then the labor tax cut multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\hat{\tau}^W_S} = \frac{\check{\sigma}\mu\kappa\psi\chi^W}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

Sales Tax Cut (Short Run, Zero Nominal Interest, $i = 0$)

The AD is:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r^e_t\right) + \chi^S\check{\sigma}\left[\hat{\tau}^S_{t+1} - \hat{\tau}^S_t\right] + \chi^A\check{\sigma}\hat{\tau}^A_t$$

The AD and AS equations when the zero bound binds can be written as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r^e_S + \chi^S\check{\sigma}(\mu-1)\hat{\tau}^S_S$$

$$\pi_S = \kappa \widehat{Y}_S + \beta\mu\pi_S + \kappa\psi\chi^S\hat{\tau}_S^S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa \widehat{Y}_S + \kappa\psi\chi^S\hat{\tau}_S^S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\left[\frac{\kappa \widehat{Y}_S + \kappa\psi\chi^S\hat{\tau}_S^S}{1-\beta\mu}\right] + \check{\sigma}r_S^e + \chi^S\check{\sigma}(\mu-1)\hat{\tau}_S^S$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \left[\frac{\check{\sigma}\mu\kappa\psi\chi^S}{1-\beta\mu} + \chi^S\check{\sigma}(\mu-1)\right]\hat{\tau}_S^S + \check{\sigma}r_S^e$$

and

$$\begin{aligned}
\widehat{Y}_S &= \frac{\frac{\check{\sigma}\mu\kappa\psi\chi^S}{1-\beta\mu} + \chi^S\check{\sigma}(\mu-1)}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}_S^S + \frac{\check{\sigma}}{(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r_S^e \\
&= \frac{\check{\sigma}\mu\kappa\psi\chi^S + \chi^S\check{\sigma}(\mu-1)(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e \\
&= \frac{[\mu\kappa\psi - (1-\mu)(1-\beta\mu)]\chi^S\check{\sigma}}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}\hat{\tau}_S^S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}r_S^e
\end{aligned}$$

The sales tax cut multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\hat{\tau}_S^S} = -\frac{[(1-\mu)(1-\beta\mu) - \mu\kappa\psi]\chi^S\check{\sigma}}{(1-\mu)(1-\beta\mu) - \check{\sigma}\mu\kappa}$$

which is the same as the one reported in the main text.

Capital Tax Cut (Short Run, Zero Nominal Interest, $i = 0$)

Recall the expression of AD:

$$[\hat{Y}_t - E_t\hat{Y}_{t+1}] = [\hat{G}_t - E_t\hat{G}_{t+1}] - \check{\sigma}\left(i_t - E_t\pi_{t+1} - r_t^e\right) + \chi^S\check{\sigma}\left[\hat{\tau}_{t+1}^S - \hat{\tau}_t^S\right] + \chi^A\check{\sigma}\hat{\tau}_t^A.$$

The AD and AS equations can be written, at the zero bound bind, as:

$$\widehat{Y}_S = \mu\widehat{Y}_S + \check{\sigma}\mu\pi_S + \check{\sigma}r_S^e + \chi^A\check{\sigma}\hat{\tau}_S^A$$

Recall the NKPC:

$$\pi_S = \kappa\widehat{Y}_S + \beta\mu\pi_S$$

which latter can be expressed for inflation as:

$$\pi_S = \frac{\kappa \widehat{Y}_S}{1-\beta\mu}$$

that can be put back into the AD equation:

$$(1-\mu)\widehat{Y}_S = \check{\sigma}\mu\frac{\kappa\widehat{Y}_S}{1-\beta\mu} + \check{\sigma}r^e_S + \chi^A\check{\sigma}\hat{\tau}^A_S$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \chi^A\check{\sigma}\hat{\tau}^A_S + \check{\sigma}r^e_S$$

And

$$\left[(1-\mu) - \frac{\check{\sigma}\mu\kappa}{1-\beta\mu}\right]\widehat{Y}_S = \chi^A\check{\sigma}\hat{\tau}^A_S + \check{\sigma}r^e_S$$

And

$$\begin{aligned}\widehat{Y}_S &= \frac{\chi^A\check{\sigma}}{(1-\mu)-\frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}\hat{\tau}^A_S + \frac{\check{\sigma}}{(1-\mu)-\frac{\check{\sigma}\mu\kappa}{1-\beta\mu}}r^e_S \\ &= \frac{\chi^A\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa}\hat{\tau}^A_S + \frac{\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa}r^e_S.\end{aligned}$$

The capital tax cut multiplier is given by:

$$\frac{\Delta\widehat{Y}_S}{\Delta\hat{\tau}^A_S} = \frac{\chi^A\check{\sigma}(1-\beta\mu)}{(1-\mu)(1-\beta\mu)-\check{\sigma}\mu\kappa},$$

which is the same as the one reported in the main text.

4.C Nonlinear Model

4.C.1 Calvo Price Setting

Recall the first-order condition of the firm:

$$\frac{p^*_t}{P_t} = \frac{\Sigma^{\infty}_{T=t}(\beta\alpha)^{T-t}\lambda_T\left(\frac{p^*_t}{P_T}\right)^{-\theta-1}Y_T\frac{\theta}{\theta-1}MC^{\text{real}}_T(i)}{\Sigma^{\infty}_{T=t}(\beta\alpha)^{T-t}\lambda_T\left(\frac{p^*_t}{P_T}\right)^{-\theta-1}Y_T\frac{P_t}{P_T}}$$

To manipulate the previous equation further we need to establish connection between firm-specific ($MC^{\text{real}}_t(i)$) and average real marginal costs (MC^{real}_t). Note that in our paper we depart from Eggertsson and Singh and allow for

DRS in production with the functional form $Y_t = N_t^{\frac{1}{\phi}}$ where $\phi > 1$; $\phi = 1$ is the case of CRS):

$$\begin{aligned}
MC_T^{\text{real}}(i) &= \frac{W_T/P_T}{MPL_T(i)} = \frac{v_l(N_t(i))/u_c(.)}{MPL_t(i)} \\
&= \frac{v_l(N_T)/u_c(.)}{MPL_T}\frac{v_l(N_t(i))}{v_l(N_T)}\frac{MPL_T}{MPL_t(i)} \\
&= MC_T^{\text{real}}\frac{v_l(N_t(i))}{v_l(N_T)}\frac{MPL_T}{MPL_t(i)} \\
&= MC_T^{\text{real}}\left(\frac{N_t(i)}{N_T}\right)^{\omega}\left(\frac{Y_T}{Y_t(i)}\right)^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{Y_t(i)}{Y_T}\right)^{\phi\omega}\left(\frac{Y_T}{Y_t(i)}\right)^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\phi\omega}\left[\left(\frac{p_t^*}{P_T}\right)^{-\theta}\right]^{\phi-1} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\phi\omega-\theta(\phi-1)} \\
&= MC_T^{\text{real}}\left(\frac{p_t^*}{P_T}\right)^{-\theta\omega_y},
\end{aligned}$$

Row 2 shows that the marginal cost has two 'specific labor' parts: one part is related to the disutility of labour and the other part is the specific marginal product of labour. Note that the specific labour market assumption does not require wage to be firm-specific. Row 3 defines the average marginal cost $MC_t = \frac{v_l(N_t)/u_c(C_t)}{MPL_t}$. In the last but one row $\theta\phi\omega$ appears only in case of specific labour market. With economy-wide labour market $\theta\phi\omega = 0$. Note that the case of CRS production function ($\phi = 1$) delivers the specific labour model of Eggertsson and Singh (2016). Row 4 used the relative demand for good i.

The last row marks a simple change in notation. In particular, the composite parameter $\omega_y \equiv \phi(1+\omega) - 1$ shows that the labour curvature parameter (ω) is rescaled after the introduction of DRS in technology. Note that when $\phi = 1$ we have $\omega_y = \omega$.

Recall the first-order condition of the firm from the appendix of our paper:

$$\Sigma_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1}\frac{p_t^*}{P_t}\frac{P_t}{p_T} = \Sigma_{T=t}^{\infty}(\alpha\beta)^{T-t}\lambda_T\left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1}Y_T MC_T^{\text{real}}(i)$$

or

$$\frac{p_t^*}{P_t} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta-1} Y_T MC_T^{\text{real}}(i)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta} \left(\frac{P_t}{P_T}\right)^{-\theta} Y_T}$$

which can be further written using the connection between firm-specific and average marginal cost as:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta-1} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{P_t}{P_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_T}\right)^{-\theta} Y_T \left[\frac{P_t}{P_T}\right]^{-\theta}}$$

which can also be written as:

$$\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} = \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{P_t}{P_T}\right)^{-\theta} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{P_t}{P_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta} Y_T \left[\frac{P_t}{P_T}\right]}$$

and let us multiply both nominator and denominator by $\frac{P_t}{P_T}$:

$$\begin{aligned}\left(\frac{p_t^*}{P_t}\right)^{1+\theta\omega_y} &= \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \left[\frac{1}{\Pi_T}\right]^{-\theta\omega_y}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\frac{1}{\Pi_T}\right)^{-\theta-1} Y_T} \\ &= \frac{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta-1} Y_T \left(\frac{\theta}{\theta-1} MC_T^{\text{real}} \Pi_T^{\theta(1+\omega_y)}\right)}{\Sigma_{T=t}^{\infty} (\alpha\beta)^{T-t} \lambda_T \left(\frac{p_t^*}{P_t}\right)^{-\theta} Y_T \Pi_T^{\theta-1}}\end{aligned}$$

which is the same as the expression in Eggertsson and Singh.

In the previous equation the average real marginal cost is defined as:

$$\begin{aligned}MC_T^{\text{real}} &= \frac{W_T/P_T}{MPL_T} = \frac{\frac{N_T^{\omega}}{(Y_T-G_T)^{-\check{\sigma}}}}{(1/\phi)N_T^{(1/\phi-1)}} \\ &= \frac{\frac{N_T^{\omega}}{(Y_T-G_T)^{-\check{\sigma}}}}{(1/\phi)N_T^{(1/\phi-1)}} = \frac{\phi Y_T^{\phi(\omega+1)-1}}{(Y_T-G_T)^{-\check{\sigma}}} = \frac{\phi Y_T^{\omega_y}}{(Y_T-G_T)^{-\check{\sigma}}}.\end{aligned}$$

The AS curve (the recursive NK Phillips curve) can be expressed as:

$$\begin{aligned}K_t &= \frac{\theta}{\theta-1}\frac{1+\tau_t^S}{1-\tau_t^W}\lambda\xi_t\phi Y_t^{1+\omega_y} + \alpha\beta E_t\left[\Pi_{t+1}^{\theta(1+\omega_y)}K_{t+1}\right] \\ F_t &= \xi_t C_t^{-\frac{1}{\check{\sigma}}} Y_t + \alpha\beta E_t\left[\Pi_{t+1}^{\theta-1}F_{t+1}\right] \\ \frac{K_t}{F_t} &= \left(\frac{1-\alpha\Pi_t^{\theta-1}}{1-\alpha}\right)^{\frac{1+\theta\omega_y}{1-\theta}}\end{aligned}$$

4.C.2 Aggregation

The production function for firm j is given by:

$$Y_t(j) = N_t^{1/\phi}(j)$$

where we abstract from technology shocks.

One derives the aggregate production function by integrating over the $j-$goods.

$$(Y_t(j))^{\phi} = N_t(j)$$

Since the workers are all the same the sum is simply, $N_t = \int_0^1 N_t(j)dj$. Plugging in from the demand function

$$\left(\left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t\right)^{\phi} = N_t(j)$$

Integrating over $j-$goods

$$N_t = \int_0^1 \left[\left(\frac{P_t(j)}{P_t}\right)^{-\theta} Y_t\right]^{\phi} dj$$

Taking variables independent from j out of the integral,

$$N_t = (Y_t)^{\phi} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$

Now expressing this equation for Y_t,

$$N_t = Y_t^{\phi} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$

$$N_t^{\frac{1}{\phi}} = Y_t \left[\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj\right]^{\frac{1}{\phi}}$$

4.C.3 Price Dispersion

Lets define price dispersion, S_t:

$$S_t^{\phi} \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta\phi} dj$$

where $1/\phi$ is the labor's share in output and θ is the elasticity of substitution between differentiated good j. Next, using the 'Calvo result' (proportion of firms changing its price), we can write price dispersion recursively as:

$$\begin{aligned}
S_t^{\phi} &\equiv \int_0^1 \left(\frac{P_t\left(j\right)}{P_t}\right)^{-\theta\phi} dj \\
&= (1-\alpha)\left(\frac{P_t^*\left(j\right)}{P_t}\right)^{-\theta\phi} + \alpha(1-\alpha)\left(\frac{P_{t-1}^*\left(j\right)}{P_t}\right)^{-\theta\phi} + \alpha^2(1-\alpha)\left(\frac{P_{t-2}^*\left(j\right)}{P_t}\right)^{-\theta\phi} + ... \\
&= (1-\alpha)\left(\frac{P_t^*\left(j\right)}{P_t}\right)^{-\theta\phi} \\
&+ \alpha\left(\frac{P_{t-1}}{P_t}\right)^{-\theta\phi}\left[(1-\alpha)\left(\frac{P_{t-1}^*\left(j\right)}{P_{t-1}}\right)^{-\theta\phi} + \alpha(1-\alpha)\left(\frac{P_{t-2}^*\left(j\right)}{P_{t-1}}\right)^{-\theta\phi} + ...\right] \\
S_t^{\phi} &\equiv (1-\alpha)\left(\frac{P_t^*\left(j\right)}{P_t}\right)^{-\theta\phi} + \alpha\left(\frac{P_{t-1}}{P_t}\right)^{-\theta\phi} S_{t-1}^{\phi} \\
S_t^{\phi} &\equiv (1-\alpha)\left(p_t^*\right)^{-\theta\phi} + \alpha\left(\pi_t\right)^{\theta\phi} S_{t-1}^{\phi} \qquad \text{(A.8)}
\end{aligned}$$

where $(1-\alpha)$ is the probability that the firm will be able to change price. Price dispersion can be written recursively as

$$S_t^{\phi} = (1-\alpha)\left(\frac{P_t^*\left(j\right)}{P_t}\right)^{-\theta\phi} + \alpha(\pi_t)^{\theta\phi} S_{t-1}^{\phi}$$

Thus, we can write the aggregate production function as,

$$N_t^{1/\phi} = Y_t S_t$$

Appendix 5

Response to Opponents

First of all I would like to thank to my opponents for their careful reading of my thesis and their valuable comments on each chapter. In what follows I addressed all the comments and provide my reactions.

Response to Opponent: Lubos Pastor, Charles P. McQuaid Professor of Finance at The University of Chicago

> *I do not fully understand why the inefficiency highlighted in the paper leads to such a large increase in macroeconomic volatility. I can see how price dispersion across firms generates inefficiencies, but I'd expect this effect on macroeconomic volatility to be modest quantitatively.*

I consider this issue highly relevant. We are working right now with my co-authors on rigorously addressing this issue. We plan to finish before submitting the paper to academic journal. Probably the best way to respond to this comment is to design a method which allows to isolate each channel at play (marginal cost, trend inflation markup and price-inflation spiral). With such method we could quantitatively evaluate the distortion to the economy due to price dispersion separately by each channel. Knowing the quantitative importance of each channel will allow us to tell which inefficiency highlighted in the paper is the one driving our result.

What we are able to do right now is to shut down the marginal cost channel of distorting the economy. The results are shown in table 2.2, column RS7.

Column RS7 of table 2.2 shows that without the marginal cost channel the model performance is only mildly distorted.

Nevertheless, it could still be the case that shutting down the marginal cost channel by using the linear production function as in RS7 decreases the overall non-linearity of the model and the upper bound on inflation discussed in section on price-inflation spiral is not binding. Thus, the third order approximation does a good job at approximating the model only because we took away the source of non-linearity. It is still possible however that with marginal cost channel in place and better approximation method the model would deliver only little distortion from price dispersion to macroeconomic variables. We are currently working on new solution method which will internalize the upper bound on inflation into the agents decision problem. Having a accurate solution method will allow us to determine exactly how much of the distortion comes from the marginal cost and trend inflation markup channel.

Response to Opponent: Doc. Mgr. Tomas Holub Ph.D.

> *Chapter 2: I think the author would need to redraft the text in either of the two directions (i) Skip the original motivation of exploring bond pricing in the DSGE setup, focus on the methodological issues associated with trend inflation under alternative specifications of price rigidities and try to publish in journal focused on macro-modelling; (ii) De-emphasize the above methodological / modelling issues, choose the specification of price rigidities that can cope with trend inflation, explore the bond pricing topics and then try to publish in a journal focused on macro-finance research. In the current version, the chapter stays half-way between the two options, which could make finding an appropriate outlet difficult . . . It is quite obvious from the text that the author achieved something else compared to his original objective. In the end there is very little about the pricing and term premium. Instead, it is a macro-modelling methodological paper. I would suggest redrafting alone on of the two options mentioned above (with the former one being clearly easier, which would allow bringing the thesis to formal defense soon.)*

The bond pricing empirical literature argues that trend inflation is the key element in explaining the level and slope of the yield curve (see for instance Cieslak & Povala (2015) and Bauer & Rudebusch (2017)). We ask what is the economics behind this empirical finding and can we confirm this finding in the theoretical model? The bond pricing models of term structure in a DSGE framework have for long struggled to explain why is the yield curve upward sloping. The most successful state of the art model which is able to match both the basic bond pricing and macro stylized facts is the Rudebusch & Swanson (2012). Even this model however still needs risk aversion of 110 (empirically justified levels are around 5) to match the level of term premia. It seems therefore natural to assume that the missing element to explain the yield curve could be the trend inflation as most models are approximated around zero steady state inflation. Motivated by this fact we introduce the trend inflation in the RS model. Contrary to what the empirical literature suggests we find out that trend inflation completely destroys the model performance in matching macro and bond prices stylized facts. For instance, volatility of inflation increases from the original 3 percent to almost 40 percents, consumption volatility is about 16 times higher. Our paper explains why is the model fit undermined by trend inflation and proposes how to restore the original performance.

The modelling features related to trend inflation are tightly linked to the macro-finance literature. I acknowledge that our results are more general and apply to all non-linearly solved models with Calvo pricing and trend inflation. However, most of such models can be found exactly in the asset pricing literature. This is because to model why financial instruments are risky one needs to solve the model non-linearly as linear solution implies certainty equivalence and thus zero risk premia by construction.

To sum up, our research question is motivated by the bond pricing literature and our results are relevant especially for this literature. Therefore, I believe that the asset pricing motivation of the chapter 2: Trend Inflation Meets Macro-Finance: the Puzzling Behavior of Price Dispersion is appropriate. Nevertheless, it is likely not well articulated in the introduction. I improve the motivation in the introduction to chapter 2 by adding new paragraph which mirrors the discussion from this response.

> *Chapter 2: Page17: Footnote 1 refers to Figure 4.1 which is not provide. Later on, the references to Tables and Figures give wrong*

> *numbers. The definition of variables reported in Table 2.1 on page 24 missing.*

I have fixed the numbering and included the model description (which contains the names of model variables) into the main text.

> *Chapter 2: What is the reason for negative (sometimes very negative) inflation means in some of the model specifications?*

The negative mean of inflation is a common feature of non-linearly solved NK models even though this number is usually not reported (most linearized models usually impose mean inflation to be zero). One exception in non-linear framework is the Andreasen *et al.* (2018) who use combination of numerical methods (new method to pruning) and adjustment to monetary policy rule (nominal term premium in Taylor rule) to generate positive mean of inflation.

In non-linear model the negative mean of inflation is driven by the precautionary saving terms (second order terms in the perturbation). Up to the first order approximation agents do not account for known variance of the shocks in their decision making problem. In case of non-linearly solved models variance of the shocks is reflected in the expectation of agents which increases equilibrium level of savings, lowers average yields and inflation through Fisher equation. The precautionary saving effect thus pushes the stochastic steady state of the model bellow the zero inflation deterministic steady state.

> *Chapter 2: Page 18: It is correctly stated in footnote 5 that the Calvo pricing mechanism is not considered to be the most realistic set up. It is also true that, nonetheless, it is the most widely used device to introduce nominal rigidities into DSGE models. However, many of the applied models that I know from the macroeconomic profession (such as the models to support inflation targeting frameworks) use Calvo pricing with inflation indexation. In reality, it seems implausible that the economic agents would stick with a non-indexation. In reality, it seems implausible that the economic agents would stick with a non-indexation version of Calvo pricing in the presence of trend inflation, as this would imply real costs for them well in access of any plausible menu costs. There should be some discussion of this in the text, as the author himself concludes that adding inflation indexation into the Calvo pricing set-up largely*

solves the problem that he identifies as the main conclusion of this chapter.

Traditionally the purpose of introducing price indexation into the model with Calvo pricing has been to match the persistence of nominal interest rate. NK DSGE models generate one period nominal interest rate which has much lower persistence than what can be empirically observed. To tackle this issue, Christiano *et al.* (2005) introduces in the macroeconomic literature price indexation and shows that it can be used to make the model generate enough persistence in the one period nominal interest rate. Nevertheless, this has been criticized because in fact there is little empirical evidence that firms index their prices.

Despite the little empirical support for price indexation I share the opinion with the opponent that if firms were in reality bounded by Calvo contracts then they would most likely index their prices in the presence of trend inflation. As the opponent in the insightful way points out the unrealistic assumption of Calvo contracts makes the assumption of price indexation realistic. This is because (as we show in the paper) the economic costs of positive trend inflation can be largely mitigated by price indexation thus it is optimal for firms to index their prices.

Chapter 2: The author states that "The economy with trend inflation and Calvo contracts produces more output than optimal." I am not sure relative to what "optimality" benchmark this holds. Is it relative to the steady state of an economy with monopolistic competition, Calvo pricing and not trend inflation? But monopolistic competition leads to a steady state output which is below the Pareto efficient level. The word "optimal" may then not be appropriate here. Or is it relative to an economy with perfect competition and no nominal rigidities?

The Lemma 1.1 proves that firms which can optimize and choose their price will produce less than firms which can change their prices (firms are otherwise identical) and thus the overall economy output will be higher with sticky prices. The optimality is defined by the absence of rigidities (by the fact that the firm cannot optimize its price at the given period). Thus by optimal we mean the ability of the firm to choose the optimal price in the given period.

I change the headline of the Lemma 1.1 to "*The economy with trend inflation and Calvo contracts produces more output than it is optimal under the flexible prices*"

> *Chapter 3: The conclusion that higher uncertainty about certain types of government spending may push the government bond yield curve lower is very interesting. But it may be highly conditional on the assumption of no sovereign default risk. This fact is only briefly mentioned on page 87. In practice, when there are sovereign solvency issues, more uncertainty about government spending plans may actually shift the yield curve up, as has recently been demonstrated nicely by the Italian case. I think this disclaimer should be clearly spelled out in the introduction, as well as conclusions of Chapter 3 (and in the relevant parts of summary Chapter 1)*

I have augmented the text as recommended.

> *Chapter 3: . . . , the government bonds are assumed to be nominal, not real. At the same time, they are by assumption the only asset that can be used by the model households as a store of value and as a hedge against various macroeconomic shocks. These modeling choices should be spelled out explicitly and explained in more detail at the beginning of the chapter, as especially the latter on seems to be far from reality.*

The underlying model is the New Keynesian monetary model which allows me to model nominal yield of bonds. Later on, in the section on attribution analysis I study each component of macroeconomic risk separately including the inflation premium.

Integral part of small scale NK DSGE models is the fact that bonds are used to intertemporally smooth consumption as opposed to RBC literature where this function is carried by capital. The fact that bonds are the only instrument to smooth consumption is very standard modeling choice in the literature.

> *Chapter 3: Given how much effort is spend in Chapter 2 arguing that trend inflation should be a part of any model trying to explain government bond pricing, the author should be explicit at the very beginning of Chapter 3 about his choice of zero steady-state inflation.*

I have added a note in the introduction to Chapter 3.

> *Chapter 3: Given how much effort is spend in Chapter 2 arguing that trend inflation should be a part of any model trying to explain government bond pricing, the author should be explicit at the very beginning of Chapter 3 about his choice of zero steady-state inflation.*

I have added the discussion into the introduction of Chapter 3 as recommended.

> *Chapter 3: It is a bit awkward notation to use the λ_t symbol both for the Lagrange multiplier in the household consumption choice as well as a parameter in the firms' production function. I also find it strange to write the latter as a time-dependent variable. I understand that in the end the firms' mark-up is time varying due to nominal rigidities; but this does not mean that the same is true for the technological parameter.*

I fixed the notation and change the Lagrange multiplier to Λ. Time varying elasticity of substitution of intermediate goods is a standard way how to introduce time varying mark-up in the model (for instance Christiano *et al.* (2005)).

> *Chapter 3: Why is the inflation target defined as time-varying in the equation (3.18)? Does it have any implications for the paper's conclusions? Some discussion of this would be welcome.*

Time varying inflation target is widely used way to capture the long-term nature of monetary policy shifts. In my model, time varying inflation target is the way to introduce the long-run inflation risk into the model. I follow here Rudebusch & Swanson (2012). They show that time varying inflation target is important element in matching the nominal term premium. Nevertheless, for instance Andreasen *et al.* (2018) or Kliem & Meyer-Gohde (2017) can match the nominal term premium in their mid-scale model even without time varying inflation target.

> *Chapter 3: I do not understand the sentence: "Higher inflation undermines the real value of bonds exactly in time of lower inflation.*

This is a typo. Correct sentence. "Higher inflation undermines the real value of bonds exactly in time of lower consumption."

> *Chapter 3: Is the reference to figure 3.11 correct? Shouldn't the text refer to figure 3.8 instead? In general, the author should guide the reader a little bit more in terms of how to interpret the results presented in figures 3.8., 3.9 and 3.11.*

I have fixed the wrong reference and elaborate on the interpretation.

> *Chapter 3: I am confused by the sentence: "If the weight on inflation is zero in the monetary policy rule bonds can protect its holders against the fluctuations in their wealth due to productive government spending. "*

This is a typo and it should be "If the weight on *output* is zero in the monetary policy rule bonds can protect its holders against the fluctuations in their wealth due to productive government spending. "

> *Chapter 3: The chapter seems unfinished. There is no comment on Tables 3.3 and 3.4 and no concluding section of the whole chapter.*

I have added the discussion related to mentioned Tables and concluded the chapter.

> *Chapter 4: I my opinion, it should be spelled out more clearly that the conclusions apply to a model with strong Ricardian features, and some of them may thus not hold universally. For example on page 161, it is stated that the demand for labor is not affected by a cut in labor tax rate. Would this hold in an OLG model, or a model with some share of hand to mouth consumers?*

Yes, it is likely that in the OLG model with hand to mouth consumers labor tax cuts would be expansionary. On the page 156 we explicitly spell out that in our model the Ricardian equivalence holds and page 168 provides a discussion stating that the conclusions about labor tax cut multiplier holds only in the model without non-Ricardian households.

> *Chapter 4: In a static model, a flatter Phillips curve should imply that a larger portion of any given shift in the aggregate demand goes into real output and a smaller portion goes to prices. The beauty of the DSGE set up is that it is not static, and the size of the aggregate demand shift depends on the circumstances. If the economy is outside the ZLB situation and monetary policy response leads to the traditional crowding-out effect, the above intuitive conclusion from a static model should be reinforced further. In this regard, the differences between fiscal multipliers for non-ZLB situations depending on the PC's slope look actually too small for me, and I would welcome some further discussion of this outcome. In the ZLB case, there is actually a dynamic crowding-in effect going through higher expected inflation and lower real interest rate. It is clear that this increases the fiscal multipliers compared to a normal situation for any given shape of the Phillips curve (and it is no surprise that the multipliers often exceed 1). Whether this intertemporal crowding-in ZLB effect implies a bigger fiscal multiplier with a steeper rather than flatter Phillips curve is likely to be an empirical issue, though. I would welcome some robustness checks e.g. with respect to: (i) the intertemporal elasticity of substitution of optimizing households; (ii) the share of hand-to-mouth consumers if included into the model; (iii) the degree to which the households are able to form rational expectations. Such robustness checks may actually bring more insight that the provided robustness check with respect to the model's non-linearities.*

The increase in government spending affects output through two channels. The first one is the standard wealth effect when increased government expenditures crowd out the private consumption. Unless for few special cases (which are discussed in Christiano *et al.* (2011)) the wealth effect results in multiplier which lies always bellow one.

The second channel works through the real interest rate which motivates (discourage) agents in the economy to save. This channel largely depends on the behavior of central bank in setting the nominal interest rate. Central bank following standard Taylor rule will response to the rise in government expenditures which leads to total demand and inflation increase by lifting up nominal interest rate. Out of the zero lower bound period (in normal times) the increase in nominal interest rate pins down the expected inflation and stabilizes the real interest rate so that equilibrium savings is at zero in the model. Nevertheless, when the ZLB binds, the nominal interest rate stays at zero and the rise in expected inflation decreases the real interest rate, discourages savings and counteracts the drop in consumption induced by the wealth effect.

The slope of the Phillips curve impacts primary the second channel. The second - real interest rate - channel is quantitatively important only when ZLB binds and this is why we see small impact of the slope of Phillips curve on fiscal multipliers in normal times.

As regards to sensitivity checks. Fiscal multipliers increase with the intertemporal elasticity of substitution (IES). IES affects the slope of the Phillips curve and in case of non-separable preferences it makes stronger the link between marginal utility of consumption with hours worked (further details are in discussed in Christiano *et al.* (2011)). How the non-Racardian households change the our results is discussed in the NK DSGE model similar to ours by **?**

> *Chapter 4: The algebra of the model is presented without any definition of the symbols that are being used.*

I have extended the the definition of variables.

Response to Opponent: István Kónya

> *My first comment concerns the general "theme" of the thesis. Reading the first chapter (which serves as an introduction) gives the reader the idea that the thesis is about asset prices. This is true only for the second model, and there is a marginal connection in case of the first. The third model has essentially nothing to do with asset prices. This is not a problem with the content, to me it is perfectly ok for a thesis to collect three different papers that are only marginally related. I suggest, however, to either choose a different theme, or just broaden it sufficiently ("there essays in monetary economics").*

In the response to question 5 of Doc. Mgr. Tomas Holub Ph.D., I argue that the first chapter is very closely related to the asset pricing literature. The second chapter is focused on fiscal policy and bond pricing. The third chapter relates to the second by studying fiscal policy. All the chapters are also thematically connected by using DSGE framework. In addition, the first two chapters are based on the same baseline model.

The dissertation is rather selection than collection of my work, and because most of my work has been done in the field of asset pricing within DSGE framework, the overall theme of my dissertation is asset prices and macroeconomics.

> *I would like to see a statement at the beginning of the dissertation that delineates the contribution of the various authors. In particular, it should be clear that the candidate's role was fundamental in at least one, and preferably all chapters.*

I add the detailed specification of my contribution in the introductory chapter where I provide the general introduction with non-technical summary of the papers.

> *I do not like that the main model setup and derivation is relegated to Appendices in all three chapters. This may be fine for a central bank working paper that wants to get to the policy issues as fast as possible. But a thesis should be reasonably self-contained.*

I have included the model derivations into the main text.

> *On page 5, I do not understand the statement "overproduction implied by the inefficient allocation of resources among firms leads to aggregate output losses" (emphases added).*

I augment the explanation, *The overproduction implied by the inefficient allocation of resources among firms leads to aggregate output losses as the same amount of goods could be produced more efficiently (with less inputs) if the low price firms decrease production and high price firm increase production.*

This means that the amount of inputs used to production, if more optimally distributed among firms, would produce higher aggregate output.

> *As I said above, I would like to see the model presented in section 2.2 and not in the Appendix.*

I have included the model into the main section.

If the emphasis on the term structure remains, there should be a detailed explanation of how long-term bonds are priced in a model with a one-period bond only.

The detailed explanation of how long-term bonds are priced in a model is described in the chapter 2. I have included the reference pointing to this section. Here I provide a bit more general discussion than in the chapter about pricing bonds in the model,

Let i_t be the nominal interest rate. Real interest rate is given by $r_t = i_t - \pi_{t+1}$ or in gross terms, $R_t = \frac{I_t}{\pi_{t+1}}$. The price of nominal ten year bond can be written recursively, $P_t^n = e^{-i_t} E_t P_{t+1}^{n-1}$ which is in logs $p_t^n = -i_t + logE_t p_{t+1}^{n-1}$. In general, the price of nominal ten year bond,

$$P_t^n = Q_{t,t+n} = \frac{1}{(1+ytm_t)^n} = \prod_{k=0}^{n-1} \frac{1}{(1+i_{t+k})^n} \tag{5.1}$$

where ytm_t is the yield to maturity on ten year bond.

$$\log P_t^n = \log Q_{t,t+n} = \log \frac{1}{(1+ytm_t)^n} \tag{5.2}$$

$$\log P_t^n = \log(1+ytm_t)^{-n} \tag{5.3}$$

$$\log P_t^n = -n\log(1+ytm_t) \tag{5.4}$$

thus

$$ytm_t^n = -\frac{1}{n}p_t^n = -\frac{1}{n}q_{t,t+n} \tag{5.5}$$

Using analogous derivation we can derive the price of inflation protected bond

$$ytm_{real,t}^n = -\frac{1}{n}p_{real,t}^n = -\frac{1}{n}q_{t,t+n}^r \tag{5.6}$$

where $q_{t,t+n}^r$ is the real stochastic discount factor.

Risk neutral bonds (under Q measure) can be in simplicity understood as buying every period one period bond and selling it at the end of the period. In finance literature risk neutral measures are computed by adjusting the probabilities, thus we can move the expectation operator in equation $p_t^n = -i_t + logE_t p_{t+1}^{n-1}$ in front of the logarithm to get $p_t^n = -i_t + E_t logp_{t+1}^{n-1}$.

This is why we can write,

$$ytm_t^{Q,n} = -\frac{1}{n}\sum_{k=0}^{n-1} i_{t+k} \tag{5.7}$$

We can derive the analogous measure for inflation protected bonds,

$$ytm_t^{Q,r} = -\frac{1}{n}\sum_{k=0}^{n-1} r_{t+k} \tag{5.8}$$

It seems that the trend-inflation version uses the calibration of the baseline model with zero inflation. A fairer comparison would be with a recalibrated model. I could imagine, for example, that with trend inflation a lower Calvo parameter would help the model fit price dispersion. There is a robustness check to handle this issue, but a serious recalibration would be more convincing.

The whole exercise underlying the first paper of this dissertation started by noticing that the trend inflation inflates the nominal term premium by a great deal. In the literature on the consumption based asset pricing, high term premia has been historically very hard to generate without setting the coefficient of relative risk aversion to unreasonably high numbers. This is why at the beginning we were trying to use the high NTP generating feature of trend inflation to match the empirical NTP with lower risk aversion Thus, at the beginning we started with a serous re-calibration of the model in a search for parameter space matching both macro and asset pricing stylized facts. Nevertheless, we concluded that there is no parameter space which allows to match both macro and finance stylized facts.

Would it be feasible to estimate the model versions? I am not sure if the price dispersion predictions need a second or third order approximation - explain.

It is essential for our argument that the model is solved (non-lineary) up to higher order approximation. In the non-linear model it is not possible to use Kalman filter which makes the Baysian methods of estimation not feasible. The particle filter is computationally extremely expensive for our model size and thus not feasible. We could use GMM estimation as in Andreasen *et al.* (2018) and some of my other research papers. I am not sure however if this would add any further insight into understanding the mechanism at play with Calvo pricing, trend inflation and higher order approximation.

> *I like the discussion on approximation errors - I guess that comes from the symmetry imposed by a local method. Would it be possible to solve a simple version (say without EZ preferences) fully non-linearly?*

We were actually considering this option. Our Review of Economic Dynamics referee asked us to solve the model with some more accurate method than third order approximation which as we show performs poorly. For now, we plan to use the new pruning method developed in Andreasen & Kronborg (2017) which they show can tackle the issue with the non-linearity given by the upper bound on inflation.

> *Lump-sum taxation is a strong assumption. Would results change with distortionary taxes calibrated to the US economy?*

I pursue this exercises in a different paper (see Kaszab & Marsal (2013)). We show that distortionary taxation amplifies inflation risk carried by bonds and thus contributes to higher term premia contained by bonds.

> *Is trend inflation calibrated to be zero? If yes, this is a bit strange in light of the previous chapter.*

Yes, in this chapter we consider the trend inflation to be zero. The reason for this is the poor model performance with Calvo prices and trend inflation. Nevertheless, the original model we used for our analysis was model with Rotemberg adjustment costs and positive trend inflation. To be in line with the more popular model in the field we changed the underlying model for the analysis to model of Rudebusch & Swanson (2012). The results were qualitatively the same in the previous model version.

> *The assumption of additively separable public expenditure is not innocuous. Provide a brief discussion.*

I have experimented with many setups of the functional form for public expenditures (among others non-separable and CES aggregator). The algebra is detailed in the appendix of that chapter. I found very moderate sensitiveness of my results with respect to function specification. What matters a lot for the results is if consumption and public consumption goods are considered to be complements or substitutes.

Moments	USdata	Model
SD(dC)	2.69	2.54
SD(N)	1.71	2.20
Mean(i)	5.72	3.61
SD(π)	2.52	1.36
SD(i)	2.71	2.88
SD(i^{real})	2.30	2.16
SD($i^{(40)}$)	2.41	0.68
Mean($NTP^{(40)}$)	1.06	1.76
SD($NTP^{(40)}$)	0.54	0.01
Mean($i^{(40)} - i$)	1.43	1.70
SD($i^{(40)} - i$)	1.33	2.44
SD(G)	2.91	2.08

Table 5.1: Model implied moments compared to moments from the data.

Does the model actually match the term structure of interest rates? While the main interest is the impact of government spending, it would be nice to see that the baseline model does a good job here.

Yes, the baseline model follows Rudebusch & Swanson (2012) which has been shown to match jointly both macro and finance stylized facts. Introducing various government spending types does not distorts the model performance. The following table summarizes the data moments for the benchmark model calibration.

I am not sure what "firm-specific" labor is. Firm-specific capital means that firms invest and accumulate their own capital stock. The analogy does not really work for labor. Should we think about a search-and-matching framework? Or firm specific human capital? Or adjustment costs to labor? Provide at least a discussion

The model setup regarding the firm-specific labor follows Woodford (2011), Ch. 3. The firm specific labor is actually a analogy to firm specific capital and each firm simply hires labor i variety ($N_t = \int_0^1 f\,(N_t(i)di)$ vs. N_t). The assumption that production factors are firm specific implies that the cost of moving them across firms is high. The factor specific approach is quite common in NK modeling framework. Factor wide markets are more often used in RBC literature and implies strategic substitutability in price setting. The important consequence of different market structures is that in firm specific labor markets the real marginal costs depend on firms decision about prices. Firm when given the opportunity to reoptimize its price will change it by a smaller amount than if its marginal cost were independent of its decisions. Everything else equal, this will translate into a smaller response of inflation to changes in the aggregate marginal cost.

The motivation for including firm-specific labor comes from the literature empirically estimating the slope of New Keynesian Phillips curve (NKPC henceforth). Gali & Gertler (1999) have shown that NKPC, fits the U.S. inflation surprisingly well. However, the estimation implies the frequency of price changes for firms to be around 5 quarters which is not in line with the microeconomic estimates. For instance, Bils & Klenow (2004) estimate the frequency of price adjustments to be 2 quarters. Introducing firm specific labor and thus higher degree of strategic complementarity between price setters allows to align the the slope of NKPC with frequency of price changes found in the data. Further, Woodford (2005) shows that firm specific labor may also help to reduce price variations and may lead to higher inflation persistence. To generate enough persistence in inflation and nominal interest rate has been a challenge for NK DSGE models.

> *I am not an expert in ZLB models, but I am puzzled by the log-linear approach. Do we need to ignore the endogenous probability of regime switch to solve the linearized system? Or do we use something like the OccBin package? Again, please provide a discussion for the general reader.*

Our solution method follows Eggertsson (2011). The only one state variable allows us to derive closed form solution to the model. Agents know the transition probabilities of the discount shock so they know the probability they are going to escape/stay at ZLB but escaping/staying at ZLB is purely exogenous and given by the transition matrix.

> *I find it strange that tax cuts lead to output decline. Wage rigidity would probably overturn this*

This result is driven by the presence of ZLB and is in line with findings of Eggertsson (2011). At the positive interest rates the tax cut is expansionary as has been documented by many studies. Nevertheless, at the ZLB it flips its sign and becomes contractionary. The explanation is as follows. The tax cuts lead to drop in output in the model because tax cuts decrease the costs of firms and this leads to deflationary pressures through lower marginal costs and thus to increase in the real interest rate. FED cannot accommodate the higher real interest rate at ZLB which discourages consumption and lowers output.

> *Given that a non-linear solution is available, wouldn't it make sense to use that as the main (or only) approach? Also, the number of grid points seems very low. If I understand, at any given exercise you have one endogenous and one exogenous state. This should allow for a much finer grid, especially around the occasionally binding constraint.*

The non-linear approach does not allow for closed form solution and thus does not allow to explain in a intuitive and convincing way the mechanism at play. Nevertheless, there is influential stream of literature arguing that some of the results about the multipliers at the ZLB are driven by the poor performance of linear approximation of the true solution. This is why we are were asked by our journal referee to test the validity of our results with respect to the solution method.

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