The present thesis is devoted to the study of various properties of Banach function spaces, with a particular emphasis on applications in the theory of Sobolev spaces and in harmonic analysis. The thesis consists of four papers. In the first one we investigate higher-order embeddings of Sobolev-type spaces built upon rearrangement-invariant Banach function spaces. In particular, we show that optimal higher-order Sobolev embeddings follow from isoperimetric inequalities. In the second paper we focus on the question when the above-mentioned Sobolev-type space is a Banach algebra with respect to a pointwise multiplication of functions. An embedding of the Sobolev space into the space of essentially bounded functions is proved to be the answer to this question in several standard as well as nonstandard situations. The third paper is devoted to the problem of validity of the Lebesgue differentiation theorem in the context of rearrangement-invariant Banach function spaces. We provide a necessary and sufficient condition for the validity of this theorem given in terms of concavity of certain functional depending on the norm in question and we find also alternative characterizations expressed in terms of properties of a maximal operator related to the norm. The object of the final paper is the boundedness of the Hardy-Littlewood maximal operator between weighted Lebesgue spaces with different weights. We focus on strengthenings of the Muckenhoupt $A_p$-condition, called “bump conditions” in the literature. These conditions are known to be sufficient for the two-weighted maximal inequality; we prove that they are however not necessary.