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Report on the Habilitation thesis

Combinatorial Structures in Hypercubes

by

Petr Gregor

From his rich collection of publications on topics of combinatorics and graph theory Petr Gregor has chosen the hypercube as the central object of his habilitation thesis. The study of the hypercube and its structure has always been part of the core of his mathematical research. The thesis encompasses 12 selected publications and a large introductory chapter.

- [Chapter 2] Queue layout

Heath and Rosenberg introduced the queue number of a graph. They showed $\mathsf{qn}(Q_n) \leq n-1$ for the hypercube Q_n. Here it is shown that $(\frac{1}{2}-\varepsilon)n \leq \mathsf{qn}(Q_n) \leq n-\log(n)$. This is a substantial improvement over previous bounds. New techniques are introduced which may lead to further progress.

- [Chapter 3] Level-disjoint partitions

The study in this chapter is motivated by broadcasting in networks. In most cases the network is assumed to be the hypercube. One of the results of the first paper it is that the hypercube has n mutually independent Hamiltonian paths with prescribed end-vertices forming a matching. In two further papers level partitions are studied for more general classes of networks. These partitions are useful in the 1-in and all-out broadcasting model. The number of level-disjoint partitions determines how many messages can be broadcast simultaneously while their maximal height determines the overall time of the broadcasting. Constructions for level-disjoint partitions are given for various classes of networks

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- [Chapter 4] Incidence colorings

Incidence colorings model 1-in or 1-out broadcasting strategies. The aim is to assign colors to incidences between vertices and edges so that adjacent incidences have distinct colors. Based on the study of Cartesian products the authors determine the exact value of incidence chromatic number of hypercubes.

- [Chapter 5] Distance magic labelings

This is about labeling the vertices of an n vertex graph with $1, \ldots, n$ such that the sum of labels of the neighbors of each vertex is the same. It is shown that for $n = 2 \pmod 4$ the hypercube Q_n admits distance magic labeling.

- [Chapter 6] Parity vertex colorings

This is about colorings obeying the condition that every path contains some occurring an odd number of times. On trees this problem is related to tree-depth. It had been conjectured that on trees the two parameters differ by at most one. This is disproved by analyzing parity vertex colorings of binomial trees, a special type of spanning tree of the hypercube.

- [Chapter 7] Gray codes

The classical binary reflected gray code corresponds to a Hamilton cycle on Q_n. Extending pathbreaking work of Mütze on the middle levels problem Gregor and Mütze study cycles in two level graphs $Q_n[k, \ell]$ which saturate one of the sides. To tackle the problem they develop strategies of 'trimming' and 'glueing' using previously known gray codes. To construct gray codes for subgraphs of Q_n they also use and construct pairwise edge-disjoint symmetric chain decompositions of Q_n.

- [Chapter 8] Linear extension diameter

The linear extension diameter of a poset P is the diameter of the graph of linear extensions. The authors determine the linear extension diameter of the subposets of $Q_n[1, k]$. They also describe all diametral pairs of extensions.

The papers have been published in good and very good journals, most prominently SIAM Discr. Math., J. Comb. Optim., and Europ. J. Comb.. As the other work of Petr Gregor the papers collected in this thesis have received quite some interest by other researchers. This can be verified by checking the citations at Google Scholar, Math Reviews or WoS.

Besides his scientific achievements Petr Gregor also has a strong record of teaching and supervising theses. Altogether this shows the excellence of the candidate. I strongly recommend accepting the thesis and grant the habilitation to Petr Gregor.

(Stefan Felsner)