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Dear Colleagues,

This is my evaluation report of the doctoral dissertation entitled VARIANTS OF GRAPH LABELING PROBLEMS by RNDr Tomášs Masařík. The dissertation is composed of three main strands, focusing on the following three areas: (1) packing colouring problems, where the thesis contains new hardness results as well as new fixed parameter tractable (FPT) algorithms for certain well structured graph classes; (2) classical colouring problems for graphs with forbidden induced subgraphs, with new polynomial-time algorithms and new dichotomy classifications; and (3) fair deletion and fair evaluation problems, with both new hardness results and new FPT algorithms, as well as a short chapter containign a useful survey of the research area of fair problems.

Individually, each strand stands on its own, with deep results at the cutting edge, extending current knowledge in each of their respective areas.

Packing colouring is an interesting variant of classical colourings, where any two vertices of colour i are required to have distance greater than i. It was first introduced by Goddard, Hedetniemi, Hedetniemi, Harris, and Rall in 2008 (under a different name), with motivation from frequency assignment problems. (Different frequencies interfere at different ranges.) It turned out the be quite difficult, e.g., even for the square grid the exact packing chromatic number is not known. The thesis focuses on complexity questions. The packing colouring problem is NP-complete in general, and Fiala and Golovach have shown NP-completeness even for chordal graphs with diameter exactly five. This raises a natural question as to what the role of diameter is for the complexity of packing colouring chordal graphs. In Chapter 2, the thesis provides a full and very pleasing answer: the packing colouring problem for chordal graphs is polynomial-time solvable when the diameter is restricted to be two, and NP-complete for any higher diameter restriction. (In fact, for chordal graphs of higher diameters, the problem is even hard to approximate.) On the other hand, for interval graphs, the thesis offers a polynomial-time packing colouring algorithm for any fixed diamater; and for proper interval graphs, an FPT algorithm parametrized by the size of the largest clique. A nice application of the logic of the last algorithm allows the author to compute in polynomial time the maximum number of coloured vertices, in any interval graph, in a packing colouring with given set of colours. These are very nice contributions to the understanding of complexity of packing colouring for some of the most popular restricted graph classes.

The area of classical k-colouring of graphs without forbidden induced subgraphs has been a hot topic in graph theory for several years now, with top research groups at Princeton (Seymour, Chudnovsky, etc.) and elsewhere driving the research forward by leaps and bounds. It is known that forbidding any induced graph other than a linear forest does not suffice to make k-colouring polynomial (for any $k > 2$). Great progress has occured recently for k-coloring graphs with forbidden induced paths of length t, with only few remaining open problems (which seem extremely difficult). Hence the focus shifted to disconnected linear forests, where two important problems were identified as having the smallest unknown complexity of 3-colouring. They were graphs without induced $P_2 \cup P_5$ and those without induced $P_3 \cup P_4$. In the thesis these two problems were shown to be polynomial-time solvable, even in the more general context of list colouring. This beautiful result completes our knowledge of complexity of 3-colouring for graph with any forbidden induced subgraph with up to seven vertices. The proof is an impresive combination of deep structural analysis, clever pre-procesing, and intricate propagation rules. In my view, this is one of the most significant contributions of the thesis, and is already having impact. In fact, Chudnovsky, Huang, Spirkl, and Zhong have since given polynomial time algorithms for list 3-colouring of $tP_3 \cup P_6$-free graphs which further extends the current work.

The last strand begins with a survey of fair cost problems, imposing a helpful classification framework (vertex/edge and deletion/evaluation) unifying several existing research directions, including fair feedback vertex edge set problems, defective colourings, minimum fair cuts, etc. Included is a helpful summary of known FPT and hardness results, and suggested research directions. Finally, the thesis closes with an FPT algorithm for any fair cost evaluation MSO_1 problem, when parametrized by the twin cover number and quantifier depth. This deep metatheorem is proved by an impressive array of tools including sophisticated reduction rules and quasiconvex optimization techniques. Then the thesis focuses on the fair vertex cover problem which is proved intractable when parametrized by tree depth and vertex feedback set, while an FPT algorithm is provided when parametrized by modular width.

I recommend acceptance of the dissertation. Dr Masařík has demonstrated he is more than capable of doing independent research, as well as collaborate with the world's top researchers and research groups.

Sincerely,

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