Abstract: This thesis consists of three parts devoted to graph labeling, hereditary graph classes, and parameterized complexity.

Packing coloring, originally Broadcasting Chromatic number, assigns natural numbers to vertices such that vertices with the same label are in distance at least the value of the label. This problem is motivated by the assignment of frequencies to the transmitters. We improve hardness on chordal graphs. We proof that packing coloring on chordal graphs with diameter 3 is very hard to approximate. Moreover, we discuss several positive results on interval graphs and on related structural graph parameters.

Hereditary graph classes are preserved under vertex deletion. We study graphs that do not contain an induced subgraph H. We prove that 3-coloring is polynomial-time solvable for $(P_3 + P_4)$-free and $(P_2 + P_5)$-free graphs and thus we have solved the last open cases for the problem on H-free graphs where H has up to 7 vertices.

Fair problems are a modification of graph deletion problems, where, instead of minimizing the size of the solution, the aim is to minimize the maximum number of neighbors in the deleted set. We show that those problems can be solved in FPT time for an MSO_1 formula parameterized by the size of the formula and the twin cover of the graph. Moreover, we define a basic fair problem, Fair Vertex Cover, and we show that it is $\mathsf{W}[1]$-hard parameterized by both treedepth and feedback vertex set. On the other hand, the problem is in FPT parameterized by modular-width. In addition, we provide a few more hardness proofs and we formulate several promising and interesting future research directions.