Supervisor's Review on the Doctoral Thesis

Author: RNDr. Jaroslav Horáček
Title: Interval linear and nonlinear systems

Contents of the thesis

The thesis consists of chapters devoted to particular problems from interval linear algebra. A typical chapter is build based on a published journal or proceedings paper. Usually, such a chapter looks as follows: a novel method (or methods) are proposed, several theoretical properties derived and a thorough numerical comparison performed. Methods are designed in a numerically verified way, that is, the numerical results are safe to rounding errors. In addition, there are provided many examples illustrating various properties of the methods.

The most relevant chapters are:

- *Square interval linear systems.* A novel method for solving systems for linear interval equations is proposed. The method iteratively cuts off parts of a given enclosure the provably contain no solution. In this way, it progressively makes the enclosure tighter.

- *Overdetermined interval linear systems.* Here, the author deals with the set of the true (not the least square) solutions of an overdetermined system of interval linear equations. He suggests several approaches, including least-square-like method and a method based on selecting and solving appropriate square subsystems (even though it might seem simple, there are interesting algorithmic points, e.g., how to utilize parallelism and the interim outputs of parallel threads).

- *(Un)solvability of interval linear systems.* The question is how to detect that a system of interval linear equations is solvable or unsolvable. The author proposes several sufficient conditions for both problems.

  An interesting result is on preconditioning. It claims that the Moore–Penrose pseudoinverse preconditioner is optimal on a specific set of matrices (which generalizes the well-known result for square systems). Other important results are those theoretically comparing the strength of the sufficient conditions presented.

- *Determinant of an interval matrix.* The author presents several novel approaches to enclose the range of the determinant of an interval matrix. He also proves inapproximability of exact computation. Further investigation is devoted to special cases, for which the determinant can be computed exactly (up to numerical accuracy).

- *Application of intervals to medical data.* This chapter is devoted to the project on mathematical processing multiple breath washout test. Interval methods are applied to solve the corresponding interval regression problem. As an output, there are several practical results. Some of them are of a negative nature: they demonstrate insufficient accuracy of the sensors and of the classical fitting models used, among others.

- *A linear approach to CSP.* In the context of constraint programming and global optimization, a novel relaxation procedure is proposed. First, nonlinear functions are linearized to interval linear functions, and then the interval problem solved by means of interval linear programming. Besides the algorithmic point of view, the author also proves that this method produces tighter enclosures than the so called Jaulin's parallel linearization.

The chapters are accompanied by an easy-to-follow introduction to interval computation and by an overview of computational complexity of the corresponding problems. One chapter is also devoted to the documentation of the interval package LIME, which assembles Matlab functions implementing the methods presented. On the whole, the thesis gives a comprehensive survey to the topic and may also serve as a readable introductory textbook.

Scientific contribution

The thesis is based on 3 journal papers and 6 proceedings papers, which is a significant base. The author contributes to all three areas: theory, methods and applications. The theoretical contribution was pointed out above for the particular chapters. The presented novel algorithms are competitive with the state-of-the-art methods, which is confirmed by exhaustive numerical experiments. The application in medicine is definitely an important step in promoting interval methods in practice.

The results were presented at about 16 international conferences and workshops. Even more, the author obtained the Young scientists award from MatTriad'13 and was invited as an invited plenary speaker to MatTriad'15. He was also the PI of two GAUK grants.

I would also like to point out the teaching activities of Jaroslav Horáček, in particular the fact that he supervised 5 bachelor students. Moreover, he motivated the students to research activities and successfully supervised some of them in this regard. In particular, P. Pelikánová was involved in his GAUK project on medicine data, J. Matějka collaborated on the work on determinants of interval matrices. The co-work with A. Szabo on parametric systems in still under development.

Questions and remarks

Compared to the extent of the thesis, it contains only a few of typos or inaccuracies. I have only some minor remarks:

- page 96: "The method `ge` works only for very small systems. Since for detection of unsolvability it must be used without preconditioning." I do not see why it must be used without preconditioning.
- page 97: I am not sure whether the function `beeck` was introduced.
- page 112: "L is upper triangular" should be "L is lower triangular"
- In Theorem 11.25, the converse implication holds true, too.

Summary

The doctoral thesis contains original research results, published also as refereed papers in scientific journals and proceedings, and obtained by an independent research work of the author.

I sincerely recommend to **accept** the thesis.

In Prague, May 15, 2019

doc. Milan Hladík, Ph.D.

Charles University, Faculty of Mathematics and Physics,
Department of Applied Mathematics,
Malostranské nám. 25, 118 00 Prague, Czech Republic,
e-mail: `hladik@kam.mff.cuni.cz`.