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Doctoral Thesis Evaluation Report

Name of the candidate:	**RNDr. Jaroslav Horácek**
Institution:	**Faculty of Mathematics and Physics**
Title of the thesis:	**Interval linear and nonlinear systems**

Dear Professor Kratochvíl,

it is my pleasure to provide the following detailed report on the doctoral thesis entitled “Interval linear and nonlinear systems” submitted for consideration by Mr. Jaroslav Horácek. My report first contains my overall assessment of the results and advancement of the work submitted, and some minor suggestions that Mr. Horácek may wish to consider when revising his work. Since my suggestions are straightforward, I do not need to review this thesis for a second time.

Detailed Report

The thesis considers linear and nonlinear systems, where the quantities of the systems are due to uncertainties but can be located between certain bounds. The underlying methods are provided by means of interval analysis. Over the last fifty years this area has advanced broadly and expanded across many branches in mathematics and its applications. The goal of this thesis is not only to present the results obtained by the candidate but also to provide a comprehensive survey. One of the merits of the work is that it could be used as a reference text on the state-of-the-art of the topics treated. Also for each problem, the diverse existing methods are not only theoretically presented but implemented and extensively tested against and compared to the newly developed methods. Always the focus is on verified methods, i.e., their results have to been guaranteed also in the presence of rounding errors.

In the first four chapters the basic means for the following chapters like interval arithmetic and prominent classes of matrices are provided. In Chapter 5, a central topic of the thesis, the solution of a system of linear interval equations, is treated. Besides di-

verse known methods, in §5.9 a new method, called the shaving method [81][1], based on the use of Farkas' lemma is presented which can be used to improve the enclosures obtained by any other method. In the next chapter, Chapter 6, known methods for the solution of overdetermined linear interval systems are presented. The new approach published in [83, 84] and explained in §6.8 is very simple but effective as the numerical examples document. The theme of Chapter 7 is the question of unsolvability of a system of linear interval equations. Based on a result in [164], some conditions for unsolvability are derived and compared in §§7.3-7.5. Chapter 8 is devoted to the enclosure of the range of the determinant on an interval matrix. This problem is only of theoretical interest when the radii of the component intervals are not rather thin. Chapter 9 reports on a joint project with the Department of Paediatrics at Charles University. As the discussion on a suitable underlying model shows, some questions are still open; however, the results of the project certainly will be very useful when the matter will be pursued further. Chapter 10 contributes to an important field, viz. the constraint satisfaction problem (CSP). The choice of an inner point for the relaxation (rather than a vertex) allows much more flexibility resulting in tighter enclosures as the ones given in [97, 99]. Chapter 11 provides a useful survey on the complexity of various interval problems in linear algebra preceded by a brief introduction to computational complexity. The last chapter presents details on the toolbox LIME which was used for most of the computations.

The thesis contributes to various interval problems mainly related to linear algebra. Theoretical statements are illustrated by careful selected examples and many computations. The interdisciplinary project has contributed to a significant real-life problem. Although here the results must be seen as somewhat preliminary, a simple and rather robust algorithm has been designed and valuable insights for the practitioner are gained.

The thesis is well written and clearly organized; the writing very competent. In my opinion, this thesis contains many significant advances in the area of interval analysis. There is no doubt that the work in particular presented in §§ 5.9, 6.8, 7.3, 8.3, 8.4, 8.6, 10.5 -10.9, and Chapter 9 is original and thoughtful research. A number of publications have resulted, besides six conference and workshop papers three papers have appeared in peer-reviewed journals (*Reliable Computing*, *Electronic Journal of Linear Algebra*, *Computers in Biology and Medicine*) which may be considered as a fine yardstick for measuring the quality of the writing and mathematics contained in this thesis. On the whole, the submitted thesis proves the author's ability for creative and also interdisciplinary research.

[1] The references are taken from the thesis.

Suggestions

p.85: introduction: Since it seems to be the first appearance of the terms **NP**-complete and **coNP**-complete, you should insert a reference to Chapter 11.

p.88, 3rd paragraph of §7.3: Why do you refer to a theorem by Frobenius? It is a simple consequence from linear algebra.
- Th. 7.4: Replace 'square' by 'rectangular'.

p.97: The caption 'beeck' (twice) is not consistent with the text.

p.107, 3rd paragraph of §8.4.2: I think that you have to assume that the union of the *k* intersecting discs is disjoint from the remaining *n-k* discs.

p.111, l.-4 of §8.4.5: If you require the strict inequality in the inequality for the absolute value of the inverse of *A* then not all *M*-matrices and inverse nonnegative matrices are included.

p.153, introduction: Where are the announced difficult examples? Do you mean Examples 10.6 and 10.7? Why are they difficult?

The thesis should be revised with respect to wording, grammar, and punctuation. E.g., at numerous places the definite or indefinite article have to be inserted. In the sequel I am listing some errors, mainly typographical errors:

p.39, l.2 of §4.8: Replace 'is' by 'in'.

p.41, l.2: Replace 'an' by 'a'.
- , l.6 of the introduction: Replace 'with' by 'by'.

p.42, l.2 of Def. 5.2: Delete 'the'.

p.44, l.-2: Replace 'another' by 'other'.

p.59, l.2 before (5.8): Use singular 'method'.

p.61, l.3: Use plural 'denote'.

p.67, l.-3 of the introduction: Use plural 'Their'.

p.68, l.-5: Insert 'of' before 'some'.

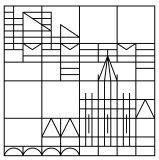
p.85, 1st l. of the 2nd paragraph: Replace 'am' by 'an'.

p.91, l.-3: Replace 'th' by 'the'.

p.124, l.-3 of the 2nd paragraph: Write 'inert'.
- , l.-4: Remove the space between 'draw' and 'back'.

p.125, 1st l.: Replace 'Figure 9.5' by 'Figure 9.2'.

p.138, 1st l. of §9.7.1: Delete 'is' before 'consists'.



p.154, l. after (10.4): Delete ‘a’.

p.178, l. after Ex. 11.14: Delete one of the two ‘can’.

p.194, l.-6: Replace ‘an’ by ‘and’.

p.201, heading of §12.5.3: Write ‘under’.

p.205, many occurrences: Use capital letter ‘in Czech’.

Sincerely yours,

Jürgen Garloff

Professor at the University of Konstanz (apl.), Department of Mathematics and Statistics

and

University of Applied Sciences / HTWG Konstanz, Institute for Applied Research