Referee Report on the Doctoral Thesis „Mathematical Modeling of Blood Coagulation Process“ written by Marek Čapek

The present thesis is devoted to numerical approximation of blood coagulation processes. Despite of its utmost importance mathematical modelling of biological processes is a very complex task. The models presented in the thesis combine the first principles, viscoelastic rheology and phase field approach to describe the time evolution of blood platelet plugs and clot growths. The author derived general three-dimensional models and performed fully three-dimensional MPI simulations applying the deal.ii software package.

The thesis of Mr. Čapek consists of 6 chapters and 4 appendices. After description of main thesis objectives and research results the candidates presents in the second chapter biological introduction of platelet plug formation and evolution of blood clots.

The third chapter is devoted to mathematical modelling using the first principles, conservation laws and relevant ingredients of non-Newtonian (viscoelastic) fluid rheology. Indeed, it is well-known that blood is a suspension of a large amount of blood cells; red blood cells, white blood cells and platelets suspended in a Newtonian solvent - the blood plasma. Experimental investigations indicate that blood is capable of stress relaxation and shear-thinning. These non-Newtonian phenomena start to be more profound in small blood vessels or if the local flow phenomena become important, as it is the case of ill patients. Chapter 3 also describes mathematical modelling of blood coagulation processes that yield the platelet plug formation models, blood clot formation models and retraction models. The focus is mostly on the platelet plug formation and blood clot coagulation. The candidate concentrates on the diffuse interface representation of the Weller model of clot growth by means of the corresponding Cahn-Hilliard-type approach. Furthermore, the model of Storti for clot growth and the Kempen model for blood clots are presented as well. At the end of chapter the candidte summarizes all mathematical models and points out the realization using the phase field approach.

Numerical methods are presented in the fourth chapter. The Navier-Stokes equations are approximated by the well-known Chorin projection algorithm for pressure/velocity decoupling. As far as I can see there are no details on how the phase field part is approximated numerically. Indeed, it is a well-known fact that phase field models have to satisfy the discrete energy dissipation law, otherwise unphysical (or non-biological) numerical solutions can be obtained. I have not find any discussion on this property of numerical schemes for the Cahn-Hilliard part. What is the total energy? Is it dissipative on the discrete level?

For the viscoelastic equations a special regularization known in the literature as the EVSS approach is applied. Transport equations are adequately approximated by the SUPG method that adds additional numerical diffusion to stabilize convective dominated flows. The presented models are implemented within the deal.ii software using MPI paralelisation technique. Chapter 5 is devoted to numerical simulations demonstrating the capability of the derived models to simulate the clot development and platelet plug formation. The candidate tried to derive some comparisons between the results obtained by the Weller model and the Storti model, though the statements are less clearly specified.

Karel Čapek presents in his thesis new original scientific results that have been obtained in a modern and highly competitive field of mathematical modelling and scientific computing. Despite of some errors in English and somehow overloaded model descriptions, it is definitely a very interesting thesis devoted to deep understanding and simulation of blood coagulation processes. Therefore, I propose to accept the manuscript as a doctoral thesis in the Mathematical and Computer Modelling at the Charles University in Prague.

In conclusion, I strongly believe that the candidate demonstrated that he is able to derive new original scientific results. The present thesis fulfils all requirements for a successful Ph.D. thesis and should be accepted for the award of the PhD.