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**Review of the doctoral thesis**  
**Inversion and Depth Range of Dipole Electromagnetic Induction Measurements in Geophysics**  
**by Fernando César Moura de Andrade**

The presented thesis is dedicated to the dipole electromagnetic profiling method, one of the geophysical techniques used to map the distribution of electrical conductivity under the Earth's surface. In the introductory chapter, the author gives an overview of the method in both the horizontal coplanar (HCP) and vertical coplanar (VCP) coils configurations, and introduces the basic concepts of apparent conductivity, depth of investigation, and the low-induction-number (LIN) approximation. The second chapter is dedicated to the task of forward modeling in the cases of homogeneous half-space, and layered Earth. His own extension of the response functions taking into account the elevation of the primary coil above the surface is introduced here, while a more detailed derivation has been published by the author and is attached in Appendix B. The third chapter deals with the inverse problem formulation, discusses the inherent non-uniqueness of the solution, and its dependence on the dimension of data and model spaces. The linear and non-linear inverse problems, their regularizations, and various methods to solve them are presented here. In the fourth chapter, different 1-D inversion schemes are tested, followed by a test of a quasi-2D inversion, which regularizes local 1-D inversions using the solution obtained from the adjacent profiles. This inversion is then applied at two sites, and the results are confronted with the images provided by the electrical resistivity tomography (ERT) method. This discussion is carried over the the fifth chapter, followed by a short summary. The appendices A, and C-E are extended abstracts presented at

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international conferences, and deal respectively with the application range of the LIN approximation, real-time 1-D inversion of multi-configuration data, 2-D VLF data inversion, and a shortened presentation of the paper already included as Appendix B.

In this review I will concentrate on the problems of forward and inverse modeling, as presented in the chapters 2-4.

1. With respect to the derivation of the mutual coupling ratio for the homogeneous half-space, I believe that the terminology used here is misleading. While the formula (2.2) for the total vertical magnetic field measured by the receiver is correct, in accordance with Ward and Hohmann (1987), and previous derivations by Wait (*J. App. Phys.* **23**, 497, 1952), the  $Q_V$  and  $Q_H$  couplings in equations (2.4) and (2.5) correspond to the ratio of *total* to primary field, not *secondary* to primary field, as defined by the author. It can be seen also from the fact, that (2.4) is obtained directly by dividing (2.2) with (2.3), and that the limit of (2.4) is 1 as  $|k| \rightarrow 0$ . I think that McNeill (1980), whose formalism this author generally follows, means by the *secondary field* the total signal observed by the receiver loop.
2. On the other hand, in the equations (2.20) and (2.21) for the case of layered Earth, the coupling ratios indeed compare the *secondary to primary* vertical field, as can be seen from eq. (539) in Keller and Frischknecht (1966). I find these differences in terminology rather confusing and would welcome a clarification and confirmation to be presented during the defense.
3. I think that the use of the term *Principle of equivalence* in Chapter 3 can be confusing, as this term is traditionally used for equivalence of electromagnetic sources. Here the author discusses the non-uniqueness of the transfer functions with respect to the conductivity and layer thickness.
4. In the presentation of the results of the inversions of the synthetic datasets in Chapter 4, the proper analysis of errors is missing. By showing only a single best model for each technique, it is difficult to judge the general applicability of the individual approaches. Given the small dimension of both model and data spaces, and the speed of the forward model, even in the non-linear case, complete exploration of the model space was achievable, and probability density function of the conductivities and layer thicknesses could be constructed.
5. The quasi-2D method in Chapter 4 represents an interesting approach to provide smooth 2-D profiles. I think that case 6 demonstrates the problem of this method when sharp lateral conductivity contrasts are present. It would be useful to test also the dependence of the results on the choice of the initial model (step 1 of the algorithm on page 34).

Notwithstanding the remarks specified above, I recommend to accept the presented thesis for the purpose of conferral of a PhD.

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