

Level Truncation Approach to Open String Field Theory

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Open bosonic string field theory is a field theory of fluctuations of a D-brane in bosonic string theory. The fluctuations are characterized by open strings which can attach to the D-brane. Since open strings can mimic an infinite variety of particle states depending on how they vibrate, a field theory of open strings is characterized by a Lagrangian containing an infinite number of spacetime fields. When given an action for any quantum field theory, perhaps the first thing one wants to do is solve the classical equations of motion to determine the space of vacua around which one can compute perturbative scattering amplitudes. In string field theory this is an especially challenging problem since the action contains an infinite number of fields. Following the work of Kostelecky and Samuel, and later Sen and Zwiebach, one can make the task manageable by ignoring all fields above a certain mass, and solving the resulting approximate equations of motion for a finite number of field variables, usually numerically. This approximation scheme is referred to as *level truncation*, and since the early 2000's has been the most important tool for extracting physics from the Lagrangian of open bosonic string field theory.

There is a general expectation that the set of classical vacua of open bosonic string field theory should correspond to the set of possible D-brane configurations in a fixed spacetime background in bosonic string theory. A D-brane is completely characterized by boundary conditions in a two dimensional quantum field theory on a disk; this is the quantum field theory of the open string worldsheet, whose states represent open strings attached to the D-brane. Technically, the two dimensional quantum field theory is a kind of boundary conformal field theory (BCFT). The BCFT has vanishing central charge, and takes the form of a tensor product of a bc system of central charge -26 with a “matter” BCFT of central charge $+26$, where twenty-six is the dimension of spacetime in bosonic string theory. Therefore, it is believed that each classical vacuum of open bosonic string field should correspond to such a boundary conformal field theory. This, however, is a highly nontrivial claim and has been an outstanding problem for a number of years. First, one has to devise a method for extracting sensible data specifying a two dimensional quantum field theory from a solution to classical equations of motion. Second, one has to show that the space of solutions really covers the set of all possible BCFTs specifying D-branes in a fixed closed string background.

The broader significance of this problem is twofold. First, it would establish that solving the equations of motion can produce new D-brane systems/BCFTs which otherwise have not been characterized. Second, it would show that open string field theory is a nonperturbative formalism which is capable of describing the physics of disparate string vacua in a single framework. Such a formalism seems necessary to understand questions of vacuum selection and cosmology in string theory, which is perhaps the central theoretical obstacle to comparing the implications of string theory to physics in the real world.

Here we give a broad summary of the contents of the thesis:

- Chapter 1 gives a compact but useful overview of boundary conformal field theory, including examples which play an important role in the thesis, in particular the free boson, minimal models, and the bc system.
- Chapter 2 starts with the definition of the string field theory action, a small detour on analytic approaches to solution of the field equations, and a broad summary of the numerical approach in level truncation. Then the discussion moves on to describing the contents of the fluctuations fields of a given D-brane, i.e. the string field. If the BCFT of the initial D-brane is known, the contents of the string field are not that difficult to describe at first passing; it is given by coefficients of a basis of mode oscillators acting on the vacuum state of the BCFT. What is more relevant for the purposes of this thesis is that the string field typically contains many more variables than are necessary to find solutions of interest. There is a lot of efficiency to be gained by taking a refined ansatz. Typically in solving the equations of motion one imposes the so-called Siegel gauge, and in this gauge one can look for solutions in a subspace spanned by $SU(1,1)$ singlets in the bc system. A practical implementation of computations in this basis is one of the significant innovations presented in this thesis. In the matter BCFT, depending on what kind of solutions you are looking for, one is typically interested fields generated by a restricted set of primary states and their Virasoro descendants. The discussion proceeds to describe various quantities of interest that can be computed once a solution is found. These include the value of the on-shell action, which for time-independent backgrounds is related to the energy of the D-brane system, and the so-called Ellwood invariant, which is related to the boundary state of the BCFT describing the D-brane system. The Ellwood invariant provides the crucial link between the classical solution and the data specifying a two dimensional quantum field theory. Another important quantity is the the out-of-Siegel gauge equation, which represents a part of the field equations which should vanish on a solution but are not accounted for when solving the gauge-fixed equations of motion. The chapter concludes with a description of conservation laws; these are formulas which relate a creation operator acting on a string vertex to a sum of annihilation operators. In different variations, conservation laws give an efficient method for computing the action and Ellwood invariant in the bases of states introduced earlier.
- Chapter 3 describes the practical implementation of numerical solution of the field equations. This includes the discussion of algorithms for computation of the kinetic terms and cubic vertices in the action; the parallelization of computations; a discussion of the homotopy continuation method as a technique for scanning the equations of motion for solutions at low truncation levels, followed by Newton's method to refine interesting solutions with higher level computations; the computation of various quantities of interest once the solution is found, including the extrapolation of results to infinite level.
- The remaining chapters describe the solutions that have been found following the previously described procedures. Chapter 4 considers the solutions when the string field is restricted to the universal sector, where all states are given by matter Virasoro descendants of ghost oscillators acting on the BCFT vacuum state; chapter 5 considers solutions when the string field consists of matter Virasoros and ghost oscillators acting on primary states of the free boson on the circle; chapter 6 considers solutions for marginal deformations in this setup where the free boson is on a circle at the self-dual radius; chapter 7 considers string field as

given by matter Virasoros and ghost oscillators acting on primaries of two free bosons which form a torus; and chapter 8 considers the string field as given by matter Virasoros and ghost oscillators acting on primaries of various minimal model BCFTs.

The following summarizes what in the referee's view are the most significant results contained in the later chapters:

- The most fundamental solution of open bosonic string field theory is the tachyon vacuum, representing the final product of the tachyonic instability of the initial D-brane, where the D-brane has disappeared and we are left with empty space. The candidate managed to compute this solution in Siegel gauge out to the “holy grail” level 30, which requires determining constant expectation values for some 85,000 fields. The significance of level 30 originates in celebrated computations which showed that the energy of the solution, which is expected to be zero, overshoots and becomes slightly negative at level 14. Based on extrapolations it was hypothesized that the energy would become increasingly negative until level 28, at which point it would begin to rise again to zero in the infinite level limit. The computation out to level 30 established that this indeed happens.
- In the context of the free boson and minimal models, the candidate has found the first solutions for backgrounds with higher energy than the initial D-brane. For many years such solutions have been intensively searched for without success, both numerically and analytically; this failure was taken as a sign that open bosonic string field theory may not be powerful enough to describe all D-branes. The candidate found that such solutions emerge in a rather surprising way: At low levels, they appear as complex solutions which would otherwise have been overlooked or discarded, but if tracked with Newton's method at a specific high level they suddenly become real and there is a drastic change and improvement in their behavior. For the tachyon lump solution on a circle of radius $1/\sqrt{2}$, the shift in behavior occurs at level 14.
- Solutions for marginal deformations, representing continuous deformations of the initial D-brane, have also long presented problems. For example, if we start with a lower dimensional D-brane, we can search for solutions representing translations of that D-brane orthogonal to its worldvolume. In level truncation studies it was found that the massless scalar representing this translation could only track the displacement over a small distance. Long ago it was proposed, based on a rather loose analogy with the lump solution of ϕ^3 theory, that solutions representing further displacement may exist, but the expectation value of the massless scalar does not give a well-defined coordinate on the moduli space of translations of the D-brane. Using the tachyon field as a coordinate instead, the candidate has confirmed that this indeed happens, though solutions representing larger displacements are increasingly difficult to accurately describe at finite levels.
- In the context of two free bosons forming a 2 dimensional torus, the candidate has constructed so-called “exotic solutions,” representing heretofore unknown or poorly understood configurations of D-branes. Through the Ellwood invariant, it is possible to extract new information about these new BCFTs. This shows that open bosonic string field theory cannot only be used to confirm old conjectures, but can be used to perform entirely new calculations.

Any one of these results would be enough to make the candidate universally known amongst practitioners of string field theory. The candidate's computations have been the backbone of much recent work on string field theory in Prague in recent years. In the referee's opinion, this contribution far surpasses what is normally seen in a PhD thesis, and more closely approximates what might be considered the results of a broad research program of a senior scientist. The thesis will almost certainly be considered the definitive reference on this subject for many years to come.

It goes without saying that the referee recommends immediate acceptance of this doctoral thesis.

Dr. Theodore Erler

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