Charles University in Prague

Faculty of Social Science

MASTER THESIS

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Equilibrium Exchange Rates and Exchange Rate Misalignments in the Visegrad Group

Institut of Economic Siences

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Study programme:	Economy and finance
Study branch:	Corporate Strategy and Finance

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Title: Equilibrium Exchange Rates and Exchange Rate Misalignments in the Visegrad Group

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Abstract:
The main objective of this diploma thesis is to estimate equilibrium exchange rates for four countries forming Visegrad Group partnership and to evaluate whether their exchange rates are overvalued, undervalued or in equilibrium with their economic development and with market. We focused on two widely used models, Behavioral Equilibrium Exchange Rate Model and Fundamental Equilibrium Exchange Rate Model and estimated several alternations of each model. Results where then compared and we derived implications for each country. We could clearly see that each country is developing differently in terms of equilibrium exchange rates. While the exchange rate of Czech Republic seems to oscillate steadily around its equilibrium value, we spotted higher misalignments for the rest of the country. Also, we saw that Hungary is on path of constant depreciation, which is related with economic issues country is struggling. Polish Zloty is characterized by overall high amplitude of movements of its exchange rate and by frequent undervaluation of the currency, implying favorable development of the economy, taking into consideration the constant growth and productivity gains. For Slovakia we saw that despite its dynamic growth most of the models were showing that the currency was overvalued. We observed also the development of equilibrium exchange rate even after adopting common euro currency in order to see the macroeconomic implications as the real exchange rate was just nominal exchange rate deflated by price ratios.

Keywords: směnné kurzy rovnováha Visegrad FEER BEER VECM

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Abstract:
Hlavním cílem títo diplomové práce je odhadnout rovnovážné směnné kurzy čtyř zemí, které tvoří partnerství Visegrádské skupiny, a zhodnotit, zda jsou jejich směnné kurzy nadhodnoceny , podhodnoceny nebo v rovnováze s jejich ekonomickým vývojem a trhem. Zaměřili jsme se na dva široce používané modely, a to model behaviorálního rovnovážného kurzu a model fundamentálního rovnovážného kurzu a odhadli několik variant každého z nich. Výsledky byly poté porovnány a my jsme odvodili důsledky pro každou zemi. Z výsledků je zřejmé, že každá země se vyvíjí jinak, pokud jde o rovnovážné směnné kurzy. Zatímco se zdá, že směnný kurz České republiky se stabilně pohybuje kolem své rovnovážné hodnoty, v ostatních zemích jsme zaznamenali vyšší nesourodost. Také jsme viděli, že Maďarsko je na cestě neustálé depreciace, což souvisí s ekonomickými problémami, se kterými se země potýká. Polský Zloty se vyznačuje celkově vysokou amplitudou pohybů svého směnného kurzu a častým podhodnocením měny, což znamená příznivý vývoj ekonomiky s přihlédnutím k neustálému růstu ekonomiky a růstu produktivity. V případě Slovenska je vidět, že navzdory jeho dynamickému růstu většina modelů ukazuje, že měna byla nadhodnocena. Také prostrednictvím deflovaného nominálniho kurzu pomerem cenových hladin jsme pozorovali vývoj rovnovážného směnného kurzu i po přijetí společné měny euro, abychom mohli vidět reálne makroekonomické důsledky.

Keywords: směnné kurzy rovnováha Visegrad FEER BEER VECM

I would like to thank professor doc.Mgr. Tomáš Holub Ph.D for his professional guidance, willingness, advice, help and time. At the same time, my thanks to professor doc.PhDr. Tomáš Havránek Ph.D. and professor PhDr. Jaromír Baxa Ph.D. for helpful advice and answers to all my questions.

Contents

Introduction

The Visegrad Group is partnership of four countries bounded by history, culture, common values and objectives. The tight links between those countries in not based purely on geographical position, it was formed through history, common enemies they needed to fight, states they were parts of and goals they wanted to achieve. Although this partnership exists formally for almost 30 years, the cooperation between those four countries lasts centuries. After fall of communist regime, all four countries had the very same objective, to become democratic states and to be integrated in European community. They decided to use their tight connection as tool to increase their bargain power and work together to achieve the common goal. They confirmed their close relationship and cooperation by creating free trade area. The joined bargaining was crucial especially in talks for joining EU and NATO. All four countries together signed the initiation agreement to EU and they joined the EU together in 2004. Even after joining global European and world market and opening their markets to the world they remain important strategic partners and cooperate closely on many political and economic fields.

Without a doubt the closest among them are Czech Republic and Slovakia, which were still one country when the Visegrad Group declaration was signed. Those countries remained close even after dissolution, which is considered as the most peaceful split of countries in human history. Long common history predestinated those countries to many common characteristics, and yet we can observe differences and unique features for each. This does apply globally to whole Visegrad Group. Those four countries remained very similar and yet very different in various factors.

The most visible difference is that Slovakia as the only country from Visegrad Group joined already in 2009 the Euro Area and adopted common euro currency while the rest of the countries keep their national currencies. However, the exchange rates of those national currencies against euro are very heterogenous, showing different strengths and weaknesses of each country. Already the exchange rate regimes differ, as it is not that long ago that exchange rate of Czech Koruna was regulated in period between years 2013 and 2017 by central bank interventions with fixed lower bound of 27 Korunas per euro,

not allowing the currency to appreciate too much and helping the economy grow.
They are just exchange rates that give us an initial view of the economic development of the country without need to look on GDP growth and other macroeconomic aspects. Usually even to laic person, strong currency evokes the assumption of strong economy. However, with all interventions,á market changes and monetary policy operations are the exchange rates we are observing the real factors we should look at? Are they the exact determinants we are searching for? What if the economy is slowing down or overheating or the appreciation of exchange rate is just temporal? All those questions lead us to the notion of equilibrium exchange rate.
What is the equilibrium exchange rate? It is the exchange rate of national currency when the whole economy is in equilibrium, when demand for currency meets its supply. However, that is very simplified definition, in order to be able to determine this rate we need to have closer look on various macroeconomic fundamentals of the country not only simple demand supply function.
That is the purpose of this diploma thesis. We will have close look on countries forming Visegrad Group. We analyze their economic development, their similarities and differences and we try to estimate their equilibrium exchange rates by most common valuation models, BEER and FEER models estimated by various approaches. The most common in scientific literature are models from Clark MacDonald, which we will use together with models created by Czech economists Komárek & Motl and Pošta. The last estimated models are based on FX valuation models computed by JP Morgan and Deutsche Bank.
The structure of this diploma thesis is as follows:

- First chapter provides overview of Visegrad Group, the economic development of the countries, common features and differences.
- Second chapter covers the theoretical background needed to understand the topic of equilibrium exchange rates.
- Third chapter develops further the models used in the diploma thesis for estimating the equilibrium exchange rates and their misalignments for each country.
- Fourth chapter summarizes the data and the methodology used in the thesis.
- Fifth chapter discusses the results of estimated models and their implications.
- Last chapter shows the missalignments of the exchange rates estimated by each model for each country

I

The Visegrad Group

So what one can imagine under the Visegrad Group? Visegrad Group was created 15^{th} February 1991 by signing the agreement of post-socialist neighbored countries Czechoslovakia, Poland and Hungary connected by common culture, history and similar economic development. After the dissolution of Czechoslovakia in 1993, both newly former countries Czech Republic and Slovakia rejoin the Visegrad Group forming already famous so called Visegrad Four. The principal aim of the Visegrad group was cooperation to empower the democratization and reformation, strengthen their international position and integrate themselves into European community. This grouping aims to represent the interests of their citizens unified in front of European structures and other strategic international alliances and partnerships. The initiating declaration of Visegrad countries clearly stated 5 objectives:

- Full restitution of state independence, democracy and freedom
- Elimination of all existing social, economic and spiritual aspects of the totalitarian system
- Creation of modern free market economy
- Full involvement in the European political and economic system, as well as the system of security and legislation

Those four confirmed their close cooperation by signing the Central European Free Trade Agreement, creating free trade area in Central Europe followed by signing the Association Agreements with the EU. Their efforts resulted in common joining of European Union and NATO in 2004.
Despite this tight connection and cooperation, countries forming Visegrad Group are still

four independent countries, with independent policies and central banks, therefore now, we should look closer on factors which are common for all four countries, and which are differentiating them among each other.

I.I Monetary policy and Inflation

The main goal of central banks of all countries from the Visegrad Group is targeting the price stability, which is in line with European Union Legislation, more concretely Treaty on Functioning of European Union. The main tool of all four countries is inflation targeting. Central banks are publicly announcing the inflation target, they are aiming to fulfill by monetary policy tools and interventions. However, each country implemented the inflation target at different point in time and each accesses the inflation targets differently. Czech National Bank implemented the inflation target already in 1998 as first of the Visegrad Countries, while Slovakia was the last country adopting the inflation target regime in 2005.

The graph I.1 presents the historical development of the inflation across the Visegrad Group together with the average inflation rate in European Union. The inflation is measured in HICP indexed to 2015 = 100 and displayed as annual average rate of change (%). The dashed line represents the most common inflation target set within the EU countries, 2% level. Setting target on this level is strategical and in line with general belief that it possibly represents the optimal level for keeping prices stable and creating conditions for sustainable economic growth. The set target should compass the upward statistical deviations that arise in the measurement of inflation and should also allow enough room for the small changes in relative prices that occur constantly in every economy with an effective price system.

The second graph shows the development of exchange rates of national currencies against euro of all countries forming the Visegrad group.

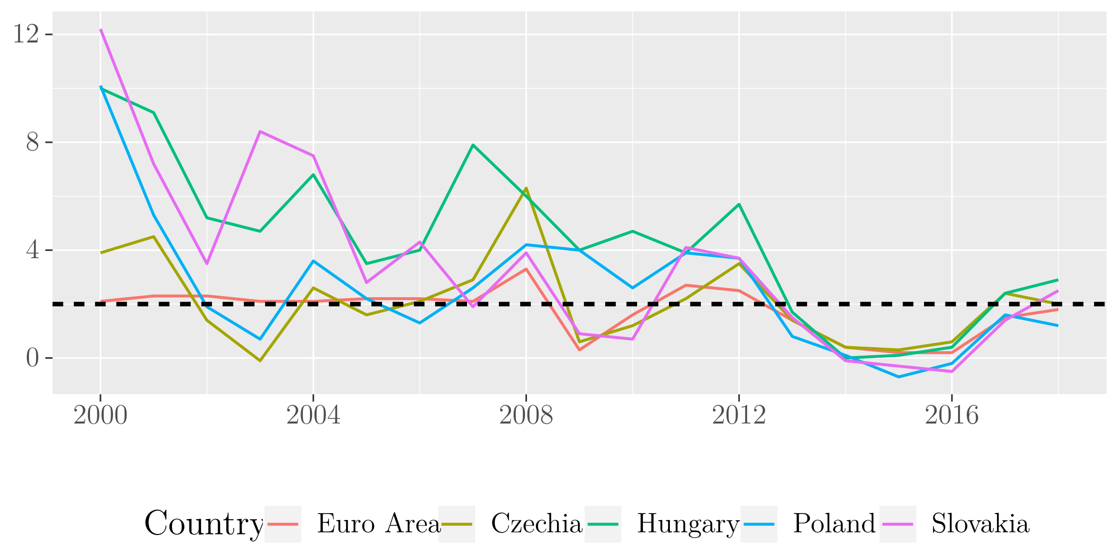

Figure I.1: Inflation development in Visegrad Group compared to European Area

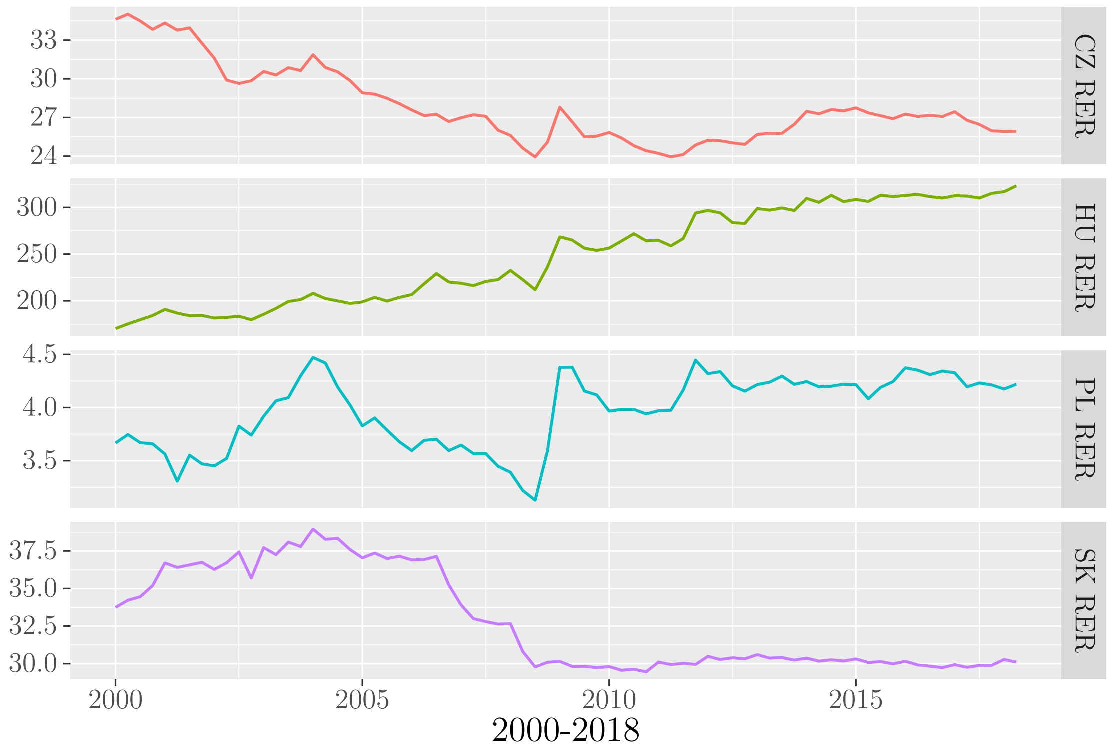

Figure I.2: Evolution of Exchange Rates in Visegrad Group

Czech National Bank (henceforth CNB) is keeping inflation at stable path targeting 2% growth with fluctuation bound of +/- 1 percentage point to ensure price stability and support the general economic goals of the Government of the country. This inflation

target was set in 2010 and it is held since then. Before the target was on 3% level with the same tolerance bound. The target band for headline consumer price inflation was changed to horizontal target between years 2002 and 2005.
The target is transparent and periodically publicly announced, focusing mainly on medium-term inflation. The independence of the CNB assures that the bank board has the full power to change the monetary instruments if the inflation deviates too much from its target. The usual tools CNB uses to control and navigate the inflation are repo rates, which are increased if there is need to lower the aggregate demand or lowered in order to reach the opposite goal. Rising repo rates are often connected with tighten monetary policy.
CNB liberated the fix exchange rate regime of Czech Koruna already in 1997 and was using interventions to keep the exchange rate against German Mark low in order to support exports. Despite the efforts, Czech Koruna was continuously appreciating, but low inflation was creating interesting environment for international investors, which started to flow into country. In 2002 CNB interrupted the unceasing appreciation by heavy intervention of 2.2 million euro and lowered interest rates. Until 2013, CNB let the exchange rate of Czech Koruna fluctuates freely in managed float regime. On th November 2013 CNB intervened against appreciating Czech Koruna and introduced the one-sided exchange rate commitment keeping the exchange rate over 27 Czech Korunas per euro to prevent deflation. The fixed lower bound of 27 Korunas per euro was then held until April 2017. After leaving the commitment, the exchange rate appreciated significantly and currently is moving under 26 Czech Korunas per euro. After the split of Czechoslovakia, the level of inflation was even above 10%. CNB then implied more strict monetary policy to diminish the inflation rate which helped to get the rate under 5% level. But inflation was still not on desired target level, which was mostly caused by volatile exchange rate of that period. Holub [2008] The inflation was stabilized between years 2003 and 2007. After crisis, during the period of economic recession, the inflation was decreasing to alarming values and interest rates were not able to pull it back. As the deflation become a real threat to the economy, CNB decided to use exchange rate as tool to stabilize the inflation because interest rates (the main tool of inflation targeting) was not enough to avoid the deflation. At the end of year 2012 the interest rates were already lowered on zero lower bound (meaning they reached 0.05 % level), but inflation was still decreasing. That was the reason to introduce the exchange rate commitment. Thanks to this act Czech economy grew by 2% already in next year, 2014.
From international prospective the relative price levels of Czech Republic comparing to

average of EU countries were after the split of the countries around 40 to 50%, but with strong economic growth already in 2011 this ratio was around 70% .

Central bank of Hungary (henceforth MNB) has the same main goal, price stability which goes hand in hand with stable sustainable economic growth. The inflation targeting regime was implemented in Hungary in 2001, few years after Czech Republic and Poland. Inflation targets were set simultaneously on a year basis up to the end of 2006. Since 2007 MNB has defined explicit inflation target on average level of 3 % increase in consumer prices, which is held until now. The reason for the slightly higher target comparing to the common inflation target across the EU of 2% is the still continuing convergence process of the Hungarian economy.

The main tool the MNB uses are changes in central bank base rate. This rate is determined by the monetary council of Hungary. After final decision the result together with voting procedure which is followed in order to set the base rate is publicly announced to keep the decision makers accountable for their vote. Together with inflation target, quarterly inflation projections are published.

Other instruments the MNB is using apart from the base rate are Forint liquidity providing swap instrument and the interest rate corridor.

Hungary has floating exchange rate regime and currency is denoted per euro. The Hungarian Forint is the weakest currency among the countries of Visegrad Group and through the period we observe continuous depreciation of the currency. Weakening currency might be favorable for exports in short-term but such long-term depreciation as is currently happening in Hungary is bringing additional risks and weakens overall country economic position. As we will see further Hungary is facing various economic issues as high debt to GDP ratio or low productivity growth. Combination of those factors can create vicious circle of economic downturn which can be dangerous when new economic crisis arise.

That is also reason why MNB introduces its second goal, the self-financing program that was added to its objectives in April 2014 as response to the high external and FX government debt. The desired outcome is to reduce country's vulnerability towards those risks using the monetary policy instruments. As we will see in further sections this commitment is MNB successfully fulfilling over the period, however, there is still long way to go.

Hungary as well as Czech Republic had to fight high inflation rates in 1990s, which was the common feature for post-socialist countries, however, the inflation in Hungary was even higher and in 1997 the inflation rate was above 18%. Even though the inflation target was set already in 2001, MNB was not very successful in targeting. The average

rate of inflation in early 2000s was around 6%. After crisis, Hungary was dealing with common threat of decreasing inflation rate. The evolution of inflation was quite volatile in crisis aftermath. The inflation started to decrease shortly after reaching the peak in 2012 when the inflation almost reached 6% level. In 2014 the inflation hit the zero bound level mainly due to cuts in energy and other utilities prices. Inflation rate then stayed extremely low in following two years caused by combination of multiple factors as subdued imported inflation, low food prices, regulated energy price and already very low inflation expectations. In third quarter of 2016 the inflation finally rose above 2% level and started converging towards its target.
Polish National Bank (NBP) has the regime of inflation targeting since 1998, adopting it as second Visegrad country, a year after CNB. The continuous target that is followed till now was set in 2004 on 2.5% level with permissible deviation band of 1 % point in both directions. The NBP is focusing on keeping the interest rates at consistent level with inflation target, by affecting the short-term nominal interest rates, which are used together with open market operations, reserve requirements or credit-deposit operations. The last mentioned, the credit-deposit operations are used to mitigate the fluctuations on open market, where central bank effectuates operations with commercial banks. The limits are usually set by Lombard rate and NBP deposit rate. Short-term (overnight) deposits, inter-bank market rates are also introduced to prevent the excess of liquidity in commercial banks.
The exchange rate policy in Poland is set on freely floating exchange rate regime since 2000 but remains as possible monetary policy tool if NBP would need to intervene in order to keep inflation within the target. Shortly after the introduction of floating exchange rate regime the currency appreciated heavily, both in nominal and real terms. One of the reasons to abandon the crawling pack and enter totally free float was persisting problem of internal inconsistency. The intervention on FX market were sterilized, which helped to rise the foreign reserves and central bank debt which was used to diminish excess liquidity surplus. The floating exchange rate helped Poland to went through turbulent converging period, and Polish Zloty become generally very stable currency. Beza-Bojanowska and MacDonald [2009]
Similarly, to the rest of Visegrad Countries, Poland was fighting high inflation rate at the beginning, but the rate diminished rapidly and in 2003 inflation rate almost hit zero level. The average level of inflation in 2000s before crisis was around 2.5%. After crisis the trend of lowering inflation hit also Poland, but inflation started to grow again shortly after. In 2012, inflation reached 3.7% due to depreciation of the zloty, high commodity

and increases in administered prices. The continuous decrease of energy prices, subdued food prices and weak domestic demand caused in the next year the fell to 0.8% and following years Poland had to fight with deflation until 2017.
Slovakia, as the only country of Visegrad group, adopted common euro currency, January 2009 and become member of the Eurozone. Since then, the monetary policy is depending on European Central Bank (ECB). The ECB aims the same goal, price stability, as the rest of the Visegrad Group using inflation target as the main tool.
From historical perspective, the euro adoption helped slovakia to stabilize the economy and the inflation. Slovakia was facing economic destabilization in early 2000s and had to drastically lowered the foreign exchange reserves in attempt to save exchange rate of Slovak Koruna. However, it was not very successful move and Slovak Koruna depreciated heavily.
Before entering the Eurozone, Slovak Koruna was in managed floating regime, which was held until 2005 when Slovakia joined the ERM II, in order to proceed with adopting the euro currency. Slovakia committed itself to continue in healthy fiscal policy and support the gradual development of real wages in line with productivity growth of the country. The ERM II helped Slovakia to stabilize the exchange rate and the sterilized parity in ERM II was revalued in 2007 to level of 35.4424 Slovak Korunas for euro and due to unceasing positive economic development, the currency was revalued again in 2008 to the final exchange rate level 30.126 Slovak Korunas per euro.
In this period, National Bank of Slovakia (NBS) also adopted inflation target as main goal of its monetary policy, however the inflation target was introduced as last among Visegrad countries. Before the target the monetary policy was following management method, which was setting continuously intermediate targets. The main goal was monetary stability, not the price stability. The main tools used by NBS were two-week Repo rates, sterilization of interest rates for one day deposits or loans. Since 2005 Slovakia had to coordinate its inflation target with ERM II conditions to follow requirements needed to adopt the euro currency. The initial target was set to 3.5% with fluctuation corridor of 0.5% .
After adopting the common euro currency, the NBS must participate in common monetary policy regime, led by ECB. The inflation target changed to common 2% level.
Inflation in Slovakia had similar development as in Poland in late 90s, however at the turn of 1999 and 2000 the inflation rose significantly above the 12% level, which was at the time the highest increase among Visegrad Group countries and despite the decreasing trend the inflation rate was quite high through whole period before the crisis. The relative

price levels were around 50 % of the European average, however rising fast especially after reforming period which came with change of political climate after the new government was named, without Vladimir Mečiar participation. The after crisis period coincides yet with ECB common monetary policy, as the euro adoption happened during the crisis.
Generally, Slovakia struggled a lot with high volatility of inflation during all observed period. After the crisis the inflation in Slovakia dropped significantly resulting in very low inflation, below 1%. The stabilization of inflation came only after the euro adoption. However, in 2011, increase in energy and food prices alongside with rise of indirect and excise taxes and adjustment in price regulations caused that inflation rose above 3%. In next years the falling energy and unprocessed food prices pulled the inflation below zero, following same development as in case of Poland.

I.II GDP Growth

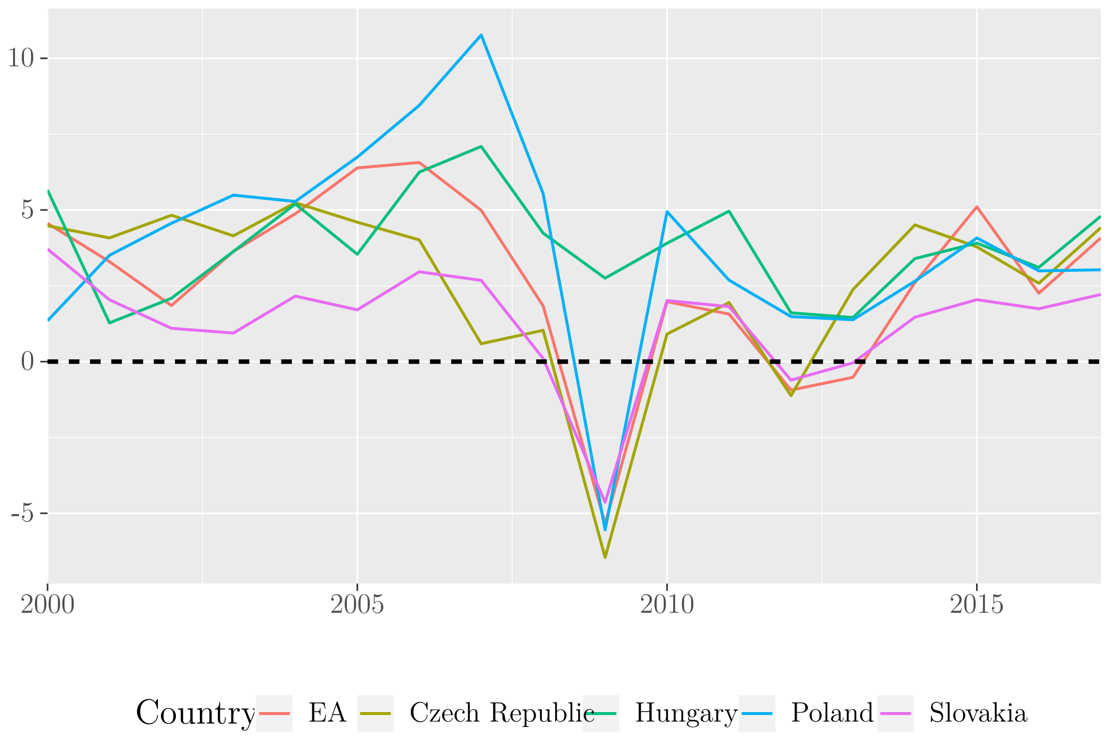

Figure I.3: Annual GDP growth of Visegrad Group and EU

The graph I.3 is displaying the annual percentage GDP growth of all Visegrad countries and compares them with average growth of EU. The GDP growth was very fluctuating in observed years, following similar path of generally strong growth in whole Visegrad

Group. Another important remark visible from the graph is continuous growth of GDP in Poland over whole period, even during the global crisis period, which hit the rest of the countries the hardest in 2009. The average growth of Poland during the period is around 4%, such growth was not observed even in Germany or United States. However, the fastest growing country within Visegrad Group is still Slovakia, which is enjoying even one of the fastest growths in whole EU. On contrary, the slowest tempo of growth had Hungary.

Til the 2009, Czech Republic was growing dynamically due to convergence process initiated in whole Visegrad Group. After the crisis Czech GDP started to growth again, however, it has not reached the level of the growth before the crises. The peak of the growth was in 2006 when the GDP growth for Czech Republic was 6.5 %. The main driver of GDP growth immediately after the crisis were net export and gross capital formation. In years 2012 and 2013 the GDP dropped again resulting in negative growth rate. Since this moment the GDP growth was positive again, rising above 5% in 2015, which was one of the highest rates in whole EU, although it contracted again shortly afterwards. During recent years the most visible impact on the growth has the strong investment increase which helped economy to grow by almost 3%. The growth was also very well supported by strong labor market conditions and rising real wages, which highlights contribution of the household consumption. However, the contribution of net exports was negative, which was pulling down the GDP growth.In next year the GDP growth is expected to moderate with estimated potential growth rates.

The GDP growth in Hungary before the crisis was evolving differently, comparing to the rest of the Visegrad countries. The economy was facing slowdown already in 2005. After crisis, Hungary followed similar path as Czech Republic and Slovakia with the significant decrease caused by crisis, hitting the all-time recorded highest decrease of -7.6% in Jun 2009. In the aftermath of the crisis the GDP begun to growth again as in all countries of Visegrad group and European Union. This growth was driven by temporary measures and factors, such increased absorption of EU funds, subsidized loan schemes and regulated utility price cuts and did not last long. Recession hit Hungary again in 2012, the most influenced factors of GDP growth were gross capital formation and domestic demand, which decreased significantly. However, the biggest issue of Hungary was external indebtedness. Despite the obvious positive contribution from EU funds, Hungary struggled still with financing, net foreign direct investment were slowing down. In recent years the GDP growth was driven mainly by investment into construction and rise of employment and wage growth.

Poland was similarly to Hungary, facing economic slowdown already before the global crisis in early 2000s. However, the GDP growth was continuously positive. Poland was the only country which preserved the growth rate even during the hardest year for the rest of the countries, 2009. In 2010 the annual growth was almost 4%, while other EU countries were only slowly recovering from the crisis. The main driver of the GDP growth in Poland was private consumption supported by improving labor market conditions and international demand, powered by domestic manufacturing. But sceptic global outlook caused the investments to be much lower, even though the investment inflows from EU funds were ongoing. The tempo of the growth peaked in 2012, when various especially infrastructure projects were finished, and the public investment slowed down. In the same year, labor market prospected worsen, influencing negatively overall consumers behavior. This was offset by outstanding increase in production and export of cars due to opening of new plants. Such increase saved the GDP growth from shrinking in 2012, but the slowdown of growth in 2013, was inevitable. At the end of the year, the domestic demand started to recover slowly again and alongside with continuous growth in exports fueled by corporate investments, the slowdown was at least partially offset.
The evolution of the GDP growth of Slovakia was very dynamic and progressive, Slovakia was the fastest growing country among Visegrad Group already before crisis. This phenomenon was mainly caused by so called catch-up effect. The highest achieved growth was 10.5% in 2007. However, the crisis hit hard Slovak economy, in 2009 it shrunk for the first time since split of the Czechoslovakia with the fall of the real GDP by almost 5%. This was caused mainly by fall of industrial production, but more GDP components suffered from the crisis, investments decreased by one third, exports together with imports decreased by almost 20%. Slovakia is trade-open country with significant degree of openness, it is natural that such decrease of exports had great impact on the GDP. Despite the critical drop in economic development caused by the crisis, Slovakia is also an example of one of the fastest recoveries in the euro area. Although, the growth was clearly slowing down, caused especially by decreasing gross capital formation and domestic demand. But throughout the period we observe Slovakia kept its position among fastest growing Countries in EU and held this place until now.

I.III Country Indebtedness

Country indebtedness is crucial factor of macroeconomic stability. High indebtedness brings negative outlook on the economy and additional risks. The Maastricht criteria are stating that the ratio of gross government debt to GDP should not rise over 60%, in order to mitigate the risk of bankruptcy. Czech Republic and Slovakia are countries with generally lowest indebtedness and yet both countries decided to adopt budget responsibility rules including the so called debt brake, In Slovakia the debt brake is implemented within the national constitution on 50% level, while in Czech Republic the critical level is on 55%. The country with the highest public debt to GDP ratio is Hungary, furthermore, its debt to GDP ratio is continuously above the required level. The whole development is displayed in the following graph I.4 and the dashed line represents the Maastricht criteria.

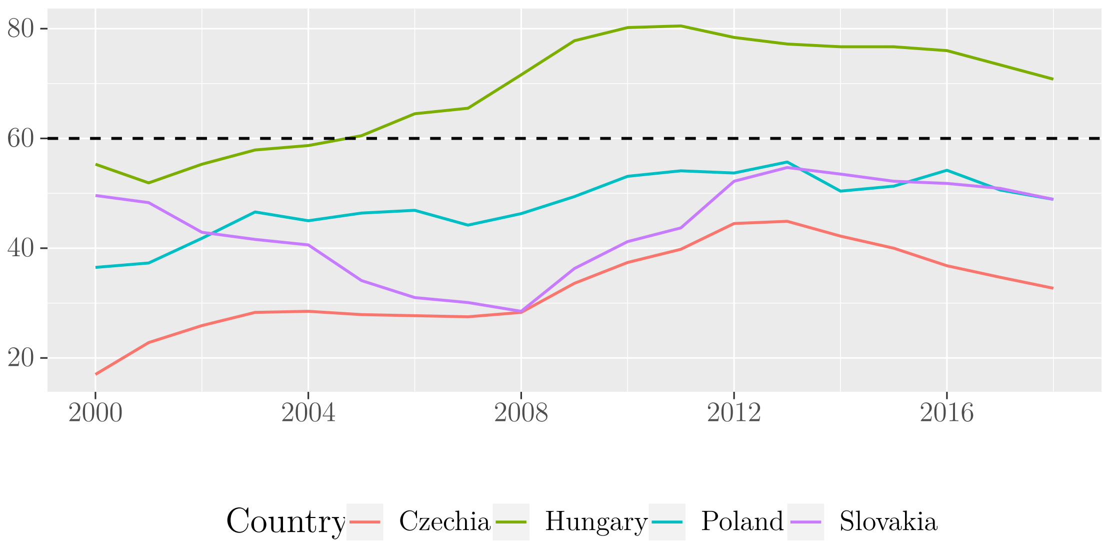

Figure I.4: Debt to GDP ratio

Even though that Czech Republic was enjoying period of long stable growth, the positive development of the economy is also accompanied by certain negative factors. One of them is rising country indebtedness. The clear example of such effect is observable on the rising government debt to GDP ratio of Czech Republic in early 2000s. The costs of economic transformation, rising investments and government consumption, resulted in increase of the government debt. The transformation period followed by steady period in terms of gross debt to GDP ratio remaining constantly under the level of 30% until the crisis. After the crisis in attempt to fight the economic downturn, the indebtedness of the country rose and continued rising until 2013, when the debt to GDP ratio reached its highest value almost 45%,however, it is still safely under the maximum level according the Maastricht criteria. Over all period Czech Republic had the best, the lowest debt to GDP ratio among all Visegrad countries.

Contrasting the situation in Czech Republic, Hungary, as it was mentioned, is the country with highest debt to GDP ratio in Visegrad Group. Until joining the EU, Hungary successfully held the debt to GDP ratio under the Maastricht criteria level, however, already in 2005 the debt to GDP ratio crossed the level of 60% and continued growing until 2011 when the ratio was above 80% which was the highest by far in Visegard Group, but also alarming in terms of Maastricht Criteria as countries that joined EU should not only fulfill the requirements before entering, but they are committed to continuously hold the requirements in order to prevent burst of economic crisis in the EU. Hungary started with strict restrictive policy on all levels together with central bank in order to shrink the indebtedness of the country. The ratio then finally started to decrease, although it is still significantly high, and last year the debt to GDP ratio was slightly above 70%.

Polish debt has very homogeneous constant development fluctuating mostly between 45 and 55%. The highest level was reached in 2013, when the ratio rose to 55.7% which is still in line with Maastricht criteria, since than except year 2016, the debt to GDP ratio was continuously decreasing and currently is under 50% level.

In case of Slovakia we should point out evident macro-economic internal disequilibrium caused by rising GDP and rising indebtedness in the same time. At first, the debt to GDP ratio was diminishing in early 2000s and this trend continued until the year 2009. However, after crisis the debt rose significantly alongside the GDP growth. The peak of the debt to GDP ratio was reached in 2013, when the debt represented almost 54% of the GDP. Since then the debt to GDP ratio is slowly unceasingly decreasing.

I.IV Labour Productivity

Labour productivity provides overview of the economic performance of the country abstracting from employment and the length of average working time. Graph I.5 shows the year-to-year changes in real labor productivity per person growing practically uninterruptedly in all Visegrad countries except the crisis period, when especially manufacturing automotive industry basically stopped, with huge drops in production. This drop hit hard especially Czech Republic, Slovakia and Hungary, while Polish productivity remained in positive numbers. Real productivity growth is linked to GDP growth, therefore the same principle that applies for GDP growth applies here as well. The significant year-to-year dynamic growth rate detected in Slovakia and Poland was caused by catching-up effect and we must keep in mind that in absolute terms the real productivity is still lower comparing to developed countries.

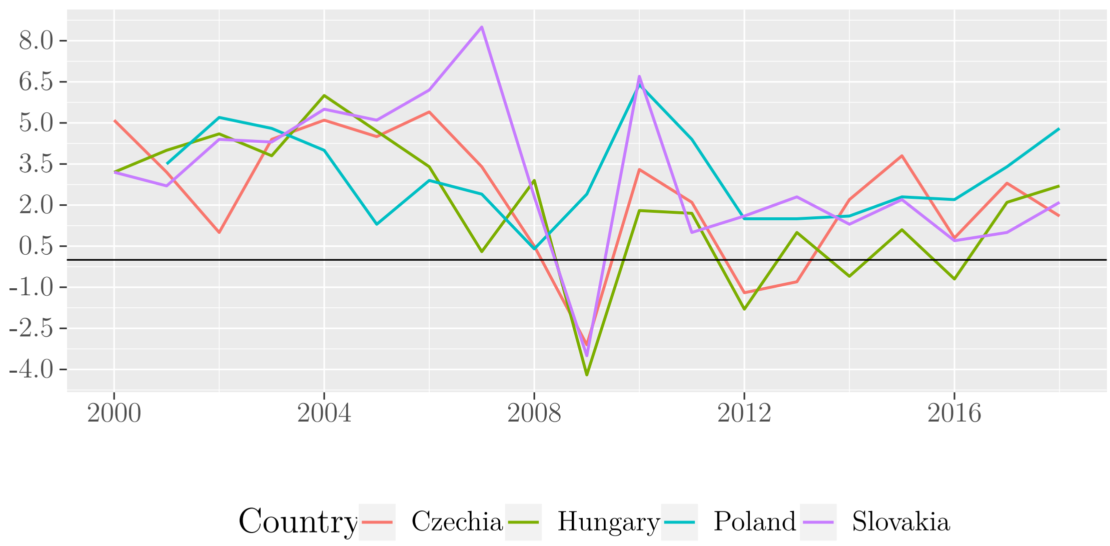

Figure I.5: Real Labour Productivity per person

Although the productivity growth of Czech Republic is not the highest among Visegrad countries, it does not mean that the economy is not performing well. On the contrary, incessant productivity growth shows progressive innovation and structural reforms of the economy. We can observe that productivity growth was more dynamic and progressive before the crisis, marking the convergence period of Czech Republic. Then the crisis affected Czech economy quite harshly, causing the fell of productivity and the growth was not resumed until 2013. Since then we observe again period of continuous growth. However, the potential risk to Czech productivity is the fact that it is driven mostly by international companies and large foreign direct investment inflows, while domestic companies are staying a bit behind.

Before the crisis, we observe also in Hungary the convergence period characterized by dynamic productivity growth. However, since the crisis stroke down the productivity of the country, Hungary was struggling with boosting the productivity growth again. Low productivity was furthermore limiting the country income convergence. Country government was performing loose macroeconomic policy to help restart the economic growth, but this behavior brings along the risk of economic imbalances. The growing labor costs in combination with low productivity were triggering rise of inflation. Hungary invested heavily into infrastructure, education, research and innovation, to prepare favorable environment supporting the boost of productivity. Another issue that is affecting the overall productivity are similarly to Czech Republic discrepancies of productivity between domestic and international companies operating on domestic market. The government eased the lending conditions hoping that the easier access to financial funds will also facilitate the expansion of companies, but this also brings additional risk of capital misallocation, as credit flows are disproportionate and directing more towards companies with more collateral, however, that does not mean that those companies do have in fact higher productivity potential.

Poland, as the only Visegrad Country was enjoying constant productivity growth across whole period and currently has the highest productivity growth among the Visegrad Group. Nonetheless, polish growth and productivity are heavily supported by automotive factories operating in the country. If Poland wishes to keep the productivity growth in long-term, the investment into education, infrastructure, research and innovation are necessary. Furthermore, Poland is also facing demographic issues and forecasts predict decline in working age population. The potential additional source for productivity growth might be hidden in corporate investments, as large-corporate sectors was quite underdeveloped in Poland with low investment to GDP ratio. Therefore, in front of Poland stands the challenge of transforming the Polish economy from manufacturer of relatively low-technology goods to producer of more advanced goods and services.

Slovakia also enjoyed convergence growth before crisis, which was outstanding comparing to the rest of Visegrad Countries. Albeit the crisis pulled productivity to negative numbers, the growth recovered quite quickly outstanding again among Visegrad countries, and yet the growth is still under its potential due to insufficient administrative capacity to manage funding limits. Slovakia is still lagging in research sector, especially in domestic public research. Additional improvements of environment such as better protection of clean air, water and forests are another important factors to preserve the current growth. Moreover, the direction of investments is straightforward into manufacturing industry, which is the dominant industry in Slovakian exports, but it is not sustainable in long-term. Slovakia similarly to Poland needs to boost investments into education and innovation sector as well as diminish the large differences between domestic and international companies. Yet already made reforms in terms of productivity are very well visible on the overall performance, especially better allocation efficiency.

I.V Current Account

The Current account provides information about the transactions of a country with the rest of the world, covering all transactions of goods, services, and income between country residents and non-residents. According to current account deficit or surplus we determine whether the country is net borrower or net lender. Current account is also important determinant of the equilibrium exchange rate. As the graph I.6 indicates, all Visegrad Group were living for long time on Current Account deficits, which were transformed into surpluses only in recent years for most of the countries, except Poland which is running whole period on current account deficit. Current account of Slovakia reached surplus shortly but then return into deficit values.

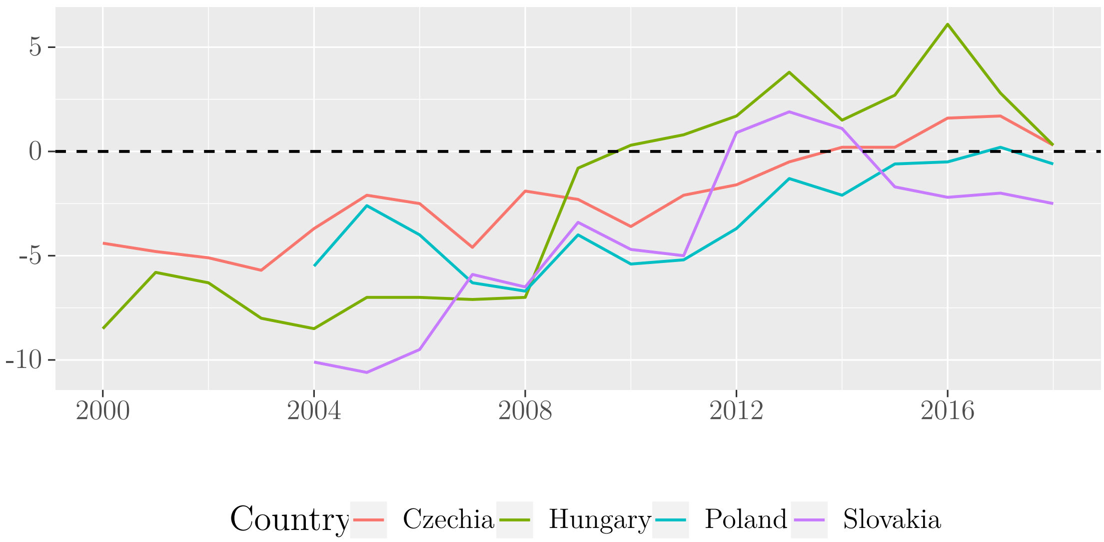

Figure I.6: Current Account to GDP ratio (%)

The current account of Czech Republic is quite balanced especially due to large primary income deficit. This deficit is long-term negative due to large profit outflows generated by international companies. We can observe constant rise of current account balance, slowly transforming Czech Republic from net borrower towards net lender.
Hungary is quite opposite example comparing to Czech Republic. Country became net lender shortly after the global crisis and kept that position during the rest of the period. Position of net lender is not favorable in terms of economic growth, with which as we saw the country was struggling. The country government, therefore made necessary steps to boost the growth which also caused the surplus of the current account fall. Financing

capacity of the country is preserved by constant flow of funds from EU. However, the challenge for Hungary is to boost the offset of private investments with net public borrowing. The balanced Current Account is expected in near future.
In case of Poland, the deficit of current account is close to balanced current account position. The overall value of current account in Poland is mainly affected by long term imports of services, which exceeds the exports.
For Slovakia the deficit of current account is connected to high export, which accelerated even further due to opening of new automotive plants. The international trade will be developed further in next chapter.

I.VI International Trade

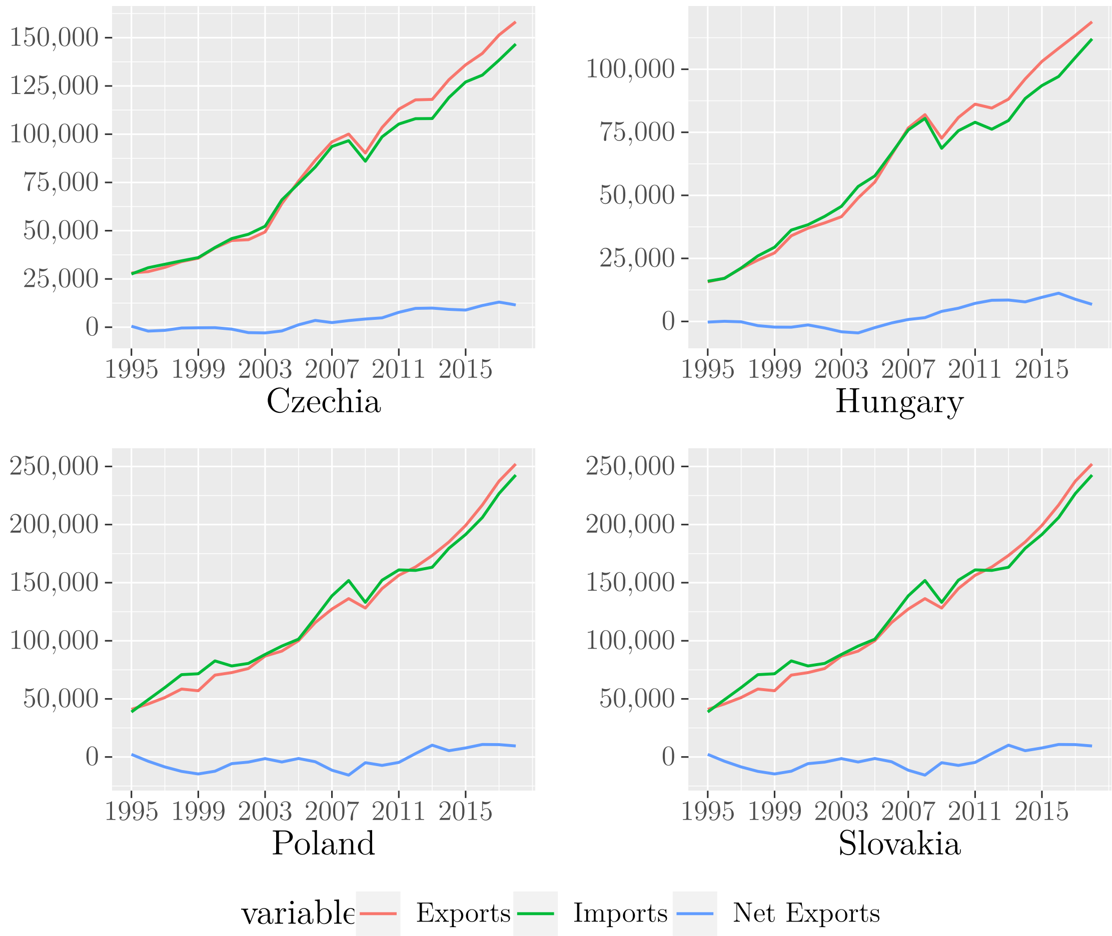

Figure I.7: International Trade Development of Visegrad Group

All Visegrad countries are small open economies, hence, it is very important to understand the development of their trade position in terms of determining equilibrium exchange rates. The graph I.7 is remarking the historical development of total exports, imports and net exports of each country of Visegrad Group. Displayed data are in millions in euro in current prices.

As the graph I.7 shows, the imports and exports of Czech Republic were following almost identical path, causing Net Export balancing always around 0. This pattern is observable until the global crisis in 2008. After that, Czech Republic was struggling for several years with negative terms of trade caused by high growth of import commodity prices. The export-oriented foreign direct investments were slowing down and the overall negative mood in the manufacturing industry was supported by continuous decrease of domestic and foreign orders. The trend finally reversed in 2014, with recovery of main trading partners. Germany is the mayor external customer for Czech manufacturing industry, thus, when the German growth transmitted successfully into Czech economy it helped restart the growth. Net Exports fell temporally in 2015 due to faster growth of imports than exports, but progressive improvement in external demand was pulling net exports up.

Evolution of imports and exports of Hungary follows similar path as Czech Republic. Post crisis, the overall recovery of Hungary was rather slow, what transmitted into worsen international trade position of the country. The improvements of the export were supported mainly by increasing production in car industry supported by increasing external demand.

In Poland the imports were slightly higher than exports during whole period before crisis and also for some years after, causing the net exports to be slightly negative for long period. This was mainly caused by constant appreciation of the currency and rising domestic demand, which supported the import demand. The exports started to grow after 2010. The growth accelerated even more in 2013 and provides important support for whole economic growth. The rise of external demand was also visible in corporate investment, which were offsetting the low level of investment spending by the government. Polish exports continue growing robustly even in recent years due to the strengthening external demand.

Slovakia, followed similar path as Poland, having the imports above exports for significant period, which resulted in long term negative net exports. In the period after the crisis, the external demand was pushing up the economic activity of the country. Exports started to grow even faster in 2012, due to the increase in exports of cars caused by new plants opening. The growth of exports slowed down in 2014 with shrinking demand from the important trading partners. In the same period, the imports grew faster than exports resulting again in negative net exports, which persist till recent years.

I.VII Unemployment

The graph I.8 reflects the historical development of unemployment rates in Visegrad group comparing to the average rate of EU. The data are in annual averages and unemployment rate is ratio of unemployed population to total active population of the country. The graph detects unceasing decrease of the unemployment rate till the crisis in Czech Republic, Poland and Slovakia. Although it is obvious that Poland and Slovakia were fighting much higher rates than Czech Republic. The unemployment rate in Hungary followed opposite path, with slow but continuous rise of unemployment. The overall unemployment rate was significantly higher in Poland and Slovakia then in Czech Republic and Hungary. After the crisis the evolution of unemployment rate was unified among Visegrad countries and followed the same path in all countries. The overall highest unemployment rate over the period was in Slovakia while the lowest unemployment rate is for long time observed in Czech Republic.

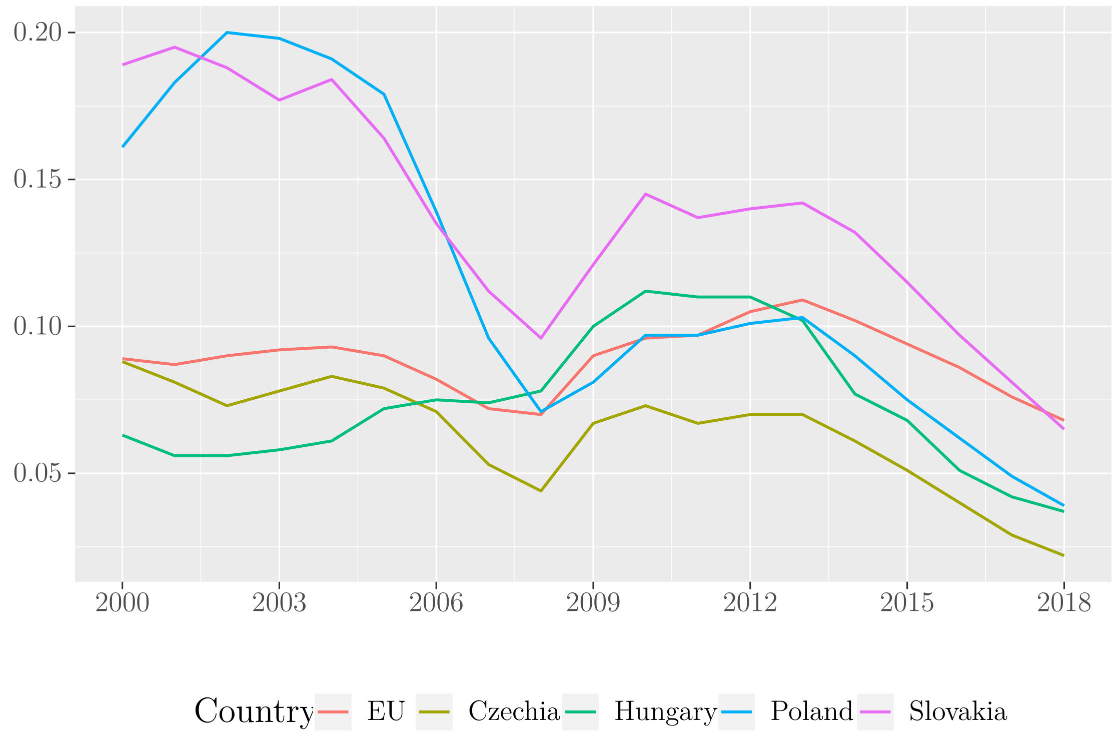

Figure I.8: Development of Unemployment rate of Visegrad Group and EU

In general, Czech Republic has the lowest rate of unemployment not only among the Visegrad countries, but also in whole EU. Despite the fact that in pre-crisis period the unemployment rate of Hungary was slightly lower, it lasted only till year 2006, when the unemployment in Hungary rose while in Czech Republic it continued decreasing. In fact, Czech unemployment was decreasing throughout whole period except the year 2009, when as consequence of crisis rose comparing to previous year by 9.2%. After the crisis, the strengthening of Czech economy reflected in improving labor market conditions. Constant fell of unemployment accompanied by strong wage growth going hand in hand with stronger growing household consumption became strong leading factor of the GDP growth. The force of the labor market is also shown by the total number of employed persons which is approaching the levels of the end of the 1980s. We could say that involuntary unemployment basically disappeared, and vacancies outnumbered applicants. However, according the OECD study Czech Republic as well as Slovakia will face the most significant risk of job losses in near future, due to automation of the production processes as both are very factory-heavy economies.

Hungary enjoyed relatively low unemployment rate for several years, albeit it was growing continuously since 2003. The growth accelerated significantly after the crisis and hit its maximum in March 2012, when the rate was almost 12%. Since then, the ratio of unemployed people started to decrease. After period of dynamic employment growth, the inevitable slowdown started to show off since 2015, as available labor reserves become exhausted, the unemployment rate remains continuously decreasing. In 2018 the unemployment rate was on historical minimum, when it decreased below 4% level.

The unemployment rate in Poland was growing even in early 2000s, until its maximum of 20% in 2002, the finally the government actions were successful, and the unemployment decreased. Another factor that helped diminish the unemployment rate was the joining of EU, especially the new access to labor markets in Great Britain and Ireland. After the crisis, the rise in unemployment was only moderate. Poland, more than with problems caused by crisis, suffered by the demographic pressures that were limiting the labor supply. Unemployment than continued rising over following years, even though country was enjoying recovery period and the GDP was growing. Unemployment begun to decrease only in 2014, when the tightening of labor market transformed into rise of nominal wages. Yet, Poland was still facing structural unemployment similarly as Slovakia.

In 2000 the unemployment rate of Slovakia was the highest among the Visegrad group, reaching almost 19% level. Ever since, the economic growth of country helped to start the growth of employment, causing regular continuous decrease of unemployed persons among active population. On the other hand, with bursting crisis the unemployment rate in Slovakia rose again significantly, crossing 12% level. Despite strong and fast recovery from the crisis the unemployment was decreasing only slowly. Slovakia had long lasting issue with long-term unemployment, which has the highest rate in the EU. The overall responsiveness of unemployment rate to economic growth was very low. This problem was caused by structural issues as large regional disparities, low labor mobility within the country and skill mismatches. Stronger growth came in 2014, when unemployment decreased to 13.2%. The improving labor market conditions due to reforms that came in force pushed the long-term unemployment finally down. Continuous decrease in unemployment, squeezed the rate below the level of 10% in 2016 and helped the unemployment rate to hit its historical minimum in 2018, 6.5%.

II

Equilibrium Exchange rate

In order to understand models and concepts used for the estimation, it is necessary to understand the meaning of the Equilibrium exchange rates, what it represents and why it is important. Various authors representations differ in already very basic question, in the definition of the Equilibrium exchange rate and whether it is long run or short run concept. Thus, various estimating approaches are used in practice. But why it is so important to know and to try to estimate the Equilibrium Exchange rate of the country and what does it mean?

One could easily conclude that if country has floating exchange rate regime, then the exchange rate of the currency is determined continuously in foreign exchange markets by the demand and supply of the currency and therefore exchange rate is always in equilibrium. However, this assumption can be made only on purely floating exchange regimes without any interventions, no monetary policy applied, and we would need to assume further that markets adjust immediately to any change and shock introduced. Once the country applies any kind of intervention in its exchange rate policy, the Equilibrium Exchange Rate musts consider this impact as it is no longer the result only of Demand-Supply evolution but also of other factors.

Exchange rate is providing us first basic intuition about the state of country's economic situation without looking further on the data of GDP and other commonly used ratios and indicators. In fact, the real effective exchange rate is derived from country's economic fundamentals that is why it can provide us with economic insights. These fundamentals might have long term or medium impact, but they also can be transitory factors influencing exchange rate in short time. The theory of equilibrium exchange rate does not provide list of concrete fundamentals that have to be used to determine exactly the exchange, the choice is rather intuitive based on country's specification and type of equilibrium exchange rate we want to examine. Typically, among commonly used fundamentals we can find GDP or GDP growth of the country, inflation or relative price levels, its international trade, foreign direct investments net foreign assets or other economic factor defining international position of the country. The equilibrium exchange rate estimations vary with the length of examined time horizon and with the choice of fundamentals. Following the Working Paper of Bank of England of Driver and Westaway [2005] we will examine basic characteristic necessary to understand for modelling equilibrium exchange rates.

II.I Exchange rate definition

The first crucial choice before even beginning of estimating equilibrium exchange rate is to set clearly what we want to examine and how and select accordingly the appropriate measure of the exchange rate with appropriate set of variables. In most research papers the real exchange rate is define by nominal rate deflated by ratio of some price level index, stating the general expression as:

$$E_{it} = \prod_{j=1}^{n} \left(\frac{P_{it} S_{it}}{P^{*}_{it}} \right)^{\varpi_{ij}} \tag{II.1}$$

where P_i denotes the price level in domestic country i, P^*_j the price level in foreign country j, S_{ij} is the nominal exchange rate between currencies of those two countries (stated as unit of domestic currency per foreign currency) and ϖ_{ij} is the weight of the country j in country i's effective exchange rate index.

When E_i rises the exchange rate appreciated, meaning the domestic country become less competitive towards foreign country.

The real exchange rate then depends also on the price index we decide to use. Each price level index focuses on different sector of the economy or different observed effect and causes different movements in real exchange rate. Naturally using different indexes gives us different real exchange rates, but it does not mean that one index is better, or another is worse. It simply means that there is no unique defined approach to measure the equilibrium exchange rate and the choice of the price level ratio should be careful and based on the country specification and economic intuition. Cerbaf [1985] The most frequently used price indexes are:

- Consumer Price Index (CPI)
- Producer Price Index (PPI)

- Relative unit labor costs
- The prices of tradable goods or output prices
- The ratio of the prices of exports to prices of imports
- The ratio of the tradable to non-tradable prices

II.II Time Horizon

We have defined the equilibrium exchange rate at its core, but in order to be able to build appropriate model we need to understand the role of the period for which we are estimating the equilibrium exchange rate. The length of the period is important for selection of fundamentals that are determining the exchange rate value. MacDonald and Clark [1998]

II.II.ii Short run Equilibrium

In case of short run equilibrium, we are working with actual values of fundamentals, omitting unexpected shocks and random effects such as market bubbles. The theory of short run equilibrium is further developed in the concept of current equilibrium exchange rate in the work Cerbaf [1985], author defines this value as exchange rate that will be applied when markets have full information and reacts rationally.

$$e_t^{ST} = \beta' Z_t + \theta' T \tag{II.2}$$

where et_t^{ST} is the exchange rate in time t considering short run, Z is vector of economic fundamentals that are used to determine the specific equilibrium we are aiming to estimate, T is vector of transitory factors specific for short run and β and θ are vectors of coefficients.

This type of equilibrium plays important role in estimating random disturbances which have power to unbalance the market, causing its deviations from the equilibrium values, indicating the presence of a bubble or shock. When markets suffer from any kind of shock, the exchange rate can accommodate this shock by its appreciation or depreciation. Short term equilibrium exchange rate gives a leash to monetary policy, which can influence short run value but has no impact in medium- and long-term values.

II.II.ii Medium run Equilibrium

In medium term we need already two exchange rate equilibrium that are characterizing the economic development of the country.
The internal balance, which means that the economy runs at its normal output capacity determined by demand-supply equilibrium, not producing any output gap, and keeping unemployment at NAIRU level (level of unemployment below which the inflation rises).
The external balance, which determines the equilibrium of external world that is influencing the country's exchange rate. Among most commonly used external factor are balance of payments, especially current accounts of the countries. The basic approach to determine the long-term equilibrium in the global economy is to put all current account equal to zero, but as we are in medium run savings do not have to necessarily equal investments in all countries. Therefore, instead of putting current accounts necessarily equaling zero, we are searching for their levels, consistent with the sustainable convergence to the equilibrium. This convergence also means that interest rates of domestic country are also still converging towards world equilibrium levels.
In medium term we expect that short run discrepancies are consumed by market and exchange rate appreciation or depreciation and what we aim to observe are real rigidity. The estimations are no longer based on current values, but on real ones instead, which are independent from monetary policy. Use of real values provides us possibility to observe trend and the convergence towards long term values, the equilibrium estimated on real values is also called flexible price equilibrium.

$$\hat{e_t} = \beta' \hat{Z}_t \tag{II.3}$$

where hats denote the trend values converging towards long term equilibrium.

II.II.ii Long run Equilibrium

In long term the whole economy, domestic and the rest of the world as well are assumed to be in equilibrium, running on their potential levels and there is no intentional tendency for change or for departing from this state. For modelling, we are considering fundamentals at their long term equilibrium values, assuming the global equilibrium. MacDonald [1997]

$$\overline{e_t} = \beta' \overline{Z_t} \tag{II.4}$$

where the overbars denote the long-term values.

II.III Modelling of Equilibrium Exchange Rate

What are the key factors for modelling the exchange rate? Doubtlessly it is its role within the economy, as exchange rate is essential tool to observe the development of price levels. Another important factor determining the exchange rate is the market situation in the country, demand and supply elasticity, business environment or presence of barriers of trade, if there are any. All those factors are influencing exchange rates indirectly through pricing policy of firms operating on the market, who the through their price setting influence the final exchange rate.
Various models introduced through time for estimating the equilibrium exchange rate depending on the need of the made research. The main focus of this diploma thesis is on BEER and FEER models, which are the most commonly used models for estimating equilibrium exchange rate, therefore special chapter is dedicated to these two models. Lee et al. [2005]
However, there are many other modelling techniques that can be used to estimate the equilibrium exchange rate. Although we did not use those models, we provide brief overview to have better general insight into the modelling of equilibrium exchange rates. Isard [2007]
The other popular methods are UIP (Uncovered Interest Rate Parity) approach, PPP (Purchasing Power Parity) approach in standard from or widen by modelling Balassa-Samuelson effect, then ITMEER, CHEER, DEER or NATREX models. Recently various papers work with Semi Structural Gap models. This model offers full set of typically unobserved variables as for example natural rate of interest. This approach was recently used in research of CNB of Quantifying the Natural Rate of Interest in a Small Open Economy Hledik and Vlcek [2018]. The gap form of model requires the decomposition of real variables into its trend component and the gap between the value and the trend, or in other words the deviation from the trend. Hansen and Roeger [2000]

II.III.iii Uncovered Interest Parity

This is very direct method used mainly for modelling short term equilibrium using as the core assumption that the interest rate differentials can determine expected change in the exchange rate, which is the explained variable in the model. As it is working with changes in the exchange rate, the stationarity assumption is crucial for modelling.

II.III.iii Purchasing Power Parity

Through PPP model we can directly estimate constant Equilibrium Exchange rate in long run depending purely ratio of price levels between domestic and foreign country. This model is rather simplistic, although it helps to understand the basic movements in the exchange rates and shows the natural course of the exchange rate in the long run.
In recent years, Deutsche Bank introduces new capital-based FX valuation model, called Cap-PPP. This model uses PPP as departing point, but it considers nominal capital indexes weighted by asset prices to obtain real effective financial exchange rates. While classical PPP model uses trade partners for computing the weighted indexes, this model focuses on the main capital partners and switch prices of goods for asset prices. The considered assets are debt and equities. Misalignments are than estimated in the same way as in PPP, by comparing the calculated exchange rate with historical average of REER, used as proxy for fair value. Kalani [2016]

II.III.iii Balassa-Samuelson

This model is working on the same principle as PPP model, however it differentiates the goods into tradable goods, which are manufactured goods that country can easily export and non-tradable goods, which are services. It explains on structural basis long-run deviations from PPP by providing an explanation, why countries that are rich (in terms of productivity) have higher non-tradable price levels. As it is very specific effect, the data transformation could possibly cause difficulties in the modelling. In order to prevent such information loss, we need to work with non-stationary data and use special estimation techniques for such modeling. However, this approach works well in case of arbitrage of markets of goods, when PPP does not hold anymore.
Balassa Samuelson is not only model itself, but all the effect presented in economy that we can approximate and use in the BEER and FEER models. The effect can be captures in three different ways through proxies. The most common is the ratio of price of non-tradeable goods relative to tradable goods. This is approximated usually by ratio of CPI and PPI indexes. However, as real exchange rate is calculated using price index ratio, we can slip into endogeneity issue as the same variable would enter both sides of equation. The issue is then, that if REER changes by change in ratio of non-tradable to

tradable goods, it will reflect the structural shift in the exchange rate that is very likely to be incorrect.
Another two very common approaches how to capture Balassa-Samuelson effect are total factor productivity and labor productivity. Egert and Lommatzsch [2004]
The original Balassa-Samuelson effects based on pure ratio of prices of tradable and non-tradable goods is function of total factor productivity and it is based on standard Cobb-Douglas production function with constant returns to scale. But total factor productivity is not observed directly, and it is hard to estimate it precisely due to data limitations. That is the reason why is the Balassa-Samuelson effect normally approximated by labor productivity ratio, which can be also captured in various ways. The widely used approximation is the real GDP at PPP per capita or calculation through indexes of production in economy and the employment, which provides more precise results. Kalani [2016]
Are we able to determine the best approach with certainty? It is very polemic question, due to varying data availability across regions and countries. Deutsche bank in their modified BEER model, called DBEER is using all three approaches and then to obtain final model and the exchange rate misalignment combine all three approaches using simple average. JP Morgan is computing relative productivity based on tailored approach for each country and its specifications. For example, for Hungary the relative productivity is stated as productivity differential in contrast with euro and the relative productivity itself is stated as GDP per employer. While the relative productivity for Czech Republic is the same productivity differential stated in contrast with investment share in GDP and in Poland the relative productivity is approximated only by the investment share in GDP.

II.III.iii Other Modelling Techniques

CHEER Model

Capital Enhanced Equilibrium Exchange Rate model combines PPP and UIP approaches with special focus on speed of convergence of fundamentals in short run.

ITMHEER Model

The acronym stands for Intermediate Term Model Based Equilibrium Exchange Rate model. The departing point for this technique is similar as in previous model extended by a risk premium and expected shifts in real exchange rate. In this model future change in the nominal exchange rate represents the explained variable.

DEER Model

Desired Equilibrium Exchange Rate model is alike FEER model, which will be explained into more detail in next chapter. The difference between those models is that DEER model defines external balance from the optimal policy, but all the other features are the same.

NATREX Model

Natural Real Exchange Rate model is also from same family of models as FEER and DEER model, however it also assumes the portfolio balance, meaning that domestic real interest rate equals the world rate.Salto and Turrini [2010]

Semi Structural Gap Model

As was already mentioned this model can work with full set of unobserved variables. In order to build this model, the real variables are decomposed into their trend component and the deviation from trend, which represents the gap in the model. Trend component is driven by fundamentals and therefore, it is independent from monetary policy (as we already saw this is key concept also for modeling medium- and long-term equilibrium exchange rate). The gap is the cyclical component, linked to business cycle. Hledik and Vlcek [2018]

III

BEER and FEER Models

Those two models are globally most frequently used models for estimating the equilibrium exchange rates and for FX valuations. They are also the main models we are using in the diploma thesis and that is why we are dedicating them whole chapter, summarizing various research papers that were later used for constructing core models of this diploma thesis.

III.I FEER Model

The Fundamental Equilibrium Exchange Rate is departing from the definition of macroeconomic balance using the assumption of equality of the current account and financial account under the conditions of full employment and low inflation. This assumption is generally connected to long term equilibrium, so the model abstracts from short-term factors and consider only variables that are most likely to persist over medium and long run. Laura and Peter [undated]

The FEER model is based on internal and external macroeconomic equilibrium, where internal balance is the level of output consistent with already mentioned conditions, while external balance is the sustainable optimal net exchange of resources in international trade, when both countries, domestic and foreign country, have reached the internal balance. As the conditions are in fact rather desirable outcome, than real situation, the FEER model is remarked as normative approach of equilibrium exchange rates. Lee et al. [2005] The main macroeconomic balance is stated by simple equilibrium of the zero sum of Current Account and Financial Account, which can be also written as:

$$CA = -FA \tag{III.1}$$

We can develop this equation further by defining equation of current account, which states that current account can be expressed as function of real exchange rate and aggregate output in domestic and foreign country:

$$CA = b_0 + b_1 q + b_2 \overline{y_d} + b_3 \overline{y_f} = -\overline{FA} \tag{III.2}$$

$where\, b_1, b_2 < 0, and\, b_3 > 0$

In the equation q denotes the real effective exchange rate, y_d states for level of domestic output and y_f denotes level of foreign output, both output levels are set at full employment in each country. Solving equation III.2 for real exchange rate will gives the FEER set up as:

$$FEER = \left(-\overline{FA} - b_0 - b_2 \overline{y_d} - b_3 \overline{y_f}\right) / \, b_1 \tag{III.3}$$

The exchange rate estimated by FEER is than considered to be the rate that would equal current account with financial account, while the full employment condition would hold. As the current account depends on external factors such as net capital flows of the country, the FEER approach is also considered to be a method of assessment of the actual value of the real exchange rate, which is then compared with real exchange rate of given period q_t to decide whether the exchange is overvalued or undervalued. MacDonald and Clark [1998]

Komarek and Motl [2012] derived modified and broader FEER model, including the equations of international trade, as they assume dependent relation between domestic and foreign demand and exchange rate:

$$\boldsymbol{Export : lnX = f(lnMEMU; lnRER; lnDPROD; lnFDI^x)} \tag{III.4}$$

$$\boldsymbol{Import : lnM = f(lnDD; lnRER; lnX)} \tag{III.5}$$

where X stands for real export, M for real import, DD is real total domestic demand, $MEMU$ is real import from Euro Area, RER is real exchange rate deflated by PPI index, $DPROD$ states for labor productivity ratio between Czech Republic and Euro Area and FDI^x as foreign direct investments, is proxy variable for inflow of investments into main industries. All variables are in logarithms and the model is estimated as VEC model (Vector Error Correction model).

To complete the FEER model, further identities are needed:

$$NX_t = X_t - M_t \tag{III.6}$$

$$NX_t^x = P_t^x NX_t + M_t \left(1 - \frac{P_t^M}{P_t^X}\right) P_t^X \tag{III.7}$$

$$Y_t = DD_t + NX_t \tag{III.8}$$

$$Y_t^{GAP} = Y_t - Y_t^{EQ} \tag{III.9}$$

$$Y_t^N = Y_t P_t^y \tag{III.10}$$

$$CA_t^{EB} = CA_t^{EQ} Y_t^N \tag{III.11}$$

$$CA_t^{GAP} = NX_t^N - CA_t^{EB} \tag{III.12}$$

where NX is real net export, NX^N is nominal net export, P^x are export prices and P^M are import prices, P^Y stands for GDP deflator, Y is real GDP, Y^N is nominal GDP, Y^EQ is potential level of GDP and Y^GAP is the gap between real and potential GDP level. CA^EQ is equilibrium level of current account balance, CA^EB is the sustainable amount of current account relative to GDP and CA^GAP is the gap between actual current account balance and the equilibrium level. Cline [2017]

The FEER model for countries forming Visegrad Group was already estimated several times in the past. These researches served us as guidelines for understanding the development in 90s and beginning of 2000s and as departing point for our estimations for next periods. Various researches done on Visegrad group on this period conclude quite significant appreciation of the exchange rates and as main reason are mentioned productivity gains shown in summary of economic development of Visegrad Countries in first chapter. As the productivity levels in all those countries were below the average EU level this gain continues throughout the covered period. This fact might also affect EMU convergence criterion used as proxy for nominal bond yields as during the focused period all countries had their currencies pegged to euro. Moreover, as those gains are specifically shown in tradable sector, there is indeed strong assumption for the presence of Balassa-Samuelson effect.
Couharde and Coudert [2003] sees those gains as problem and possible challenge of FEER model. Estimates from the FEER model are sensitive to trade elasticity, which under optimizing assumption are constant over the years, which is obviously not the case of the

Visegrad group as huge productivity gains are observed because the countries had undergone rapid and very important structural transformation. However, under the FEER model's assumptions, the productivity and capital flows changes are neglected.
Another important note we should remember from their research is the misalignment sensitivity towards trade openness. The sensitivity is inversely related to the degree of openness, the larger the degree the less sensitive the misalignment to the imbalance of the country involved, meaning that only a small adjustment of the exchange rate is needed to reduce the imbalances of a very open economy.

III.I.i Deutsche Bank FEER Model

In order to compare standard academic estimation techniques of FEER model we also estimated the FEER model used by Deutsche Bank.Gautam Kalani [2017] In this model, the current account assumption is simplified, using as proxy for sustainable current account, its rolling averages. This model is further adjusted by impact parameter for the difference between the latest cyclically-adjusted current account balance and its long-term average level. This parameter is introduced in two different ways:

- As high elasticity parameter, price elasticity of exports, which is estimated using only ratio of exports over GDP.
- Or as impact parameter depending on both, import and exports, in this case the parameter is estimated by equation where are introduced both ratios, exports over GDP and imports over GDP.

III.II BEER Model

The Behavioral Equilibrium Exchange Rate approach on contrary, as it is implied by its name, studies the behavior of the real equilibrium exchange rate. Egert et al. [2005] For estimating BEER model, we must first define economic fundamentals and determine their equilibrium levels. The behavior is explained by reduced-form equation:

$$q_t = \beta_1^{'} Z_{1t} + \beta_2^{'} Z_{2t} + \tau^{'} T_t + \varepsilon_t \tag{III.13}$$

In the equation Z_1 denotes a vector of economic fundamentals that have continual effects over long term period, Z_2 is a vector of economic fundamentals that have impact on real exchange rate over the medium term period, such that can coincide with business cycles and $\beta_1 and \beta_2$ are the coefficients of those vectors. T states values of vector of transitory factors that are influencing real exchange rate in short time and τ is its coefficient. As we see, the real exchange rate is in this approach explicitly explained in short, medium and long term period. If we ignore the short term period and take into account only medium and long term we would obtain the current equilibrium rate Melecky and Komarek [2005]:

$$q_t^{'} = \beta_1^{'} Z_{1t} + \beta_2^{'} Z_{2t} \tag{III.14}$$

From this equation we derive further the current misalignment of exchange rate from the equilibrium, which is equal to the short term period component of equation III.13:

$$cm_t = q_t - q_t^{'} = q_t - \beta_1^{'} Z_{1t} + \beta_2^{'} Z_{2t} == \tau^{'} T_t + \varepsilon_t \tag{III.15}$$

The current misalignment, as it implies the equation III.15, depends only on short time changes, counting with the fact that medium and long term fundamentals should not change over short run. However, in long run those variables can change indeed. This change is then presented as total misalignment of real exchange rate:

$$tm_t = q_t - \beta_1^{'} \overline{Z_{1t}} + \beta_2^{'} \overline{Z_{2t}} \tag{III.16}$$

This equation III.16 can be further developed, when we add and subtract the real exchange rate from both sides of the equation.

$$tm_t = (q_t - q_t^{'}) + [\beta_1^{'}(Z_{1t} - \overline{Z_{1t}}) + \beta_2^{'}(Z_{2t} - \overline{Z_{2t}})] \tag{III.17}$$

As we see the first part of the equation III.17 is the expression of current misalignment, and we can rewrite the equation as:

$$tm_t = \tau^{'} T_t + \varepsilon_t + [\beta_1^{'}(Z_{1t} - \overline{Z_{1t}}) + \beta_2^{'}(Z_{2t} - \overline{Z_{2t}})] \tag{III.18}$$

The second part of the equation III.18 shows the impact of the deviations of fundamentals from their long-term values.

The departing point of BEER model is interest rate parity condition adjusted by risk expressed as expectations:

$$E_t[\Delta s_{t+k}] = -(i_t - i_t^*) + \pi_t \tag{III.19}$$

where s_t is the price of home currency expressed in foreign currency, i_t is the nominal interest rate, π_t is the risk premium, $t + k$ is the maturity horizon of the bonds.

This equation III.19 might be transformed into current equilibrium exchange rate by deducting the expected inflation differential $E_t(\Delta p_{t+k} - \Delta p^*_{t+k})$ from the interest differential and from the exchange rate.

$$q_t = E_t[q_{t+k}] + (r_t - r^*_t) - \pi_t \tag{III.20}$$

where $r_t = i_t - E_t(\Delta p_{t+k})$ denotes the ex-ante real interest rate. The whole equation III.20 can be then divided into three parts. The first part presents the expectations of the real exchange rate in time $t + k$, the second part shows the real interest differential and the third part is the risk premium, which is here negative meaning, that higher risk premium stands for depreciation of the real exchange rate, that continuously generates the expectations for future appreciation.

We develop further the risk premium and explain it through time-varying component and the time horizon, $\pi =_t +k.$, while, the time-varying component is assumed to be function of relative supply, ratio of domestic to foreign debt. As we see there is positive relation between the risk premium and the time-varying component. MacDonald and Clark [1998]

$$\lambda_t = g(gdebt_t/gdebt^*_t) \tag{III.21}$$

Further we need to express the expectations of exchange rate, as this variable is unobservable, we can assume that expectations of the exchange rate are based on macroeconomic equilibrium and we express the expectation by long term fundamentals Giannellis and Papadopoulos [2007]:

$$\hat{q}_t = E_t[q_{t+k}] = E_t[\beta' Z_{1t}] = \beta' Z_{1t} \tag{III.22}$$

Various approaches for estimating BEER model were carried out. We will depart from mostly academic researches to models used in practice for FX Valuation.

III.II.ii JP Morgan BEER Model

JP Morgan uses for BEER models for FX trading strategies. Anezka Christovova [2019] The BEER estimates are used determine the fair value of the exchange rate. The main assumption of the model is that time-varying equilibrium exchange rate expressed as function of fundamental variables such as productivity, terms of trade, interest rates, net foreign assets etc. Model is estimated by OLS regression separately for each currency. The common set of variables as relative productivity growth, terms of trade, interest rate differentials, external balances and fiscal positions are all in line with exchange rate theory. JP Morgan departs from estimating panel regression and constructs model for each country instead. The choice of proxies for fundamental set of variables differ from country to country. The reason for this is the belief that common set of variables in panel regression tend to brush over important structural differences across regions and countries with different stages and exchange regimes. To capture the key theoretical drivers of exchange rates, models poses common structure, by five factors that are impacting BEER estimations, relative productivity growth, terms of trade, interest rate differentials, external balances and fiscal positions. However, in the final estimation not necessarily all factors are incorporated in all models. JP Morgan does this mainly due to limited data availability, the missing factor is then replaced by variable with highest possible explanatory power. Therefore, these BEER models are tailored on the conditions of each country, as was already mentioned in case of Balassa-Samuelson effect and its approximations

Further JP Morgan ranks and divides independent variables according their importance for BEER model relationship. The ranking is based on the absolute value of the standardized shocks (the size of the standard deviation moves in the explained variable generated by a one standard deviation shock to the independent variable). This relationship is later used for conclusions of the model. From this ranking we can see that JP Morgan assumes as most important independent variable for countries of EMEA EM, where are grouped also our Visegrad Group countries, is relative productivity growth.

Slovakia does not enter the estimations of JP Morgan as it has common euro currency.

The real effective exchange rate is based on CPI.

The approximations of Balassa-Samuelson effect are already explained in previous chapter that is why we will not mention it further here, but we will compare the approximations of the rest of the factors for Czech Republic, Hungary and Poland.

The differences between models for our countries are in different proxies for same variables that are entering the models. For example, the external balances are defined variously according the data for each country. While in Czech Republic this variable is approximated by ratio of Trade balance to GDP and Net International assets to GDP, in case of Hungary, it is Current Account balance to GDP and for Poland is used ratio of External debt to GDP. Fiscal accounts are computed by the same manner in Hungary and Poland, using ratio of government debt to GDP, while in model for Czech Republic this variable is absent.

Although the productivity growth differential is marked as significant variable for Visegrad Group, it is not present in JP Morgan estimation for Polish Zloty. The reason for

this is that it was included in previous models and first draft estimations, but the prediction power was very low and hence it was wrongly signed and insignificant in long term regressions causing undervaluation of Zloty.
In case of interest rate differentials, for Hungary this variable is stated as 3 months HUF nominal rate and 3 months EUR real rates. These variables where added when standard variables resulted insignificant in first estimations and after adding them, the model was stabilized gaining back the predictive power. The assumption behind this behavior is the excessively easy monetary policy pursued by the NBH, that has become the dominant driver of the HUF, introducing a wedge between the currency and its fundamentals, which is likely to persist until the easy monetary policy will maintain. But in general, the model has lower power than the rest of the powers as the easy policy of NBH was introduced only in recent years and therefore cannot explain the evolution of the exchange rate equilibrium in previous years. The estimations for Czech Republic are based on CNB estimation for BEER and FEER models using real exchange rate rather than broader real effective exchange rate.

III.II.ii DBEER Model

This model introduced by Deutsche Bank was already mentioned in section of Balassa-Samuelson approximations in previous chapter. Gautam Kalani [2017] It is special case of BEER model introduced by the Deutsche Bank for purpose of FX valuation. The final BEER exchange rate is computed as simple average of three DBEER models, while the models differ by the approximation of Balassa-Samuelsson effect. The model is simplified and estimated based on three ratios:

- Terms of Trade stated as ratio between export prices and import prices. The ratio captures the income wealth and substitution effects from the shocks that economy went through.

- Balassa-Samuelson effect defined in three already mentioned ways, using total factor production, GDP at PPP per capita and ratio of prices of tradable and non-tradable goods, approximated by ratio between CPI and PPI indices.

- Country Openness determined as net exports over GDP. The economical intuition behind is that the more open the country, the lower non-tradable prices and therefore, lower real exchange rates.

The model is used for panels and countries that are more economically alike are grouped into regional panels. Our Visegrad group countries are grouped in panel of EM commodity importers together with Turkey, Israel, India, Philippines or Mexico. Panel regression is then stated as:

$$REER_{it} = alpha_i + betaTOT + deltaPROD + phiOPEN + epsilon \qquad \text{(III.23)}$$

The real effective exchange rate is than real broad trade weighted exchange rate. Model is based on fixed effects, because REER is presented in form of indexes and estimated by dynamic ordinary least squares. The theoretical background suggests that terms of trade and Balassa-Samuelson should have positive impact on real exchange rates, while openness has a negative impact.
In our estimations we depart from panel estimation and we simplified the model using only the Balassa-Samuelson effect approximated by productivity ratios.

IV

Methodology

IV.I Data

For purposes of this diploma thesis, we are using time series data set, consisting of series Visegrad Group countries, Czech Republic, Hungary, Poland and Slovakia. All countries are within European Union and their national currencies are pegged against euro, which we use as referential currency and the benchmark series are therefore, for Euro Area and not for whole European Union. Euro Area consists of countries, where national currency was replaced by common euro currency. Euro Area is formed by 19 countries: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia and Spain
Data are measured quarterly, covering mostly the period between years 2000 and 2018. Main source of data were public databases from Eurostat, European Central Bank (ECB), International Monetary Fund (IMF) and The Organization for Economic Co-operation and Development (OECD). Data are further uniformly indexed, where the average value of 2015 is set to 100 and normalized, so the 0 is equal 100, this transformation allows us to compute logarithms even from negative numbers.
As the main goal of this diploma thesis is to understand the development and the misalignments of equilibrium exchange rates of Visegrad countries, the explained variable is real exchange rate. The variable for each country was computed in two ways according research papers we are using as lead to set up the models.
First form of real effective exchange rate was calculated from nominal exchange rates quarter averages against euro deflated by price level ratio of domestic price level against Euro Area price level, using the Harmonized Index of Consumer Prices (HICP). This calculation allows us to observe exchange rate also in Slovak Republic, where is already

implemented common euro currency since 2009 and therefore the nominal exchange rate convention is stabilized on level of 30.126 Slovak Korunas per euro. The HICP values were obtained with monthly frequency, and transformed into quarter values, by period averages. Both, nominal exchange rates values and HICP values were obtained from Eurostat database. The second from of real exchange rate was computed for purposes of BEER model according to paper of Pošta in the similar way but using PPI instead of HICP. The exchange rate computed with PPI deflator was used only in this model and in the rest of the models we preferred the real exchange rate deflated by HICP ratio.
The choice of HICP over PPI is strategical. First thing we considered is the fact that PPI measurement differs across countries, which is bringing discrepancies into the models. Furthermore, as all four countries are small open economies, PPI is less convenient, as the domestic currency is not used to determine the pricing of exports.

As explanatory variables were chosen further listed time series. Not all variables are for all countries of Visegrad group, the main reason was data availability or the model we were using. For example, JP Morgan is estimating tailored BEER models according the characteristics of each country, that is why variables that are entering model might change according the country the model is currently estimating.
Brent crude oil prices obtained in euros measured quarterly were used as proxy variable for terms of trade. The time series were obtained from ECB measured at historical closes, average of observations through quarter periods.
Another proxy for terms of trade introduced in the models is the ratio of Exports to Imports.
Nominal bond yields are stated as Maastricht criterion bond, which are EMU convergence criterion for long-term interest rates (central government bond yield on the secondary market, gross of tax, with around 10 years' residual maturity). Data were obtained directly in quarter frequency from Eurostat.
For obtaining the real interest rate differentials, real interest rates were computed from nominal bond yields deflated by inflation. Then we obtained real interest rate differentials by deducting Euro Area differentials from real interest rates of each country.
Further we used government debt measured quarterly as percentage of Gross domestic product as well as government consumption. Both obtained directly in proportional form to GDP measured quarterly from Eurostat. The government consumption was further used to compute the ratio of share of domestic government consumption on total Euro Area government consumption.

Due to limited data availability of net foreign assets we used international investment position instead. Data were obtained directly in quarterly measure from Eurostat.
In order to analyze Balassa-Samuelson effect the relative productivity ratio was used, computed as index of total industrial production, production in manufacturing, employment in total industry and employment in manufacturing in each country of Visegrad Group and in the euro area, respectively. However, some papers also state as suitable form of approximating this effect by ratio between HICP and PPI index. It is also the case of Pošta BEER estimations, but using this ratio is creating endogeneity problem, of entering the same variable on both side of equation, that is why we used in the end the proxy for BS computed from.
As proxy variable for country openness was used ratio of total exports to nominal GDP both obtained directly measured quarterly from Eurostat.
Total domestic demand was calculated according ECB standards using GDP expenditure approach, by extracting external balance of goods and services from the GDP, both quarter time series. Data were taken in national currencies therefore, they were recalculated by exchange rates for given periods to obtain the values in euros. Data were obtained from OECD, measured quarterly.
Further we used ratio of External Debt to GDP for polish model of JP Morgan, data were obtained directly in the form of ratio from Eurostat.
Three months Libor rate is used as proxy for external real rates for Poland and for Hungary, data are from European Commission, measured monthly and transformed to quarter measure by averaging the quarters.
In order to get proxy for internal rate for Hungary we use three-month money market interest rate obtained also from European Commission, measured monthly and we transform them into quarterly measured data.

IV.II Data Handling

In this section will be introduced all test performed on the data used in empirical research of the diploma thesis and commented their results. The theoretical explanation of the tests is in annex of the diploma thesis, while here the focus is on the results and their impact on the models.

IV.II.ii Stationarity testing

In the first place we need to test the data that was used in the empirical research for stationarity due to their time series character in order to prevent spurious regression problem. Basic characteristic of time series that are stationary, is their independence from time at which the series observations were made. Any time trend and seasonality that have an impact on values of observations are causing the time series to be non-stationary.

Before testing the data for stationarity, we can have a look on autocorrelation function (ACF) and partial autocorrelation function (PACF). However, this can be misleading as ACF for a unit root process can be slowly decreasing over time towards zero, even though shocks to a unit root process will remain in the system indefinitely. Because of that, the process might be considered as persistent, but stationary. Therefore, we need hypothesis testing to state exactly whether the time series has or has not a unit root. Hyndman et al. [2014] In order to test stationarity, we have to first determine whether presented trend in time series is stationary or a unit root, meaning that stationary trend can be reduced by removing the trend component of the time series. For unit root testing we used Augmented Dickey-Fuller Test.

Augmented Dickey-Fuller Unit Root Test

The Augmented Dickey Fuller Test (ADF Test) was introduced for situations when time series have complex dynamic structure in contrast to simple autoregressive model of first order.

The ADF test has the null hypothesis that data are stationary and alternative hypothesis states for otherwise. Results for all variables according the countries are shown further in Appendix, the result of the test shows that we have high $p - values$ and for all variables the $p - value$ is higher than 0.05 which confirms the intuition that time series data are containing unit root and are non-stationary.

As we were modeling equilibrium exchange rate development and studying closely its misalignments, we intuitively expected this information, but we needed the confirmation in order to understand the movements of values of variables. Hence, to be build suitable models with relevant power we need to choose strategically the estimation and modelling technique. Therefore, the preliminary models were estimated by dynamic linear regression models for time series and for results VAR models were chosen, especially Vector Error Correction Models (VECM). This model is from VAR models family, it is special case for cointegrated variables, in order to be sure about cointegration we performed Johannsen test of cointegration as next. Pfaff [2008]

IV.II.ii Lag Selection

In order to build VEC model we need to know the number of lags that should be introduced in the model to obtain coherent results. For Lag selection we had look on four most frequently used information criterion, Akaike (AIC), Hannan Quinn (HQ), Schwarz (SC) and Final Prediction Error (FPE) Information Criterion. When selecting the lags for model we chose set up of deterministic regressors including both, constant and the trend as we are working with time series. Hamilton [1994]

The results are displayed in Annex in tables **??** and **??**. In models we were mostly using the lags indicated by Akaike Information Criterion. Those lags were then used constructing VEC models and proceeding with Johansen Cointegration. Most frequent value of lag introduced in model was 5, giving the nature of the data (as we are dealing with quarterly measured data), we can conclude that our lags are representing period of one year and quarter, which means that real exchange rate is impacted by fundamental variables values going back for one year and quarter. This seems logical as it takes some time for country and its agents to adjust and behave according the economic situation.

Johansen Test for Cointegration

This test serves to find whether cointegration is presented in time series through the Eigen value decomposition. The null hypothesis is defined as $r = 0$ and states that cointegration is not presented, while alternative hypothesis implies a cointegrating relationship and is stated as $r > 0$. In all models we construct the cointegration was presented, however in some of the cointegration was rather weak. The test was always done based on the lag selection from previous subsection. The best cointegrating relationship was chosen according the diagnostics of the model. We needed to make sure serial correlation is not presented in the model as well as heteroscedasticity and we tested models for normality. Arize and Ndubizu [1992]

V

Models

After conducting all necessary data handling operations and estimating all models, we can proceed with examination of final models. We will introduce the results of the estimations of the FEER and BEER models according the main research papers used for constructing the models and we will comment nested models we tried to estimate basing on the best results for each country. For BEER models we tried to use several variables that resulted as significant in various BEER models and for FEER models we tried to enrich the best FEER model by additional variables.

V.I FEER model

In total we estimated three FEER models for each country according the papers of MacDonald, Komárek and Deutsche Bank. The models were first estimated by DOLS technique but as the autocorrelation and serial correlation were still presented and as we were modeling non-stationary data, we did our core estimation using VEC models with Johansen Cointegration procedure in order to deal with autocorrelation and build effective models.

For each country the results of DOLS models are in Attachments and we comment them just briefly as they are considered as preliminary results and departing point for constructing VEC Models. Those models are examined into more depth for each country.

The DOLS estimations were based on papers from Deutsche Bank and their FX Valuation models, which is simplified version of MacDonald approach we estimated as well, and then the third model is based on paper from Komárek and Motl. The DOLS model from DB is simplified and serves well for preliminary results. The reason for this is that the main two models DB uses for FX Valuation is DBEER model, which we aimed to estimate among BEER models and then Cap-PPP model, which is classical PPP model based on capital flows, this model was specified into more details in previous chapter so we will not explain it further. Therefore, the FEER model approach is used more as final check and not as the core estimation technique. Although Deutsche Bank uses DOLS as main estimation technique, we estimated those models also as VEC models in order to compare them with the rest of the FEER Models. The equation for this model is stated as:

$$RER_i = GDP_i + GDP_{EA} + SustainableCA_i \tag{V.1}$$

where Sustainable Current Account is computed as rolling averages of historical current account values.

The models from DB and MacDonald are much alike, because DB uses MacDonald model as departing point. The difference between them is that while in model from MacDonald we use Demand, for models from DB we use GDP, therefore the difference should be very small. The MacDonald model equation is then:

$$RER_i = D_i + D_{EA} + SustainableCA_i \tag{V.2}$$

The FEER model estimated by DOLS based on paper from Komárek consists from several steps and it is more linked to the underlying economic theory. We estimated first Import and Export functions which equations are:

$$EXP_i = RER_i + IMP_{EA} + Prod_i + FDI_i \tag{V.3}$$

$$IMP_i = RER_i + EXP_{EA} + Demand_i \tag{V.4}$$

in Export equation we are using Productivity ratio, later used as proxy for Balassa-Samuelson. In Import equation the variable of demand stands for domestic demand of the country we are aiming to estimate the Import.

Those two equations are then combined by common relationship forming net exports and put equal to current account together with values for supposed sustainable current account. Then the relationship for real exchange rate was derived and the values for exchange rate were computed.

The main problem faced while estimating DOLS models was too high R-squared, which suggests biased model or present of correlation, which we expected since we were modelling time series, where correlation is very usual feature. Furthermore, we did not differentiate the data to preserve as much information as possible, which implies that used

data were non-stationary. That is why we proceed with VEC models. With this technique we estimated all three models for each country. For clear interpretation of the VEC models we needed to adjust the model output and we calculated the standard errors and t-statistics for each model. We computed t-statistics for the speed of adjustment, and $t - statistics$ for the coefficients that describe the long-run relationship, β.
Results of those models are reported in tables in following subsections.

V.I.i Czech Republic

Preliminary results according the DOLS technique are reported in table VI.13. First two models are MacDonald models, first one is based on DB assumption and using GDP, second one is original model from MacDonald based on demand of Czech Republic and Euro Area. As we expected the DOLS models have very high R-squared suggesting the presence of correlation, also when we tested the models this suspicion was confirmed, and it showed that VEC model might gave us more coherent results. Third model is based on Komárek and Motl approach, using fitted values for Net Export. This variable seems to be highly insignificant, which would be contradicting to underlying economic theory. Possible explanation for that might be hidden in underlying estimated equations for Import and Export as those models, had similar nature to first two FEER models, meaning the correlation was presented in the models, which is causing inconsistency of the models.
The models that are our priority concern are VEC models. Results of our best models are reported in tables below. First model is constructed according DB and according the table V.1 we can conclude that variable for domestic GDP resulted insignificant in short run and lagged exchange rate has only weak significance while Sustainable Current Account values and GDP of Euro Area are strongly significant, suggesting that with increase of foreign GDP the equilibrium exchange rate appreciates and depreciates with rising sustainable current account.
In long-term relationship all variables expect trend resulted to be significant, the insignificance of trend might be result of using seasonally adjusted data and even though in cointegration test the trend resulted to be present, it is obvious that its impact is very low. The rest of the variables are strongly significant and in line with economic theory, showing that increase in domestic GDP or sustainable current account result in appreciation of equilibrium exchange rate while increase in foreign GDP causes depreciation of the exchange rate.

Table V.1: DB FEER Model Czech Republic

	CZRER.d	NGDP_CZ.d	NGDP_EA.d	CZSustCA.d	
alpha	0.019	−0.004	−0.012	−0.307	
alpha.t	1.343	−0.313	−4.395	4.713	
	CZRER.l5	NGDP_CZ.l5	NGDP_EA.l5	CZSustCA.l5	trend.l5
ect1	1	−2.440	7.498	−2.481	0.001
beta.t		−2.831	2.358	−7.954	0.095

Next model is MacDonald FEER model, the only difference between this and previous model is the switch from GDP to Demand of the economy. In short run the only significant variable is sustainable current account and it seem to have positive impact in equilibrium exchange rate. In long run, all variables resulted significant with very high coefficients suggesting very significant impact on the equilibrium exchange rate as the table shows V.2. The highest impact seems to come from the demand from Euro Area countries, this result seems to be logical, as Czech Republic is small open economy and exporter, who has tight connection to European market. Although, this variable is insignificant in short term, it is not worrying us as FEER models are used mainly to estimate long term relationships.

Table V.2: MacDonald FEER Model Czech Republic

	CZRER.d	CZ_D.d	EA_D.d	CZSustCA.d
alpha	−0.008	0.002	−0.009	0.278
alpha.t	−0.501	0.091	−1.274	5.262
	CZRER.l5	CZ_D.l5	EA_D.l5	CZSustCA.l5
ect1	1	−6.979	17.457	−2.649
beta.t		−4.846	5.195	−6.331

Last estimated model from FEER family models for Czech Republic was according the approach of Komárek and Motl. For this model we estimated 2 equations before computation the equilibrium exchange rate itself, because in order to estimate it we needed the equations for export and import, which we used then in the final computation of equi-

librium exchange rate. The results of import and export equation are presented below in table V.3. In case of import, in short run the only significant variable is domestic demand suggesting positive relationship, that with rising demand imports are rising, what is in line with economical intuition and logic. However, long run suggests contradictory results, showing that the domestic demand in long run has negative, although very small, impact on imports. This result might be explained by possible reasons. First is economical, that in long run the demand might shift towards domestic goods and services, causing decrease of demand for imported goods. Another possible explanation is rather technical, as during estimating this model we were struggling with presence of serial correlation and even though we successfully reject the presence of serial correlation on 5% level we cannot reject in on 10% significance level and so we have to count with possible that there is still some serial correlation presented in the model.
Export equations suggests that in long run the most important significant variables are exchange rate and productivity ratios states as Balassa Samuelson effect. However, as it is obvious from too low or too high $t - statistic$ the model is not working well suggesting that VEC model might not be the right approach to model exports and that serial correlation might be still presented in the models causing the loss of efficiency.

Table V.3: Komárek Import and Export equations for Czech Republic

Import		IMP_CZ.d	CZRER.d	EXP_CZ.d	CZ_D.d	
	alpha	0.328	−0.250	0.399	0.266	
	alpha.t	3.225	−5.166	4.163	4.173	
		IMP_CZ.l7	CZRER.l7	EXP_CZ.l7	CZ_D.l7	trend.l7
	ect1	1	3.345	−0.438	1.009	−0.014
	beta.t		7.718	16.254	−170.923	7.790
Export		EXP_CZ.d	CZRER.d	IMP_EA.d	CZFDI.d	BS_CZ.d
	alpha	−0.001	−0.048	0.033	0.033	0.006
	alpha.t	−0.033	−2.684	1.843	4.197	1.671
		EXP_CZ.l5	CZRER.l5	IMP_EA.l5	CZFDI.l5	BS_CZ.l5
	ect1	1	1.728	−2.215	−5.3215	15.845
	beta.t		2.499	0.214	−0.505	−132.968

V.I.i Hungary

The same approach as for Czech Republic was taken for all countries of Visegrad Group. The FEER model seems to work for Hungary even in short run, when sustainable current account and domestic GDP resulted significant and Euro Area GDP weakly significant. We can observe shifts in results when moving from short run toward long run. While domestic GDP and Sustainable Current account have negative coefficients resulting in appreciation of exchange rate in speed of adjustment, the coefficients turn to be positive in long term, suggesting the depreciation. This can possibly mean the internal macroeconomic imbalances within the economy. As we know, Hungary is very indebted country which is struggling with rising external debt, this might be the possible explanation for the depreciation pressures caused by rising GDP.

Table V.4: DB FEER Model Hungary

	HURER.d	NGDP_HU.d	NGDP_EA.d	HUSustCA.d	
alpha	−0.002	−0.070	−0.009	−0.013	
alpha.t	−0.060	−2.072	−1.485	−5.593	
	HURER.l5	**NGDP_HU.l5**	**NGDP_EA.l5**	**HUSustCA.l5**	**trend.l5**
ect1	1	4.408	−4.380	27.070	−0.030
beta.t		7.801	−3.734	6.586	−4.550

Next model, based on original MacDonald approach is as well working well for Hungary in both, short term and long term period. In short run the Euro Area Demand is significant together with sustainable current account, but both variables have very low impact. Fur us, more important is the long run relationship. We can again observe the change of behavior of Euro Area Demand and sustainable account, which are in line with results from previous model. Although the domestic demand seems to be insignificant in terms of speed of adjustment, in long-run the significance increased as well as the coefficient, showing that the increase of domestic demand causes the depreciation of the equilibrium exchange rate and increase in Euro Area demand cause its appreciation. Furthermore, trend resulted to be highly significant and downward sloping showing long term depreciation path of the Hungarian forint.

Table V.5: MacDonald FEER Model Hungary

	HURER.d	HU_D.d	EA_D.d	HUSustCA.d	
alpha	−0.090	0.056	0.097	−0.014	
alpha.t	−1.121	0.422	3.618	−2.528	
	HURER.l5	HU_D.l5	EA_D.l5	HUSustCA.l5	trend.l5
ect1	1	1.594	−1.684	8.887	−0.014
beta.t		6.121	−4.686	3.047	−3.996

The last FEER model for Hungary is model based on Import and Export equations. The results for Import show that in short run the lagged import and exchange rate are significant the rest of variables are insignificant. In long run all variables resulted to be significant, showing all negative impact on the import. When we translate this, the relation means that the imports are rising with depreciating exchange rate, with rising exports and with rising domestic demand. The relationship with exchange rate and with domestic demand seem to be a little contradictory. It might suggest in case of exchange rate that rising imports are also rising the external debt of the country which pushes the exchange rate to depreciate and in case of demand, that although the domestic demand is rising, people prefer domestic production over imports.

Long run relationship is providing coherent results for Hungary, showing that exports are rising with depreciating exchange rate, decreasing imports decreasing productivity ratio and rising foreign direct investments. The equation for Exports for Hungary seems to work better than for Czech Republic, showing the differences between the countries.

Table V.6: Komárek Import and Export Equations Hungary

Import		IMP_HU.d	HURER.d	EXP_HU.d	HU_D.d	
	alpha	−0.819	0.709	−0.107	−0.456	
	alpha.t	−2.120	2.143	−0.330	−0.932	
		IMP_HU.l5	HURER.l5	EXP_HU.l5	HU_D.l5	constant
	ect1	1	0.071	−0.772	−0.321	−1.439
	beta.t		37.209	2.225	−9.859	−11.946
Export		EXP_HU.d	HURER.d	IMP_EA.d	BS_HU.d	HUFDI.d
	alpha	0.091	-0.077	0.124	0.008	-1.306
	alpha.t	2.325	-2.040	4.854	0.847	-1.841
		EXP_HU.l3	HURER.l3	IMP_EA.l3	BS_HU.l3	HUFDI.l3
	ect1	1	1.456	-2.437	-1.991	0.101
	beta.t		4.250	1.185	−51.908	−8.463

V.I.i Poland

DB FEER model for Poland resulted to be weak in short run, as almost all variables are insignificant, except GDP from Euro Area. In long-run the results differ, domestic GDP resulted to be insignificant and EA GDP is only weakly significant the only variable that seems to be significant over time is sustainable current account. This model includes also constant, which results to be significant.

Table V.7: DB FEER Model Poland

	PLRER.d	NGDP_PL.d	NGDP_EA.d	PLSustCA.d	
alpha	−0.015	0.029	0.040	−0.005	
alpha.t	−0.271	0.637	6.478	−0.364	
	PLRER.l5	NGDP_PL.l5	NGDP_EA.l5	PLSustCA.l5	constant
ect1	1	0	−1.154	−3.609	20.719
ect2	0	1	−2.439	−2.529	18.267
beta.t		0	−1.042	−2.918	2.447

When we changed GDP for demand, we can observe that this model is more suitable in terms of significance of variables. Demand from Euro Area seems to be insignificant in speed of adjustment comparing to the rest of the variables which are at least weakly significant. But its significance for long-term relationship rises, suggests similar development to the Czech Republic. In long run we see all variables significant. In this model the constant was replaced by trend component which is significant and decreasing suggesting long term appreciation of the exchange rate. Overall the model suggests that Polish Zloty depreciates with rising both demands, domestic and from Euro Area.

Table V.8: MacDonald FEER Model Poland

	PLRER.d	PL_D.d	EA_D.d	PLSustCA.d	
alpha	−0.428	0.764	0.049	0.335	
alpha.t	−1.148	1.785	0.519	7.003	
	PLRER.l5	**PL_D.l5**	**EA_D.l5**	**PLSustCA.l5**	**trend.l5**
ect1	1	1.084	0.852	−3.401	−0.022
beta.t		12.819	4.461	−12.964	−25.584

The last FEER model estimated for Poland, according Komárek and Molt. As we see the model is working quite well in both cases, Imports and Exports. For the imports long run suggests that exports are insignificant, and their impact is negligible on the imports, while exchange rage and domestic demand are significant showing that with depreciating exchange rate the polish markets is less interesting for importers and imports decreases. The model shows that rising domestic demand also cause lowering imports, which again might relate to preference of domestic products.
In terms of exports we see that exchange rate is only weakly significant and, therefore, suggests that more important impact have imports, productivity ratio and foreign direct investments. Rising imports are negatively impacting exports as well as productivity ratio while increase in foreign direct investments seem to support the exports.

Table V.9: Komárek Import and Export Equations Poland

Imports		IMP_PL.d	PLRER.d	EXP_PL.d	PL_D.d	
	alpha	−0.738	−0.157	−0.486	0.272	
	alpha.t	−2.022	−0.477	−1.590	0.760	
		IMP_PL.l5	PLRER.l5	EXP_PL.l5	PL_D.l5	
	ect1	1	−0.012	−0.589	−0.511	
	beta.t		28.260	−0.212	−16.636	
		EXP_PL.d	PLRER.d	IMP_EA.d	BS_PL.d	PLFDI.d
	alpha	-0.037	0.038	-0.023	0.007	-0.054
	alpha.t	-1.704	1.701	-1.686	1.993	-5.178
		EXP_PL.l2	PLRER.l2	IMP_EA.l2	BS_PL.l2	PLFDI.l2
	ect1	1	-0.712	-2.342	-12.236	15.808
	beta.t		−1.623	−9.547	−4.059	5.088

V.I.i Slovak Republic

It was very interesting to estimate equilibrium exchange rate for Slovakia, as it is the only country from Visegrad Group which is no longer using its own national currency but adopted common euro currency in 2009 instead. Therefore, to compute real exchange rate we used nominal rate of Slovak koruna and since 2009, when the euro was adopted the nominal exchange rate is constant and not changing, but thanks to the development of price levels over time we were still able to observe the real exchange rate, although it is no longer used. Also, due to euro adoption valuation models are normally no longer estimated for Slovakia and it is therefore usually incorporated in models for euro and therefore for global models for all Euro Area.

From the estimated DB model using GDP and sustainable current account it seems that while all variables are significant in speed of adjustment, they lose their significance for long-term relationship, and the only significant variable remains sustainable current account. Results are implying that even exchange rate of Slovak Koruna, which is no longer used does not depend on performance of the Slovak economy nor on the EA GDP development.

Table V.10: DB FEER Model Slovakia

	SKRER.d	NGDP_SK.d	NGDP_EA.d	SKSustCA.d	
alpha	−0.032	0.124	0.015	−0.048	
alpha.t	−1.287	5.402	1.684	−2.140	
	SKRER.l2	NGDP_SK.l2	NGDP_EA.l2	SKSustCA.l2	constant
ect1	1	0.075	0.113	3.558	−20.657
beta.t		0.427	0.209	3.125	−3.805

We obtain better results when we focus on the demand functions. The table V.11 provides the results for both, for speed of adjustments and for long term relationship. In short run the only significant variables exchange rate itself and sustainable current account. In long run the demand from Euro Area gained significance, suggest that higher demand is causing the exchange rate to depreciate.

Table V.11: MacDonald FEER Model Slovakia

	SKRER.d	SK_D.d	EA_D.d	SKSustCA.d	
alpha	−0.142	−0.022	−0.032	−0.081	
alpha.t	−4.443	−0.358	−1.468	−3.108	
	SKRER.l5	SK_D.l5	EA_D.l5	SKSustCA.l5	trend.l5
ect1	1	−0.244	3.366	−1.561	−0.013
beta.t		−0.573	2.829	−1.253	−4.138

Last FEER model estimated generally and for Slovakia is traditionally model based on Import and Export equations. Import equations seem to give better results for short run then for long run showing that in short run the imports are rising with rising domestic demand and exports, but decreasing with depreciating exchange rate, which is logical as the country become less interesting for importers. In long run the only relation that holds from short run are the exports. The other variables, although they are significant, they are showing contradictory results, implying that imports should increase with depreciating currency and with lowering domestic demand. Which does not seem logical. However, the export equations work better and provides more reasonable results. In long term the export is positively related with exchange rate, meaning that depreciation of the exchange rate causes the increase of exports, which is in line with economic intuition as exports are cheaper and more attractive for foreign markets. Rising import is on contrary pulling exports down together with production ratio and foreign direct investments. This we can understand as the growth of economy is decreasing the competitive advantage of the country on the market.

Table V.12: Komárek Import and Export Equations Slovakia

Imports		IMP_SK.d	SKRER.d	EXP_SK.d	SK_D.d		
	alpha	0.039	−0.205	0.331	0.377		
	alpha.t	0.269	−3.146	2.070	3.389		
		IMP_SK.l1	SKRER.l1	EXP_SK.l1	SK_D.l1		
	ect1	1	0.510	−0.779	−0.224		
	beta.t		22.423	4.879	−17.457		
Exports		EXP_SK.d	SKRER.d	IMP_EA.d	BS_SK.d	SKFDI.d	
	alpha	0.243	-0.054	-0.087	0.056	0.016	
	alpha.t	2.592	-1.605	-1.511	2.650	0.397	
		EXP_SK.l5	SKRER.l5	IMP_EA.l5	BS_SK.l5	SKFDI.l5	constant
	ect1	1	1.011	-0.589	-4.633	-5.244	18.967
	beta.t		7.595	-5.065	-11.527	−4.898	4.095

V.II BEER model

After conducting analysis of FEER models, based on fundamental equilibrium we will have look on behavioral approach conducted in BEER models. Models were built using logarithms of the seasonally adjusted data. We estimated four BEER models, first version is according to paper from Clark and MacDonald MacDonald and Clark [1998], second according to the article of Vít Pošta Posta [2010], third model is based on DBEER approach from Deutsche Bank and fourth version is based on JP Morgan BEER model. All four models were estimated by DOLS estimation technique and by VECM technique and compared, DBEER Kalani [2016] and JP Morgan BEER model are originally in reports estimated only by DOLS, but to have comparable results we estimated also their VEC models. Anezka Christovova [2019] Furthermore, we anticipated also in BEER models the presence correlation and data are still non-stationary, that is why VEC models

might give us more stable results.
Equations of each model are identical for each country, except the models estimated according JP Morgan. The reason for this is further explained in theoretical part of BEER models.
First estimated BEER model for all countries is according MacDonald and the equation states real exchange rate as:

$$RER_i = GOVC_i + EMU_i + EMU_{EA} + BS_i + ToT_i \tag{V.5}$$

In this equation real exchange rate is defined through government consumption per GDP, Maastricht convergence criterion used as proxy for long term interest rates, Balassa-Samuelson ratio computed through productivity index and Terms of Trade.
Second estimated model is according the paper from Pošta which uses interest rate differentials and oil prices as proxy for terms of trade.

$$RER_i = IRdiff_i + GOVC_i + GOVDebt_{EA} + BS_i + Oilp_i + IIP_i \tag{V.6}$$

Than we moved to models used for FX valuation according JP Morgan and Deutche Bank. Those models are more simple comparing to previous ones. Equation for JP Morgan differs from country to country as they are tailored. The equation for DBEER model constructed according the DB FX valuations is:

$$RER_i = Openness_i + BS_i + ToT_i \tag{V.7}$$

For reporting is taken similar approach as for FEER models, results from BEER DOLS estimations are reported in Appendix, while results for VEC models are shown below in tables and commented.

V.II.ii Czech Republic

First BEER model is according MacDonald. As we see in table V.13 In short run except long interest rates approximated by Maastricht convergence criterion are all variables significant, which applies also for long run, however, in long run we can observe that Czech long interest rates are gaining significance, although staying still weakly significant. The model shows that in short term, government consumption presses real exchange rate up, causing depreciation as well as productivity ratios, while the rest of the variables are causing exchange rate to appreciate. we can also observe that in long term the domestic long term interest rates change the direction of the influence and cause real exchange rate to appreciate. This phenomenon might relate to one sided bound of Czech Koruna that was introduced as response to falling interest rates and inflation. Similarly, to long term interest rates production ratio (stated as Balassa Samuelson effect in the model), seems to change the impact over time and pressure to Czech Koruna to depreciate. We also see that the trend component is significant in long term and it has decreasing character, which corresponds with appreciation of the Czech Koruna over time.

Table V.13: MacDonald BEER Model Czech Republic

	CZRER.d	CZGOVCperGDP.d	CZ_EMU.d	EA_EMU.d	BS_CZ.d	CZTOT.d	
alpha	0.054	−0.046	0.077	−0.051	−0.008	−0.011	
alpha.t	4.488	−4.702	0.659	−0.693	−3.401	−1.378	
	CZRER.l4	**CZGOVCperGDP.l4**	**CZ_EMU.l4**	**EA_EMU.l4**	**BS_CZ.l4**	**CZTOT.l4**	**trend.l4**
ect1	1	7.689	−0.217	−0.015	29.193	−7.226	−0.013
beta.t		10.136	−1.505	−0.073	8.491	−3.499	−2.706

Next model is estimated according Pošta. We did small adjustment of the model, because originally in the model was also supposed to be incorporated Balassa Samuelson effect computed as ratio between CPI and PPI. However, as we are deflating nominal exchange rates by price ratios, we were creating directly issue of endogeneity, when same variable entered both sides of equation. Another adjustment we did was to switch Net Foreign Assets for International Investment Position. The full dataset for Net Foreign Assets were available only for Czech Republic and Hungary, while for Poland and Slovakia were not available at all.

This model is using the highest number of variables. Overall, the model seems fine, although we see that in terms of significance some variables were dropped out. We can see that oil prices, which are in this model used as proxy for terms of trade, variable we have not used until now is significant in both periods, causing depreciation of the exchange rate. We see that for all variables, their impact changed passing from short run to long run. Interest rate differentials have very weak significance in both periods suggesting only low impact on the exchange rate. Also, the international investment position lost significance in long term. From the rest of the variables we can conclude that productivity ratio has appreciating influence on exchange rate, which is opposite of what suggested previous model. However, government consumption has in both models depreciating effect, although in this model the impact is smaller.

Table V.14: Pošta BEER Model Czech Republic

	CZRER.d	oil_pPPI.d	CZIR_DIFF.d	PROD_CZ.d	CZGOVCperGDP.d	CZDebtperGDP.d	CZIIPperGDP.d	
alpha	0.030	−0.890	−0.007	0.018	−0.051	0.277	0.390	
alpha.t	0.691	−3.249	−0.989	1.544	−1.347	3.407	2.118	
	CZRER.l2	oil_pPPI.l2	CZIR_DIFF.l2	PROD_CZ.l2	CZGOVCperGDP.l2	CZDebtperGDP.l2	CZIIPperGDP.l2	trend.l2
ect1	1	0.325	0.375	−6.281	0.743	−0.381	−0.022	−0.004
beta.t		9.195	0.236	−7.013	2.773	−5.996	−0.747	4.366

Next model is DBEER. This model is usually estimated on regional panel data. We adjusted the model for our needs and estimated additional VEC model, although in original set up this model is estimated by DOLS. We can observe several differences between the DOLS estimation and VEC model. The most visible change is the relevance of country Openness. While in our DOLS model the country openness resulted as insignificant, in VEC model it gained significance in short term and although the significance in long term relationship is weak, it is higher than in DOLS model. Also, if we have closer look on terms of trade, we can see that the variable gained significance by switching the models, although it has no significance on α level. Another important observation is the change of impact of Balassa-Samuelson effect, which in short term despite being significant has only very low impact and toward appreciation. In long term the direction and the power of the variable changed, the BS effect became more significant and pressing on depreciation of the real exchange rate. While terms of trade are pushing the exchange rate to depreciate the country openness has opposite effect. However, we also see that coefficient for Balassa-Samuelson effect and terms of trade are very high suggesting possible further issue with correlation.

Table V.15: DBEER Model Czech Republic

	CZRER.d	BS_CZ.d	CZTOT.d	CZ_op.d	
alpha	0.001	−0.001	−0.0001	−0.020	
alpha.t	0.612	−2.167	−0.051	−5.597	
	CZRER.l5	BS_CZ.l5	CZTOT.l5	CZ_op.l5	constant
ect1	1	98.469	−95.688	9.825	−1.151
beta.t		3.271	−4.874	2.834	−1.078

Last BEER model is built according JP Morgan. This model is originally estimated by DOLS as the previous one, but we wanted to compare the results more precisely with the rest of the models, that is why we have decided to estimate also as VEC model. The variables that are in the model are chosen strategically, according the best model results from JP Morgan. However, we can observe different results comparing this model with original DOLS model. When looking on net exports in DOLS estimation is this variable significant and has strong impact on the real exchange rate development, but according VEC model this holds only in long run while in terms of speed of adjustment it is insignificant.

Table V.16: JP Morgan BEER Model Czech Republic

	CZRER.d	BS_CZ.d	CZ_Invest.d	CZNXperGDP.d	CZIIPperGDP.d	
alpha	0.028	−0.044	0.174	0.001	−0.280	
alpha.t	0.565	−3.958	3.009	0.503	−1.297	
	CZRER.l2	**BS_CZ.l2**	**CZ_Invest.l2**	**CZNXperGDP.l2**	**CZIIPperGDP.l2**	**trend.l2**
ect1	1	5.054	−1.258	−45.193	0.122	0.002
beta.t		5.051	−3.557	−5.199	5.001	0.874

V.II.ii Hungary

First BEER model for Hungary is also according to paper from MacDonald. Similarly, to Czech Republic this model works well, the only insignificant variable is Terms of Trade, the rest of the variables are significant on both levels. We can observe that long term interest rates are pressing real exchange rate to depreciate in short term and appreciate in long. As we know Hungarian National Bank was easing its policy quite significantly in recent years in order to get inflation under control and stop falling interest rates, and lowering inflation helped to the exchange rate to appreciate. We can also observe that the impact of long term interest rates changes over time causing appreciation in long term relationship. The opposite change is observable for Balassa-Samuelson effect which is in long term having depreciation effect.

Table V.17: MacDonald BEER Model Hungary

	HURER.d	HUGOVCperGDP.d	HU_EMU.d	EA_EMU.d	HUTOT.d	BS_HU.d	
alpha	−0.037	−0.094	0.309	0.641	0.002	−0.067	
alpha.t	−0.772	−2.033	2.198	3.590	0.052	−6.451	
	HURER.l3	**HUGOVCperGDP.l3**	**HU_EMU.l3**	**EA_EMU.l3**	**HUTOT.l3**	**BS_HU.l3**	**trend.l3**
ect1	1	1.013	−0.328	−0.169	−0.352	8.570	−0.021
beta.t		3.118	−3.263	−2.311	−0.800	9.541	−11.359

The BEER model according Pošta, has in total five cointegrated vectors, which is resulting into five error term introduced for each variable. However, as we see the model is overall weak, we obtained very small basically zero coefficients for many variables, together with low or 0 $t-statistics$, resulting into insignificant variables and over all weak estimation, although this model seemed to work better estimated by DOLS technique. The reason might be weak cointegration, causing the model to be overparametrized.

Table V.18: Pošta BEER Model Hungary

	HURER.d	oil_pPPI.d	HUIR_DIFF.d	PROD_HU.d	HUIIPperGDP.d	HUGOVCperGDP.d	HUDebtperGDP.d	
alpha	−0.735	1.174	−0.045	−0.014	0.269	−0.444	−1.381	
alpha.t	−4.883	1.341	−1.416	−0.306	1.026	−2.312	−4.740	
	HURER.l2	**oil_pPPI.l2**	**HUIR_DIFF.l2**	**PROD_HU.l2**	**HUIIPperGDP.l2**	**HUGOVCperGDP.l2**	**HUDebtperGDP.l2**	**trend.l2**
ect1	1	0	0	0	0	0.843	−0.313	−0.009
ect2	0	1	0	0	0	22.120	−0.792	0.016
ect3	0	0	1	0	0	0.066	−0.061	−0.0001
ect4	0	0	0	1	0	0.687	−0.140	−0.001
ect5	0	0	0	0	1	10.055	−1.532	0.008
beta.t		0	0	0	0	9.213	−12.832	−46.858

DBEER Model estimated as VEC model has similar results as DOLS model. Balassa-Samuelson results insignificant, although in speed of adjustments we can conclude weak significance rather than total insignificance. We can see that terms of trade and country openness resulted both strongly significant. While country openness is resulting in appreciation of the exchange rate, which is in line with economic intuition, the model suggests that in long run terms of trade are causing the depreciation of the exchange rate.

Table V.19: DBEER Model Hungary

	HURER.d	BS_HU.d	HUTOT.d	HU_op.d	
alpha	−0.039	0.004	−0.022	0.032	
alpha.t	−3.533	1.304	−3.080	2.021	
	HURER.l5	BS_HU.l5	HUTOT.l5	HU_op.l5	constant
ect1	1	−3.231	10.121	−4.580	−6.283
beta.t		-0.712	4.133	−4.259	−31.557

Last estimated model according JP Morgan FX Valuation approach is also estimated as VEC model, although the original one is estimated by DOLS. This time VEC estimation is in line with DOLS model, all variables are significant in long run. It is interesting to observe that short interest rates are working in opposite direction through time, while in short term the European short term interest rate pushes currency to appreciate and domestic short term interest rate has depreciation pressures, in long term those variables switches. Also, current account which as insignificant in short term gain significance in long term relationship, however the coefficient in long run is too high and with Portmanteau test $p - value$ close to 1 we must assume correlation effect.

Table V.20: JP Morgan BEER Model Hungary

	HURER.d	BS_HU.d	TMEUR.d	HUFTM.d	CA_HU_perGDP.d	HUDebtperGDP.d	
alpha	−0.681	−0.093	−0.097	7.625	−0.001	−0.677	
alpha.t	−5.399	−2.197	−3.216	0.980	−0.457	−2.510	
	HURER.l4	BS_HU.l4	TMEUR.l4	HUFTM.l4	CA_HU_perGDP.l4	HUDebtperGDP.l4	trend.l4
ect1	1	0.633	1.353	−0.029	96.216	−0.311	−0.013
beta.t		3.321	5.543	−8.578	5.211	−9.689	−23.455

V.II.ii Poland

Although the MacDonald seemed to work better for Czech Republic and Hungary, however, it is weaker in case of Poland and Polish Zloty. Government consumption and domestic long term interest rates resulted insignificant with zero coefficients, suggesting that the real exchange rate depends more on foreign fundamentals, but terms of trade are significant, which is the ratio of domestic export to total net export, but again as those fundamentals are directly related to foreign trade it only supports our theory that Polish Zloty is depending on international position of the country, which is logical as Poland is open economy and it was its exports that helped Poland to preserve economic growth in post crisis period.

Table V.21: MacDonald BEER Model Poland

	PLRER.d	PLGOVCperGDP.d	PL_EMU.d	EA_EMU.d	PLTOT.d	BS_PL.d	
alpha	−0.140	−0.033	−0.195	−0.808	0.200	−0.021	
alpha.t	−1.529	−1.026	−0.994	−2.561	4.567	−1.550	
	PLRER.l5	**PLGOVCperGDP.l5**	**PL_EMU.l5**	**EA_EMU.l5**	**PLTOT.l5**	**BS_PL.l5**	**constant**
ect1	1	0	0	0.138	0.600	−2.796	−1.360
ect2	0	1	0	0.099	1.168	−2.021	0.038
ect3	0	0	1	0.125	5.470	−3.375	−1.104
beta.t		0	0	4.028	1.787	−6.831	−14.288

Model from Pošta seems to work very well for Polish Zloty, certainly better than for Hungary. In long term the Debt and international investment position resulted insignificant and government consumption is only weakly significant. Oil prices are here as well as in case of Czech Republic significant and pressing the exchange rate to depreciate in both terms.

Table V.22: Pošta BEER Model Poland

	PLRER.d	oil_pPPI.d	PLIR_DIFF.d	PROD_PL.d	PLGOVCperGDP.d	PLDebtperGDP.d	PLIIPperGDP.d	
alpha	−0.192	0.534	0.008	0.003	−0.034	−0.287	−0.106	
alpha.t	−4.997	2.884	2.374	0.350	−2.185	−5.200	−3.442	
	PLRER.l2	**oil_pPPI.l2**	**PLIR_DIFF.l2**	**PROD_PL.l2**	**PLGOVCperGDP.l2**	**PLDebtperGDP.l2**	**PLIIPperGDP.l2**	**trend.l2**
ect1	1	−0.341	−21.084	2.428	−1.253	−0.019	0.103	−0.001
beta.t		-4.698	−5.936	1.873	−1.308	−0.145	0.499	−0.477

DBEER model estimations are also rather week for Polish Zloty. The model consists of three variables and country openness as well as Balassa-Samuelson effect are insignificant over long period. Only terms of trade seem to be significant, which is in line with the DOLS estimations. It can be caused by possibly still presented serial correlation as we were able to reject the presence on 5% level, however not on 10% level.

Table V.23: DBEER Model Poland

	PLRER.d	BS_PL.d	PLTOT.d	PL_op.d	
alpha	−0.239	−0.016	0.205	−0.109	
alpha.t	−2.681	−1.252	4.074	−1.078	
	PLRER.l6	BS_PL.l6	PLTOT.l6	PL_op.l6	constant
ect1	1	−0.055	−0.718	−0.026	−1.421
beta.t		-0.144	−1.998	−0.190	−15.440

JP Morgan for Polish Zloty is working well for long-term relationship than for short-term. We observe that in log term investment per GDP has appreciation pressures together with external debt and total government debt. Both debt ratio resulted to be significant in both periods.

Table V.24: JP Morgan BEER Model Poland

	PLRER.d	TMEUR.d	PLInvestperGDP.d	PLExtDebtperGDP.d	PLDebtperGDP.d	
alpha	−0.351	−0.017	−0.024	−1.221	−0.459	
alpha.t	−2.381	−0.623	−0.215	−6.809	−1.932	
	PLRER.l2	TMEUR.l2	PLInvestperGDP.l2	PLExtDebtperGDP.l2	PLDebtperGDP.l2	trend.l2
ect1	1	0.678	−0.240	−0.078	−0.366	−0.005
beta.t		1.439	−1.735	−2.464	−5.015	−3.601

V.II.ii Slovak Republic

BEER models for Slovakia are working in similar way as for Polish Zloty, also MacDonald model which is working very well for Czech Republic and Hungary, works poorly for Slovak Koruna. In long-term relationship we can see that only terms of trade and Balassa-Samuelson effect are significant, the rest of variables and error correction terms are extreme values, close to 0 or close to 1, which suggests that cointegration procedure was possibly overfitted, however, we tried to estimate the model in various ways running cointegration procedure several times and this was the best model we obtained. This model works better estimated by DOLS technique which would suggest that cointegration is very weak in model and we do not need to model it specifically, but we can use more general form of model, such as DOLS estimator.

Table V.25: MacDonald BEER Model Slovakia

	SKRER.d	SKGOVCperGDP.d	SK_EMU.d	EA_EMU.d	SKTOT.d	BS_SK.d	
alpha	−0.006	−0.037	−0.650	−0.629	−0.030	−0.123	
alpha.t	−0.156	−0.700	−1.522	−1.976	−0.518	−5.844	

	SKRER.l5	SKGOVCperGDP.l5	SK_EMU.l5	EA_EMU.l5	SKTOT.l5	BS_SK.l5	constant
ect1	1	0	0	0.029	−4.804	5.137	−3.419
ect2	0	1	0	0.036	2.202	−1.253	−0.016
ect3	0	0	1	−1.291	−0.847	1.519	0.339
beta.t		0	0	0.278	−9.222	14.263	−117.003

Model according Pošta has almost the same results as previous model, we see the estimation is poor and error correction terms are 0. Again, when we check, the DOLS estimation works much better for the model, suggesting the cointegration vectors are not presented in the model.

Table V.26: Pošta BEER Model Slovakia

	SKRER.d	oil_pPPI.d	SKIR_DIFF.d	PROD_SK.d	SKGOVCperGDP.d	SKDebtperGDP.d	SKIIPperGDP.d	
alpha	−0.079	−0.778	0.026	−0.150	0.079	−0.019	0.548	
alpha.t	−1.609	−1.498	1.873	−2.626	1.020	−0.110	4.402	

	SKRER.l2	oil_pPPI.l2	SKIR_DIFF.l2	PROD_SK.l2	SKGOVCperGDP.l2	SKDebtperGDP.l2	SKIIPperGDP.l2	constant
ect1	1	0	0	0	−7.119	−1.261	1.115	−4.360
ect2	0	1	0	0	8.551	0.096	−1.873	−3.875
ect3	0	0	1	0	0.048	0.012	0.029	4.615
ect4	0	0	0	1	8.755	1.197	1.787	0.936
beta.t		-0	0	0	−46.542	−64.759	12.881	−0.904

Last model for Slovakia from BEER models is DBEER model. JP Morgan model is not estimated for Slovakia as JP Morgan is not estimating valuation for Slovakia, due to the common euro currency adopted in 2009. From all BEER VEC models estimated for Slovakia this gives the best results. In long term the country openness resulted insignificant, while Balassa-Samuelson effect together with terms of trade are significant and while production ratio shown as Balassa-Samuelsson is causing depreciation of the real exchange rate, the terms of trade are working in opposite direction, pushing the exchange rate to appreciate.

Table V.27: DBEER Model Slovakia

	SKRER.d	BS_SK.d	SKTOT.d	SK_op.d	
alpha	0.008	−0.076	0.016	−0.035	
alpha.t	0.358	−6.248	0.549	−0.568	
	SKRER.l5	BS_SK.l5	SKTOT.l5	SK_op.l5	trend.l5
ect1	1	5.501	−4.177	−0.232	−0.001
beta.t		7.024	−3.998	−0.924	−0.640

V.III Nested models

After estimating various models according various research papers, there is obvious issue that several models were not giving meaningful results. Therefore, we gathered the variables for each country that were resulting significant in most of the models and we tried to estimate nested models. First, the nested models were estimated by DOLS but, the best models were then also estimated by VEC models. Through the models, we see that in terms of modelling, we could divide Visegrad countries into two groups according the fit of estimations. We saw similarities between Czech Republic and Hungary and then Poland and Slovakia. Overall FEER models seemed to fit better all countries, while BEER VEC models were working very well for Czech Republic and Hungary but poorly for Slovakia and Poland, suggesting weak cointegrated relations.
In nested models we tried to combine various BEER approaches and obtain best model, while for FEER models we tried to add more variables into best FEER models.

V.III.iii FEER Models

We tried to enrich the FEER models by variables that are entering BEER models and create hybrid models that might work well for predicting the real exchange rate misalignments. All models were at first estimated by DOLS and later we tried VEC models.
Best nested model based on FEER assumptions for Czech Republic was obtained when adding Balassa-Samuelson effect into FEER model estimated according MacDonald. All variables were significant, and test went well, and we could reject the presence of serial correlation. The models suggest that the real exchange rate is depreciating with increasing productivity ratio and domestic demand and appreciating with rising foreign demand, which is logical.

Table V.28: Nested FEER Model Czech Republic

	CZRER.d	BS_CZ.d	CZ_D.d	EA_D.d	
alpha	-0.147	-0.125	0.088	0.044	
alpha.t	-0.982	-4.598	0.450	0.662	
	CZRER.l5	BS_CZ.l5	CZ_D.l5	EA_D.l5	trend.l5
ect1	1	1.577	1.415	−2.840	0.00001
beta.t		3.917	12.035	−8.102	0.020

For Hungary, the best nested model based on FEER model also contains demand functions enriched by long term interest rates. In long run however the domestic long term interest rates resulted insignificant with very low, almost 0 coefficient and 0 $t-statistics$. Also, sustainable current account resulted as insignificant in long run, while being significant in terms of speed of adjustments.

Table V.29: Nested FEER Model Hungary

	HURER.d	HU_EMU.d	EA_EMU.d	HUSustCA.d	HU_D.d	EA_D.d
alpha	0.028	0.074	-0.273	-0.005	-0.136	-0.033
alpha.t	0.919	0.815	-2.331	-2.738	-2.730	-2.861
ect1	1	0	0.230	2.102	1.132	-1.233
ect2	0	1	-0.596	-3.211	-0.600	1.293
beta.t		0	2.095	0.904	3.431	-2.385

For Poland the best FEER models was DB model based on GDP, we tried to incorporate in model Investment position and external debt of Poland, this model resulted as best nested FEER model. This model was estimated also as VEC models resulting in quite decent model with significant coefficients except the case for long relationship for External Debt.

Table V.30: Nested FEER Model Poland

	PLRER.d	PLExtDebtperGDP.d	PLInvestperGDP.d	NGDP_PL.d	NGDP_EA.d	
alpha	-0.377	-0.705	-0.691	0.330	-0.029	
alpha.t	-2.112	-3.123	-5.204	1.903	-1.135	
	PLREER.l5	**PLExtDebtperGDP.l5**	**PLInvestperGDP.l5**	**NGDP_PL.l5**	**NGDP_EA.l5**	**constant**
ect1	1	0	0.703	−0.884	1.801	−5.578
ect2	0	1	−0.433	−4.731	7.539	−16.337
beta.t		0	10.857	−6.296	7.321	−9.260

For nested model for Slovakia we used as for Poland, FEER Model estimated on GDP, we further enriched the model by government consumption resulting in best nested FEER model for Slovakia. This model was also estimated as VEC model, however, the results are not strong, as we see issue with very low coefficients for government consumption in contrast with very high coefficients and $t - statistic$ for other variables.

Table V.31: Nested FEER Model Slovakia

	SKRER.d	SKGOVCperGDP.d	NGDP_SK.d	NGDP_EA.d	SKSustCA.d	
alpha	-0.270	-0.301	0.268	0.088	0.028	
alpha.t	-4.123	-2.483	3.690	4.407	0.422	
	SKRER.l5	**SKGOVCperGDP.l5**	**NGDP_SK.l5**	**NGDP_EA.l5**	**SKSustCA.l5**	**trend.l5**
ect1	1	0	-0.720	11.190	-7.315	-0.060
ect2	0	1	0.693	-6.899	2.012	0.034
beta.t		0	-12.589	23.855	-11.682	-25.301

V.III.iii BEER Models

After estimating nested BEER models first by DOLS, we tried to estimate them also as VEC models, we found cointegration relation for Poland and Slovakia, in this case the cointegration relationship was stronger than in models estimated according the research papers resulting in best VEC BEER models for those two countries.

The best models are reported in following tables. For Czech Republic the best nested model based on BEER models, is the combination of variables including Sustainable Current Account which is typical variable for FEER Models rather than BEER models. However, all variables in model resulted to be significant, except the oil prices, but even them are at least weekly significant. The results are showing that in long run the sustainable current account with international investment position have appreciation effect while productivity ratios with oil prices are pushing the currency to depreciate.

Table V.32: Nested BEER Model Czech Republic

	CZRER.d	CZSustCA.d	BS_CZ.d	CZIIPperGDP.d	oil_pPPI.d
alpha	0.007	0.450	-0.007	0.234	-0.252
alpha.t	0.305	4.028	-1.489	2.554	-1.806
	CZRER.l5	**CZSustCA.l5**	**BS_CZ.l5**	**CZIIPperGDP.l5**	**oil_pPPI.l5**
ect1	1	-1.561	5.184	-0.318	0.127
beta.t		-8.224	4.223	−4.475	1.691

For Hungary, the best model is composed of foreign long term interest rates, Balassa-Samuelson effect and Terms of Trade, suggesting that the real exchange rate is very dependent on foreign economy, rather than on domestic factors. Again, the nature of variables that are in the model is suggesting that cointegration does not have to be presented, which was confirmed by conducting Johansson cointegration procedure.

In nested model of Polish Zloty, we observe higher relevance of domestic factors, such is government debt or government consumption. Other variables in the nested models are terms of trade and external debt. However, the terms of trade are showing only weak significance.

Table V.33: Nested BEER Model Poland

	PLRER.d	PLGOVCperGDP.d	PLTOT.d	PLDebtperGDP.d	PLExtDebtperGDP.d
alpha	0.161	-0.137	0.013	0.347	-0.547
alpha.t	1.461	-3.421	0.207	2.085	-3.899
	PLRER.l5	**PLGOVCperGDP.l5**	**PLTOT.l5**	**PLDebtperGDP.l5**	**PLExtDebtperGDP.l5**
ect1	1	2.262	0.245	-0.343	-0.169
beta.t		6.746	1.597	-6.763	−7.751

Last nested model was built for Slovakia and yet non-existing Slovak Koruna. Best result was obtained combining interest rate differentials with government debt, government consumption and Balassa-Samuelson effect. The nature of the data is again showing us the possible presence of cointegration; however, we were facing the same issue with the results as with nested FEER model.

Table V.34: Nested Model Slovakia

	SKRER.d	SKGOVCperGDP.d	SKDebtperGDP.d	SKIR_DIFF.d	BS_SK.d	
alpha	-0.270	-0.301	0.268	0.088	0.028	
alpha.t	-4.123	-2.483	3.690	4.407	0.422	
	SKRER.l5	**SKGOVCperGDP.l5**	**SKDebtperGDP.l5**	**SKIR_DIFF.l5**	**BS_SK.l5**	**trend.l5**
ect1	1	0	-0.332	18.186	1.131	0.002
ect1	1	0	-0.720	11.190	-7.315	-0.060
ect2	0	1	0.693	-6.899	2.012	0.034
beta.t		0	-12.589	23.855	-11.682	−25.301

After estimating all models for all countries, we should have at least brief look on the diagnostics of the main estimated models.

V.IV Diagnostics

V.IV.iv Portmanteau Test for Serial Correlation

Serial correlation is the relationship between observations of the same variable over time period. If the serial correlation is non zero, that means that there is some dependent relationship between the variable and its lagged values, in the way that past observations can affect the present and the future observations. We assume this effect in our data as we are dealing with time series and with fundamental economic variables, which are often depending on expectations, which are made according the historical development. This relation is also reason why we chose to construct VEC model in first place, as those models can deal with the issue of serial correlation by cointegrating vectors. Therefore, when We test all models, we assume that serial correlation will be no longer presented in VEC Models, but we can expect it to be presented in our DOLS models.

According Portmanteau test, under H0 we assume that there is serial correlation presented in the models, that is why our aim is to have p-value above 0.05 level. As tables below shows, most of the VEC models estimated for purpose of this diploma thesis have the

$p - value$ above this level, therefore we can conclude we did in deed successfully treated serial correlation in most of our VEC Models.

Table V.35: Portmanteau Test of BEER Models

Model		Chi-Squared	DF	p-value
MacDonald	Czech Republic	206.98	180	0.08206
	Hungary	403.324	474	0.992
	Poland	444.809	402	0.069
	Slovakia	396.870	402	0.563
Pošta	Czech Republic	580.573	693	0.999
	Hungary	702.72	693	0.3906
	Poland	666.103	693	0.762
	Slovakia	661.411	693	0.801
DBEER	Czech Republic	190.68	180	0.2786
	Hungary	180.31	180	0.4795
	Poland	255.91	228	0.09887
	Slovakia	186.79	180	0.3489
JP Morgan	Czech Republic	342.28	355	0.6766
	Hungary	402.14	438	0.8895
	Poland	322.04	355	0.8948

Results for FEER VEC models:

Table V.36: Portmanteau Test of FEER Models

Model		Chi-Squared	DF	p-value
DB	Czech Republic	190.910	180	0.275
	Hungary	183.022	180	0.423
	Poland	184.56	180	0.392
	Slovakia	253.96	228	0.1144
MacDonald	Czech Republic	206	180	0.089
	Hungary	204.82	180	0.09903
	Poland	197.096	180	0.182
	Slovakia	224.65	180	0.0133
Komárek Import	Czech Republic	173.8	148	0.07231
	Hungary	204.43	189	0.1024
	Poland	228.27	180	0.008625
	Slovakia	228.27	180	0.008625
Komárek Export	Czech Republic	238.38	228	0.3051
	Hungary	274.19	330	0.988
	Poland	309.82	280	0.1064
	Slovakia	343.66	355	0.6573

V.IV.iv Autoregressive Conditional Heteroskedasticity

Usually when constructing models, we are assuming homoscedasticity (unless constructing ARCH models), which means that we expect the variance of errors to be constant. Heteroskedasticity is violation of this assumption even though the mean value could be constant.
This test is based on Ljung-BOx statistics of squared series and it computes the Lagrange multiplier. This test has under H0 assumption of heteroscedasticity; therefore, we are aiming models which p-value is higher than 0,05 in order to be able to reject the assumption of the heteroscedasticity. As the table states the results below, all p-values are higher than 0.05, which means we can reject the hypothesis of presence of heteroscedasticity and we can assume homoscedastic character of the variance of errors of our models.

From table above **??** it is obvious we have heteroscedasticity issue in several models. Although heteroscedasticity has no impact on unbiases of the models, it will cause us efficiency of them. This can be potentially solved with other estimation technique, however VEC Models were strategical choice in order to model non-stationary data. Furthermore, with time series we often expect heteroscedasticity issue, as data are usually serially correlated and depending on their historical values, we can assume that the variance will not be homoscedastic.

Table V.37: ARCH Multivariate Test BEER Models

Model		Chi-squared	DF	p-value
MacDonald	Czech Republic	865.437	882	0.648
	Hungary	885.771	882	0.458
	Poland	896.565	882	0.359
	Slovakia	915.091	882	0.214
Pošta	Czech Republic	1,574.827	1,568	0.447
	Hungary	1,603.222	1,568	0.262
	Poland	1,512	1,568	0.841
	Slovakia	1,512	1,568	0.841
DBEER	Czech Republic	240.686	200	0.026
	Hungary	211.623	200	0.273
	Poland	300.420	200	0.00001
	Slovakia	244.835	200	0.017
JP Morgan	Czech Republic	527.701	450	0.007
	Hungary	836.442	882	0.862
	Poland	481.579	450	0.147

Table V.38: ARCH Multivariate Test for FEER Models

Model		Chi-squared	DF	p-value
DB	Czech Republic	213.962	200	0.237
	Hungary	229.069	200	0.078
	Poland	220.933	200	0.148
	Slovakia	292.944	200	0.00002
MacDonald	Czech Republic	171.485	200	0.929
	Hungary	192.692	200	0.632
	Poland	222.489	200	0.132
	Slovakia	328.474	300	0.124
Komárek Import	Czech Republic	302.492	200	0.00000
	Hungary	142.688	100	0.003
	Poland	224.506	200	0.113
	Slovakia	225.465	200	0.105
Komárek Export	Czech Republic	238.719	200	0.032
	Hungary	268.113	225	0.026
	Poland	479.352	450	0.164
	Slovakia	455.667	450	0.417

V.IV.iv Normality

For testing normality, we checked three most common tests. Jarque-Bera test (for the goodness of the fit), test for skewness (third moment) and test for kurtosis (fourth moment).

Unfortunately, as we see from the test, we did not achieve to have residuals of all models normally distributed. Even though this is unfavorable result, it is quite frequent when modelling time series. The issue of time series is that usually we are dealing with heavy tails data, especially as we are modeling exchange rates.

Table V.39: Normality Tests for BEER VECM Models

Models		Czech Republic			Hungary			Poland			Slovakia		
	tests	Chi-squared	df	p-value	Chi-squared	df	p-value	Chi-squared	df	p-value	Chi-squared	df	p-value
Macdonald	JB-Test	100.16	12	0	21.576	12	0.04255	30.758	12	0.002144	32.101	12	0.001335
	Skewness	14.318	6	0.02628	9.3737	6	0.1536	18.111	6	0.00596	7.5716	6	0.2712
	Kurtosis	85.843	6	0	12.203	6	0.0576	12.647	6	0.04901	24.529	6	0.0004172
Pošta	JB-Test	102.56	14	0	22.492	14	0.06906	54.661	14	0	36.578	14	0.0008543
	Skewness	43.934	7	0	11.654	7	0.1125	14.61	7	0.04134	12.258	7	0.09237
	Kurtosis	58.628	7	0	10.838	7	0.1458	40.051	7	0	24.32	7	0.001001
DBEER	JB-Test	63.181	8	0	19.772	8	0.01123	34.554	8	0	18.113	8	0.0204
	Skewness	13.064	4	0.011	6.8618	4	0.1434	14.413	4	0.006086	4.8554	4	0.3025
	Kurtosis	50.117	4	0	12.91	4	0.01172	20.14	4	4.6e-05	13.257	4	0.01008
JP Morgan	JB-Test	188.84	10	0	951.11	12	0	326.625	10	0			
	Skewness	47.305	5	0	102.76	6	0	76.476	5	0			
	Kurtosis	141.54	5	0	848.35	6	0	250.17	5	0			

Table V.40: Normality Tests for FEER VECM Models

Model		Czech Republic			Hungary			Poland			Slovakia		
	tests	Chi-squared	df	p-value	Chi-squared	df	p-value	Chi-squared	df	p-value	Chi-squared	df	p-value
DB	JB-Test	10.115	8	0.2571	6.5448	8	0.5864	9.2917		0.3183	681.84	8	0
	Skewness	8.2493	4	0.08286	5.3188	4	0.2561	1.8855	0.7568	70.455	4	0	
	Kurtosis	1.8656	4	0.7605	1.226	4	0.8738	7.4062	4	0.1159	611.38	4	0
MacDonald	JB-Test	5.9571	8	0.652	5.9571	8	0.652	12.637	8	0.125	11.684	8	0.1659
	Skewness	2.1531	4	0.7076	2.1531	4	0.7076	7.3082	4	0.1205	4.4869	4	0.3441
	Kurtosis	3.804	4	0.4332	3.804	4	0.4332	5.329	4	0.2552	7.1974	4	0.1258
Komárek Import	JB-Test	100.61	8	0	20.411	8	0.00888	15.835	8	0.0448	18.873	8	0.01555
	Skewness	18.987	4	0.0079	11.888	4	0.01821	9.5877	4	0.04798	5.5008	4	0.2397
	Kurtosis	81.62	4	0	8.5229	4	0.07419	6.2473	4	0.01814	13.372	4	0.009592
Komárek Export	JB-Test	105.83	8	0	60.461	10	0	24.66	8	0.001775	161.39	10	0
	Skewness	19.682	4	5e-04	15.028	5	0.01025	14.668	4	0.005441	30.483	5	0
	Kurtosis	86.149	4	0	45.433	5	0	9.9917	4	0.04057	130.91	5	0

V.IV.iv Exogeneity Test

This test provides insight on the speed of adjustment of the variable. In order to be able to test exogeneity we build test matrices for each model. Testing matrix of each model has the number of rows corresponding to the number of variables that enters the model. Than always the row that corresponds to the variable we want to test contains only 0, and the rest of rows contains 0 or 1 in the way that diagonal line of matrix is composed of 1 and everywhere else are 0. The order of variables must be in line with rows of columns and always the row for tested variable contains only 0. In order to test the variables, we add to matrix cointegrated vector of the model we want to test. We then estimate restricted VAR model to observe the relationship between variables. The results are reported in attachments; however, we could not reject the hypothesis of exogeneity in all models.

VI

Misalignments

After conducting, examining and presenting our best models, we shall now compare them. For each country we created graphs showing the misalignments over time and the percentage misalignments from real exchange rate. We will now compare all models for each country and tried to determine which models suits the best for each country.
For each country we composed two graphs, one, where is the evolution of estimated equilibrium exchange rates around the real exchange rate over time and below is graph of percentage misalignments computed as:

$$Misalignments = \frac{RealExchangeRate - EstimatedEquilibriumExchangeRate}{EstimatedEquilibriumExchangeRate} \quad \text{(VI.1)}$$

VI.I Czech Republic

In graph VI.1 we can see all the models estimated for Czech Republic and below the misalignments. We can observe that especially at the beginning of the observed period most of the models are suggesting that the exchange rate was overvalued, except model according the Komárek which was suggesting quite the opposite. We must recall from first chapter that this was convergence period, in other words period of dynamic growth of the economy, when country begun to develop the market economy, it was newly created democratic state, opening to the global markets. This is important fact for understanding the models. As the period is characterized by huge growth of GDP and significant gains in productivity, models that are based on those fundamentals are suggesting also very progressive development of the exchange rate. However, Czech Republic only started to open its market and its international trade was on very low level, that is explanation why FEER model based on Komárek might see currency as overvalued as the export at that stage has not been yet developed to the extent of later period. Another very interesting conclusion that models brought out is that year before crisis, in 2007 all models agreed on the undervaluation of the currency and suggest that the Czech Koruna was in fact stronger than its market value.

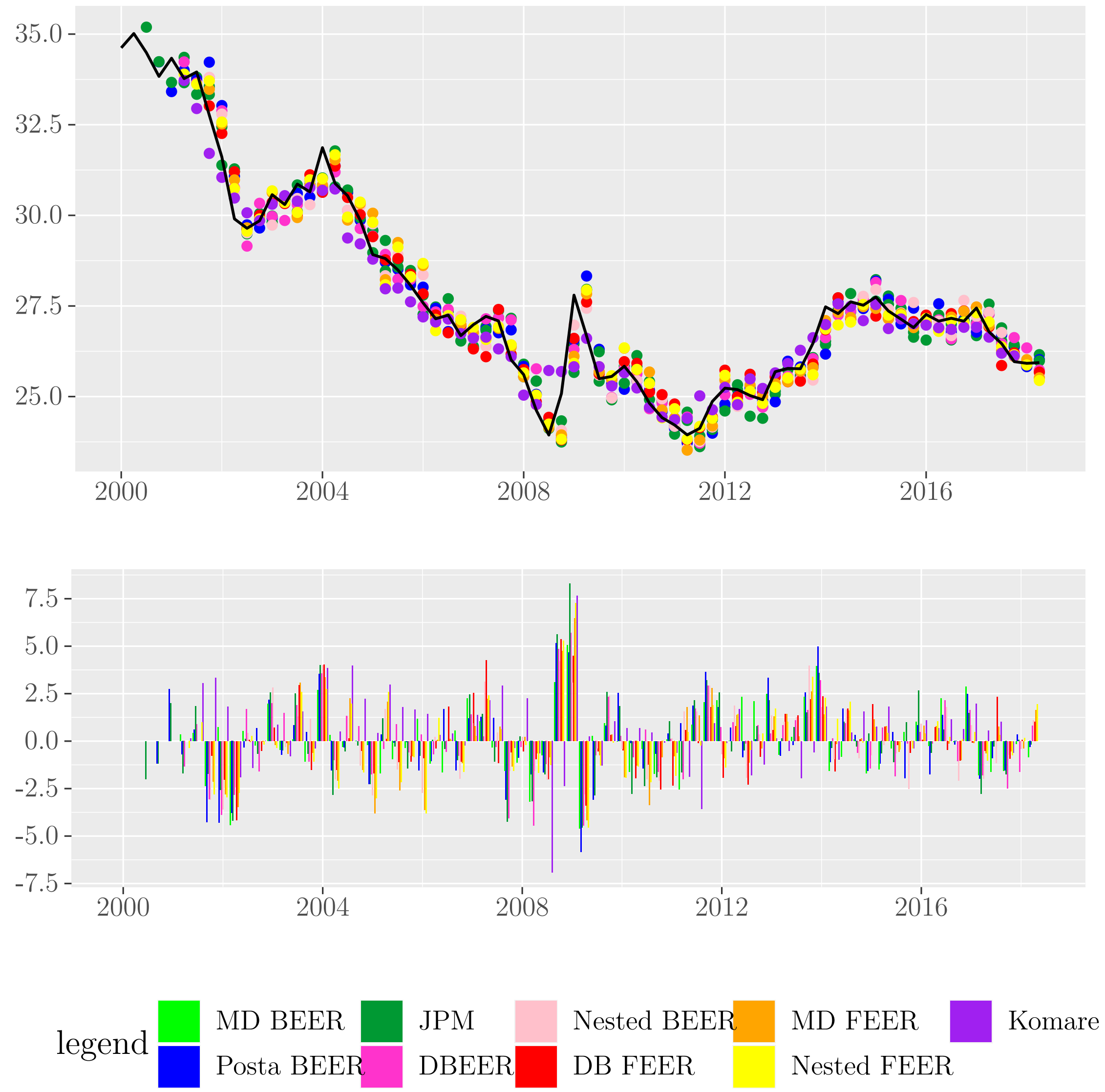

Figure VI.1: Model Comparison for Czech Republic

The change is visible during the 2008 when the currency seemed to be overvalued the first half of the year, which is confirmed by all models, but already in second half of the year they show undervaluation of the Czech Koruna. The undervaluation then persists also during the first quarter of 2009, in the period when the crisis hit Czech economy the hardest. The undervaluation is showing that in fact the economic situation of the country was not as bad as the general development showed, and exchange rate demonstrated higher resilience. Although, from pragmatic point of view it was convenient to have undervalued currency as it gives to the country advantage on global market and it makes its exports more attractive. As we know it was indeed export that was the main driver of the growth after the crisis. This also corresponds with the evolution according the model of Komárek, which is based on export and import equations. This model showed high undervaluation in first quarter of 2009, the misalignment was 8% and the model continued showing undervaluation until 2010 when the economic growth finally passed zero level and become positive again.

After crisis, when economy started to slowly recover and grow again, the models are showing that the equilibrium exchange rate was weaker than the market rate and that exchange rate was in fact overvalued again. The only model that almost throughout all observed period showed that the exchange rate of Czech Koruna was undervalued is model according Komárek and Motl. During the period that the Czech economy was struggling with very low inflation almost getting into negative numbers, when the one-sided commitment was introduced, models agreed on the fact that was commonly known that Czech Koruna was forced to depreciate but the real exchange rate was in fact stronger. During the period that this commitment was held models depart from unanimous development. However, we can observe clear similarities within the groups of models. While FEER models are in accordance that the exchange rate was undervalued, BEER models are especially in 2015 suggesting overvaluation. In April 2017, when CNB left the commitment we observe that just before leaving the commitment, in first quarter of 2017 most of the models thought the exchange rate is undervalued by most of the models, BEER models especially are showing over 3% undervaluation. In that time it was most common belief that Czech Koruna was indeed significantly undervalued and that leaving the commitment would lead to heavy appreciation of the currency. However, the next quarter of 2017, when the commitment was no longer held, all models except Komárek were showing that the currency is actually overvalued and, again, the most significant misalignment were shown by BEER models. In both cases the highest misalignments showed models by McDonald and Pošta. It is clear this period was challenging in terms of clearly determined the equilibrium because again, at the end of the 2017 all models agreed on undervaluation of the exchange rate. However, the misaligments of this period are rather low, usually under 3% level, which suggests that the exit of exchange rate commitment at the end led only to corrections of exchange rate and not to significant changes in its value, which was also confirmed by the development of the exchange rate.

VI.II Hungary

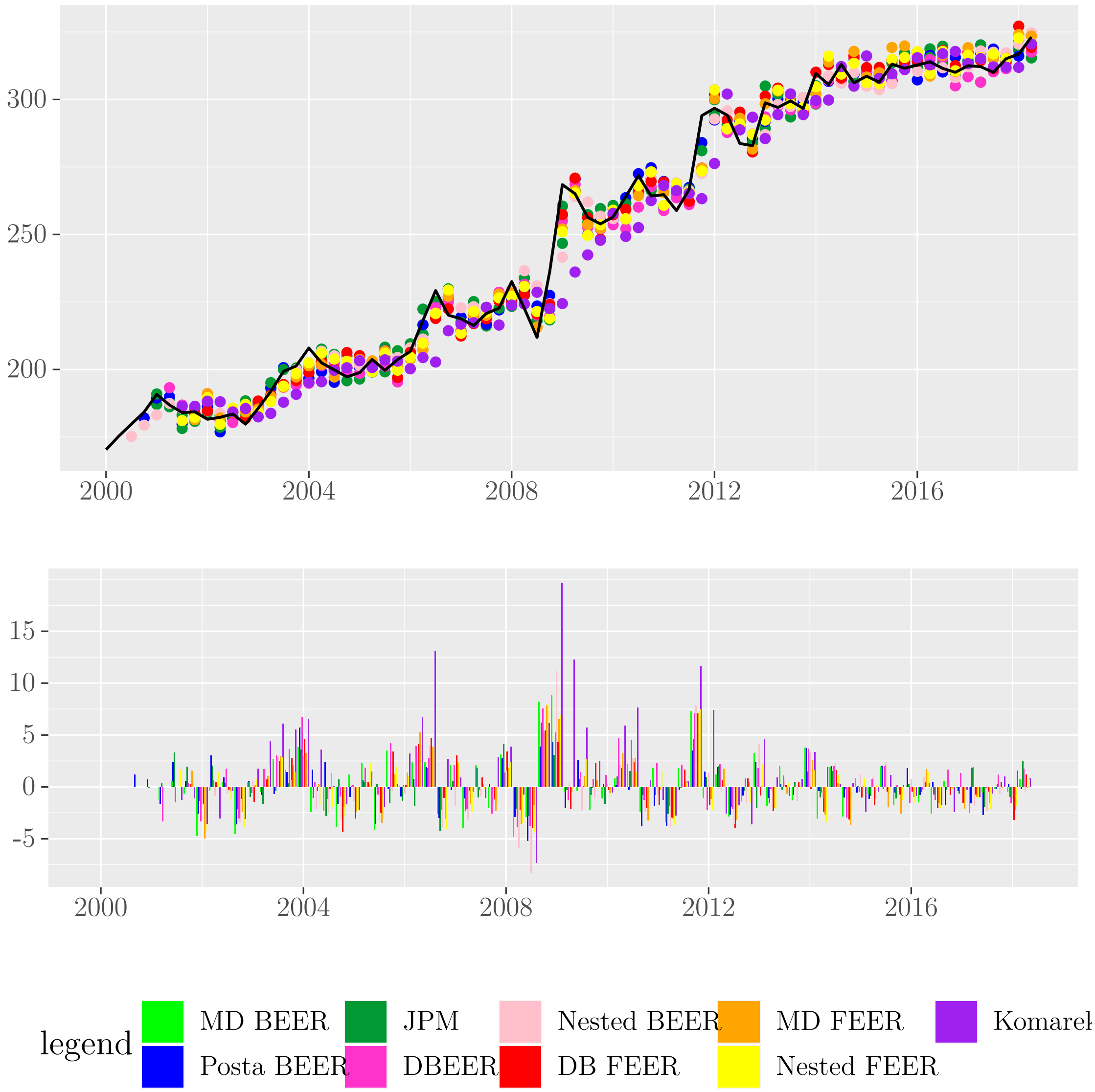

Figure VI.2: Model Comparison for Hungary

The situation is different for Hungary, what is clearly visible from graph VI.2. We know already that Hungary was not growing as dynamically as the rest of the Visegrad countries, furthermore, it had already problems with the indebtedness and low productivity. All those factors are clearly visible in equilibrium exchange rate estimations. Yet between the years 2001 and 2003 the models agree that the Hungarian currency was overvalued and that it was much, familiarly speaking, cheaper in terms of its value. In the period before joining the European Union, Hungary was decreasing its debt as much as possible to comply with all requirements of entry and we can see that shortly before entering

the EU, all models are confirming that the currency was in fact undervalued and it was stronger than its market values suggests. This persists also during the years 2005 and 2006, however already in the end of 2006 we can see that all models except Komárek are showing that the Hungarian currency was overvalued again. We know that Hungarian economy was going through slow down already in this period, since 2005. The strongest overvaluation is shown by JP Morgan, which states that the exchange rate should be in fact more than 4% weaker. In second half of 2007 both models from Deutsche Bank and from MacDonald were the only models suggesting that the equilibrium exchange rate is in fact higher than the market shows and currency is still undervalued.
In the 2008 all models arrived to this conclusion but what is interesting, all models agreed on this for the first quarter, then all models unanimously agreed on overvaluation throughout the 2008 and then again, all modest unanimously agreed on undervaluation at the end of 2008 and at the beginning of 2009. The models were clearly showing that economy was doing better than markets assumed. It is necessary to point out that the amplitude of misalignments differ from one model to another. While at the beginning of 2009 Komárek was showing undervaluation of almost 20%, Pošta a little more skeptical, saying that the misalignment is around 4%. This observation does not apply only to this particular year, although it is most visible in this year, but generally we can conclude that Komárek was suggesting much higher undervaluation comparing to the rest of the model, while always when showing overvaluation, it was very modest, around 1%. On the other side stands DBEER model, which has higher upper amplitude, while suggesting only low overvaluation over time. When we linked it with the underlying fundamentals, we know DBEER is based on GDP values while Komárek is working with Export and Import and international trade fundamentals. This observation is in line with the relatively slow growth of the economy comparing to the rest of the countries forming Visegrad Group. However, the position of Hungary on international market and its high external debt are clearly supporting the exchange rate, what is shown by model from Komárek. After the crisis the general mood shown in models is that the Hungarian Forint is most of the time overvalued, although it is not confirmed by all models and results rather differ. We cannot even roughly say that overall BEER models suggest overvaluation of the currency more often than FEER models, and FEER models are more often suggesting the undervaluation, but this is just roughly speaking.
However we can observe clearly the second recession in Hungary in 2012. The actual evolution of equilibrium exchange rate between two recession in 2008 and 2012 is departing from values of real exchange rate. We can even say that models are foreseeing the second

recession as they mostly suggests overvaluation of the exchange rate in this period. The highest overvaluation is visible then at the end of 2011 when the misalignments of all models are moving between 7 and 11%. After the second recession the depreciating trend is very obvious and in line with all models, as the exchange rate fluctuates mostly around the equilibrium values and misalignments are quite low. Majority of the models are, however, showing constant overvaluation of the exchange rate, which is persistently pushing the rate to depreciate. The most visible overvaluation is showing model according Pošta, this model accounts with the country indebtedness, what as we know was and still is the most appealing issue.

VI.III Poland

For Poland unfortunately, several variables such as Current Account and international investment position were measured only since year 2004, therefore only two models are working also before this year and those models are BEER models according McDonald and DBEER, this is also visible from following graph VI.4. Both models are signaling same absolute values in terms of overvaluation or undervaluation, however in terms of size of misalignments, DBEER is showing higher amplitude in undervaluation while Macdonald has higher amplitude for overvaluation. After 2004 all models are working which is giving us space to be able to compare them all.

Till 2007 most of the models are suggesting overvaluation of the exchange rate. Traditionally, Komárek is among the models that are saying the opposite and showing that the models is undervalued, but this is supported also by DB FEER model and Macdonald FEER model. In 2007 and till the end of the year 2008 all models suggest overvaluation of Polish Zloty, while Komárek is showing the highest misalignment, around 17−18%.

Figure VI.3: Model Comparison for Poland

We know the crisis hit hardest Visegrad Group during the year 2009, in terms of misalignments we see that at the end of 2008 and beginning of 2009 the exchange rate of Polish Zloty cross over its equilibrium rates and all models shows its undervaluation. This observation is very interesting as it shows that Poland, the only country that kept GDP growing even through the crisis period could in fact have stronger exchange rate due to its macroeconomic development that it had. Especially the first quarter of 2009, the undervaluation presented by all models is significantly high, moving between 7 and 10%. Another important discovery is that model by Komárek continued to show the undervaluation since this year basically until the second quarter of 2017. We know from economic outlook from chapter two, that the growth of the country is heavily supported

by the export, more concretely by international demand for domestic production. Therefore the results from Komárek model are in line with this economic development of the country.
But another important source of growth was the domestic demand which was growing with improving labor conditions. This is clearly projected in DB FEER model, which is based on domestic demand and which is also suggesting undervaluation of the exchange rate.
We can observe that, through all post crisis period until the end of 2012, almost all models are showing that the exchange rate should be more expensive and that is undervalued. Again, the highest misalignment is shown by Komárek model, which shows misalignment above 10%. Second model after is Macdonald with misalignments around 5%. The rest of the models are quite consistent with misalignments between 4 and 2%. At this time most of the countries were suffering from the second recession, when economy was slowing down again and as it was mentioned already Komárek model was showing undervaluation of the currency throughout this period- We know that continuous growth of economy in this period was mostly driven by exports, which gives us again the economic intuition behind those results.
The trend of undervaluation than continues through the rest of the observed period, although, BEER models are often in this period suggesting overvaluation, especially BEER model constructed by MacDonald and Pošta. We know that in this period domestic conditions worsen slightly, with worsening situation on labor market and finishing strategic development projects. However, we must point out that overvaluation during this period is very small, usually under 1%. From the case of Poland we see that undervalued exchange rate can indeed help economy to grow faster and boost its growth. Although, one most not forget that it depends also on the choice of the model used. The highest undervaluation is shown by JP Morgan and MacDonald BEER models, while FEER models are overall more skeptical and holding on assumption of overvalued exchange rate.

VI.IV Slovakia

Slovakia is the last analyzed country. We had similar issue due to data availability as in case of Poland, however the situation was a bit better in case of Slovakia and we can observe the exchange rate misalignments throughout all period.

Figure VI.4: Model Comparison for Slovakia

The interesting fact on case of Slovakia is that the models were quite often moving in same direction, suggesting all together the overvaluation or undervaluation of the exchange rate, the difference was in the amplitudes of the misalignments. In the beginning of the period all models conclude that the exchange rate was rather undervalued than overvalued, which relates to convergence period and dynamic growth of the economy. Although we can observe short periods of overvaluation, but usually the misalignments are very modest around 1%, the only exception is the last quarter of year 2002, when all models concluded overvaluation and DB FEER model was showing the highest misalignment value around 4.5%. The situations changed in 2004, when the misalignments started to differ from models, but overall showing rather overvaluation then undervaluation. Significant variance can be observed especially after joining the EU/. During the end of the year 2006 and whole first half of 2007 models again unified and were all suggesting overvaluation of the currency. The highest misalignments were shown by FEER models from Komárek and MacDonald. Than we can again observe frequently changing misalignments oscillating around real exchange rate. The only model that is showing constant overvaluation throughout 6 years, since 2006 until the end of 2012 is model from Komárek. It is clearly visible from the graph, that while the rest of the models are moving in line with real exchange rate and agree with market development of fast appreciation of the exchange rate. The appreciation according Komárek is more gradual and slower over time.

We should recall that since 2008 the exchange rate of Slovak Koruna was fixed on level 30.126 Slovak Korunas per euro and stayed on this level until the euro adoption, therefore, since already 2007, even though the Slovak Koruna was still used, we are observing real exchange rate in terms of nominal exchange rate deflated by price levels ratios. We can see that the misalignments are quite smaller since the Slovak Koruna was fixed, however, during the crisis period we can still observe high variance, especially during the year of 2009.

Slovakia was also fighting with inflation and was struggling with successful inflation targeting, in which it finally succeed only after euro adoption, this might be also an explanation why the misaligments become smaller, as the price levels starting to develop more systematically and inflation was already in targeting regime.

Rest of the models are showing overvaluation during the most difficult period of the crisis, at the end of year of 2009. This is also the period of euro adoption, exchange rate was fixed for long time and so the currency could not naturally depreciate and ease the difficulties the economy was facing. It is clear that the fixed rate was too strong for Slovakia. Exports at this time shrunk by 20% and weaker currency might have prevented such fall and keep Slovakia on more balanced path.

After that during the post crisis period in years 2009 and 2010 we can see that while BEER models are rather in favor of undervaluation of the currency, the FEER models are suggesting the opposite. Slovakia was country with one of the fastest recovery after crisis, the main driver was external demand and rising exports. We can conclude that euro adoption helped Slovakia and supported its trade position by opening country to inflows from abroad. Despite this outstanding growth and fast recovery we also know that Slovakia is still under its potential due to underdeveloped research sector, which is crucial for further development, as growth based only on manufactured exports is not

sustainable in long term, this might be also possible explanation for constantly overvaluation showed by models.
However, both types of models are showing quite small misalignments in both directions. Only after 2012 are models again quite unified in terms of undervaluation, also the misalignments are having quite small variance, all moving around 1%. This holds only till the beginning of 2013, soon after the currency become slightly overvalued according most of the models. Even though the overvaluation is rather small, the highest point of this period is in second half of 2014, when MacDonald BEER model shows overvaluation on level above 2%. During years 2013 and 2014 we can see that FEER models, except Komárek are showing undervaluation of the exchange rate while BEER models and Komárek are rather inclining to overvaluation. After that for years 2015 and 2016 the models are almost all together agreeing on overvaluation, although the misalignments are smaller than 1 %. Since the second half of 2016 and through 2017 the currency is stated as undervalued, and the highest misalignment is shown again by MacDonald suggesting almost 2% undervaluation together with DB FEER model.

Conclusion

The main aim of this thesis was to try to estimate equilibrium exchange rate for each country of Visegrad Group and to understand their differences and misalignments. As we saw in chapter one, although those countries are tightly connected and working closely together, their development differs and each country is unique with its special features, which caused that each country responded differently to different model approaches. Models, which were not working for one country seemed to work well for another country and vice versa.

The results were presented for each country in two graphs and linked with the economical development of each country. We observe that while the highest missaligments are reported by model according Komárek and Motl the lowest misaligments are estimated by nested model, which are copying the movements of the real exchange rates the most. After analyzing the data, constructing the models and analyzing the results, it is time to ask the questions. What are the conclusions? Which models is the best? Which country is in biggest disequilibrium? Which exchange rate we assume as generally undervalued or overvalued?

There are many question and for sure there are not all mentioned above. In order to address the conclusions of this diploma thesis we must answer the partial question and then we should be able to understand the big picture.

First of all, we have to remind ourselves that there is no best model, because those models are different from each other, comparable yes, some of them more, some less and some are using the same theoretical principles as departing point. However, each of them is construct in way that it focuses on different sector of the economy, on different set of fundamental variables and so each of them understands and explains the equilibrium exchange rate differently. Already the two main groups of models we estimated are differing based on examining fundamental characteristics or behavioral characteristics of the economy. While BEER models are observing equilibrium exchange rate in terms of behavioral composition, FEER model is examining purely the economical fundamentals and it is up to author of the certain research which component he or she wants to examine and how.

The overall lowest misalignments were observed undoubtedly for Czech Republic. The most turbulent period in terms of oscillating around equilibrium was clearly around the time when CNB was leaving one-sided commitment. Undervaluation was detected mostly by Komárek model while overvaluation was reported frequently by DB FEER model. Those models are both from FEER group of models and yet their results are the most contrasting. Why is it so? Possible explanation can be found in the fundamentals used to construct the models. While DB FEER is using GDP of domestic and foreign country and Sustainable Current Account, model by Komárek is more complex, taking into consideration two equations, Import and Export and determining the equilibrium exchange rate by further computations. Generally, not just in case for Czech Republic but for whole Visegrad Group, model construct basing on approach from Komárek had the highest amplitude of them all.

BEER models are working quite consistently for Czech Republic suggesting balanced development of exchange rate, of course except the periods when the CNB intervened and depreciated the exchange rate, but even after Czech Koruna seems to continue on balanced path.

The amplitude of misalignments of Hungarian Forint is way higher, showing imbalances within the economy. The overall highest misaligments were observed around the crisis period, when Komárek shows the highest misaligment of almost 20 %. Hungarian currency is depreciating over time, which is suggesting rather weak economy situation and development of the country. We saw that this situations is caused by various factors, but mainly due to high indebtedness and relatively low productivity. Those factors are also projected in the development of equilibrium exchange rate. Almost all models were showing most of the time overvaluation of the exchange rate, suggesting that the real exchange rate should be even weaker than markets show, which is very dangerous for long term development of Hungary. We saw that government and MNB are trying to put in place set of reforms and regulations in order to help the economy and boost the growth, but there is still long way to go. The most optimistic view on the equilibrium exchange rate had both DBEER models, showing the undervaluation the most, although still the ratio of times when the overvaluation was reported is higher. Furthermore, while misaligments showing overvaluation are high, the average of the highest misalignments stating overvaluation is 9%, the misalignments of undervaluation are quite low and the average is around 5 %. This shows that even if the undervaluation were reported, they were less frequently and in lower amplitudes.

Polish Zloty was in terms of amplitude similar case to Hungary. Both, the highest and the lowest misalignment were reported by model by Komárek, the highest misalignment, the highest undervaluation was reported in 2012, it is not coincidence that in the very same year the GDP growth of the country peaked, as we recall from chapter 2. Furthermore, we know that this period was marked also by significant export growth, which is well noted by model by Komárek showing constant undervaluation of the exchange rate and demonstrating that the exchange rate should be in fact even stronger.

Slovakia is our last and very specific case as we were observing misaligments of de facto non-existing currency. What is interesting from our analysis, is that Slovak currency seemed to be more often overvalued, rather then undervalued. Even more interesting is that Komárek is suggesting the most frequently the overvaluation, considering the openness of the country and its increasing exports, we would expect a different results, while undervaluation is mostly reported by DBEER which is in fact taking into consideration country openness, terms of trade and Balassa-Samuelson effect. Slovakia was the fastest growing country among Visegrad group and yet we do not observe significant undervaluation over time, implying that the progressive growth was in line with the development of the country.

To sum it up, as we observed most of the Visegrad countries are characterized by strong and fast growth and development. Taking into consideration this fact, we could conclude that the most appropriate model for those counties are then models which are taking into account this dynamic development, however the possible threat of those model is that they can possibly overvalued the currency and that is why we should always compare various models in order to get general outlook of the development of the exchange rate. On the other hand, this does not imply that the models based on overall international position are bad, as those countries are small open economies, mostly exporters and so it is important to observe the development of their currencies in those terms too. The question is what is more important for us.

The drawback of this thesis is that during our analysis VEC models were showing not very significant results for some models, implying that other estimation technique would suit better the analysis. We tried to estimate DOLS and VAR models, however various recent researchers are showing that DSGE or semi-structural models might be the answer for even more precise estimations.

Bibliography

Robert Habib Michael Harrison Jonny Goulden Anezka Christovova, Jonathan Cavenagh. EM Currencies are still cheap. Globalemerging market research, JP Morgan, 2019.

Augustine C. Arize and Gordian Ndubizu. Cointegration, error correction representation and the import demand function with implications in international finance and accounting research. *Review of Quantitative Finance and Accounting*, 2(4):359–376, Dec 1992. ISSN 1573-7179. doi: 10.1007/BF00939017. URL https://doi.org/10.1007/BF00939017.

Égert Balázs and Lahrèche-Révil Amina. Estimating the equilibrium exchange rate of the central and eastern european acceding countries: The challenge of euro adoption. *Review of World Economics*, 139(4):683–708, Sep 2003. ISSN 1610-2886. doi: 10.1007/BF02653109. URL https://doi.org/10.1007/BF02653109.

Czech National Bank. Monetary policy inflation targeting, 2019. URL https://www.cnb.cz/en/monetary-policy/inflation-targeting/.

Ray Barrell, Dawn Holland, and Katerina Smidkova. Estimates of Fundamental Real Echange Rates for the Five EU Pre- Accession Countries. Macroeconomics 0303016, University Library of Munich, Germany, March 2003. URL https://ideas.repec.org/p/wpa/wuwpma/0303016.html.

Agnes Benassy-Quere, Sophie Bereau, and Valerie Mignon. How Robust are Estimated Equilibrium Exchange Rates? A Panel BEER Approach. Working Papers 2008-01, CEPII research center, March 2008. URL https://ideas.repec.org/p/cii/cepidt/2008-01.html.

Joanna Beza-Bojanowska and Ronald MacDonald. The Behavioural Zloty/Euro Equilibrium Exchange Rate. CESifo Working Paper Series 2568, CESifo Group Munich, 2009. URL https://ideas.repec.org/p/ces/ceswps/_2568.html.

Cerbaf. The exchange rate system : John Williamson, study no. 4, Institute for International Economics, Washington, DC (MIT Press, Cambridge, MA, 1983) pp. 85. *Journal of Banking & Finance*, 9(2):333–333, June 1985. URL https://ideas.repec.org/a/eee/jbfina/v9y1985i2p333-333b.html.

William R. Cline. Estimates of Fundamental Equilibrium Exchange Rates, May 2017. Policy Briefs PB17-19, Peterson Institute for International Economics, May 2017. URL https://ideas.repec.org/p/iie/pbrief/pb17-19.html.

Ccile Couharde and Virginie Coudert. Exchange Rate Regimes and Sustainable Parities for CEECs in the Prospect of joining the EMU. Post-Print halshs-00119109, HAL, September 2003. URL https://ideas.repec.org/p/hal/journl/halshs-00119109.html.

Mark De Broeck and Torsten Sloek. Interpreting real exchange rate movements in transition countries. BOFIT Discussion Papers 7/2001, Bank of Finland, Institute for Economies in Transition, March 2001. URL https://ideas.repec.org/p/bof/bofitp/2001_007.html.

Rebecca L Driver and Peter F Westaway. Concepts of equilibrium exchange rates. Bank of England working papers 248, Bank of England, January 2005. URL https://ideas.repec.org/p/boe/boeewp/248.html.

Balas Egert and Kirsten Lommatzsch. Equilibrium exchange rates in the transition : the tradable price-based real appreciation and estimation uncertainty. BOFIT Discussion Papers 9/2004, Bank of Finland, Institute for Economies in Transition, August 2004. URL https://ideas.repec.org/p/bof/bofitp/2004_009.html.

Balazs Egert, Laszlo Halpern, and Ronald MacDonald. Equilibrium Exchange Rates in Transition Economies: Taking Stock of the Issues. William Davidson Institute Working Papers Series wp793, William Davidson Institute at the University of Michigan, October 2005. URL https://ideas.repec.org/p/wdi/papers/2005-793.html.

Christabel Charles Gautam Kalani, Oliver Harvey. Introducing Cap-PPP: Deutsche Bank's new capital-based FX valuation model. Deutsche bank research, Deutsche Bank, October 2017.

Nikolaos Giannellis and Athanasios Papadopoulos. Estimating the Equilibrium Effective Exchange Rate for Potential EMU members. Working Papers 0719, University of

Crete, Department of Economics, March 2007. URL https://ideas.repec.org/p/crt/wpaper/0719.html.

James D. Hamilton. *Time Series Analysis*. Princeton University Press, 1 edition, January 1994. ISBN 0691042896. URL http://www.amazon.com/exec/obidos/redirect?tag=citeulike07-20&path=ASIN/0691042896.

Jan Hansen and Werner Roeger. Estimation of real equilibrium exchange rates. European Economy - Economic Papers 2008 - 2015 144, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission, September 2000. URL https://ideas.repec.org/p/euf/ecopap/0144.html.

Tibor Hledik and Jan Vlcek. Quantifying the Natural Rate of Interest in a Small Open Economy - The Czech Case. Working Papers 2018/7, Czech National Bank, July 2018. URL https://ideas.repec.org/p/cnb/wpaper/2018-7.html.

Tomas Holub. Causes of Deviations from the CNBs Inflation Targets: An Empirical Analysis. *Czech Journal of Economics and Finance (Finance a uver)*, 58 (09-10):425–433, December 2008. URL https://ideas.repec.org/a/fau/fauart/v58y2008i9-10p425-433.html.

Rob J. Hyndman, George Athanasopoulos, and OTexts.com. *Forecasting : principles and practice / Rob J Hyndman and George Athanasopoulos*. OTexts.com [Heathmont?, Victoria], print edition. edition, 2014. ISBN 9780987507105.

P. Isard. *Equilibrium Exchange Rates: Assessment Methodologies*. IMF Working Papers. INTERNATIONAL MONETARY FUND, 2007. ISBN 9781451913125. URL https://books.google.sk/books?id=dNbk2gf8_IoC.

Gautam Kalani. Special Report: FX Valuations:A framework and an Introduction to our new DBeer model. Deutsche bank research, Deutsche Bank, 2016.

Lubos Komarek and Martin Motl. Behavioralni a fundamentalni rovnovazny menovy kurz ceske koruny¡BR¿[Behavioural and Fundamental Equilibrium Exchange Rate of the Czech Koruna]. *Politicka ekonomie*, 2012(2):147–166, 2012. URL https://ideas.repec.org/a/prg/jnlpol/v2012y2012i2id835p147-166.html.

PISCITELLI Laura and WESTAWAY Peter. FEER Computation: A Model Based Approach. EcoMod2003 330700122, EcoMod, undated. URL https://ideas.repec.org/p/ekd/003307/330700122.html.

Jaewoo Lee, Hamid Faruqee, and Tamim Bayoumi. A Fair Exchange? Theory and Practice of Calculating Equilibrium Exchange Rates. IMF Working Papers 05/229, International Monetary Fund, December 2005. URL https://ideas.repec.org/p/imf/imfwpa/05-229.html.

Ronald MacDonald. What Determines Real Exchange Rates? The Long and Short of it. IMF Working Papers 97/21, International Monetary Fund, February 1997. URL https://ideas.repec.org/p/imf/imfwpa/97-21.html.

Ronald MacDonald and Peter B. Clark. Exchange Rates and Economic Fundamentals; A Methodological Comparison of BEERs and FEERs. IMF Working Papers 98/67, International Monetary Fund, May 1998. URL https://ideas.repec.org/p/imf/imfwpa/98-67.html.

Martin Melecky and Lubos Komarek. The Behavioral Equilibrium Exchange Rate of the Czech Koruna. International Finance 0504010, University Library of Munich, Germany, April 2005. URL https://ideas.repec.org/p/wpa/wuwpif/0504010.html.

Bernhard Pfaff. *Analysis of Integrated and Cointegrated Time Series with R*. Springer Publishing Company, Incorporated, 2nd edition, 2008. ISBN 0387759662, 9780387759661.

Vit Posta. Estimation of the behavioral equilibrium real exchange rate of the czech koruna. *TRENDY EKONOMIKY A MANAGEMENTU / TRENDS ECONOMICS AND MANAGEMENT*, IV(07):48–56, November 2010.

Matteo Salto and Alessandro Turrini. 427 - Comparing alternative methodologies for real exchange rate assessment - Matteo Salto and Aless. European Economy - Economic Papers 2008 - 2015 427, Directorate General Economic and Financial Affairs (DG ECFIN), European Commission, September 2010. URL https://ideas.repec.org/p/euf/ecopap/0427.html.

Bulent Ulasan. Real Exchange Rate Misalignment and Economic Growth : An Update. Working Papers 1819, Research and Monetary Policy Department, Central Bank of the Republic of Turkey, 2018. URL https://ideas.repec.org/p/tcb/wpaper/1819.html.

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List of Abbreviations

Table VI.1: Abbreviations

	Shortcut	Explanation
1	BEER	Behavioral Equilibrium Exchange Rate
2	BNP	Polish Central Bank
3	BS	Balassa Samuelson
4	CA	Current Account
5	CNB	Czech Central Bank
6	CR	Czech Republic
7	CZ	Czechia
8	CZK	Czech Koruna
9	D	Demand
10	DB	Deutsche Bank
11	EA	Euro Area
12	ECB	European Central Bank
13	EMU	Maastricht Convergence Criterion
14	ER	Exchange Rate
15	EU	European Union
16	EUR	Euro Currency
17	EXP	Export
18	ExtDebt	External Debt
19	FA	Financial Account
20	FDI	Foreign Direct Investment

Table VI.2: Abbreviations

	Shortcut	Explanation
21	FEER	Fundamental Equilibrium Exchange Rate
22	FX	Exchange Rate
23	GOVC	Government Consumption
24	GOVDEBT	Government Debt
25	HICP	Harmonized Index of Consumer Prices
26	HU	Hungary
27	HUF	Hungarian Forint
28	IIP	International Investment Position
29	IMP	Import
30	Infl	Inflation
31	IR	Interest Rate
32	IRR	Interest Rate Differential
33	MNB	Hungarian Central Bank
34	NFA	Net Foreign Assets
35	NPS	Central Bank of Slovakia
36	NX	Net Export
37	OilP	Oil Prices
38	OP	Openness
39	PL	Poland
40	PLN	Polish Zloty
41	PPI	Producers Price Index
42	PPP	Purchase Power Parity
43	Prod	Productivity
44	RER	Real Exchange Rate
45	SK	Slovakia
46	SKK	Slovak Koruna
47	SustCA	Sustainable Current Account
48	TOT	Terms of Trade

Attachments

Table VI.3: ADF test on Dataset Czech Republic

	Critical Value	Lags	p-value
CZ_HICP	-2.95282998962026	4	0.186983290003969
czk_aver	-1.82539573759646	4	0.646597742520808
EA_EMU	-1.14901558163907	4	0.908560065368895
CZ_EMU	-2.00904489144905	4	0.571686742127093
CZEARIR_IND	-2.72342953563173	4	0.280635756515896
Prod_EA	-2.40389176408309	4	0.410765689326093
Prod_CZ	-1.28254027437694	4	0.867900418109686
ProdMan_EA	-3.63633968690438	4	0.0363016087599395
ProdMan_CZ	-1.37160729715172	4	0.831590991784869
BS_CZ	-2.73994572797798	4	0.273768557693445
CZ_D	-2.55325907651069	4	0.349874000607136
EA_D	-2.60859877889989	4	0.327313991479866
CZ_op	-3.10891971118446	4	0.12335111651673
CA_CZ_perGDP	-3.2690828949151	4	0.0834486977292304
CZIIPperGDP__1	-0.744960706280618	4	0.962161816198264
EA_grossdebt	-2.61041597227608	4	0.326573187005267
CZ_grossdebt	-2.58600302427961	4	0.336525469107377
CZDebtperGDP	-2.34879529287903	4	0.433226541834882
NGDP_EA	-1.48845958691758	4	0.783954510021372
NGDP_CZ	-1.88549329459369	4	0.622098126949166
GOVC_EA	-1.64486971757935	4	0.720191717252608
GOVC_CZ	-2.6038847541833	4	0.329235730051651
EXP_EA	-3.08079536276467	4	0.134816403275717
EXP_CZ	-1.55526958400983	4	0.75671847370166
IMP_EA	-2.79167673461361	4	0.25267968421785
IMP_CZ	-1.55073660776896	4	0.758566405312289
NX_EA	-2.02645726431041	4	0.564632179245654
NX_CZ	-3.95118758079166	3	0.0180654295716254
oil_p	-2.000821891553	4	0.575082800019162
CZNXperGDP	-3.30108019549152	4	0.0783011268514278
CZGOVCperGDP	-4.05405543075787	4	0.0120511717087813
CZInvestperGDP	-3.33476965943134	4	0.0728813289203123
CZTOT	-3.06204601851035	4	0.142459837541642
CZSustCA	-3.28119214483694	4	0.0817406699555617
CZFDI	-3.80554797673482	4	0.0232924618485453

Table VI.4: ADF Test Hungary

	Critical Value	Lags	p-value
HU__HICP	0.674868846193374	4	0.968301607726579
HUF_Aver	-3.41417991923968	4	0.0601061905985065
HU_EMU	0.993207300944687	4	0.932811359227393
EA_EMU	-1.14901558163907	4	0.908560065368895
HUEARIR_IND	-2.58358732921082	4	0.337609562350449
Prod_EA	-2.40389176408309	4	0.410765689326093
Prod__HU	-1.6159601277394	4	0.731977118736487
ProdMan_EA	-3.63633968690438	4	0.0363016087599395
ProdMan_HU	-1.22503070259018	4	0.891345005059037
BS_HU	-1.86211695516868	4	0.631627820966702
EAD	-2.60859877889985	4	0.327313991479881
HU_D	-3.13608443408805	4	0.112277034615551
HU_op	-3.80325296209064	4	0.0233962773481315
CA_HU	-1.53916697215097	3	0.750176334109347
CapA_HU	-2.57894268789646	4	0.339893238001846
CA_HU_perGDP	-1.93304481338593	4	0.60271308056016
CapA_HU_perGDP	-2.35338914378781	4	0.431353793808474
NFA_HU	-2.57985453135258	3	0.340398330091926
NFA_EA	-2.65704530867401	3	0.309543954539436
HU_DebtperGDP	-2.4887479704632	4	0.376172861613047
EAgrossdebt	-2.61041597227608	4	0.326573187005267
HU_grossdebt	0.767092082085843	4	0.960223188324646
NGD_EA	-1.48845958691758	4	0.783954510021372
GOV_EA	-1.64486971757935	4	0.720191717252608
CAPFORM_EA	-2.19396279461512	4	0.496346190536029
EXP_EA	-3.08079536276467	4	0.134816403275717
IMP_EA	-2.79167673461361	4	0.25267968421785
NX_EA	-2.02645726431041	4	0.564632179245654
NGDP_HU	-3.46426990665698	4	0.0520479558145143
GOVC_HU	-3.97131541458288	4	0.0157939348047551
CAPFORM_HU	-1.72376164209818	4	0.688030313045991
EXP_HU	-1.85934398538502	4	0.632758261155719
IMP_HU	-2.0622563784525	4	0.550038166142479
NX_HU	-1.61708276422326	3	0.726625716587663
oilp	-2.000821891553	4	0.575082800019162
HUTOT	-1.7494486097871	4	0.677558658871952
HU_NXperGDP	-1.57592021093317	4	0.74829995477653
HUGOVCperGDP	-3.12804150090023	4	0.115555849612626
STMEUR	-2.23741520621161	3	0.478852045103245
TMEUR	-2.45253221084037	4	0.390936726114809
HUSustCA	-3.0990880133382	4	0.127814326816692
HUFDI	-3.08285596332609	4	0.134646606988366

Table VI.5: ADF test on Dataset Poland

	Critical Value	Lags	p-value
EA_HICP	-2.0834927033675	4	0.541380879181614
PL_HICP	-1.71348327007634	4	0.692220436169449
PL_EMU	-2.67330142780725	4	0.301058697165511
EA_EMU	-1.14901558163907	4	0.908560065368895
PLEARIR_IND	-4.35856220368722	4	0.01
Prod_EA	-2.40389176408309	4	0.410765689326093
Prod_PL	-1.01427716813198	4	0.929612942479378
ProdMan_EA	-3.63633968690438	4	0.0363016087599395
ProdMan_PL	-0.888755886155171	4	0.949225642788254
BS_PL	-2.62125849363994	4	0.322153080456609
PL_Demand	-3.02704976477947	4	0.156726553290066
EA_D	-2.60859877889989	4	0.327313991479866
PL_op	-2.97195628512673	4	0.179186186250823
CA_PL_perGDP	-1.71853103336596	3	0.688320131866792
NFA_PL	-1.9739579953842	3	0.58514908396845
NFA_EA	-2.65704530867401	3	0.309543954539436
EA_grossdebt	-2.61041597227608	4	0.326573187005267
PL_grossdebt	-2.70608484982238	4	0.287572421597074
NGDP_EA	-1.48845958691758	4	0.783954510021372
GOVC_EA	-1.64486971757935	4	0.720191717252608
EXP_EA	-3.08079536276467	4	0.134816403275717
IMP_EA	-2.79167673461361	4	0.25267968421785
NX_EA	-2.02645726431041	4	0.564632179245654
NGDP_PL	-2.15373059150254	4	0.512747414797171
GOVC_PL	-1.94545667280098	4	0.59765321125113
EXP_PL	-2.09747242687299	4	0.535681847993072
IMP_PL	-2.04783092451538	4	0.555918905619496
NX_PL	-3.65423894685551	2	0.0461257895103205
oil_p	-2.000821891553	4	0.575082800019162
TMEUR	-2.45253221084037	4	0.390936726114809
PLDebtperGDP	-2.83349224279472	4	0.235633003344996
PLNXperGDP	-2.17963881001714	4	0.50218556460777
PLGOVCperGDP	-2.52256416345124	4	0.362387214247356
PLInvestperGDP	-3.54823471686065	4	0.0438759700085413
PLTOT	-2.47953037551676	4	0.379930544020887
PLExtDebtperGDP	-0.595790316900582	3	0.97411726291575
PLSustCA	-1.56001992660417	3	0.75231331182714
PLFDI	-3.67036594874583	3	0.0350802533019998

Table VI.6: ADF Test Dataset Slovakia

	Critcal Value	Lags	p-value
EA_HICP	-2.0834927033675	4	0.541380879181614
SK_HICP	-1.98151440910213	4	0.582953767182171
SK_EMU	-1.42105736995941	4	0.810861700932081
EA_EMU	-1.14901558163907	4	0.908560065368895
SK_EA_RIRIND	-4.53243780337762	4	0.01
ProdEA	-2.40389176408309	4	0.410765689326093
ProdSK	0.655667110496491	4	0.96998360980234
ProdManEA	-3.63633968690438	4	0.0363016087599395
ProdManSK	-1.32475390031935	4	0.850691438924031
BS_SK	-4.26646321980068	4	0.01
EA_D	-2.60859877889985	4	0.327313991479881
SK_D	-2.60859877889989	4	0.327313991479864
SK_op	-2.99031557290073	4	0.171701764002964
CASK_perGDP	-3.67554020638144	4	0.0329315503454744
CapASK_perGDP	-4.20597222611439	4	0.01
NFA_SVK	-1.89020848637646	3	0.616213956085793
NFA_EA	-2.65704530867401	3	0.309543954539436
EA_grossdebt	-2.61041597227608	4	0.326573187005267
SK_grossdebt	-2.02380162325682	4	0.565714788725308
NGDP_EA	-1.48845958691758	4	0.783954510021372
GOVC_EA	-1.64486971757935	4	0.720191717252608
GOVCSK_	-2.6038847541833	4	0.329235730051651
CAPFORMEA	-2.19396279461512	4	0.496346190536029
CAPFORMSK	-1.84280741773989	4	0.639499625870408
EXPEA	-3.08079536276467	4	0.134816403275717
IMPEA	-2.79167673461361	4	0.25267968421785
NXEA	-2.02645726431041	4	0.564632179245654
NGDPSK	-1.392283801031	4	0.823161923754177
GOVCSK	-1.21834271177734	4	0.894071458712866
CAPFORMSK	-2.05105757908168	4	0.554603514438777
EXPSK	-1.17107795829505	4	0.905112819016398
IMPSK	-1.06823260171837	4	0.921182405981505
NXSK	-3.65423894685551	2	0.0461257895103205
oilp	-2.000821891553	4	0.575082800019162
SK_NXperGDP	-3.5657021880189	2	0.0547635849973745
SK_GOVCperGDP	-3.18531476003282	4	0.0969249099046303
SK_TOT	-4.17499081287367	4	0.01

Table VI.7: LAG Selection for FEER Models

Model		AIC(n)	HQ(n)	SC(n)	FPE(n)
MacDonald	Czech Republic	5	5	5	5
	Hungary	5	5	5	5
	Poland	5	5	5	5
	Slovakia	5	5	4	5
DB	Czech Republic	5	4	2	5
	Hungary	5	5	2	5
	Poland	5	2	1	2
	Slovakia	4	2	1	4
Komarek Import	Czech Republic	5	4	4	5
	Hungary	5	4	4	5
	Poland	5	5	5	5
	Slovakia	5	5	4	5
Komarek Export	Czech Republic	2	2	1	2
	Hungary	3	2	1	5
	Poland	5	2	1	2
	Slovakia	5	1	1	5

Table VI.8: LAG Selection for BEER Models

Model		AIC(n)	HQ(n)	SC(n)	FPE(n)
MacDonald	Czech Republic	5	1	1	4
	Hungary	5	1	1	5
	Poland	5	1	1	2
	Slovakia	5	1	1	5
Pošta	Czech Republic	5	1	1	2
	Hungary	5	5	1	5
	Poland	5	2	1	5
	Slovakia	5	2	1	5
DBEER	Czech Republic	5	1	1	5
	Hungary	5	5	1	5
	Poland	5	1	1	5
	Slovakia	5	1	1	5
JP Morgan	Czech Republic	2	1	1	1
	Hungary	4	1	1	1
	Poland	2	1	1	2

Table VI.9: DOLS BEER Models Czech Republic

	Dependent variable:			
	CZRER			
	(MacDonald)	(Posta)	(DBEER)	(JP Morgan)
CZ_EMU	0.023			
	(0.022)			
EA_EMU	−0.055			
	(0.038)			
CZIR_DIFF		3.911***		
		(0.756)		
PROD_CZ		−0.453*		
		(0.262)		
oil_pPPI		−0.124***		
		(0.014)		
CZGOVCperGDP	−0.711***	−1.003***		
	(0.169)	(0.123)		
BS_CZ	−2.376***		−1.443***	−1.747**
	(0.566)		(0.323)	(0.681)
oil_p	−0.102***			
	(0.031)			
CZDebtperGDP		0.261***		
		(0.033)		
CZInvestperGDP			0.022	
			(0.115)	
CZNXperGDP			5.581**	
			(2.432)	
CZIIPperGDP__1			−0.146***	
			(0.009)	
CZTOT				−0.800*
				(0.442)
CZ_op				0.050
				(0.099)
Constant	3.743***	−13.943***	−22.469**	3.282***
	(0.111)	(3.478)	(11.221)	(0.031)
Observations	73	73	74	74
R^2	0.782	0.899	0.904	0.586
Adjusted R^2	0.765	0.892	0.898	0.568
Residual Std. Error	0.047 (df = 67)	0.032 (df = 67)	0.032 (df = 69)	0.066 (df = 70)
F Statistic	47.929*** (df = 5; 67)	119.496*** (df = 5; 67)	162.466*** (df = 4; 69)	32.973*** (df = 3; 70)

Table VI.10: DOLS BEER Models Hungary

	Dependent variable:			
	HURER			
	(MacDonald)	(Posta)	(DBEER)	(JP Morgan)
HU_EMU	0.253***			
	(0.072)			
EA_EMU	−0.366***			
	(0.047)			
HUIR_DIFF		3.567*		
		(2.050)		
PROD_HU		3.266***		
		(1.133)		
oil_pPPI		−0.013		
		(0.062)		
HUGOVCperGDP	−0.617***	−2.187***		
	(0.213)	(0.364)		
BS_HU	1.286**		2.975***	−0.536
	(0.643)		(0.604)	(0.612)
oil_p	0.183***			
	(0.034)			
HUDebtperGDP		−0.316***		
		(0.116)		
TMEUR			−0.027**	
			(0.011)	
CA_HU_perGDP			273.583***	
			(26.288)	
HUTOT				2.099***
				(0.281)
HU_op				0.468***
				(0.113)
Constant	4.766***	−10.930	−1,254.361***	5.769***
	(0.161)	(9.447)	(121.047)	(0.024)
Observations	73	73	57	74
R^2	0.896	0.654	0.840	0.875
Adjusted R^2	0.888	0.628	0.831	0.870
Residual Std. Error	0.068 (df = 67)	0.125 (df = 67)	0.082 (df = 53)	0.075 (df = 70)
F Statistic	115.722*** (df = 5; 67)	25.324*** (df = 5; 67)	92.630*** (df = 3; 53)	163.712*** (df = 3; 70)
Note:				*p<0.1; **p<0.05; ***p<0.01

Table VI.11: DOLS BEER Models Poland

	Dependent variable:			
	PLRER			
	(MacDonald)	(Posta)	(DBEER)	(JP Morgan)
PL_EMU	0.050			
	(0.054)			
EA_EMU	0.052			
	(0.035)			
PLIR_DIFF		0.293		
		(0.486)		
PLGOVCperGDP	−0.671	-1.706^{***}		
	(0.427)	(0.239)		
TMEUR			0.150	
			(0.529)	
PLInvestperGDP			-0.509^{***}	
			(0.075)	
PLExtDebtperGDP			0.235^{***}	
			(0.019)	
PLDebtperGDP		0.479^{***}	0.221^{***}	
		(0.045)	(0.029)	
BS_PL	0.321	1.506^{***}		−0.053
	(0.343)	(0.115)		(0.296)
PLTOT	1.275^{***}			0.636^{***}
	(0.266)			(0.192)
oil_p	-0.055^{**}			
	(0.026)			
oil_pPPI		0.102^{***}		
		(0.020)		
PL_op				0.050
				(0.070)
Constant	1.589^{***}	0.022	0.028	1.487^{***}
	(0.131)	(2.281)	(2.458)	(0.053)
Observations	73	73	58	74
R^2	0.531	0.793	0.852	0.461
Adjusted R^2	0.488	0.778	0.841	0.438
Residual Std. Error	0.063 (df = 66)	0.041 (df = 67)	0.033 (df = 53)	0.066 (df = 70)
F Statistic	12.454^{***} (df = 6; 66)	51.451^{***} (df = 5; 67)	76.491^{***} (df = 4; 53)	19.932^{***} (df = 3; 70)

Note: *p<0.1; **p<0.05; ***p<0.01

Table VI.12: DOLS BEER Models Slovakia

	Dependent variable:		
	SKRER		
	(MacDonald)	(Posta)	(DBEER)
SK_EMU	0.051		
	(0.054)		
EA_EMU	−0.048		
	(0.074)		
SKIR_DIFF		−3.900***	
		(0.901)	
SKGOVCperGDP	−0.570***	−0.477***	
	(0.165)	(0.140)	
SKDebtperGDP		0.138***	
		(0.018)	
BS_SK	−0.957***	−0.383***	−1.037***
	(0.248)	(0.071)	(0.242)
SKTOT	0.534*		−0.144
	(0.282)		(0.259)
oil_p	−0.119***		
	(0.026)		
oil_pPPI		−0.105***	
		(0.017)	
SK_op			0.054
			(0.078)
Constant	3.895***	21.870***	3.434***
	(0.099)	(4.142)	(0.013)
Observations	71	71	74
R^2	0.770	0.854	0.619
Adjusted R^2	0.749	0.842	0.603
Residual Std. Error	0.050 (df = 64)	0.040 (df = 65)	0.062 (df = 70)
F Statistic	35.809*** (df = 6; 64)	75.751*** (df = 5; 65)	37.975*** (df = 3; 70)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table VI.13: DOLS FEER Models Czech Republic

	Dependent variable:		
	CZRER		CA
	(DB)	(MacDonald)	(Komarek)
NGDP_CZ	-0.822^{***}		
	(0.041)		
NGDP_EA	1.214^{***}		
	(0.094)		
CZ_D		-0.647^{***}	
		(0.072)	
EA_D		0.780^{***}	
		(0.166)	
fitNX			0.0003
			(0.001)
CZSustCA	0.045^{***}	0.048^{*}	0.001^{***}
	(0.016)	(0.025)	(0.0002)
Constant	1.285^{***}	3.051^{***}	4.600^{***}
	(0.216)	(0.114)	(0.001)
Observations	72	72	72
R^2	0.960	0.877	0.256
Adjusted R^2	0.958	0.871	0.235
Residual Std. Error	0.019 (df = 68)	0.034 (df = 68)	0.0004 (df = 69)
F Statistic	545.138^{***} (df = 3; 68)	161.374^{***} (df = 3; 68)	11.895^{***} (df = 2; 69)

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table VI.14: DOLS FEER Models Hungary

	Dependent variable:		
	HURER		CA
	(DB)	(MaDonald)	(Komarek)
HU_D	-0.387^{***}		
	(0.083)		
EA_D	1.649^{***}		
	(0.155)		
NGDP_HU		-0.612^{***}	
		(0.113)	
NGDP_EA		2.104^{***}	
		(0.211)	
fitNX			-0.0004
			(0.0003)
HUSustCA	4.204^{***}	2.049^{***}	0.034^{***}
	(0.681)	(0.731)	(0.002)
Constant	-13.699^{***}	-10.600^{***}	4.449^{***}
	(3.139)	(2.958)	(0.008)
Observations	72	72	72
R^2	0.943	0.956	0.859
Adjusted R^2	0.940	0.954	0.855
Residual Std. Error	0.049 (df = 68)	0.043 (df = 68)	0.0002 (df = 69)
F Statistic	372.676^{***} (df = 3; 68)	495.897^{***} (df = 3; 68)	210.728^{***} (df = 2; 69)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table VI.15: DOLS FEER Models Poland

	Dependent variable:		
	PLRER		CA
	(DB)	(MacDonald)	(Komarek)
PL_Demand	−0.027		
	(0.182)		
EA_D	0.170		
	(0.519)		
NGDP_PL		−0.306	
		(0.191)	
NGDP_EA		0.937*	
		(0.497)	
fitNX			0.0005**
			(0.0002)
PLSustCA	−1.199***	−0.646	−0.007***
	(0.444)	(0.468)	(0.001)
Constant	6.970***	1.511	4.639***
	(2.061)	(3.320)	(0.003)
Observations	58	58	58
R^2	0.214	0.269	0.731
Adjusted R^2	0.170	0.228	0.721
Residual Std. Error	0.077 (df = 54)	0.074 (df = 54)	0.0001 (df = 55)
F Statistic	4.900*** (df = 3; 54)	6.608*** (df = 3; 54)	74.737*** (df = 2; 55)

Note: *p<0.1; **p<0.05; ***p<0.01

Table VI.16: DOLS FEER Models Slovakia

	Dependent variable:		
	SKRER		CA
	(DB)	(MacDonald)	(Komarek)
SK_D	−0.227*		
	(0.120)		
EA_D	−0.144		
	(0.263)		
NGDP_SK		−0.341***	
		(0.052)	
NGDP_EA		0.468***	
		(0.165)	
fitNX			0.001***
			(0.0005)
SKSustCA	1.308***	0.465	−0.032***
	(0.355)	(0.307)	(0.002)
Constant	−2.633	0.679	4.752***
	(1.638)	(1.505)	(0.010)
Observations	72	72	72
R^2	0.762	0.829	0.785
Adjusted R^2	0.752	0.821	0.778
Residual Std. Error	0.049 (df = 68)	0.042 (df = 68)	0.0003 (df = 69)
F Statistic	72.631*** (df = 3; 68)	109.520*** (df = 3; 68)	125.664*** (df = 2; 69)
Note:			*p<0.1; **p<0.05; ***p<0.01

Table VI.17: Exogeneity Test BEER Model

Model		p-value
Macdonald	Czech Republic	0
	Hungary	0.520
	Poland	0.010
	Slovakia	0.160
Posta	Czech Republic	0.560
	Hungary	0
	Poland	0.010
	Slovakia	0.070
DBEER	Czech Republic	0.590
	Hungary	0.150
	Poland	0.060
	Slovakia	0.760
JP Morgan	Czech Republic	0.730
	Hungary	0
	Poland	0.170

Table VI.18: Exogeneity Test FEER Model

Model	p-value	
MacDonald	Czech Republic	0.027
	Hungary	0.960
	Poland	0.490
	Slovakia	0.180
DB	Czech Republic	0.320
	Hungary	0.360
	Poland	0.410
	Slovakia	0
Komarek Import	Czech Republic	0.790
	Hungary	0.130
	Poland	0.220
	Slovakia	0
Komarek Export	Czech Republic	0.090
	Hungary	0.180
	Poland	0.100
	Slovakia	0.010

Table VI.19: Nested BEER models

	Dependent variable:			
	CZRER	HURER	PLRER	SKRER
	(1)	(2)	(3)	(4)
CZSustCA	0.054**			
	(0.026)			
BS_CZ	−0.720***			
	(0.250)			
CZIIPperGDP	−0.110***			
	(0.014)			
oil_pPPI	−0.040**			
	(0.017)			
EA_EMU		−0.121***		
		(0.021)		
BS_HU		1.952***		
		(0.562)		
HUTOT		1.866***		
		(0.254)		
PLGOVCperGDP			−0.746***	
			(0.205)	
PLTOT			0.614***	
			(0.086)	
PLDebtperGDP			0.348***	
			(0.026)	
PLExtDebtperGDP			0.210***	
			(0.017)	
SKGOVCperGDP				−0.510***
				(0.120)
SKDebtperGDP				0.139***
				(0.018)
SKIR_DIFF				−4.427***
				(0.854)
BS_SK				−0.926***
				(0.105)
Constant	3.200***	5.753***	0.933***	23.905***
	(0.141)	(0.018)	(0.053)	(3.940)
Observations	72	74	58	71
R^2	0.892	0.896	0.889	0.839
Adjusted R^2	0.886	0.892	0.880	0.829
Residual Std. Error	0.032 (df = 67)	0.068 (df = 70)	0.029 (df = 53)	0.041 (df = 66)
F Statistic	138.348*** (df = 4; 67)	201.164*** (df = 3; 70)	105.653*** (df = 4; 53)	85.747*** (df = 4; 66)
Note:				*p<0.1; **p<0.05; ***p<0.01

Table VI.20: Nested FEER models

	Dependent variable:			
	CZRER	HURER	PLRER	SKRER
	(1)	(2)	(3)	(4)
BS_CZ	−0.359			
	(0.383)			
CZ_D	-0.719^{***}			
	(0.067)			
HU_EMU		0.160^{***}		
		(0.049)		
EA_EMU		-0.105^{***}		
		(0.035)		
HUSustCA		4.633^{***}		
		(0.776)		
HU_D		-0.311^{***}		
		(0.084)		
EA_D	1.040^{***}	1.480^{***}		
	(0.191)	(0.155)		
PLExtDebtperGDP			0.302^{***}	
			(0.016)	
PLInvestperGDP			-0.426^{***}	
			(0.048)	
NGDP_PL			-0.631^{***}	
			(0.064)	
SKGOVCperGDP				-0.455^{***}
				(0.104)
NGDP_SK				-0.399^{***}
				(0.048)
NGDP_EA			0.921^{***}	0.619^{***}
			(0.146)	(0.151)
SKSustCA				0.337
				(0.274)
Constant	3.273^{***}	-15.857^{***}	-0.987^{**}	0.834
	(0.006)	(3.596)	(0.409)	(1.336)
Observations	74	72	58	72
R^2	0.889	0.951	0.924	0.867
Adjusted R^2	0.884	0.947	0.918	0.859
Residual Std. Error	0.034 (df = 70)	0.047 (df = 66)	0.024 (df = 53)	0.037 (df = 67)
F Statistic	187.204^{***} (df = 3; 70)	254.263^{***} (df = 5; 66)	161.388^{***} (df = 4; 53)	108.999^{***} (df = 4; 67)

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table VI.21: Portmanteau Test of Nested Models

Model		Chi-Squared	DF	p-value
Nested BEER	Czech Republic	470.97	402	0.0993
	Hungary	297.02	355	0.988
	Poland	296.76	280	0.235
	Slovakia	308.93	280	0.113
Nested FEER	Czech Republic	351.42	280	0.0239
	Hungary	429.5	438	0.605
	Poland	300.95	280	0.1862
	Slovakia	299.97	280	0.197

Table VI.22: ARCH Multivariate Test Nested Models

Model		Chi-squared	DF	p-value
Nested BEER	Czech Republic	883.79	882	0.4767
	Hungary	491.98	450	0.08378
	Poland	417.77	450	0.850
	Slovakia	449.77	450	0.4941
Nested FEER	Czech Republic	417.65	450	0.8606
	Hungary	880.01	882	0.5126
	Poland	455.74	450	0.4158
	Slovakia	526.96	450	0.07