**Report on the Ph.D.-Thesis**

# Investigation of geometrical and physical properties of exact spacetimes

## by Ondřej Hruška

The general field of the present thesis is exact solutions in Einstein's general theory of relativity. More specifically it is devoted to a detailed study of the class of non-expanding Plebański-Demiański geometries.

Let me first describe the content of Mr. Hruška's thesis in some detail. It's main body is comprised of 4 chapters, whereby the first three consist of the already published papers [8][1], [7], and [11], respectively in form of their arXiv-manuscripts.

Chapter 1 provides a *general study of the non-expanding Plebański-Demiański class* of metrics. These spacetimes, which belong to the Kundt class are electrovacuum solutions of algebraic type $D$ and have hitherto not been analysed in detail—in contrast to their expanding counterparts. The metric contains 2 discrete geometric parameters called $\varepsilon_0$, $\varepsilon_2$, and 5 continuous, physical parameters including a cosmological constant $\Lambda$, and electromagnetic charges. After discussing the coordinate representations induced on the background spacetimes (that is Minkowski- and (anti-)de Sitter space in sections 1.5 and 1.6, respectively) by various choices of the geometric parameters, the physical meaning of the two remaining parameters $n$ and $\gamma$ is analysed in detail. If only $n$ is non-vanishing one obtains the $B$-metrics [9], respectively their cosmological analogues if in addition $\Lambda \neq 0$. The most vivid interpretation of $n$ is given as the "strength" of a tachyon in the context of the $BI$-metrics (i.e., $\varepsilon_2 = 0$, $\varepsilon_0 = \pm 1, 0$). The meaning of $\gamma$ as a so called anti-NUT parameter (cf. [35]) is revealed through a study of the general vacuum case which is discussed prior to the analysis of the most general case with all parameters non-vanishing.

Chapter 2 takes up and extends the topic of subsection 1.6 by providing an exhaustive study of *new representations of (anti-)de Sitter space* using Plebański-Demiański coordinates. In contrast to the well-known coordinate representations which correspond to $3+1$ foliations, the new representations are warped-products of 2-spaces of constant curvature. In total, this approach leads to 3, respectively 8 new coordinate systems for (parts of) de Sitter, respectively anti-de Sitter space. These 11 representations are then all thoroughly analysed and vivid graphical descriptions are given. As a first application a discussion of the field of a tachyon is given.

This topic is taken up and detailed in the 3rd chapter: The (anti-)de Sitter space in the respective Plebański-Demiański coordinates is regarded as a background geometry for the $BI$-metric with non-vanishing $\Lambda$, which together with a corresponding $AII$-metric (belonging to the expanding Plebański-Demiański, class see [9,17]) describes the *geometry of a tachyon.* In this way a construction of Gott [10] is extended to the case of a non-vanishing cosmological constant, that is to a superluminal particle that e.g. moves around the "neck" in the de-Sitter hyperboloid with infinite speed. In the weak field limit (i.e., the tachyon-mass parameter vanishing) the relevant parts of the background spacetimes are covered by two disconnected $AII$-regions and one $BI$-region, which are joined along a Mach-Cherenkov shock wave generated by the particle. This structure is also vividly illustrated via a boost which "slows down" the tachyon to finite superluminal speed, and also to the speed of light in which case the the $BI$-region collapses to coincide with the shock wave leaving the two separated $AII$-regions behind (see the Figures in Sec. 3.7).

[1]References refer to the cumulative bibliography on pp. 145ff of the thesis.

This very construction is, however, only formal in case the tachyon-mass parameter is finite. Another peculiar issue concerns the global structure of the $BI$-metric and its possible extension to negative values of the coordinate $\rho$ (see Sec. 3, eqs. (27) and (39)) as suggested in [10] (in the case $\Lambda = 0$). As a first step to shed light on both these issues the author, in chapter 4, analyses the *geodesics of the $BI$-metric*. The upshot is that, while there are geodesics that cross from positive values of $\rho$ to negative ones, they need to be of a special form with no motion in one of the directions. This, of course, does not allow to finally settle the above points but provides a starting point for a more detailed analysis that may be a rewarding research question for the near future.

Let me now come to my assessment of the present thesis. Chapters 1–3 contain new and interesting results that significantly contribute to the physical understanding of the geometries considered. In fact, these parts have already been prominently published. Chapter 4, on the other hand, prepares interesting future lines of research.

Although most parts of this thesis have been published by the candidate together with his advisor Jiří Podolský and chapter 1 also with Jerry Griffiths as an additional co-author, the overall form and content of this work clearly shows the candidate's ability to pursue original research and to tackle technically demanding tasks. The introductory parts as well as the interludes connecting the published articles furthermore demonstrate the candidate's ability to produced helpful and technically precise synopses. To this reviewer there remains no doubt that the candidate is able to contribute new and interesting results to his field.

Summing up, the present thesis is of very high quality and I am convinced that the candidate well deserves to be awarded a Ph.D. degree in (Theoretical) Physics.

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