We study nonlinear evolutionary partial differential equations that can be viewed as a generalization of the heat equation where the temperature gradient is bounded but the heat flux is apriori only a measure. We consider this system in spatially periodic setting and use higher differentiability techniques to prove the existence and uniqueness of weak solution with integrable heat-flux for all values of the material parameter a. Under some more restrictive assumptions on a, we prove higher integrability of the heat flux.