We study the complexity of the homeomorphism relation on the classes of metrizable compacta and Peano continua using the notion of Borel reducibility. For each of these two classes we consider two different codings. Metrizable compacta can be naturally coded by the space of compact subsets of the Hilbert cube with the Vietoris topology. Alternatively, we can use the space of continuous functions from the Cantor space to the Hilbert cube with the topology of uniform convergence, where two functions are considered as equivalent iff their images are homeomorphic. Similarly, Peano continua can be coded either by the space of Peano subcontinua of the Hilbert cube, or (due to the Hahn-Mazurkiewicz theorem) by the space of continuous functions from $[0, 1]$ to the Hilbert cube. We show that for both classes the two codings have the same complexity (the complexity of the universal orbit equivalence relation). Among other results, we also prove that the homeomorphism relation on the space of nonempty compact subsets of any given Polish space is Borel bireducible with the above mentioned equivalence relation on the space of continuous functions from the Cantor space to the Polish space.