Advisor's Review of "Minkowski-Weyl Theorem", a bachelor thesis by Nathan Chappell

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Summary of Results

This work describes a detailed proof of the Minkowski-Weyl theorem, which is a fundamental theorem in the theory of polyhedra and polytopes and it roughly states that a polyhedron can equivalently be described by a finite set of linear inequalities or two finite sets of points/vectors. The work consists of three parts (or two depending on the perspective).

In the first part, the author describes a proof of the theorem in detail. This proof is not new and can be considered a detailed presentation of the proof presented in the textbook "Lectures on Polytopes" by Günter M. Ziegler. Often proofs of this theorem are quite terse because a typical reader of such material has relatively advanced knowledge. One purpose of this thesis was to present a proof in enough detail so that it may be accessible to an average undergraduate student in their first year of studies.

In the second part, the author implements this proof as a computer program that converts between the two representation of polyhedra. This algorithmic problem is significant and subject of a few PhD theses due to the complexity of making any such program work in practice. For the purpose of this thesis, the task was to quite literally write down the given proof of the theorem as a computer program.

In the third part, the author attempts to make a framework for testing the correctness of the program. Program testing is a highly non-trivial task so one should not expect to see much of it in a bachelor thesis.

Evaluation

Strengths

- The work is structured and presented in a coherent fashion.
- The work is non-trivial (for the standards of a bachelor thesis). In fact, the part on testing the correctness of the program was not advised by me to be a part of the thesis. This part was done completely on the

author's initiative and the ideas presented there show a good amount of thought and effort.

- The Minkowski-Weyl theorem is an "intuitively obvious" theorem that is quite non-trivial to prove. So even though the proof presented here is not new, it demonstrates amply that the author can understand a complicated topic relatively in detail.
- Figures are amply used in a helpful way for presenting the ideas in the thesis.

Weaknesses

I do not see any significant weaknesses in the thesis. One may perhaps (justifiably) complain about notational issues and some presentation issues. A big problem when writing a proof is to find the right balance between formality and informality. A proof that is too informal cannot be verified easily for correctness and a proof that is too formal will not be read by anyone except the reviewer. This thesis is written in a way that is far from perfect in this regard, but within various constraints of the program I find it within acceptable range.

Conclusion

The author of thesis has aptly demonstrated the capability to understand a complicated theorem and present it relatively well. The initial goal of presenting the proof in full detail while making it accessible to a first-year undergraduate student may not quite have been achieved but explaining a technically nuianced theorem such as the Minkowski-Weyl theorem in a way that is both complete and not tiresome to read is a very challenging task and the present thesis does this task reasonably well.

I recommend that this thesis be accepted as a bachelor thesis with the grade "2".

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